CS 415: Computer Vision

Mini Project 3

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1 Question 1

K-means and mean shift are both clustering algorithms that could be used for segmenting an image. K-means, given k (number of clusters), does the following steps:

- Select initial centroids at random
- Assign each object to the cluster with the nearest centroid
- Compute each centroid as the mean of the object assigned to it
- Repeat previous two steps until no changes

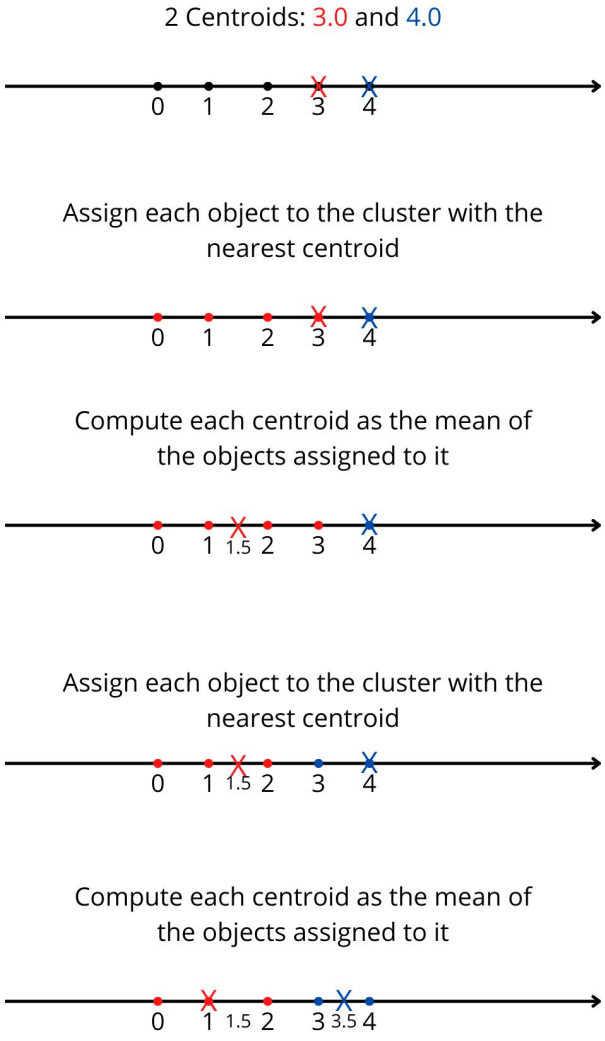
As distance measures, K-means traditionally uses Euclidean, but others can be used. In mean shift clustering, instead we estimate modes of probability density function. Given a kernel, for each point:

- Center a window on that point
- Compute the mean of the data in the search window
- Center the search window at the new mean location
- Repeat previous two steps until convergence

The difference between K-means and mean shift clustering is that K-means specify the number of clusters while in mean shift it is determined by the algorithm. Another difference is the complexity of the algorithm, K-mean has linear complexity while mean shift has quadratic complexity.

2 Question 2

We want to cluster 5 1D points 0,1,2,3,4 using the K-mean algorithm. We have initial two initial centroids 3.0 and 4.0. The first two iteration of the algorithm are listed below.



3 Question 3

A 5x5 greyscale image is given below. We want to compute the Hessian matrix for the 3x3 matrix W.

1	1	1	1	1
1	1	0	2	1
1	2	2	1	1
1	2	1	0	1
1	1	1	1	1

Table 1: 5x5 input image W

The Hessian matrix is defined as:

$$H = \sum_{(x,y)\in W} \begin{bmatrix} I_x(x,y)^2 & I_x(x,y)I_y(x,y) \\ I_x(x,y)I_y(x,y) & I_y(x,y)^2 \end{bmatrix}$$

The gradient is defined as:

$$\nabla A = [\tfrac{\partial A}{\partial x}, \tfrac{\partial A}{\partial y}]$$

We need to perform two convolutions of the image with two different filters. The two filters are listed below.

$$H_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mathrm{H}_y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Those two matrices are obtained from the finite difference formula, listed below for a general function f.

$$\tfrac{\partial f}{\partial x}[x,y] \simeq F[x+1,y] - F[x,y]$$

$$\tfrac{\partial f}{\partial y}[x,y] \simeq F[x,y+1] - F[x,y]$$

The calculus are reported below

$$I_x = W * H_x = \\ = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 & 1 \\ 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \\ = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$I_y = W * H_y = \\ = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 & 1 \\ 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \\ = \begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$I_x^2 = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad I_y^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 4 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$I_x I_y = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

The final result is:

$$H = \begin{bmatrix} 10 & 7 \\ 7 & 10 \end{bmatrix}$$

4 Programming

4.1 P1

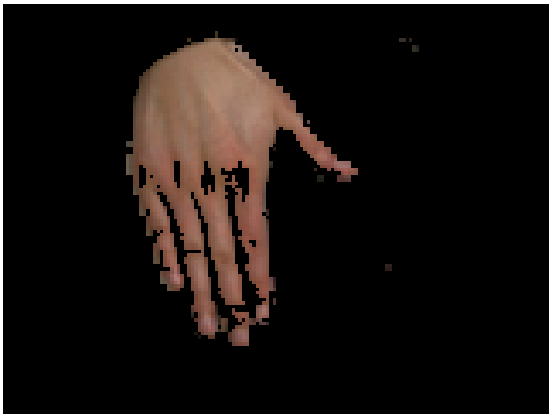
In this section we want to implement an histogram-based color segmentation. The testing image is listed below.

Figure 1: Test image

We will use ten training images representing all the color variation of skins. The testing images are listed below

Using this images we calculate the HS histogram and then we use this model to segment the test images. The result is listed below.

(a) Mask



(b) Segmentation

4.2 P2

In addition to building a skin color model based on the histogram, an alternative approach is to model the skin color via a 2D Gaussian distribution, with mean μ and covariance matrix Σ. d=2 is the dimension of the random vector.

$$p(x|\mu,\Sigma) = \frac{1}{(2\pi)^d|\Sigma|}exp(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu))$$

We use the same training data used in P1 to build a Gaussian-based skin color model. The mean and covariance obtained are:

$$\mu = \begin{bmatrix} 11.33281079 \\ 132.58649617 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 4.96395384 & -26.58454086 \\ -26.58454086 & 2217.35021386 \end{bmatrix}$$

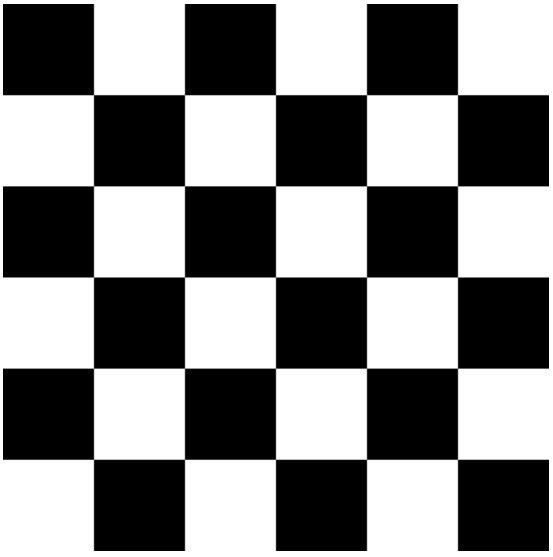
Now we calculate the skin probability for each pixel on the testing image based on the estimated Gaussian distribution and we set threshold= 0.000002. We obtain the following results.

(a) Mask

(b) Segmentation

4.3 P3

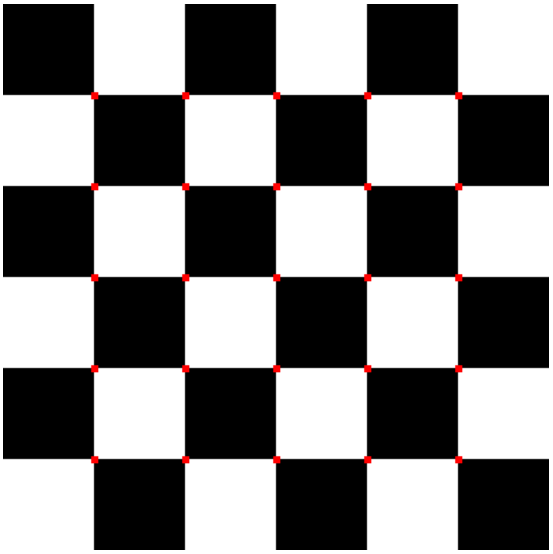
In this section we use the Harris corner detector implemented in OpenCV and we use it on two input images, checkerboard.png and toy.png.



(a) checkerboard.png

(b) toy.png

The results obtained are listed below.



(a) checkerboard

(b) toy