Discrete Mathematical Models

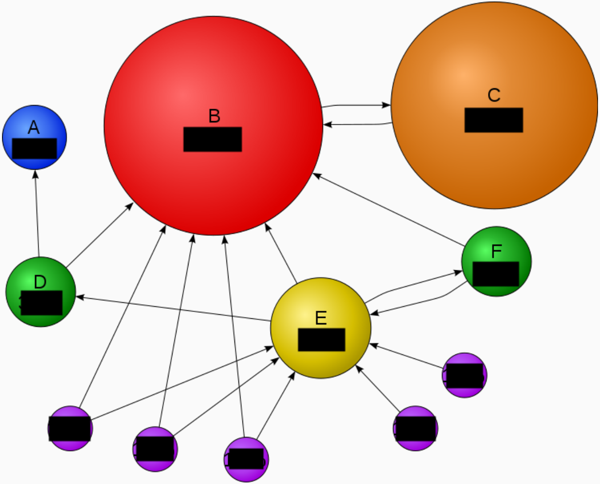
Lecture 30

Kane Townsend

Semester 2, 2024

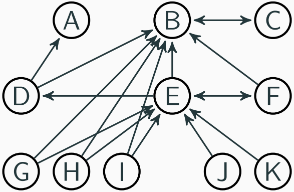
Wikipedia Example

Wiki WebGraph



`https://en.wikipedia.org/wiki/PageRank` We will fill in PR. It will differ slightly to seen on wiki due to a small difference in our algorithm which is common to do for large n).

Example 5



At left is the Wikipedia example we mentioned in the previous lecture consisting of miniweb of 11 pages and 17 hyperlinks. Colours, variable sizes and PageRanks have been removed and the bottom five vertices have been labelled G to K, following the given labelling of the top six vertices. The layout is similar to that in the original.

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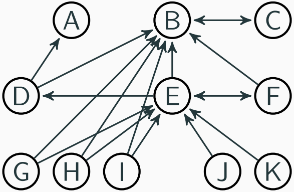

Using the steady state method we have been discussing, we will derive the PageRanks given on the Wikipedia diagram.

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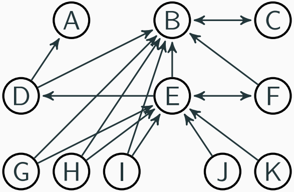

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Step 1: Construct G^+ from G

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Let n_i be the number vertices to which i is adjacent in G^+, that is

$$n_i = |\{j \ : \ (i,j) \in E(G^+)\}|$$

The $p_{i\,j}$ of a transition from vertex j to i is given by

$$p_{i\,j} = \begin{cases} {}^1/_{n_i} & \text{if } n_i \neq 0 \text{ and } (j,i) \in E(G^+) \\ 0 & \text{otherwise} \end{cases}$$

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For our Wikipedia example we get

$$T = \begin{bmatrix} 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/10 & 0 & 1 & 1/2 & 1/3 & 1/2 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 1/10 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/10 & 0 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/10 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 1/2 & 1/2 & 1 & 1 \\ 1/10 & 0 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$M = (\alpha/n)U + (1-\alpha)T =$$
$$\begin{bmatrix}
3/220 & 3/220 & 3/220 & 193/440 & 3/220 & 3/220 & 3/220 & 3/220 & 3/220 & 3/220 & 3/220 \\
217/2200 & 3/220 & 19/22 & 193/440 & 49/165 & 193/440 & 193/440 & 193/440 & 193/440 & 3/220 & 3/220 \\
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217/2200 & 3/220 & 3/220 & 3/220 & 49/165 & 3/220 & 3/220 & 3/220 & 3/220 & 3/220 & 3/220 \\
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$$\mathcal{P}R = \begin{bmatrix} 5135730/169545563 \\ 3195978661779342/8292828011094295 \\ 165932254119056/482651365725065 \\ 6644106/169545563 \\ 5317726843699199/65574887090606070 \\ 6644106/169545563 \\ 2748522/169545563 \\ 2748522/169545563 \\ 2748522/169545563 \\ 2748522/169545563 \\ 2748522/169545563 \end{bmatrix} \approx \begin{bmatrix} 0.0303 \\ 0.3854 \\ 0.3438 \\ 0.0392 \\ 0.0811 \\ 0.0392 \\ 0.0162 \\ 0.0162 \\ 0.0162 \\ 0.0162 \\ 0.0162 \end{bmatrix} \ (4d.p.)$$

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A computationally simpler method starts from the defining equation attempts to find PR by iteratively calculating the chain of probability vectors $P_0, P_1, \ldots$ where P_0 is arbitrary and $P_k = MP_{k-1}$ for $k \geq 1$ (so $P_k = M^k P_0$).

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Each iteration takes a weighted average of teleporting and hyperlinking.

Example 4C

In Example 4B we used the equation-solving method to find PR.

For $T = \begin{bmatrix} 0 & 0 & 0 & 1/3 \\ 1/2 & 0 & 1/2 & 1/3 \\ 0 & 1 & 0 & 1/3 \\ 1/2 & 0 & 1/2 & 0 \end{bmatrix}$ and $\alpha = 0.1$ we found $PR = \begin{bmatrix} .10 \\ .30 \\ .37 \\ .23 \end{bmatrix}$ to 2d.p.

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Results of the first ten iterations, rounded to 2d.p. Calcs used 15d.p.

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Pretty good after just 2 iterations! Within 1%-point after 4 iterations.

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END OF SECTION D3

End of Course Notes :')

My research in graph theory

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A graph G is called **geodetic** if for all vertices $u, v \in V(G)$, there exists a unique geodesic from u to v

Geodetic graphs

Examples: Trees, complete graphs, odd cycle graphs etc.

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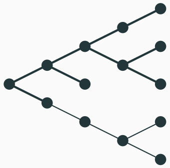

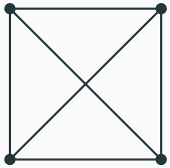

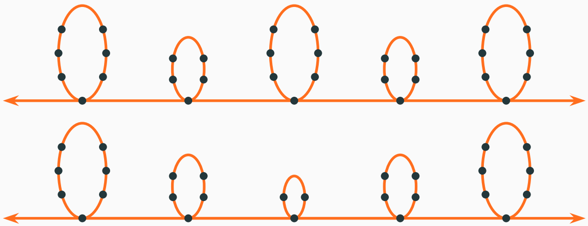

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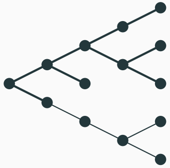

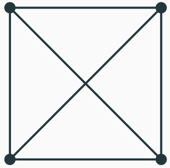

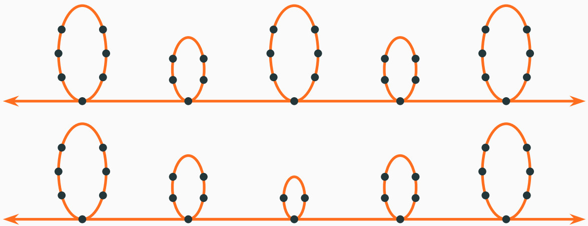

Question: Characterise all geodetic graphs (Ore in The Theory of Graphs (1962)).

More geodetic talk

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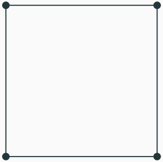

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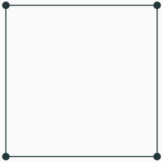

Given a graph there are minimal spanning trees of G consisting of geodesics from v to every other vertex in G. However, in a geodetic graph G, such a minimal spanning tree is unique $v \in V(G)$.

Combining geodetic graphs with bridges and cut vertices

Given two geodetic graphs G_1 and G_2 we can join them at a single vertex or via a single edge and produce a new geodetic graph. Also, if a graph G is geodetic then one can subdivide the edges and obtain a new geodetic graph.

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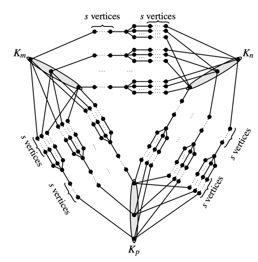
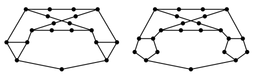
Recently my previous supervisor's colleagues (Weißand his PhD student Stober) used the NAUTY program to generate geodetic graphs (with no cut vertices) of up to 25 vertices (`https://arxiv.org/pdf/2308.08970`). There are 12969219563 geodetic graphs with 19 vertices, only 19 of them have no cut vertices. Of those 19, only 2 are not subdivisions of a smaller geodetic graph, one is K_{19} and other is F_3.





(a) F_3 (b) F_5

Geodesics in a geodetic graph cannot stay close forever

Suppose we have two long geodesics in a geodetic graph G travelling next to each other (never touching) such that they are at most $M > 0$ away from each other. Then by the generalised pigeonhole principle there are many times they are some they are distance $0 < m \leq M$ arbitrarily many times.

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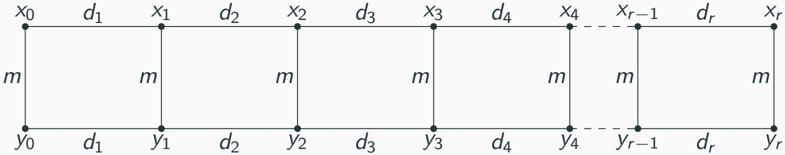

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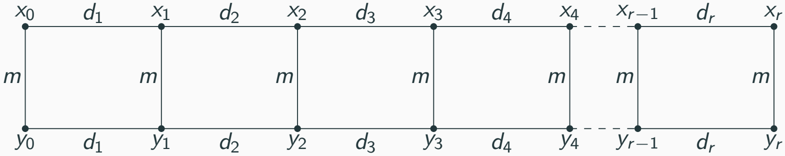

We get a contradiction!

Vertex transitive graphs

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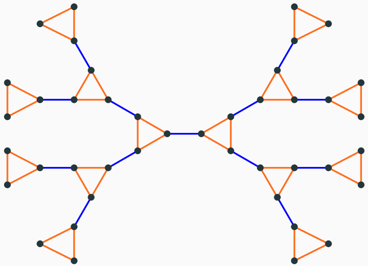

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Conjecture: Let G be a transitive geodetic graph. Then every vertex is a cut vertex? Then there is a bound on simple circuits in G?

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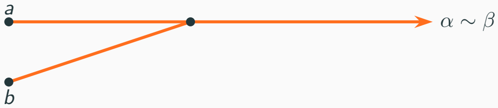

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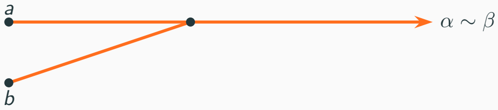
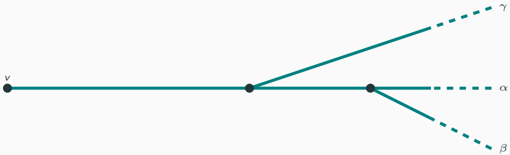
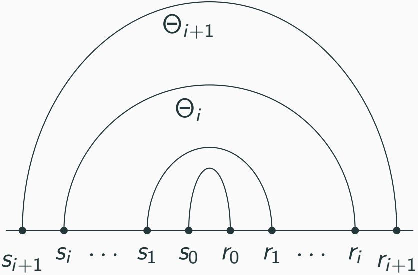


Step 2: Define a topology/metric on the geodesic boundary of a geodetic graph which is dependent on a basepoint vertex.



Proof of theorem outline (cont.)

Step 3: Prove that geodetic graphs cannot contain an 'infinite onion' (proof by contradiction using the topological properties of the geodesic boundary of a geodetic graph).



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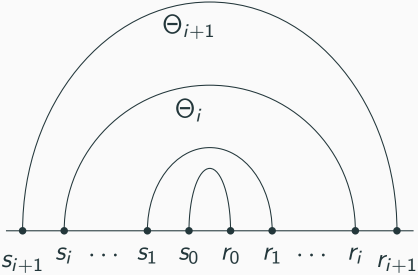

Step 4: Prove that a transitive geodetic graph cannot have unbounded size of isometric simple circuits as this implies that you can construct an infinite onion.

References to prepints

https://arxiv.org/pdf/2311.03730

https://arxiv.org/pdf/2211.13397