Discrete Mathematical Models

Lecture 27

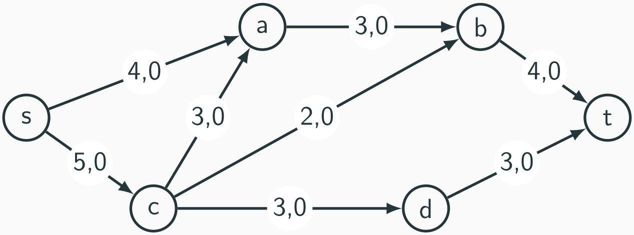
Kane Townsend

Semester 2, 2024

Maximal flow (cont)

Vertex labelling algorithm, Example 2: Stage 1: F_0 to F_1

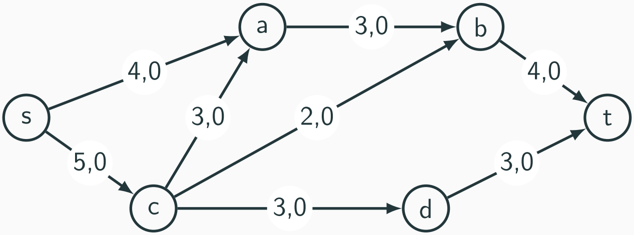
Flow F_0



Vertex labelling algorithm, Example 2: Stage 1: F_0 to F_1

Flow F_0

Levels



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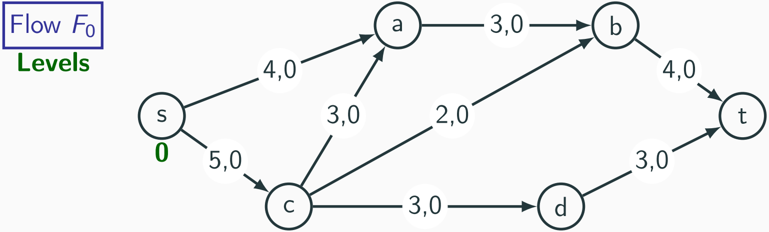

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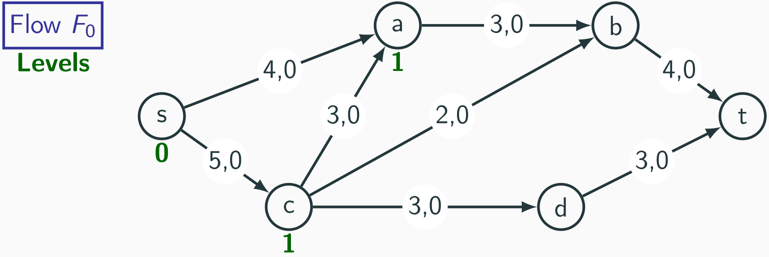

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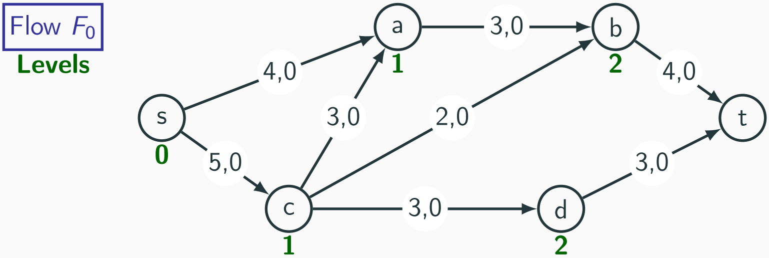

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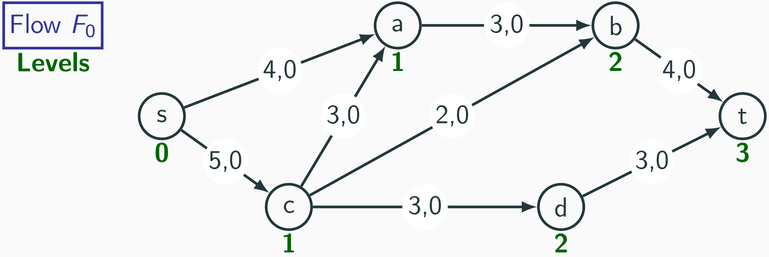
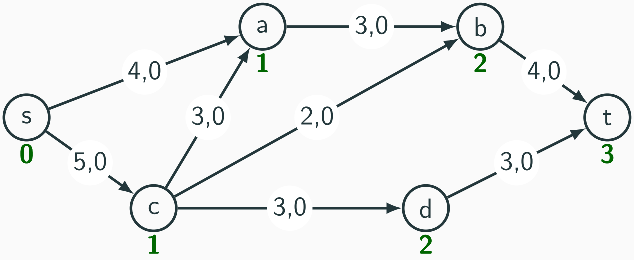


Vertex labelling algorithm, Example 2: Stage 1: F_0 to F_1

Flow F_0

Levels

Labels



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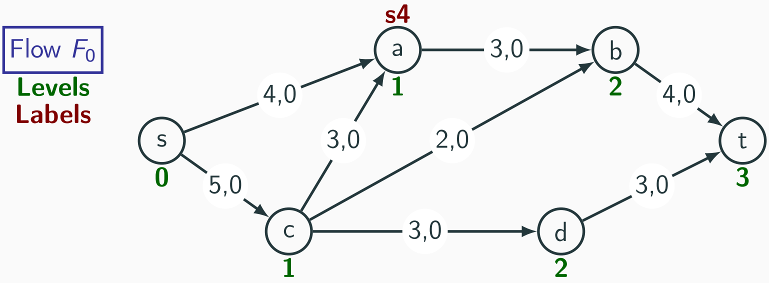

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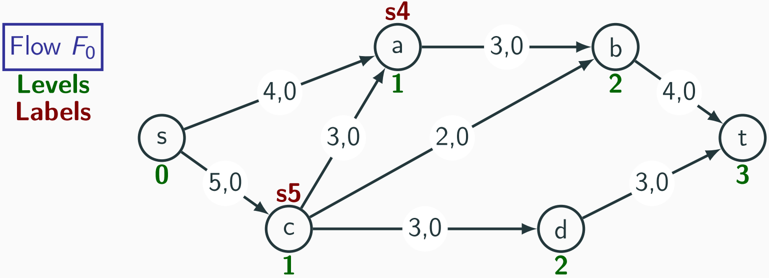

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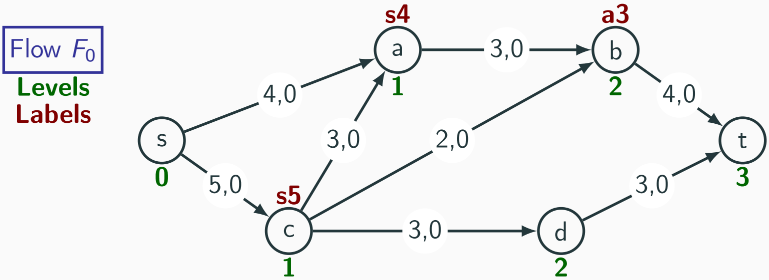

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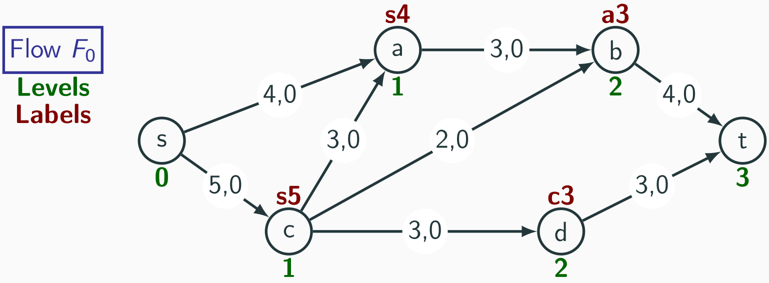

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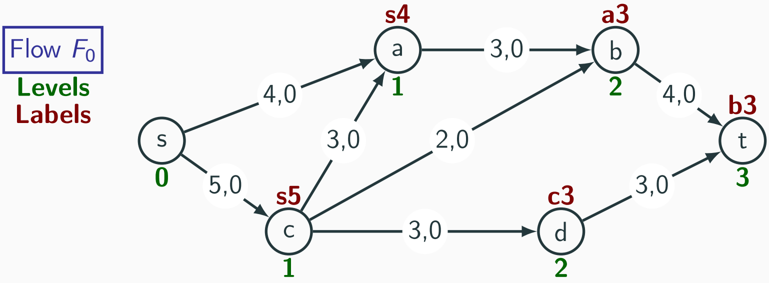

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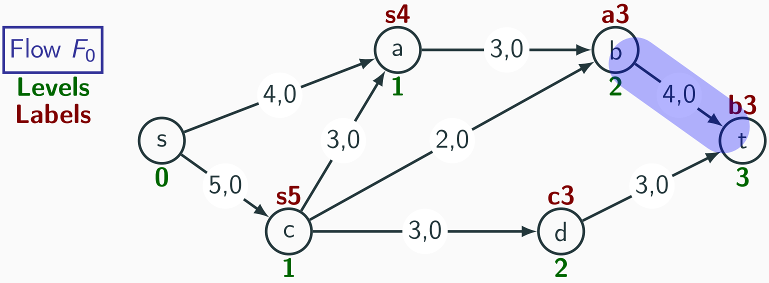

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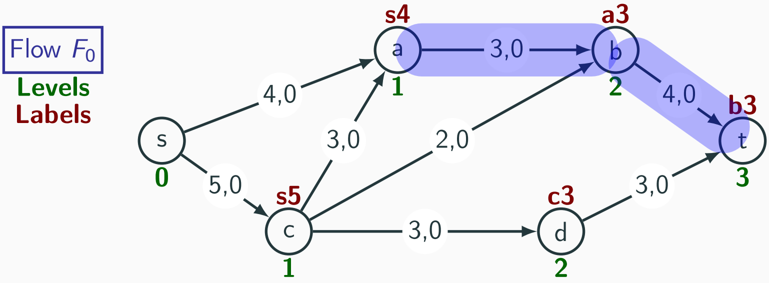

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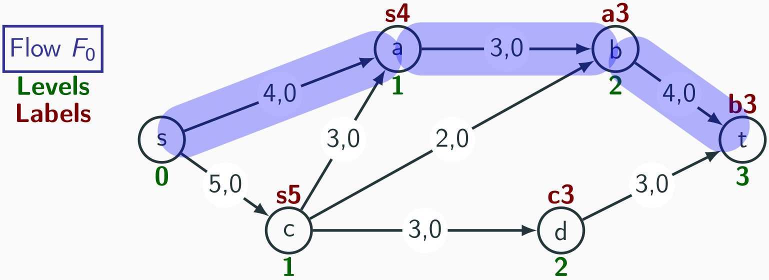


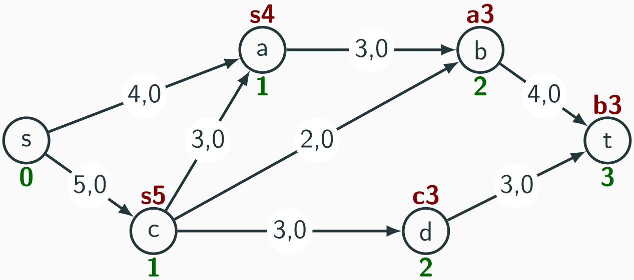
The incremental flow f_1

Vertex labelling algorithm, Example 2: Stage 1: F_0 to F_1

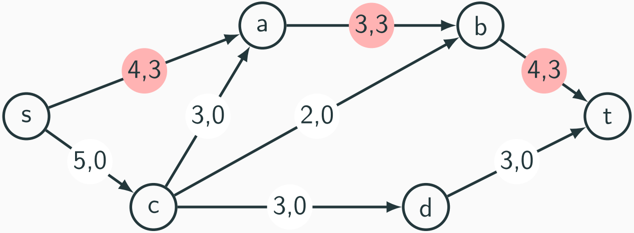
Flow F_0

Levels

Labels

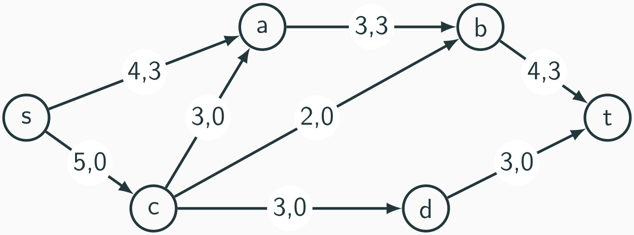


Flow F_1



Vertex labelling algorithm, Example 2: Stage 2: F_1 to F_2

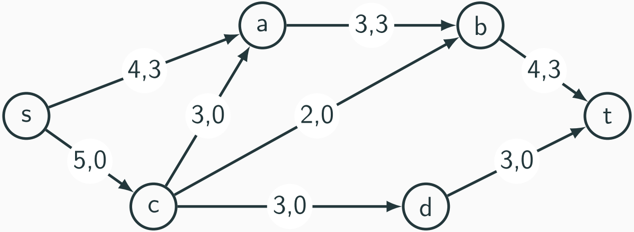
Flow F_1



Vertex labelling algorithm, Example 2: Stage 2: F_1 to F_2

Flow F_1

Levels



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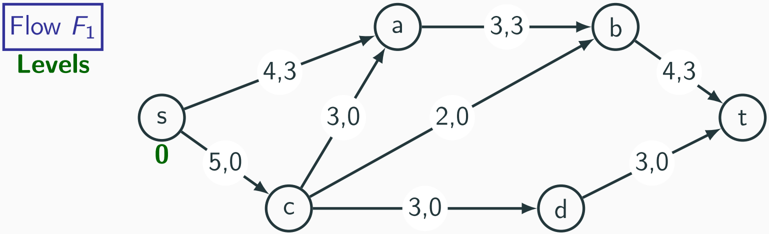

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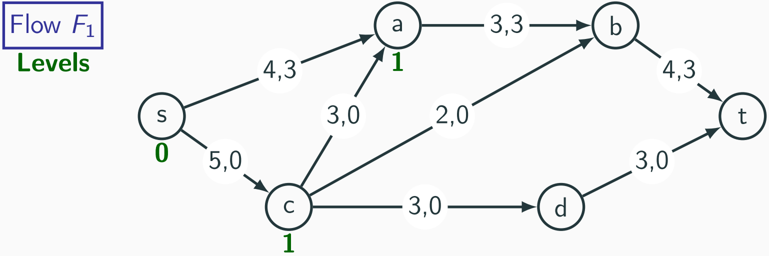

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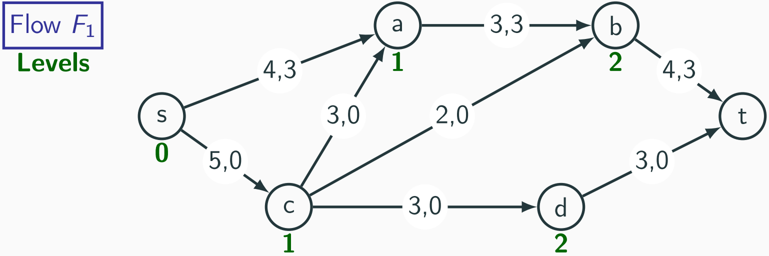
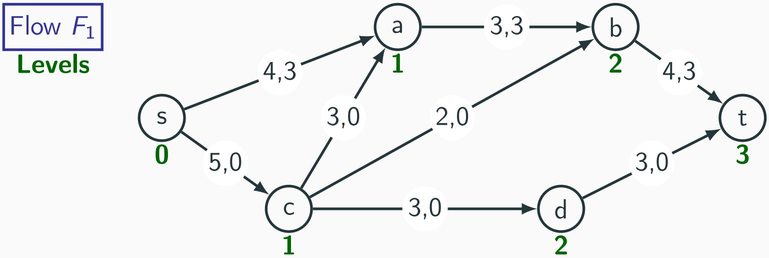


Vertex labelling algorithm, Example 2: Stage 2: F_1 to F_2

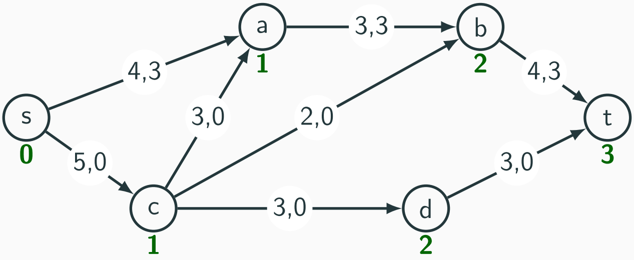


Vertex labelling algorithm, Example 2: Stage 2: F_1 to F_2

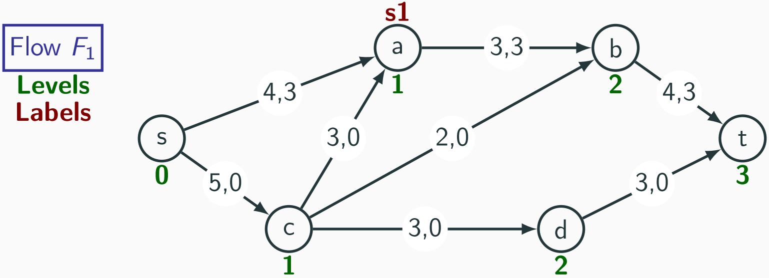
Flow F_1

Levels

Labels



Vertex labelling algorithm, Example 2: Stage 2: F_1 to F_2



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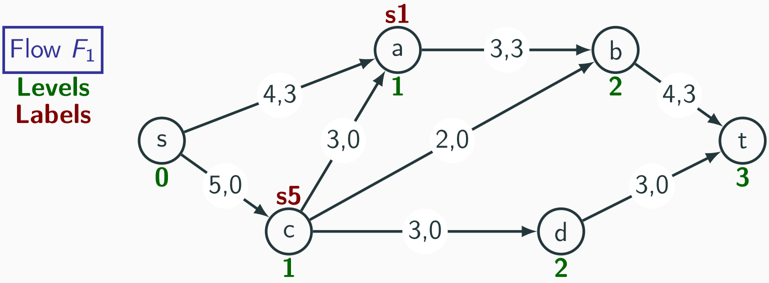

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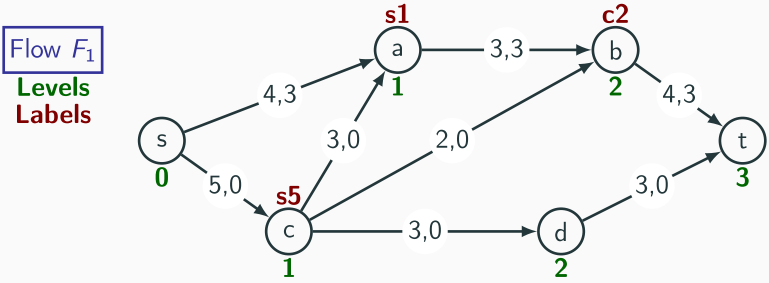

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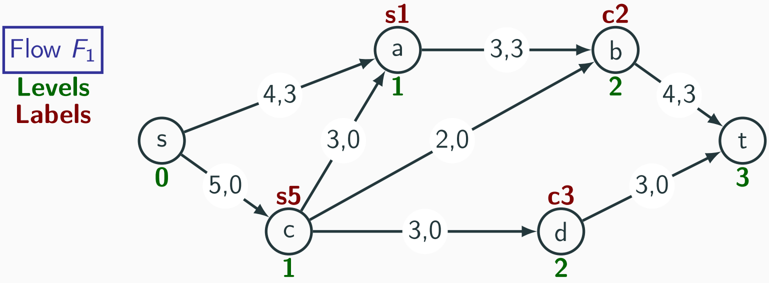

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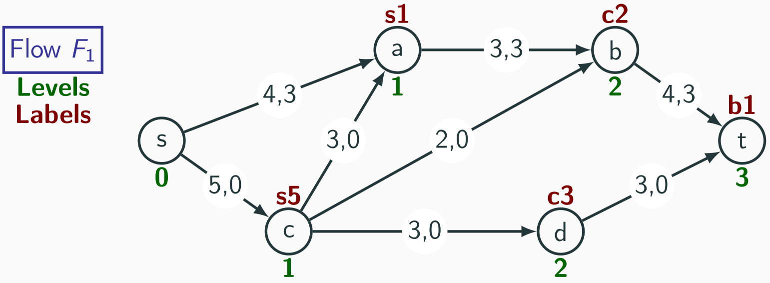

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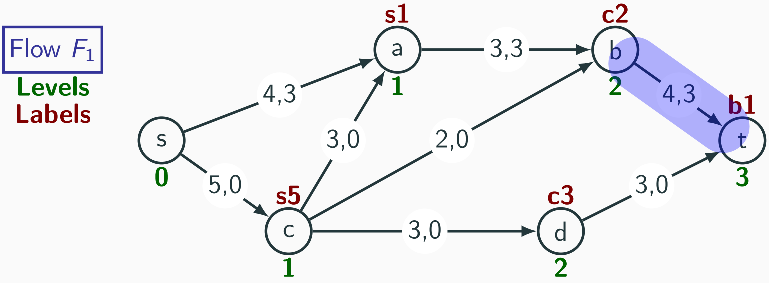

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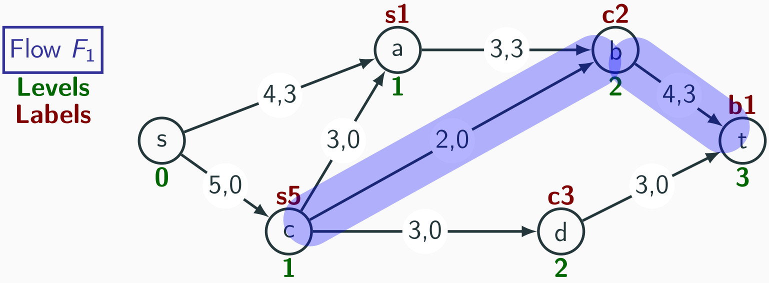

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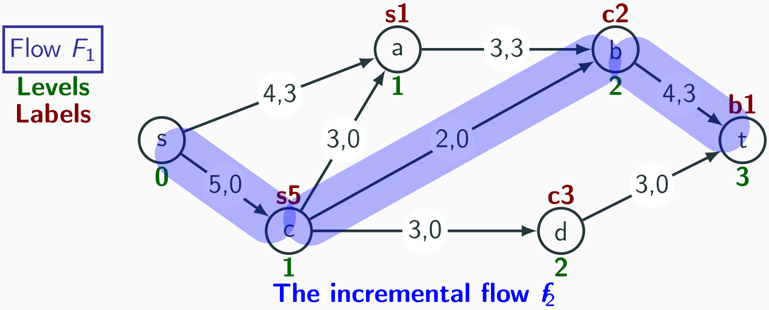

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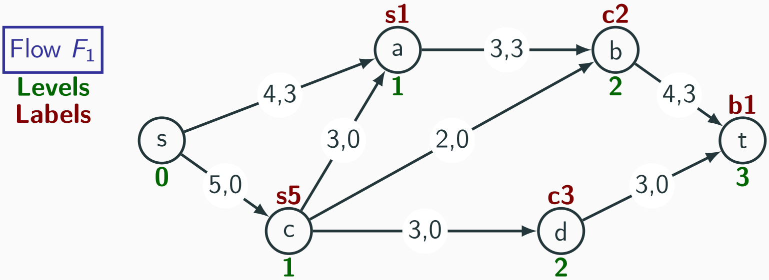
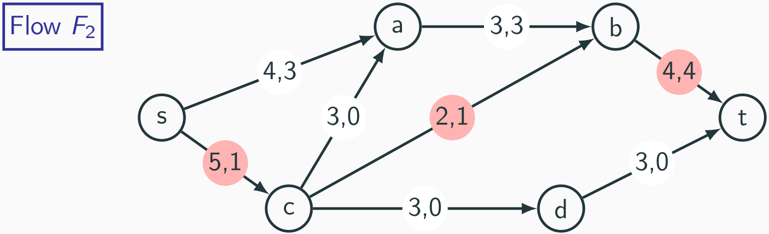
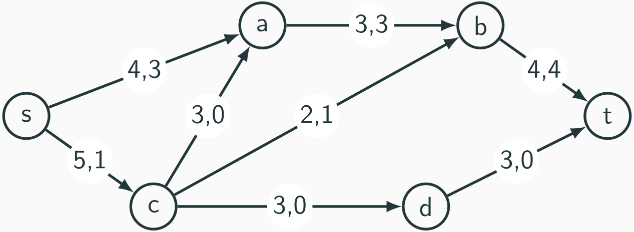




Vertex labelling algorithm, Example 2: Stage 3: F_2 to F_3

Flow F_2



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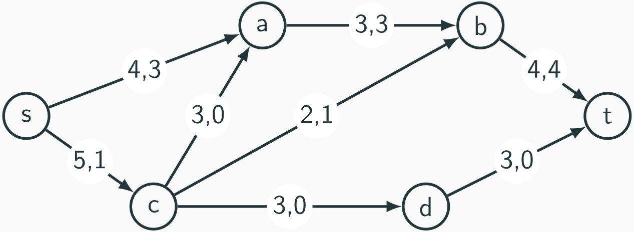
Flow F_2

Levels



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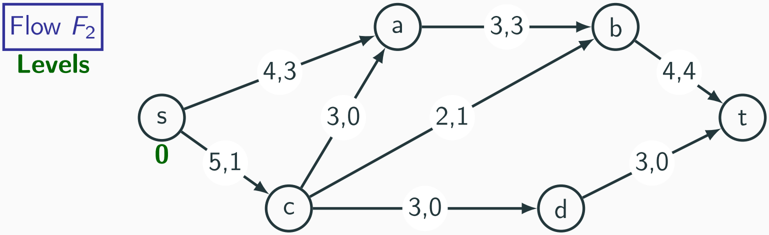

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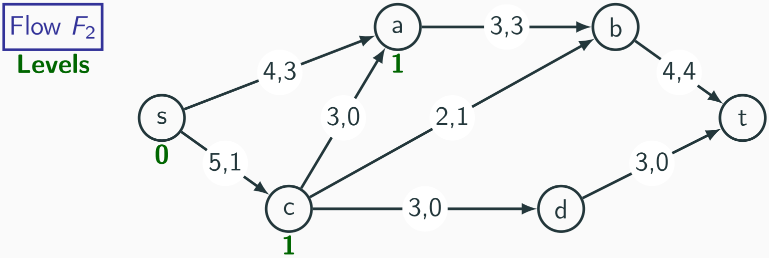

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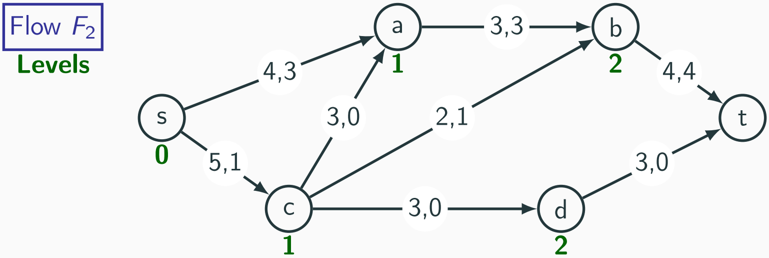

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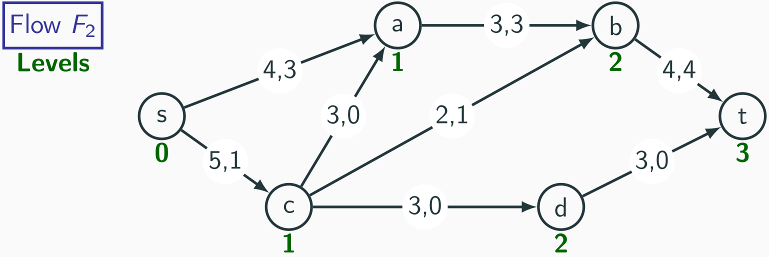

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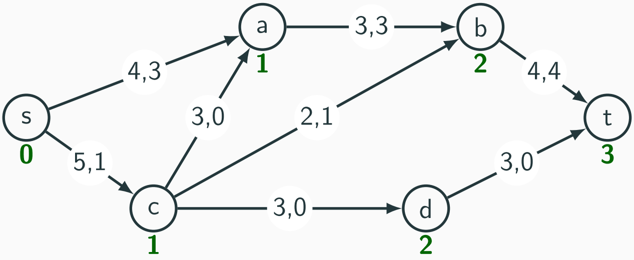
Flow F_2

Levels

Labels



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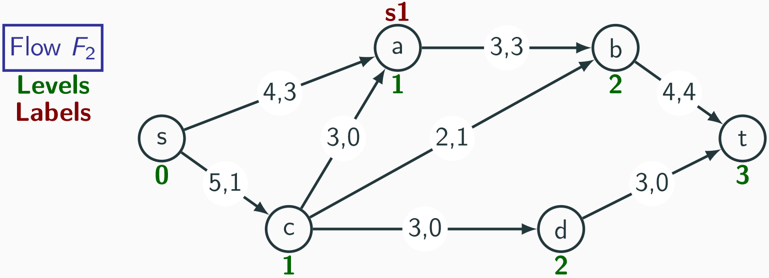

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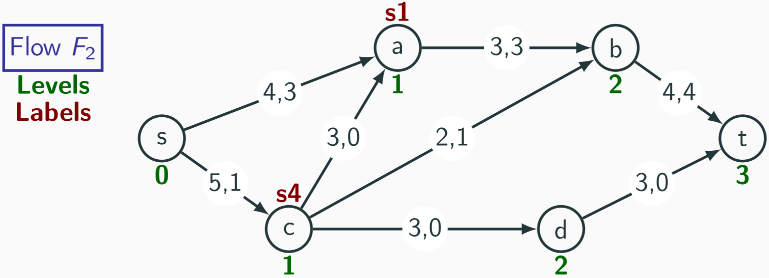

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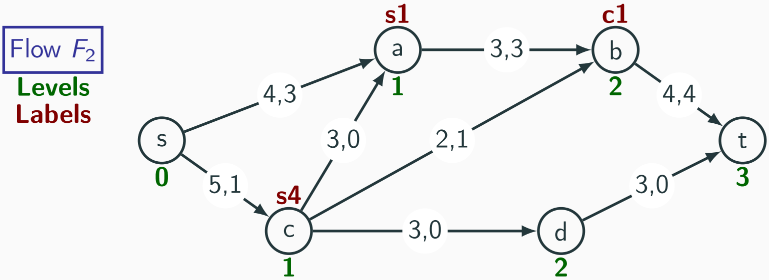

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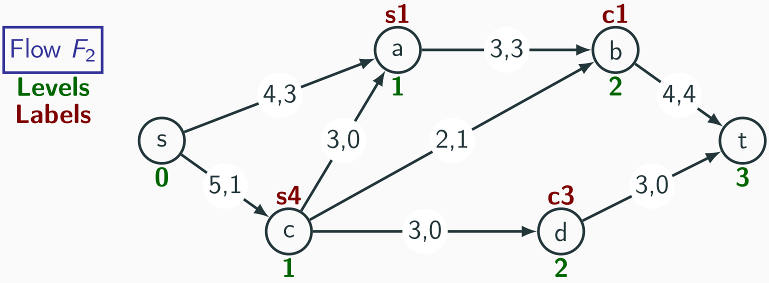

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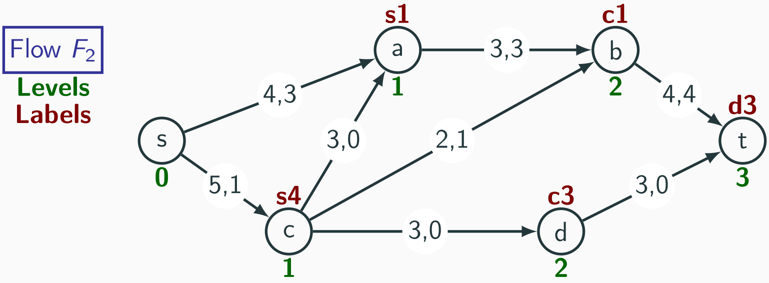

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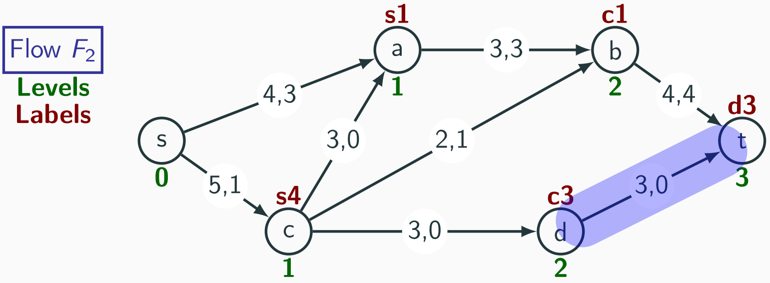

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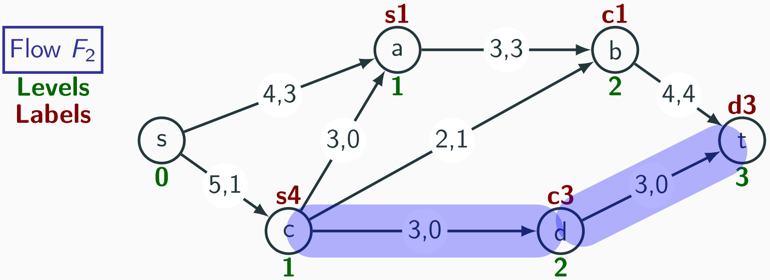

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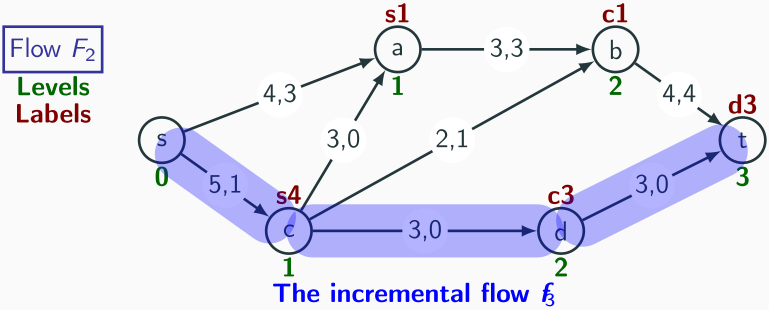

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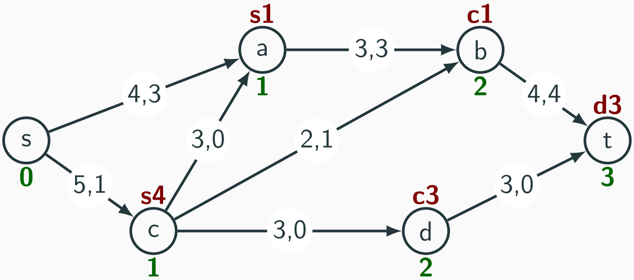
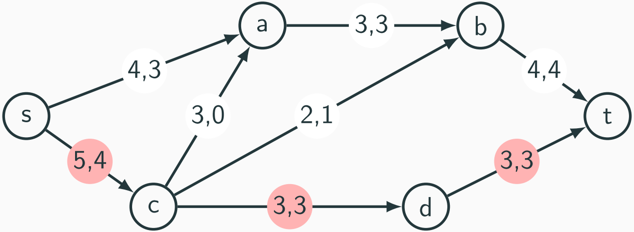
Flow F_2

Levels

Labels

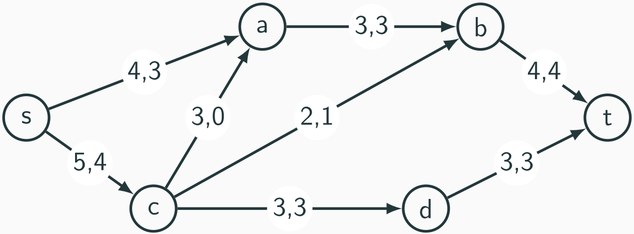


Flow F_3



Vertex labelling algorithm, Example 2: Stage 4: F_3 is $F_{\mathbf{max}}$

Flow F_3



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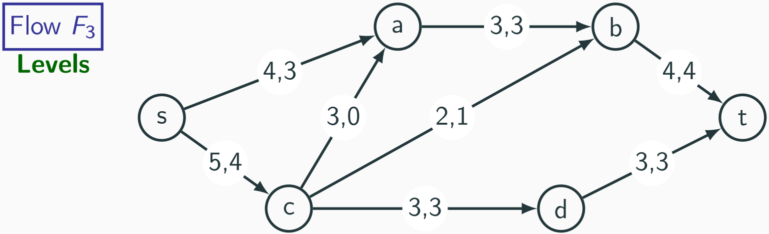

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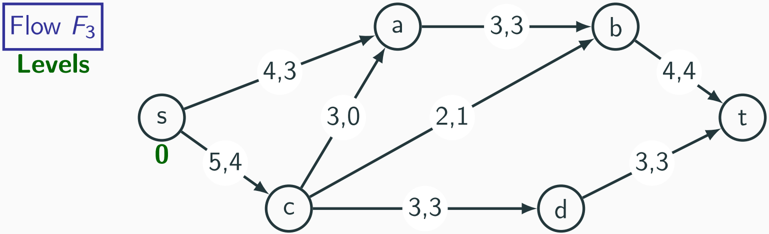

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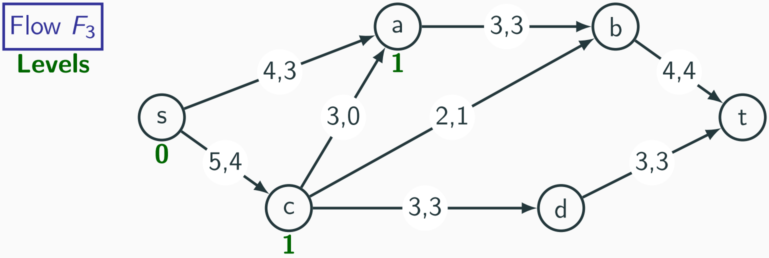

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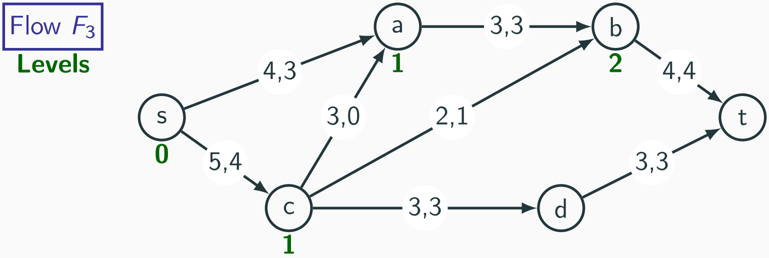

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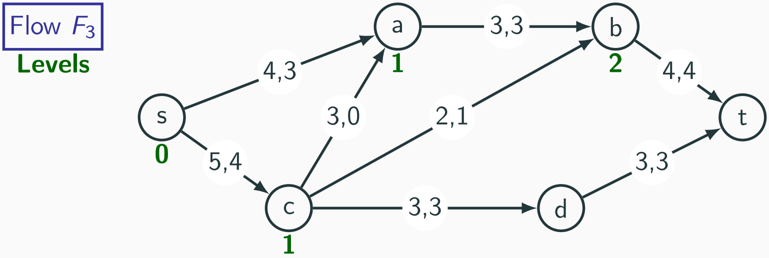

No level can be assigned to t !

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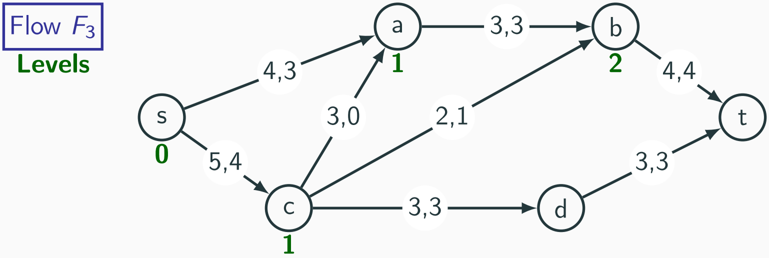
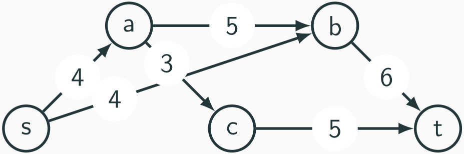


No level can be assigned to t !

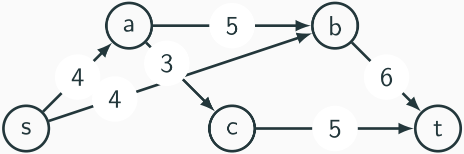
So the algorithm terminates with $F_{\max} = F_3$.

Cuts

Consider again Example 1 at right.



Cuts



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Since the total capacity of edges leading from the source is $4 + 4 = 8$ it is clear that the maximum flow value cannot be more than 8.

Cuts

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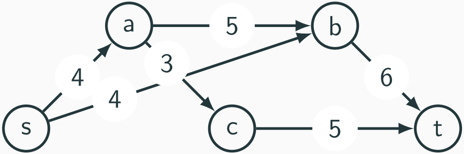

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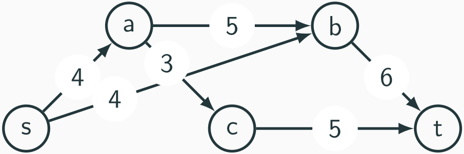
In fact the maximum flow value *is* 8 as a flow with that value is easily found. (We found it using the vertex labelling algorithm, but that isn't really needed on such a simple example.)

Cuts



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The set of edges leading out of the source is an example of a network *cut*, so called because removing the edges would 'cut' the network in two, separating the source from the target.
The $4 + 4$ is the 'capacity' of the cut.

Cuts

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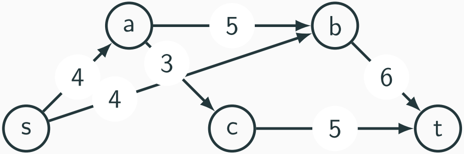

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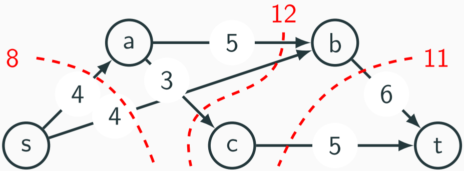
A cut may be indicated by drawing a (red) line through its edges.

Cuts

Consider again Example 1 at right.



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Some examples, with values, are shown on the digraph above.

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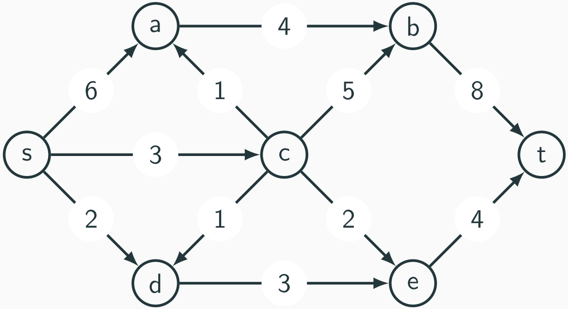
Though highly plausible, this theorem is little tricky to prove, and the proof will be omitted, as will the proof that the vertex labelling algorithm always finds a maximum flow.

Max flow min cut: Class example 1

What is the maximum flow value for this transport network?



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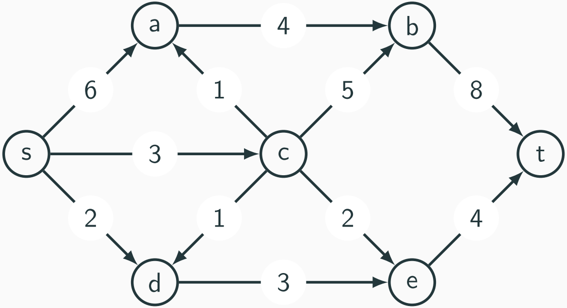

Answer:

Max flow min cut: Class example 1

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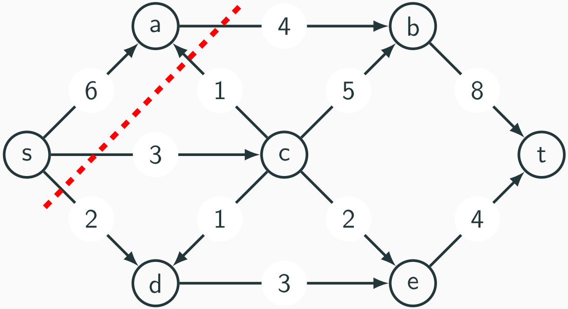

Answer: After some searching we find the minimum cut shown, for which $S = \{s, a\}$ and $T = \{b, c, d, e, t\}$. This is given by cut $K = \{(s, d), (s, c), (a, b)\}$

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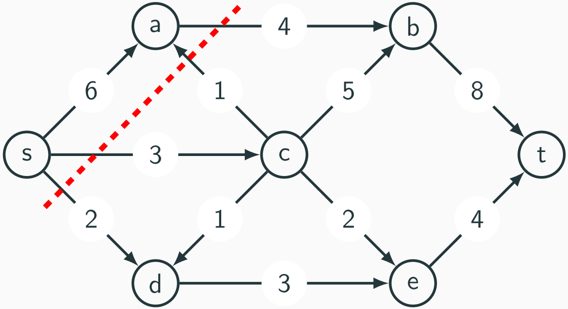

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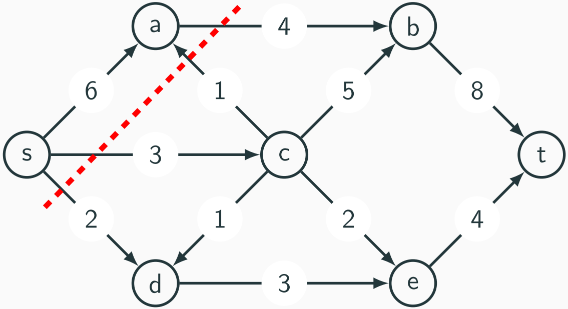

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Note: Make sure $K = E(D) \cap (S \times T)$

Max flow min cut: Class example 2

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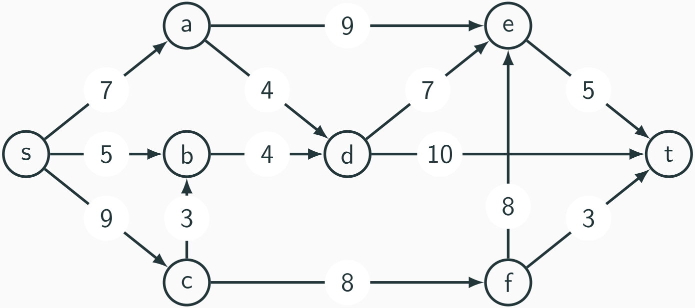

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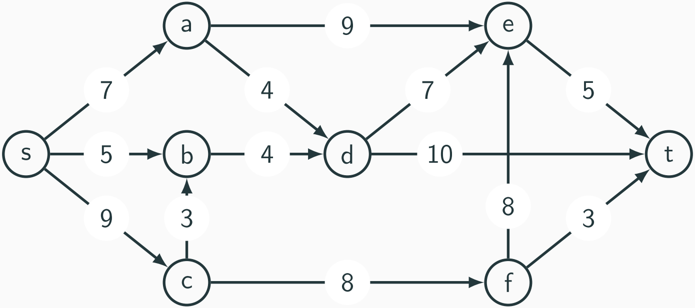

Hint:

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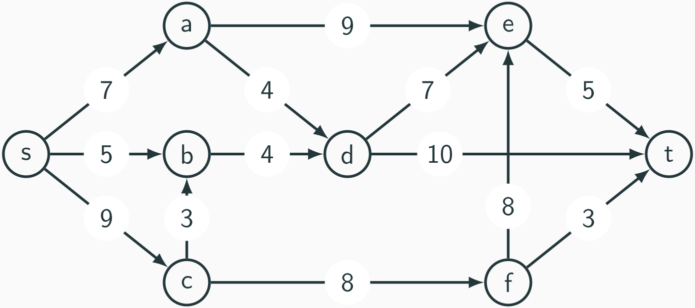

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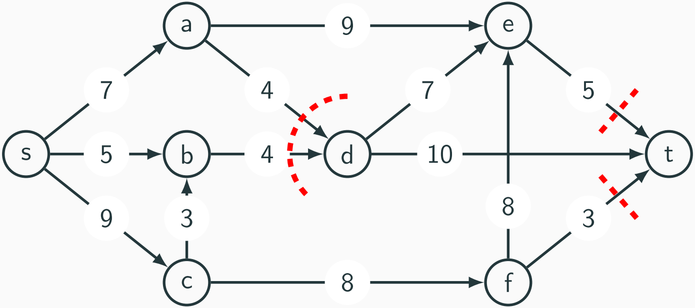

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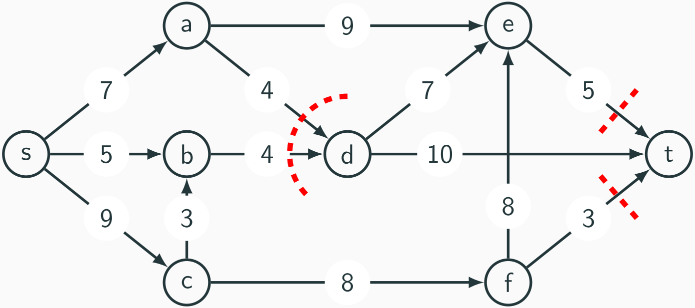

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The capacity of this cut is 4+4+5+3=16, so this is the max flow.

Virtual flows

Introduction to virtual flows

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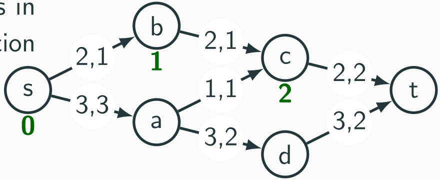

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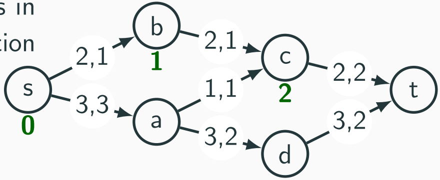

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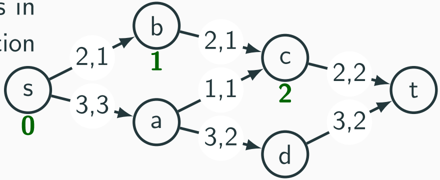

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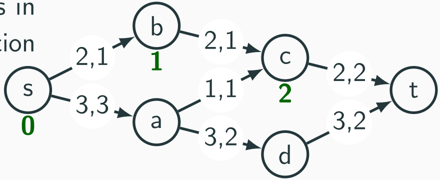

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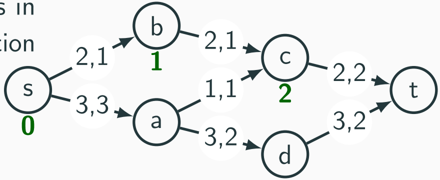

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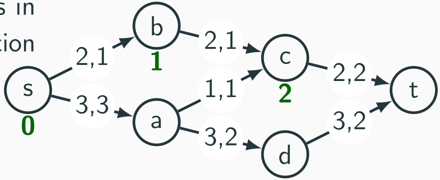

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First, the definition and an explanation of how the algorithm is modified.

The complete vertex labelling algorithm

Let (u,v) be a (directed) edge in a transport network D, and suppose there is currently a flow of $f > 0$ along this edge. The vertex labelling algorithm can reduce this flow to $g < f$ by introducing a **virtual flow** of $f - g$ in the opposite direction, *i.e.* from v to u.

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For vertices u,v of D, where D has capacity and flow functions C, F:

$$\boxed{S((\mathrm{u,v})) = \begin{cases} C((\mathrm{u,v})) - F((\mathrm{u,v})) & \text{if } (\mathrm{u,v}) \in E(D) \\ F((\mathrm{v,u})) & \text{if } (\mathrm{v,u}) \in E(D) \\ 0 & \text{otherwise} \end{cases}}$$

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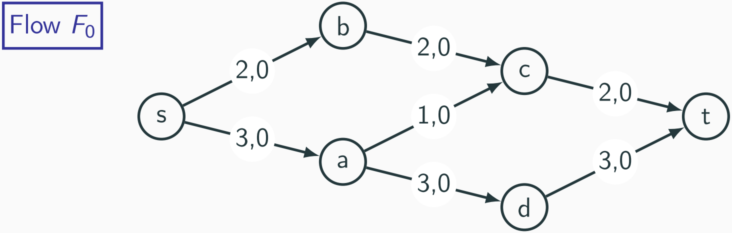
When $(\mathrm{v,u}) \in E(D)$, $S((\mathrm{u,v}))$ is called a **virtual capacity**.

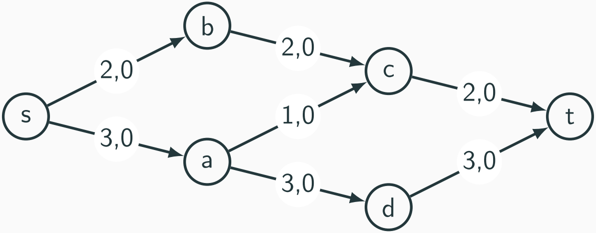
Vertex labelling algorithm, Example 3



Vertex labelling algorithm, Example 3

$\boxed{\text{Flow } F_0}$

Levels



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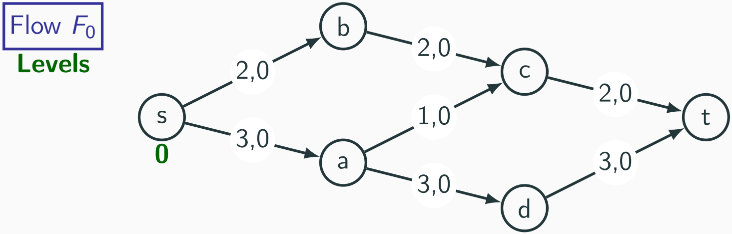

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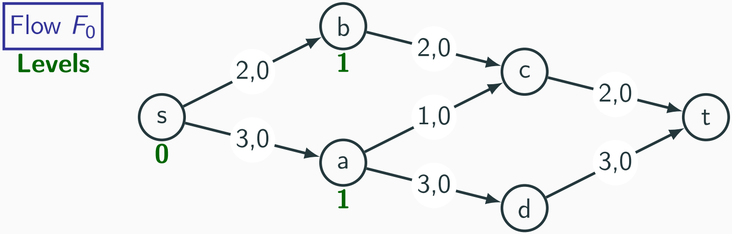

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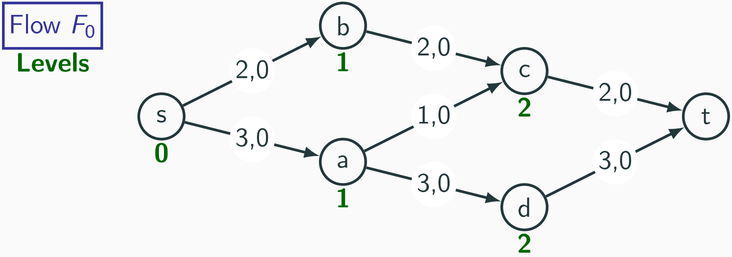

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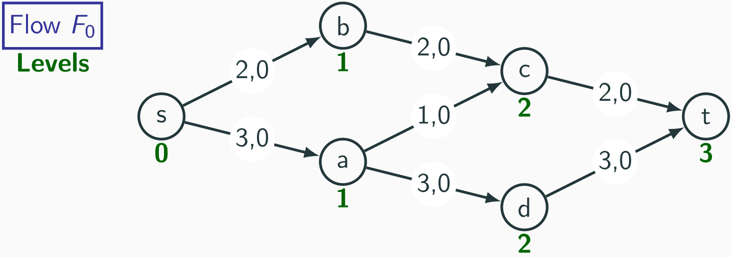

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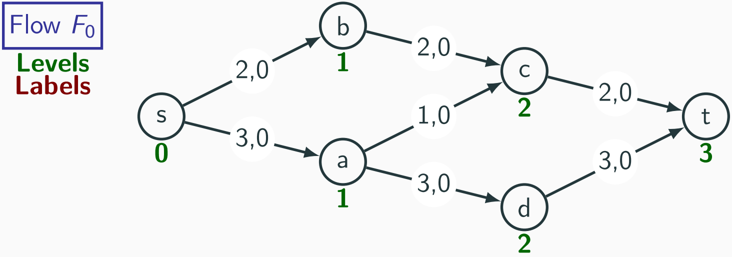

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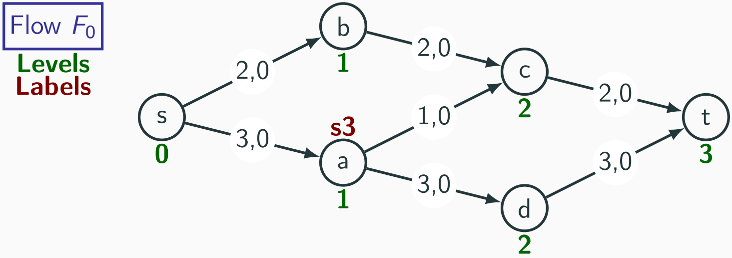

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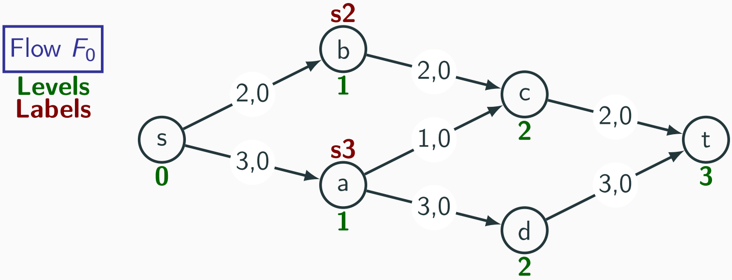

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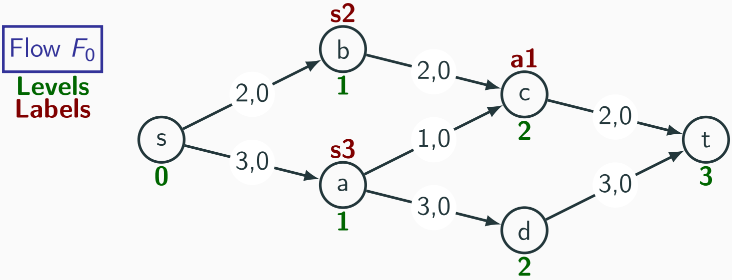
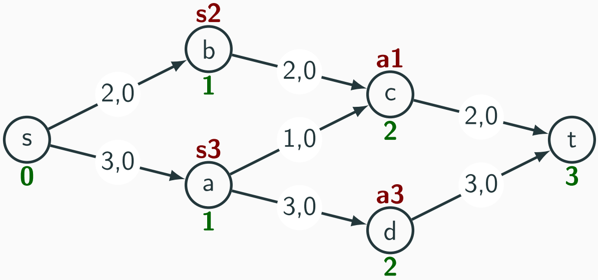


Vertex labelling algorithm, Example 3

Flow F_0

Levels
Labels



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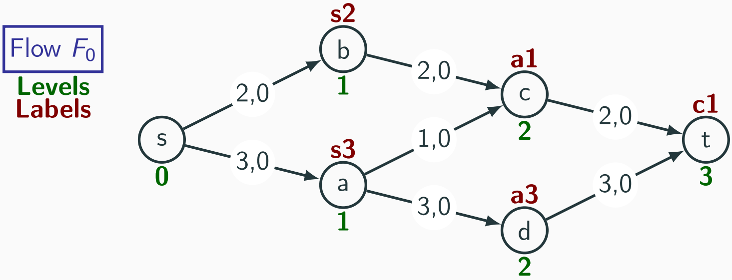

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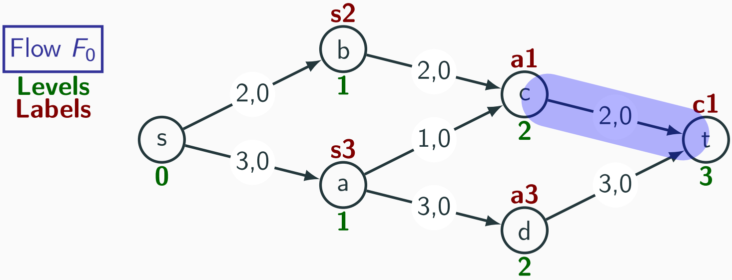

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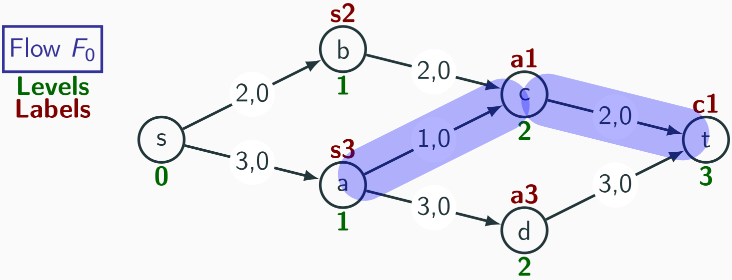

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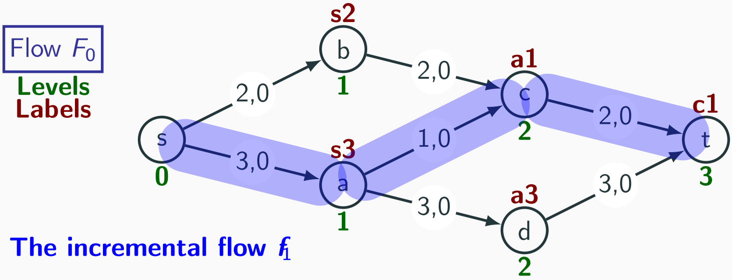
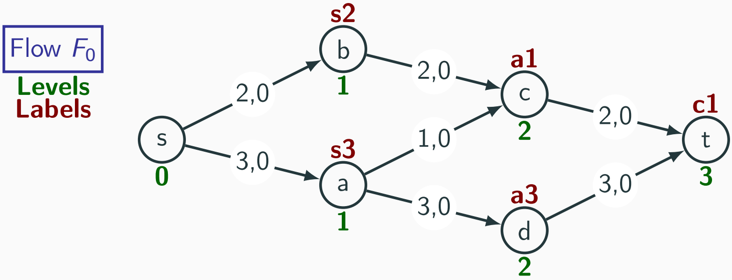
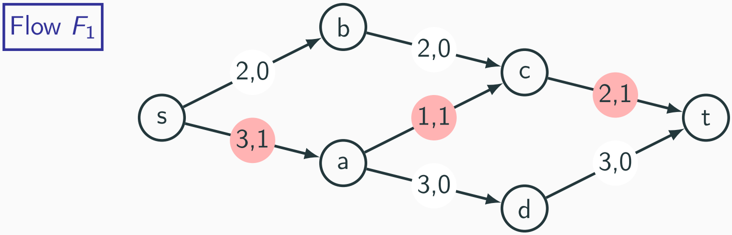


The incremental flow f_1

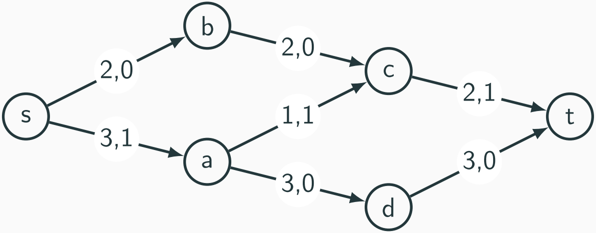
Vertex labelling algorithm, Example 3





Vertex labelling algorithm, Example 3

Flow F_1



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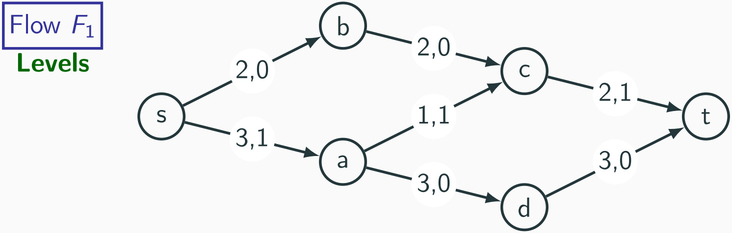

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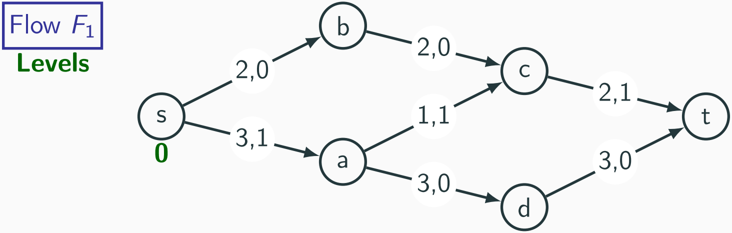

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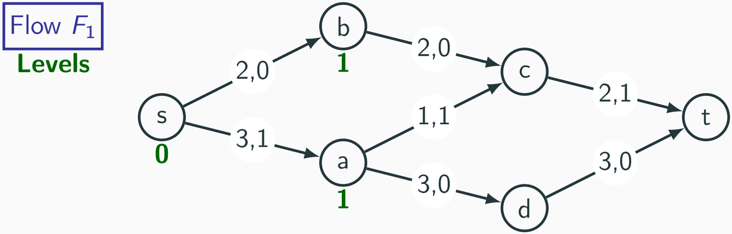

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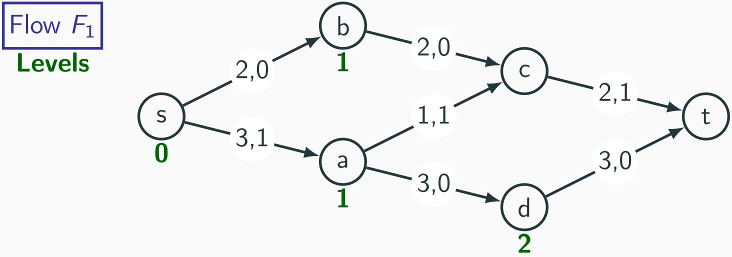
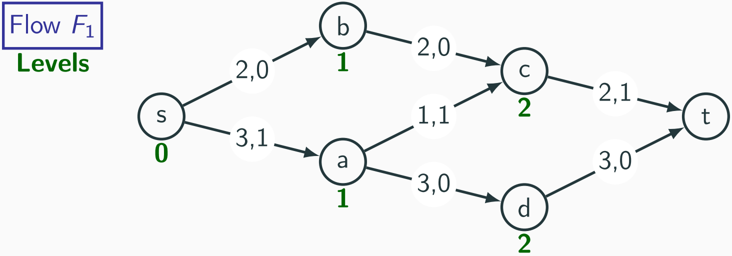
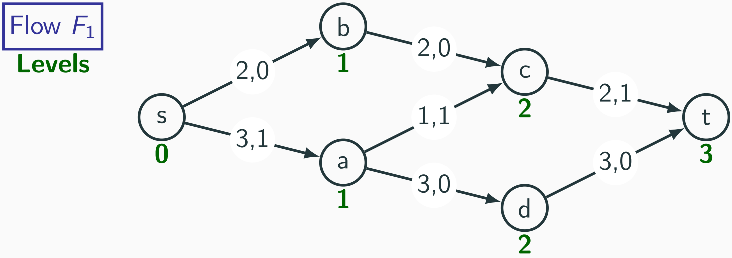


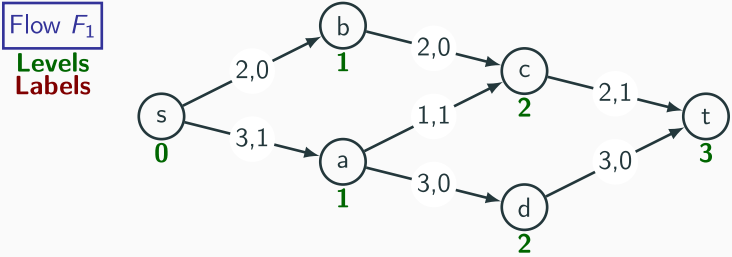
Vertex labelling algorithm, Example 3



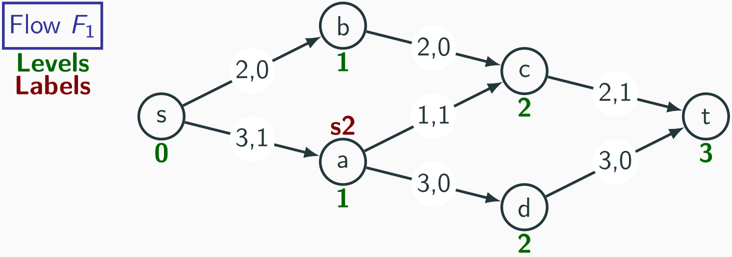
Vertex labelling algorithm, Example 3



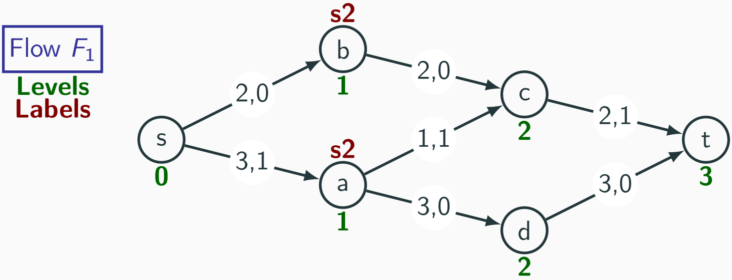
Vertex labelling algorithm, Example 3



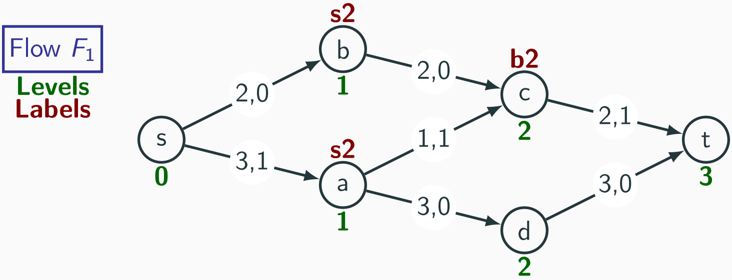
Vertex labelling algorithm, Example 3



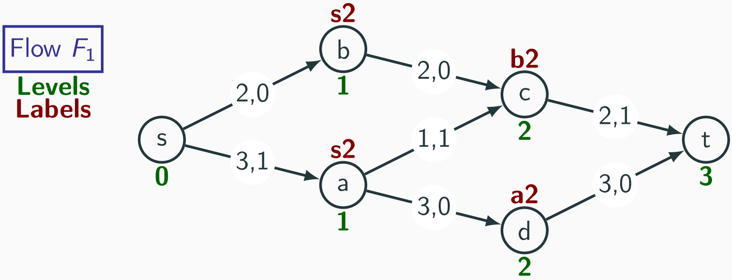
Vertex labelling algorithm, Example 3



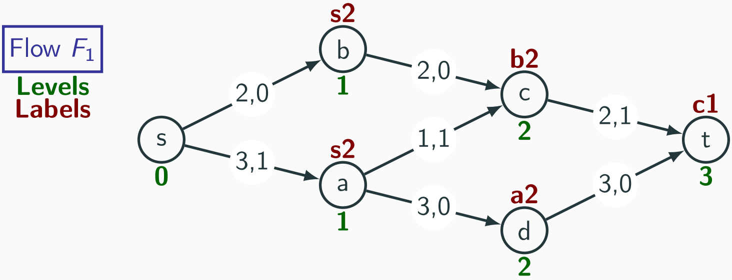
Vertex labelling algorithm, Example 3



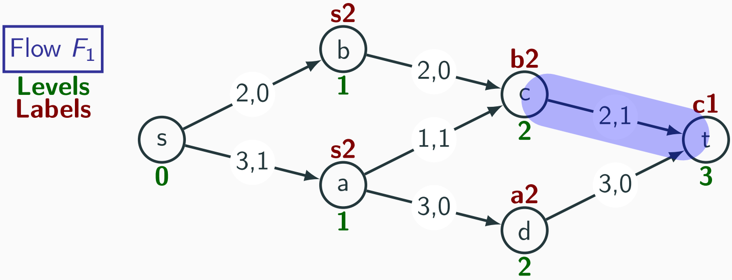
Vertex labelling algorithm, Example 3



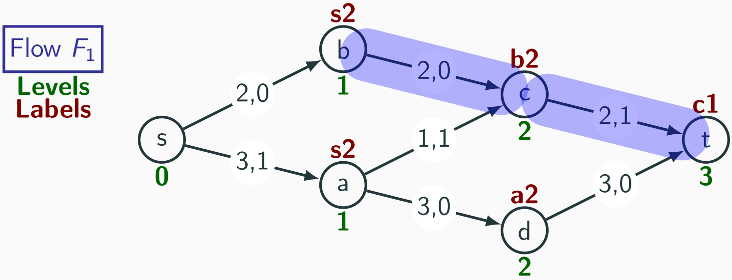
Vertex labelling algorithm, Example 3



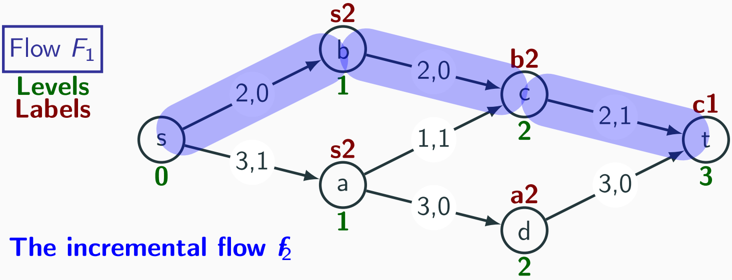
Vertex labelling algorithm, Example 3



Vertex labelling algorithm, Example 3

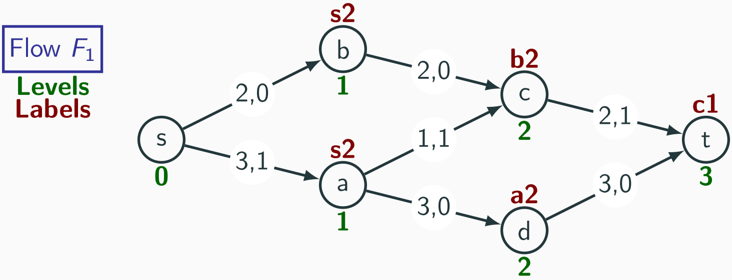


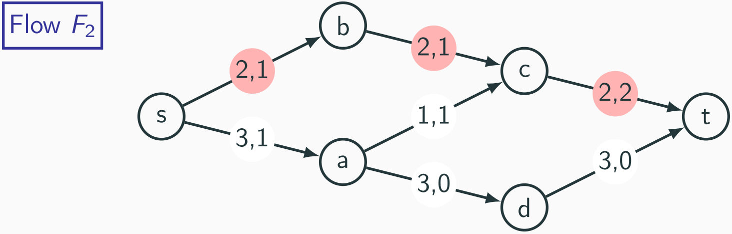
Vertex labelling algorithm, Example 3



The incremental flow f_2

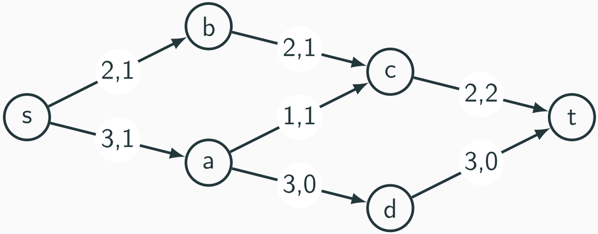
Vertex labelling algorithm, Example 3



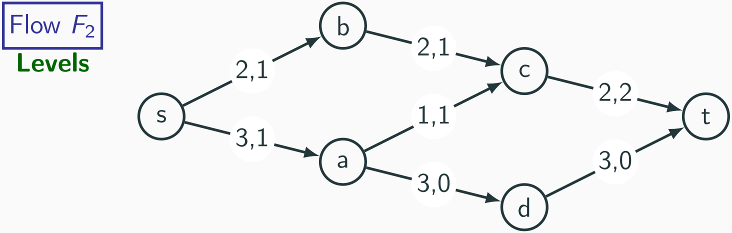


Vertex labelling algorithm, Example 3

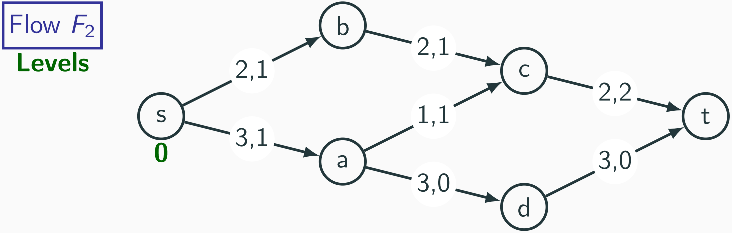
Flow F_2



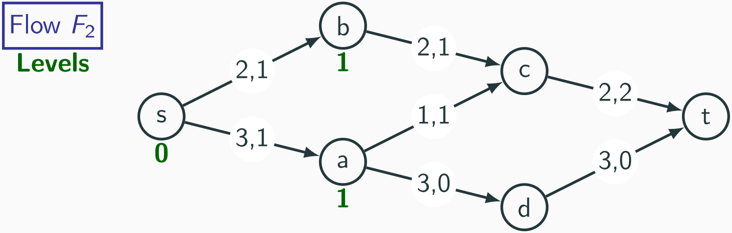
Vertex labelling algorithm, Example 3



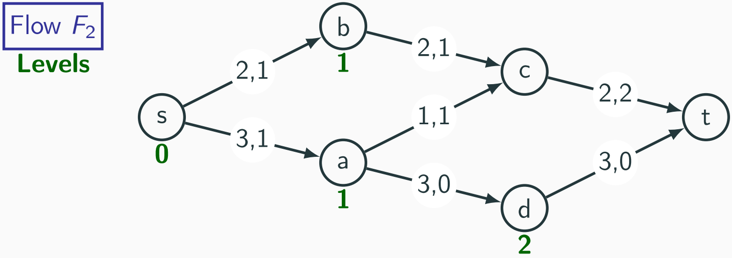
Vertex labelling algorithm, Example 3



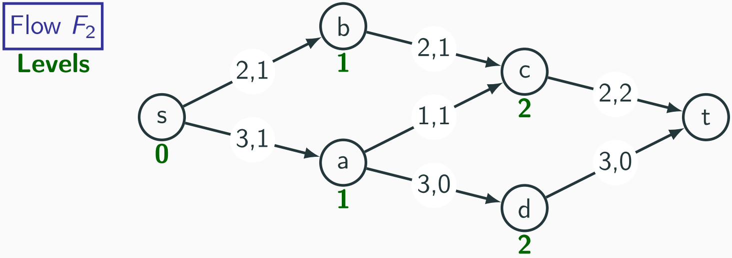
Vertex labelling algorithm, Example 3



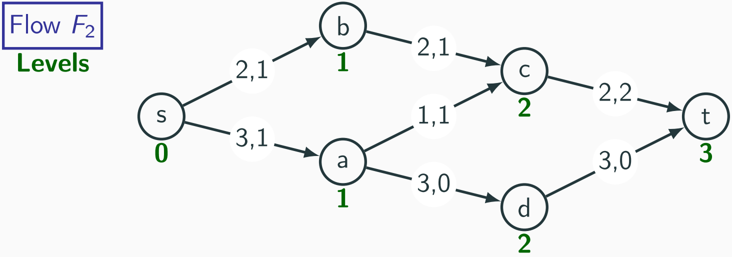
Vertex labelling algorithm, Example 3



Vertex labelling algorithm, Example 3



Vertex labelling algorithm, Example 3

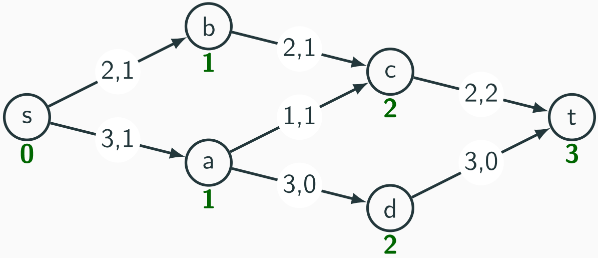


Vertex labelling algorithm, Example 3

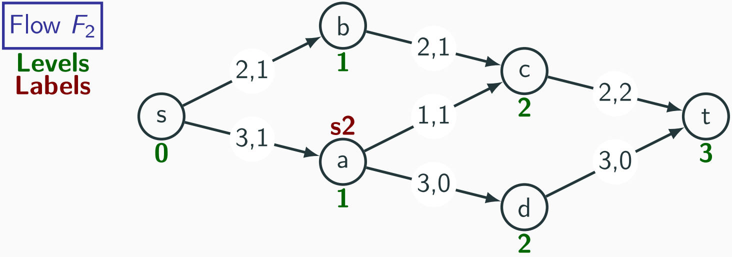
Flow F_2

Levels

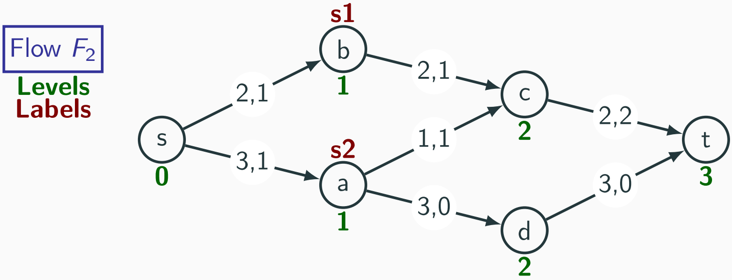
Labels



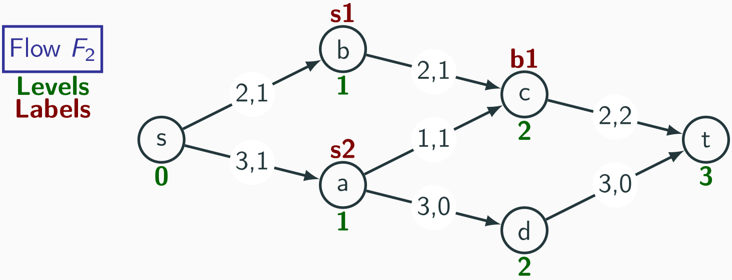
Vertex labelling algorithm, Example 3



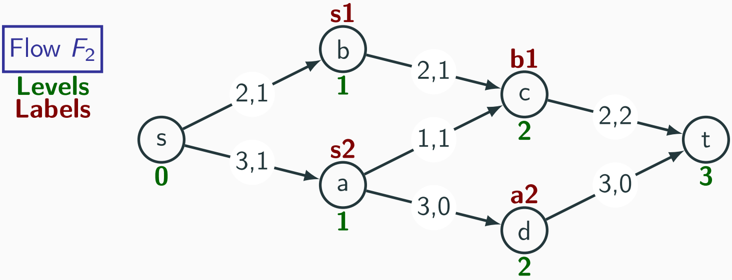
Vertex labelling algorithm, Example 3



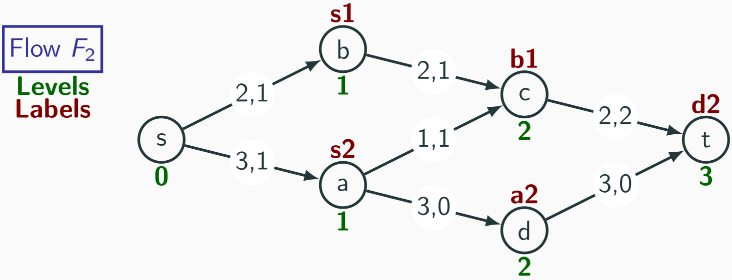
Vertex labelling algorithm, Example 3



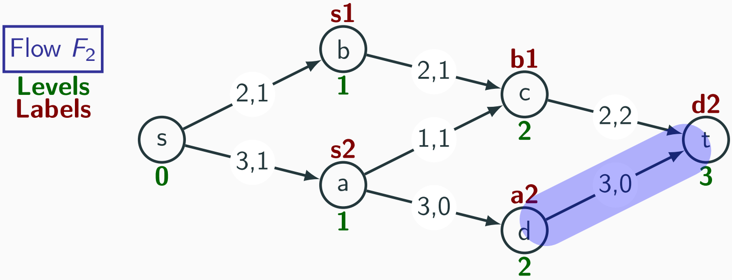
Vertex labelling algorithm, Example 3



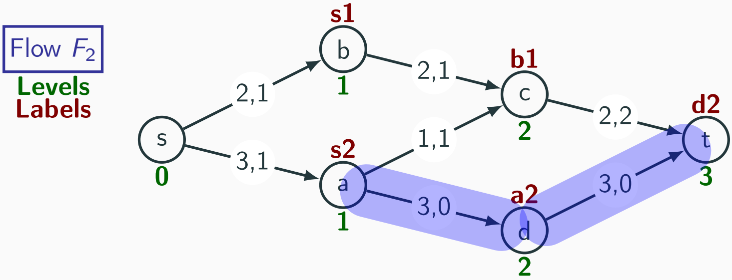
Vertex labelling algorithm, Example 3



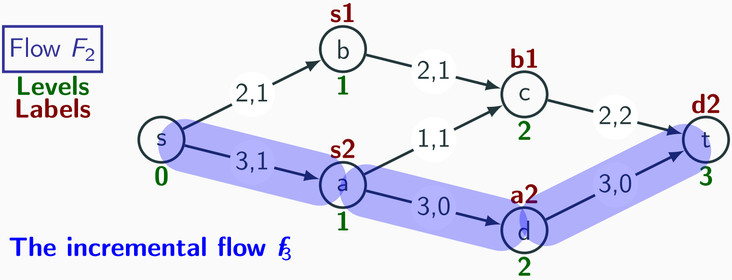
Vertex labelling algorithm, Example 3



Vertex labelling algorithm, Example 3



Vertex labelling algorithm, Example 3

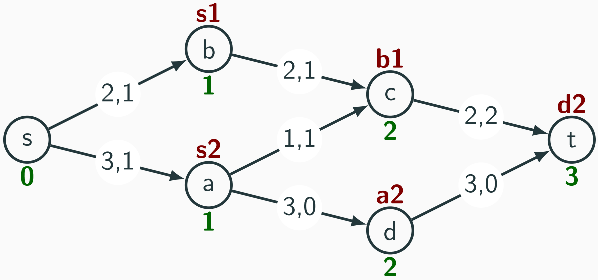


The incremental flow f_3

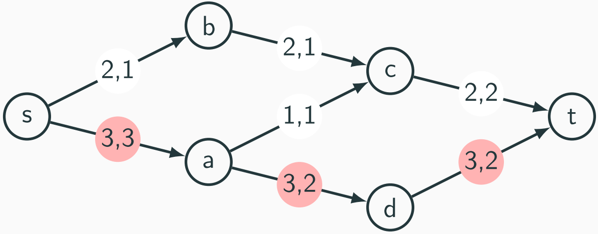
Vertex labelling algorithm, Example 3

Flow F_2

Levels
Labels

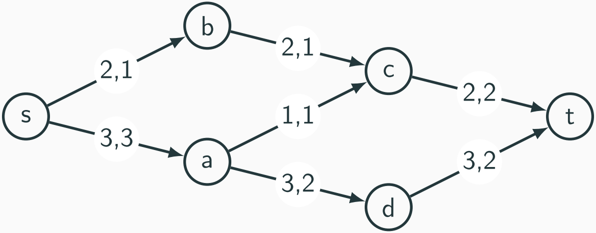


Flow F_3



Vertex labelling algorithm, Example 3

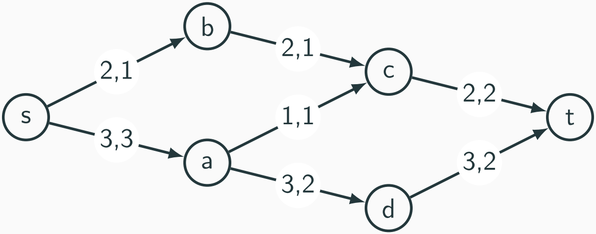
Flow F_3



Vertex labelling algorithm, Example 3

Flow F_3

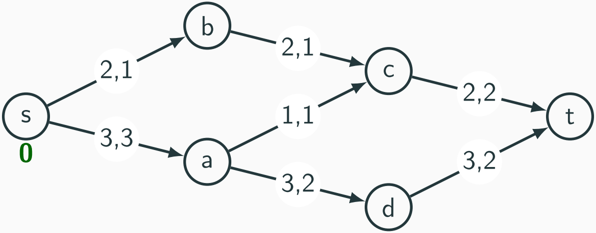
Levels



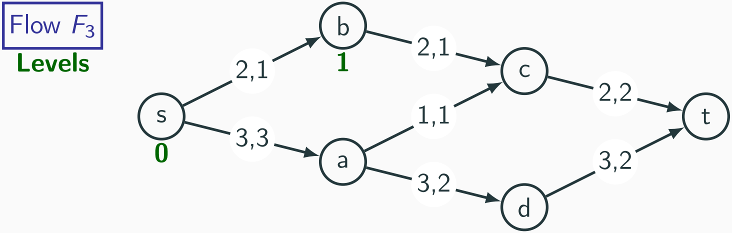
Vertex labelling algorithm, Example 3

Flow F_3

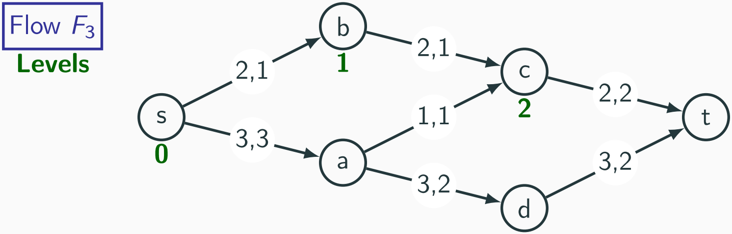
Levels



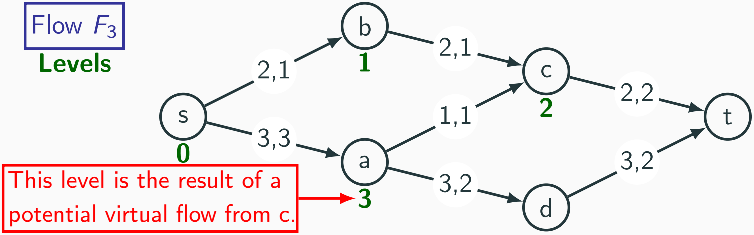
Vertex labelling algorithm, Example 3



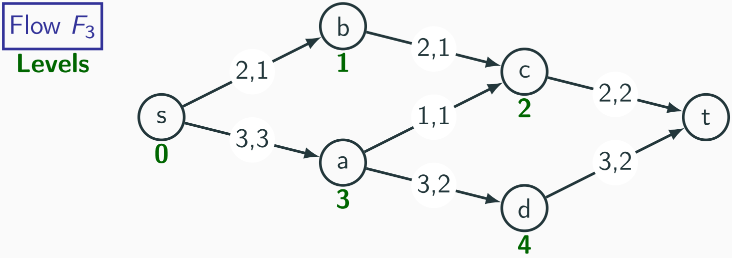
Vertex labelling algorithm, Example 3



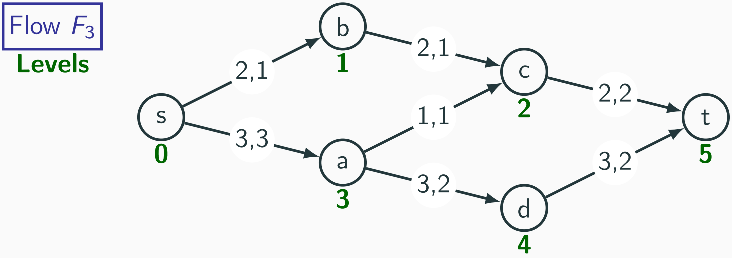
Vertex labelling algorithm, Example 3



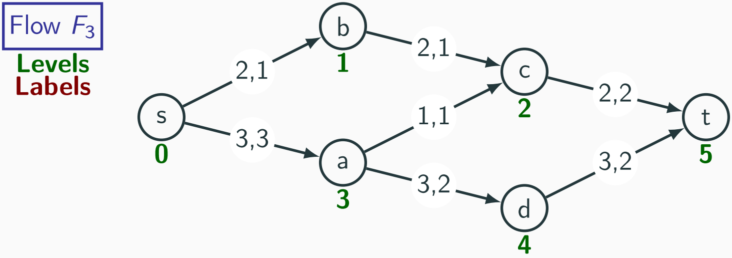
Vertex labelling algorithm, Example 3



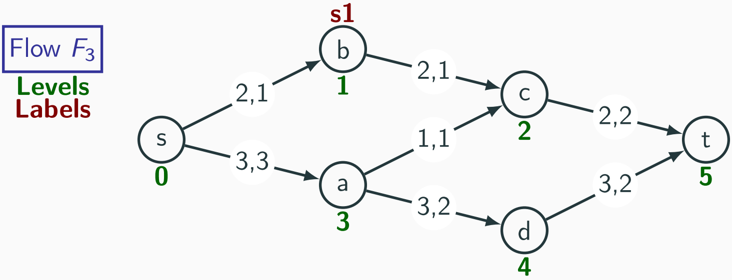
Vertex labelling algorithm, Example 3



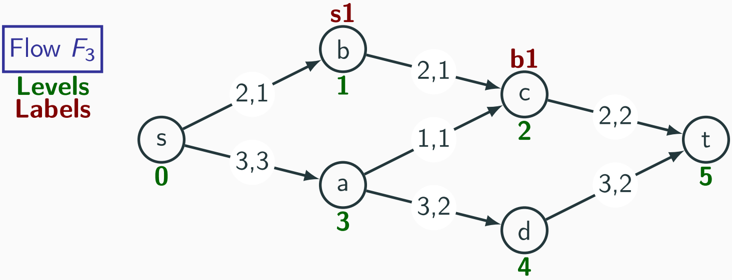
Vertex labelling algorithm, Example 3



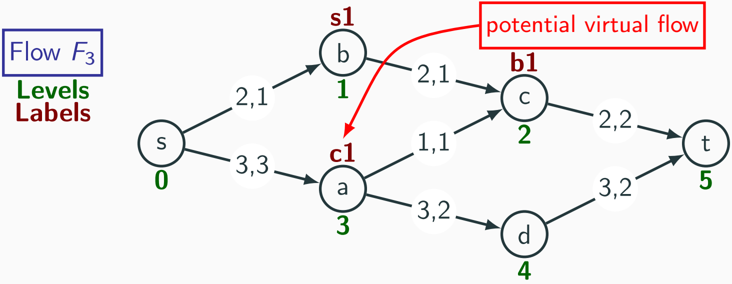
Vertex labelling algorithm, Example 3



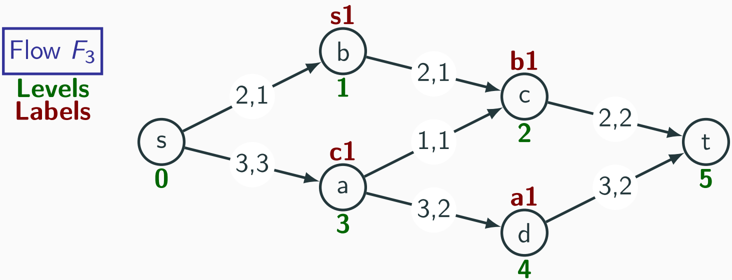
Vertex labelling algorithm, Example 3



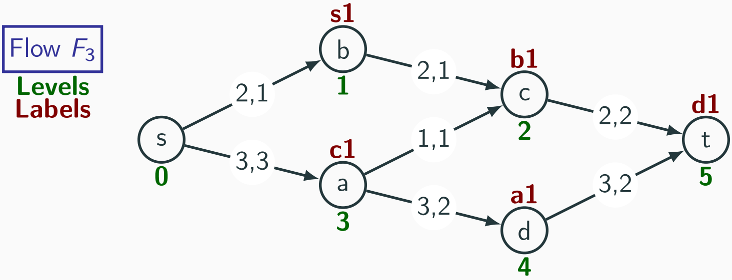
Vertex labelling algorithm, Example 3



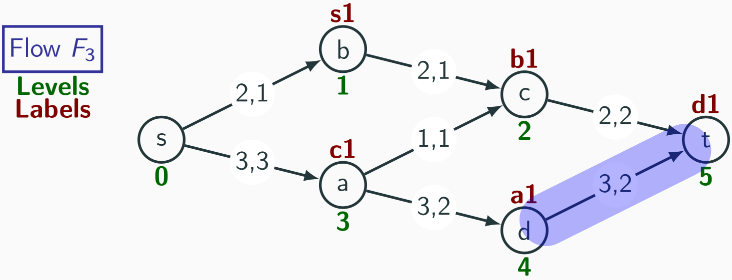
Vertex labelling algorithm, Example 3



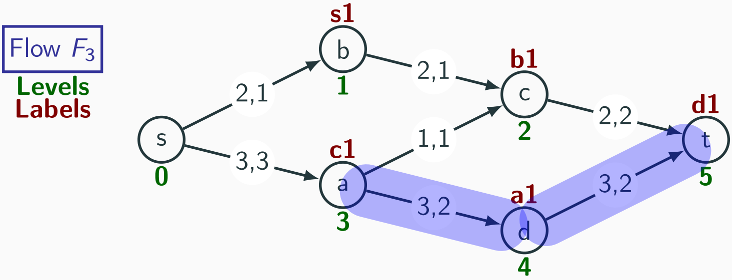
Vertex labelling algorithm, Example 3



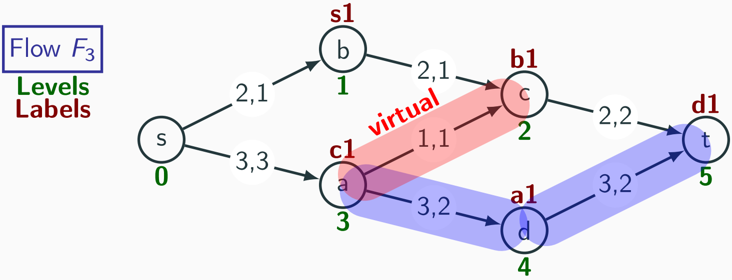
Vertex labelling algorithm, Example 3



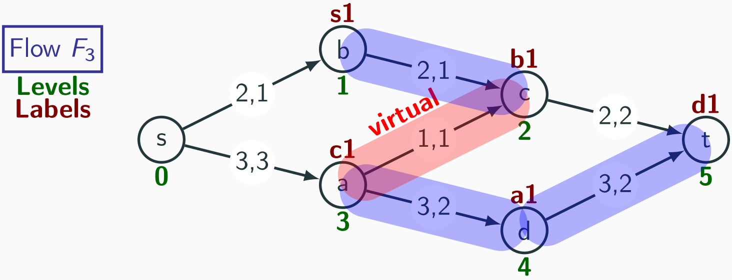
Vertex labelling algorithm, Example 3



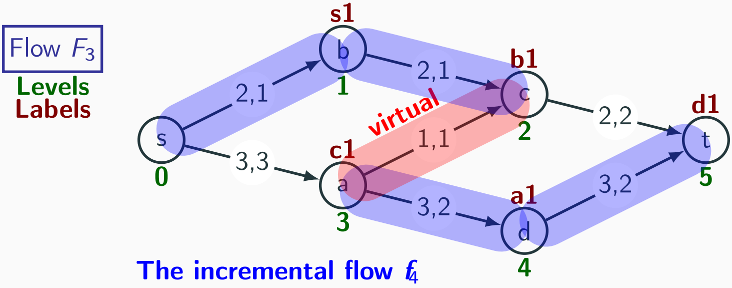
Vertex labelling algorithm, Example 3



Vertex labelling algorithm, Example 3



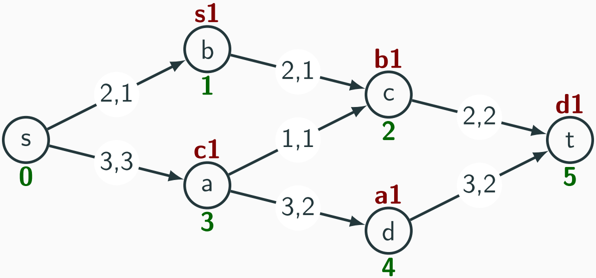
Vertex labelling algorithm, Example 3



Vertex labelling algorithm, Example 3

Flow F_3

Levels
Labels



Flow $F_4 = F_{\max}$