Discrete Mathematical Models

Lecture 0

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Semester 2, 2024

An acknowledgment of country

We acknowledge and celebrate the First Australians on whose traditional lands we live, work and study. We pay our respects to the elders past and present. In particular, we acknowledge the Ngunnawal and Ngambri people, the Traditional Owners of the land upon which the University's Acton campus is located.

Preface to the course

Credits

This course has been developed by a number of mathematicians over more than a decade. Contributing authors include:

- Judy-anne Osborne
- Pierre Portal
- Malcolm Brooks
- Adam Piggott
- and now me!

The mathematics in this course has been developed over millennia.

Learning Outcomes

Upon successful completion of MATH1005, students will have the knowledge and skills to:

1. Recall, invent, interpret examples of motivation for mathematical constructs used in discrete mathematics as models of processes in the world.
2. Recognise, define, explain and use terminology and notation from discrete mathematics.
3. Identify the logical structure of a statement, and then identify the logical structure of an argument that may be used to prove or disprove the statement.
4. Perform mathematical calculations in discrete mathematics using methods presented in the course.
5. Write simple proofs/construct explicit counterexamples for statements relating to discrete mathematics topics covered in the course.

Learning Outcomes

Upon successful completion of MATH6005, in addition to the knowledge and skills attained by students completing MATH1005, students will have the knowledge and skills to:

6. Use their deep knowledge and understanding of the material presented in the course to formulate responses to complex concrete and abstract problems.
7. Communicate their understanding and skills in discrete mathematics with colleagues and non-experts and apply their knowledge in an occupational situation.

Schedule

WEEK/SESSION	SUMMARY OF ACTIVITIES	ASSESSMENT
1	A1 Logic: Statements and Predicates. Valid Arguments.	No workshops in Week 1.
2	A2 Sets: Set Operations and identities. Russell's Paradox.	Workshop with a quiz.
3	A3 Relations and Functions: Definition & Properties of Relations and Functions.	Workshop with a quiz.
4	B1 Numbers: N,Z,Q,R. Base n. Computer & Modular Arithmetic.	Workshop with a quiz.
5	B2 Sequences and Induction: Implicit to Explicit Seq Def by Induction. Sorting. B3 Matrices: Matrix & Vector Operations. Linear Functions.	Workshop with a quiz.
6	C1 Counting: Cardinality. Permutations & Combinations. Stars & Bars. Pigeonhole Principle.	Workshop with a quiz. The first assignment is due.
7	C2 Probability: Probability Properties. Distributions. Random Variables.	No workshops.
8	C3 Markov Processes: Markov States &Transition Matrices. Steady State.	Workshop with a quiz.
9	D1 Graph Theory: Graphs & Digraphs. Degree. Euler & Hamilton Graphs. Trees.	Workshop with a quiz. The second assignment is due.
10	D2 Weighted Graphs: Minimum Span. Travelling Sales Person Problem. Shortest Path. Max Flow. Matching.	Workshop with a quiz.
11	D3 Random Walks: Graph 'Walking'. Webgraphs & PageRank Algorithm.	Workshop with a quiz.
12	Revision	Workshop with a quiz. The third assignment is due.

Reference materials

The lecture notes for the course are intended to be self-contained. Notes will be available through the course Wattle page immediately after the lecture, if not before. For most students, these notes, the workshop materials and the web will be sufficient.

If you would like an alternative presentation of the material, our optional text is Susanna S. Epp. *Discrete Mathematics with Applications: Metric Version*. Cengage Learning, 5th edition, 2019. ISBN: 9780357114087.

- An ebook copy of the text is available through the ANU library.
- From the publisher: "Students who wish to purchase their own eBook copies they can do so through https://au.cengage.com/ Use the coupon code WOW10 to receive a discount at checkout."

Other notes

- Check the Wattle page often, particularly the announcements.
- MATH1005 Assessment: workshop quizzes, workshop participation and three assignments, a final exam.
- MATH6005 Assessment: workshop quizzes, three assignments, a final exam.
- All students should sign up for a workshop.
- No class activities (lectures, workshops, etc) will take place on the following ACT public holidays: Monday Week 10.
- No workshops in Week 7.

Learning design

Lectures (3 hours per week): I will present the course material as comprehensibly as I can. Many new ideas, definitions, examples and results will be introduced.

Processing ($\sim$3 hours per week) Spend time reviewing lecture material and workshop sheets. Unpack definitions, work through examples yourself, try other examples.

Workshops (1.5 hours per week) A short quiz, then questions to try with your classmates and a demonstrator to assist.

Assignments ($\sim$2 hours per week) A chance to hone your skills and communicate your understanding.

Exams Summative assessment to test the extent to which you have obtained the learning outcomes.

About the Quizzes

A quiz will take place at the beginning of each of your workshops.

- Three questions.
- They will require you to recall definitions, results and do basic calculations.
- Each question is marked discretely (0 or 1), so you must answer accurately ($0/3 = 0, 1/3 = 0.33, 2/3 = 0.67, 3/3 = 1$).
- We take your best 8 of 10 workshop marks towards your final mark for Workshop quizzes.

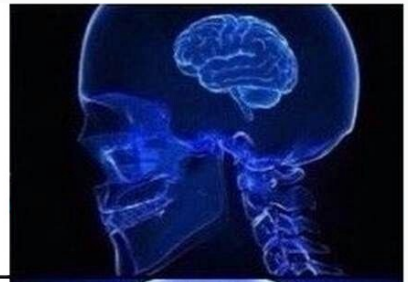
Example (not course related):

- State Pythagoras' Theorem.
- Give a definition of a circle.
- Give the prime decomposition of 314159.

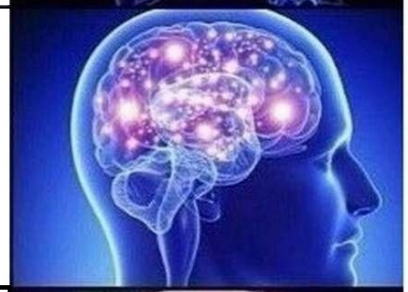
State Pythagoras' Theorem

State Pythagoras' Theorem

$a^2 + b^2 = c^2$



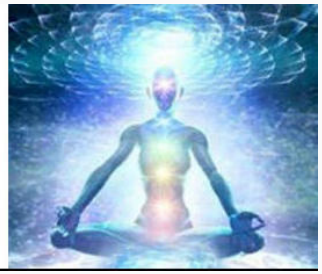
If a, b, c are lengths of the sides of a triangle then $a^2 + b^2 = c^2$.



If a, b, c are lengths of the sides of a right-angled triangle then $a^2 + b^2 = c^2$.

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Cosine rule with angle 90°.



Why study discrete mathematical models?

What are discrete mathematical models?

Discrete mathematical models are abstract representations of processes and objects, the steps or units of which can be indexed by the positive integers $\{1, 2, 3, 4, \dots\}$. In particular, we avoid continua (like the open interval $(0, 1)$).

Discrete mathematics is the study of discrete mathematical models.

Sudoku?

Q: Is completing a Sudoku puzzle an exercise in discrete mathematics?

Example: Which webpages are the most interesting?

Problem: A search on the internet can produce an overwhelming number of "hits." We need an effective and efficient method to order the hits from a web search so that the most interesting pages appear first.

Google's PageRank algorithm is an excellent solution to this problem that has made its inventors very wealthy. The algorithm combines a simple discrete mathematical model of the internet with some linear algebra discovered circa 1910. We will learn this algorithm in our course.

Computing and discrete mathematical models

Discrete mathematics and computer science go hand-in-hand because ...

Q: Describe an abstract model of the memory of a computer. That is, how do you think about memory?

Q: Do you think about computers working in continuous time or discrete time ("clock cycles")?

Q: Can the interval $(0, 1)$ be modeled inside a computer?

Discrete mathematics and computer science

In 1974, A. M. Turing Award laureate Knuth wrote:

> *"Discrete mathematics, especially combinatorial theory, has been given an added boost by the rise of computer science, in addition to all the other fields in which discrete mathematics is currently being extensively applied."*

Donald E. Knuth . Computer science and its relation to mathematics. *Amer. Math. Monthly*, 81:323-343, 1974.

Some optional activities

- (Optional) Have a read of Knuth's paper referenced on frame #15 (the paper is available in Wattle), and take notes.
- (Optional) Play Sudoku at Sudoku.com. Ask yourself the following questions: Can you describe your strategy? Can you guarantee that your strategy will work every time? Is your strategy "the best"? Are these questions examples of discrete mathematics?