INTEGRATION OF EXPONENTIAL FUNCTION

1. $\int e^u \, du = e^u + c$
2. $\int a^u \, du = \frac{a^u}{\ln a} + c$

Example

1. $\int \frac{dx}{e^{2x}}$

Solution

$\Rightarrow \int e^{-2x} \, dx \quad \Rightarrow -\frac{1}{2} \int e^{-2x} \, -2dx$

let $u = -2x$

$du = -2dx$

$nf = -\frac{1}{2}$

$\Rightarrow \boxed{-\frac{1}{2} e^{-2x} + C}$

2. $\int e^{\sin 4x} \cos 4x \, dx \quad = \int e^u \, du = e^u + C$

Solution:

let $u = \sin 4x$

$du = \cos 4x \, (4) \, dx$

$nf = \frac{1}{4}$

$\frac{d}{dx} \sin u = \cos u \, du$

$\Rightarrow \frac{1}{4} \int e^{\sin 4x} \, 4 \cos 4x \, dx \Rightarrow \boxed{\frac{1}{4} e^{\sin 4x} + C}$

3. $\int \frac{(e^x - 4e^{-x})}{e^x} dx$

$\int e^u \, du = e^u + C$

note:

$a^m \cdot a^n = a^{m+n}$

$\Rightarrow \int (e^x - 4e^{-x}) e^{-x} \, dx$

$\int (e^x \cdot e^{-x} - 4e^{-x} \cdot e^{-x}) \, dx$

$\int (e^0 - 4e^{-2x}) \, dx$

$\int (1 - 4e^{-2x}) \, dx$

$\int dx - 4 \int e^{-2x} \, dx$ let $u = -2x$

$du = -2 \, dx$

$\int dx - 4\left(-\frac{1}{2}\right) \int e^{-2x} \cdot -2 \, dx$ $\quad nf = -\frac{1}{2}$

$\int dx + 2 \int e^{-2x} \cdot -2 \, dx$

$1 + 2e^{-2x} + C$

4. $\int 2^{4x}\,dx$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

let $u = 4x$

$du = 4dx$

$\frac{1}{4}du = dx$

$\frac{1}{4}\int 2^{4x}\,4dx \Rightarrow \frac{1}{4}\cdot\frac{2^{4x}}{\ln 2} + C$

note: $a\ln b = \ln b^a$

$\Rightarrow \frac{2^{4x}}{4\ln 2} + C$

$\Rightarrow \frac{2^{4x}}{\ln 2^4} + C \Rightarrow \boxed{\frac{2^{4x}}{\ln 16} + C}$

5. $\int \sqrt[3]{4^{2x}}\,dx$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$\Rightarrow \int (4^{2x})^{1/3}\,dx$

$= \int 4^{\frac{2x}{3}}\,dx \Rightarrow \frac{3}{2}\int 4^{\frac{2x}{3}}\,\frac{2}{3}dx$

let $u = \frac{2x}{3}$

$du = \frac{2}{3}dx$

$\frac{3}{2}du = dx$

$\Rightarrow \boxed{\frac{3}{2}\left[\frac{4^{\frac{2x}{3}}}{\ln 4}\right] + C}$

$a^m \cdot b^m = (ab)^m$

6. $\int 3^x 2^x \, dx$

$\int 6^x \, dx$

$$\frac{6^x}{\ln 6} + C$$

INTEGRATION OF HYPERBOLIC FUNCTION

1. $\int \cosh u \, du = \sinh u + c$
2. $\int \sinh u \, du = \cosh u + c$
3. $\int \operatorname{sech}^2 u \, du = \tanh u + c$
4. $\int \operatorname{csch}^2 u \, du = -\coth u + c$
5. $\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + c$
6. $\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + c$
7. $\int \tanh u \, du = \ln|\cosh u| + c$
8. $\int \coth u \, du = \ln|\sinh u| + c$

EXAMPLE

1. $\int \sinh(3x-1)\,dx \Rightarrow \frac{1}{3}\int \sinh(3x-1)\,dx$

let $u = 3x-1$

$du = 3\,dx$

$\frac{1}{3}\,du = dx$

$\Rightarrow \boxed{\frac{1}{3}\cosh(3x-1) + C}$

2. $\int (\cosh 4x + \sinh 2x)\,dx$

$\int \cosh 4x\,dx + \int \sinh 2x\,dx$

let $u = 4x$

$du = 4\,dx$

$\frac{1}{4}\,du = dx$

let $u = 2x$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$\frac{1}{4}\int \cosh 4x\ 4\,dx + \frac{1}{2}\int \sinh 2x\ 2\,dx$

$\boxed{\frac{1}{4}\sinh 4x + \frac{1}{2}\cosh 2x + C}$

3. $\int \operatorname{sech} \frac{1}{4}x \tanh \frac{1}{4}x \, dx = 4 \int \operatorname{sech} \frac{1}{4}x \tanh \frac{1}{4}x \, dx$

let $u = \frac{1}{4}x$

$du = \frac{1}{4} dx$

$4 du = dx$

$= \boxed{-4 \operatorname{sech} \frac{1}{4}x + C}$

4. $\int \frac{\operatorname{sech}^2(\ln x) \, dx}{x} \rightarrow \frac{\operatorname{sech}^2(\ln x)}{1} \cdot \frac{dx}{x}$

$\Rightarrow \int \operatorname{sech}^2(\ln x) \frac{dx}{x} \Rightarrow \boxed{\tanh(\ln x) + C}$

let $u = \ln x$

$du = \frac{dx}{x}$