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Principal Component Analysis (PCA) –

- Eigen Value
- Eigen Vector

⇒ Trade off information & Complexity

Covariance matrix –

$$\boxed{A = \begin{bmatrix} Cov(x,x) & Cov(x,y) \\ Cov(y,x) & Cov(y,y) \end{bmatrix}}$$

$$\boxed{Cov(x,y) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n-1}}$$

$$Var(x) = \boxed{Cov(x,x) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

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$A\,x = \lambda\,x$

matrix   parameter   eigen values

Suppose, $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$\Rightarrow \left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow \boxed{[A - \lambda I]X = 0}$ Eigen equation

$\det [A - \lambda I]$ — (1)

Suppose, $A = \begin{pmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{pmatrix}$

Put A in eqn (1)

$\Rightarrow \det \begin{pmatrix} 0.6-\lambda & 0.2 \\ 0.4 & 0.8-\lambda \end{pmatrix} = 0$

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$\Rightarrow (0.6-\lambda)(0.8-\lambda) - (0.4 \times 0.2) = 0$

$\Rightarrow \lambda^2 - 1.4\lambda + 0.4 = 0$

$\Rightarrow$ 0.4, 0.1 — Eigen values.

Case 1 – $\lambda_1 = 0.4$

$$(A - \lambda I)X = 0$$

$$\Rightarrow \begin{pmatrix} 0.6-\lambda & 0.2 \\ 0.4 & 0.8-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} 0.2 & 0.2 \\ 0.4 & 0.4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

matrix Elimination method –

$$0.2x_1 + 0.2x_2 = 0$$
$$0.4x_1 + 0.4x_2 = 0$$

$$\Rightarrow \boxed{x_1 = -x_2}$$

~~let~~ $x_1 = t$

Eigen vector – $\begin{pmatrix} -t \\ t \end{pmatrix} = t\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.