21K91A6762

1. In a survey among few people, 60% read Hindi newspaper, 40% read English newspaper and 20% read both. If a person is choosen at random & if he already reads English newspaper find the probability that he also reads Hindi newspaper.

sol:- H : The event that a person reads Hindi newspaper

E : The event that a person reads English newspaper

Given

$P(H) = 0.60$

$P(E) = 0.40$

$P(H \cap E) = 0.2$

We have to find the condn probability that a person reads the Hindi newspaper given that he reads the English newspaper, $P(H|E)$

W.K.T, $P(H|E) = \frac{P(H \cap E)}{P(E)}$

$\Rightarrow P(H|E) = \frac{0.20}{0.40} = 0.50$

So, the probability is 50% or 0.50

2. You are given a set of cards numbered from 1 to 15. You choose two cards at random such that the sum of the numbers on the cards is even. What is the probability that both the cards you choose have odd numbers?

ans:- Given,

The total no. of cards = 15.

odd cards (1, 3, 5, 7, 9, 11, 13, 15) = 8 cards.

even cards (2, 4, 6, 8, 10, 12, 14) = 7 cards.

Total no. of ways to choose 2 from 15 cards = ${}^{15}C_2 = 105$

The total number of ways to choose 2 cards such that their sum is even :

- ways to choose 2 odd cards from 8 = ${}^{8}C_2$

$${}^{8}C_2 = \frac{8 \times 7}{2 \times 1} = 28$$

- ways to choose 2 even cards from 7 : ${}^{7}C_2$

$${}^{7}C_2 = \frac{7 \times 6}{2 \times 1} = 21$$

- Total way to choose 2 ~~even~~ cards from 15 cards to get an even sum = 28+21 = 49.
- The conditional probability that both cards are odd given the sum is even

$$P(\text{both odd cards} \mid \text{sum is even}) = \frac{P(\text{both odd cards and sum is even})}{P(\text{sum is even})}$$

$$P(\text{sum is even}) = \frac{49}{105}$$

$$P(\text{both odd cards and sum is even}) = \frac{28}{105}$$

$$P(\text{both odd cards} \mid \text{sum is even}) = \frac{\frac{28}{105}}{\frac{49}{105}} = \frac{28}{49} = \frac{4}{7}$$

$\therefore$ The desired event's probability $= \frac{4}{7} = 0.571$

3. Let E and F be events of an experiment such that $P(E) = 3/10$, $P(F) = 1/2$, $P(F|E) = 2/5$. Find:

(i) $P(E \cap F)$

(ii) $P(E|F)$

(iii) $P(E \cup F)$

Given,

$P(E) = 0.3$, $P(F) = 0.5$, $P(F/E) = 0.4$

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(i) $P(E \cap F)$

W.K.T. $P(F|E) = \frac{P(F \cap E)}{P(E)}$

$P(E \cap F) = P(F|E) \cdot P(E)$

$= \frac{2}{5} \times \frac{3}{10} = 0.4 \times 0.3$

$P(E \cap F) = 0.12$

(ii) $P(E|F)$

w.k.t. $P(E|F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{0.12}{0.5} = \frac{3/25}{\cancel{5}/\cancel{10}_2} = \frac{6}{25}$

$P(E|F) = 0.24$

(iii) $P(E \cup F)$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.3 + 0.5 - 0.12$

$P(E \cup F) = 0.68$

BAYES THEOREM

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BAYES THEOREM

1. Three persons A, B and C have applied for a job in a private company. The chance of their selections is in the ratio 1:2:4. The probabilities that A, B and C can introduce changes to improve the profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

sol:- Let the events be described as:

N : No change takes place

A : Person A gets appointed

B : Person B gets appointed

C : Person C gets appointed.

The chances of selection of A, B, and C are in the ratio of 1:2:4.

Hence, $P(A) = \frac{1}{7}$, $P(B) = \frac{2}{7}$, $P(C) = \frac{4}{7}$

probabilities of A, B and C introducing changes to improve profits of company are 0.8, 0.5 and 0.3 respectively. Hence probability of no changes on appointment of A, B and C are 0.2, 0.5 and 0.7.

Hence,

$P(N|A) = 0.2 = \frac{2}{10}$ $\quad P(N|B) = 0.5 = \frac{1}{2}$ $\quad P(N|C) = 0.7 = \frac{7}{10}$

The required probability is

$$P(C|N) = \frac{P(N|C) \cdot P(C)}{P(N|A) \cdot P(A) + P(N|B) \cdot P(B) + P(N|C) \cdot P(C)}$$

$$= \frac{\frac{7}{10} \cdot \frac{4}{7}}{\frac{2}{10} \cdot \frac{1}{7} + \frac{5}{10} \cdot \frac{2}{7} + \frac{7}{10} \cdot \frac{4}{7}}$$

$$= \frac{\frac{28}{70}}{\frac{2}{70} + \frac{10}{70} + \frac{28}{70}} = \frac{28}{40} = \frac{7}{10}$$

$\therefore$ if no change takes place, the probability that it is due to appointment of C is $\frac{7}{10}$.

2. A new virus test has been developed. The tests accuracy is as follows:
If a person is infected, the test correctly identifies it with probability. $P(T/I) = 0.98$.
If a person is not infected, the test incorrectly identifies them as infected with probability $P(T/N) = 0.03$
The overall prevalence of the virus in the population is 1%.
If a person test positive, what is the propability that they are actually infected?

sol: Let us define the even

I: Person is infected

T: Person tests positive.

$P(I/T) = ?$

$P(I/T) = \dfrac{P(T/I) \cdot P(I)}{P(T)}$ {∵ if a person tests +v, the prob that they are actually infected}

W.K.T,

$P(T/I) = 0.98$

$P(T/N) = 0.03$

$P(I) = 1\% = 0.01$, $P(T) = P(T/I) \cdot P(I) + P(T/N) \cdot P(N)$

$P(I/T) = \dfrac{P(T/I) \cdot P(I)}{P(T/I) \cdot P(I) + P(T/N) \cdot P(N)}$

$P(N) = 1 - P(I) = 1 - 0.01 = 0.99$.

$P(I/T) = \dfrac{0.98 \times 0.01}{0.98 \times 0.01 + 0.03 \times 0.99}$

$= \dfrac{0.0098}{0.0098 + 0.0297} = \dfrac{0.0098}{0.03395}$

$P(I/T) \approx 0.2487$.

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3. There are three identical cards except that both the sides of the first card are coloured red, both sides of the second card are coloured blue and for the third card one side is coloured red and the other side is blue. One card is randomly selected from these three cards and put down the visible side of the card is red. What is the probability that the other side is blue

RR = All red

BB = All Black

RB = Red & Black

R = upturned side of chosen card is red

$P(R|RB) = \frac{1}{2}$ $\quad P(RB) = \frac{1}{3}$ $\quad P(BB) = \frac{1}{3}$ $\quad P(RR) = \frac{1}{3}$

$P(R|RR) = 1$ $\quad P(R|RR) = 1$ $\quad P(R|BB) = 0$

Using the results we have to find $P(RB|R)$.

$$P(RB|R) = \frac{P(RB \cap NR)}{P(R)}$$

$$= \frac{P(R|RB)\,P(RB)}{P(R|RB)\,P(RB) + P(R|BB)\,P(BB)\,P(R|RR)\,P(RR)}$$

$$P(RB|R) = \frac{\frac{1}{2} \times \frac{1}{3}}{\left(\frac{1}{2}\times\frac{1}{3}\right) + \left(0 \times \frac{1}{3}\right) + \left(1 \times \frac{1}{3}\right)} = \frac{1}{3} = 0.3333$$

The required probability is $= \frac{1}{3} = 0.3333$.