1 a)

$$\min_{\omega, b, s} \frac{1}{2}\|\omega\|_2^2$$

$$y_i(\omega^T x_i + b) \geq 1 - s_i \qquad s_i \geq 0 \quad \forall i = [1, n]$$

$$\Rightarrow y_i(\omega^T x_i + b) - 1 + s_i \geq 0$$

$$s_i \geq 0$$

$$L(\omega, b, s, \alpha, \beta) = \frac{1}{2}\|\omega\|_2^2 - \sum_i \alpha_i [y_i(\omega^T x_i + b) - 1 + s_i]$$

$$- \sum_i \beta_i s_i$$

b) $$\frac{\partial L}{\partial \omega} = \omega - \sum_i \alpha_i y_i x_i = 0$$

$$\Rightarrow \omega = \sum_i \alpha_i y_i x_i$$

$$\frac{\partial L}{\partial b} = +\sum_i \alpha_i [y_i] = 0$$

$$\frac{\partial L}{\partial s} = +\sum_i \alpha_i + \sum_i \beta_i = 0$$

c) $\omega = \sum_i \alpha_i y_i x_i$

$\sum_i \alpha_i y_i = 0$

$\sum_i \beta_i + \sum_i \alpha_i = 0 \Rightarrow \sum_i (\alpha_i + \beta_i) = 0$

$\alpha_i \geq 0 \quad \beta_i \geq 0$

$$L(\omega, b, \xi, \alpha, \beta) = \frac{1}{2}\left(\left(\sum_i \alpha_i y_i x_i\right)^2\right)^{\cancel{1/2}\ \cancel{2}}$$

$$- \sum_i \alpha_i y_i \sum_j \alpha_j y_j x_j x_i$$

$$- \cancelto{0}{\sum_i \alpha_i y_i}\, b + \sum_i \alpha_i - \sum_i \alpha_i \xi_i$$

$$\sum_i \beta_i \xi_i$$

$$= \frac{1}{2} \sum_i (\alpha_i y_i x_i)^2 - \sum_{ij} \alpha_i y_i \alpha_j y_j x_j x_i$$

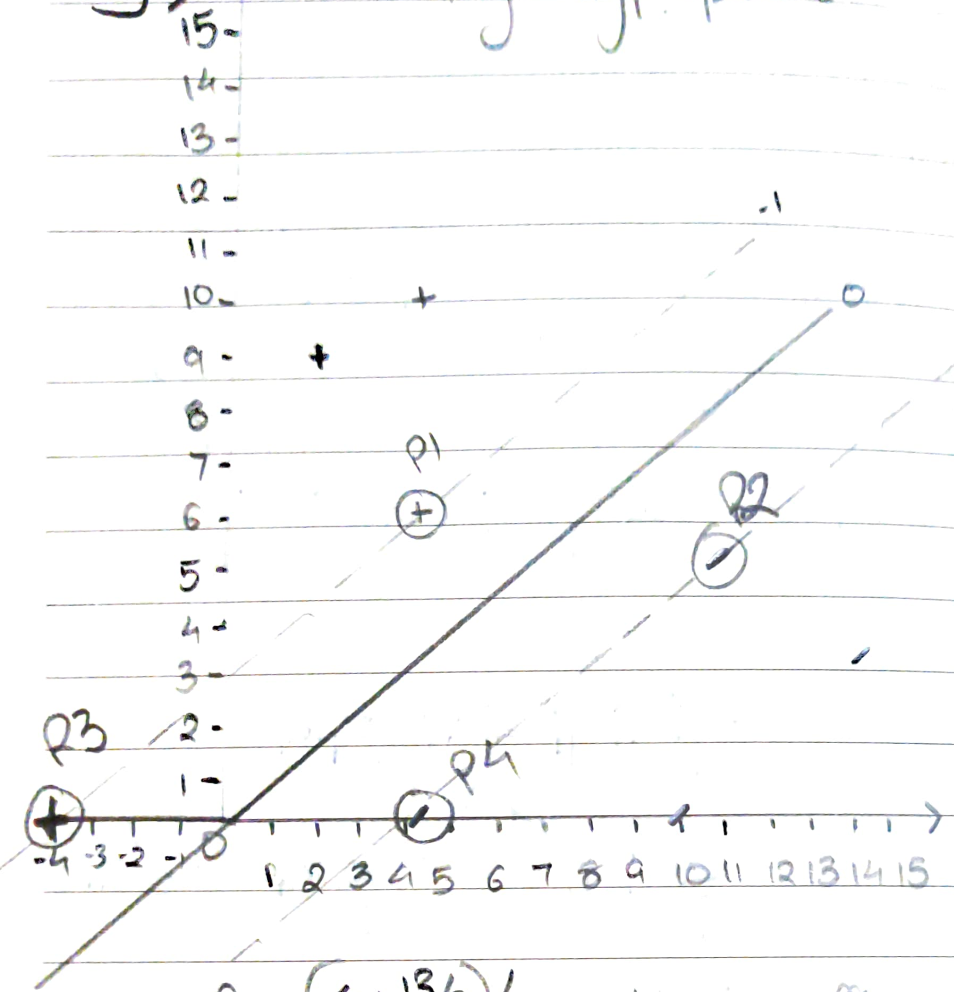
$$+ \sum_i \alpha_i - \sum_i \cancelto{0}{(\alpha_i + \beta_i)}\, \xi_i$$

$$= \frac{1}{2} \sum (\alpha_i y_i x_i)^2 - \sum_{ij} \alpha_i y_i \alpha_j y_j x_j x_i$$

$$+ \sum_i \alpha_i$$

2] a)

My hyperplane



$d = (4 + 13/3)/2$ for my diagram above

$= 0.1\bar{6}$

Created by measuring slope & eq. of P3-P1 & P2-P4
took the centre of two

b) Points => $(4,6,+1)$, $(11,5,-1)$ $(4,0,1)$

$(4,6,+1)$

$(+1)(w_1 4 + 6w_2 + b) = 1$

$\Rightarrow 4w_1 + 6w_2 + b = 1$ (1)

$(11,5,-1)$

$-(11w_1 + 5w_2 + b) = 1$

$\Rightarrow 11w_1 + 5w_2 + b = -1$ (2)

$(-4, 0, 1)$

$$-4\omega_1 + b_1 = 1 \quad -③$$

(let $\omega_1 = a$, $\omega_2 = b$ & $b_1 = c$

③ gives $a = -\left(\frac{1-b}{4}\right) = \frac{b-1}{4}$

Plugging in ① gives

$$2c + 6b - 1 = 1 \Rightarrow c = -3b + 1 \quad -④$$

Plugging in ②

$$5b - \left(\frac{-11 + 15(-3b) + 1}{4}\right) = -1$$

$$\Rightarrow \frac{-25b + 4}{4} = 1$$

$$\Rightarrow b = \frac{8}{25}$$

$$\Rightarrow c = \frac{1}{25}$$

$$\Rightarrow a = \frac{-6}{25}$$

Thus, $\omega_1 = \frac{-6}{25}$, $\omega_2 = \frac{8}{25}$, $b = \frac{1}{25}$

c) i) removing $(\frac{10}{3}, -1)$ might not change ω^* as (there is still $(11/5, 1)$ to

(ii) keep the solution same but removing $(11/5, -1)$ will change ω^* (might shift right)

3] gradient of hingeloss

$$\frac{1}{2}\alpha^T K\alpha + \frac{\lambda}{n}\sum_{i=1}^{n} \max(0, 1 - y_i(K\alpha)_i)$$

is

$$\frac{\cancel{2}}{\cancel{2}} K\alpha + \frac{\lambda}{n}\sum_{i=1}^{n} \nabla\max(0, 1 - y_i(K\alpha)_i)$$

1 if $1-y_i(K\alpha)_i > 0$

0 if $1-y_i(K\alpha)_i \leq 0$