1) Solve the following

(a) $x(n) = x(n-1) + 5 \quad n < 1 \quad x(1) = 0$

using back Substitution

$$x(n) = x(n-1) + 5 \longrightarrow (1)$$

where

$n = n-1$

find $x(n-1) = x(n-1-1) + 5$

$$= x(n-2) + 5 \longrightarrow (2)$$

Sub (2) in (1)

$$x(n) = (x(n-2) + 5) + 5$$

$$\Rightarrow x(n-2) + 10 \longrightarrow (3)$$

find $x(n-2) = x(n-2-1) + 5$

$$= x(n-3) + 5 \rightarrow (4)$$

Sub (4) in (3)

$$x(n) = (x(n-3) + 5) + 10$$

$$= x(n-3) + 15$$

here the Sequence observed is

$$x(n) = x(n-k) + 5k.$$

1) Solve the following

(a) $x(n) = x(n-1) + 5 \quad n < 1 \quad x(1) = 0$

using back substitution

$$x(n) = x(n-1) + 5 \longrightarrow ①$$

where

$n = n-1$

find $x(n-1) = x(n-1-1) + 5$

$$= x(n-2) + 5 \longrightarrow ②$$

Sub ② in ①

$$x(n) = (x(n-2) + 5) + 5$$

$$\Rightarrow x(n-2) + 10 \longrightarrow ③$$

find $x(n-2) = x(n-2-1) + 5$

$$= x(n-3) + 5 \rightarrow ④$$

Sub ④ in ③

$$x(n) = (x(n-3) + 5) + 10$$

$$= x(n-3) + 15$$

here the sequence observed is

$$x(n) = x(n-k) + 5k.$$

$n - k = 1 \rightarrow$ base condition

$n - k + k = 1 + k$

$n - 1 = k$

$$x(n) = x(1) + 5(n-1)$$

$$\Rightarrow 0 + (5n - 1) \Rightarrow 5n - 5$$

$O(n) \rightarrow$ Time complexity

b) $x(n) = 3x(n-1) \quad x(1) = 4$

$$x(n) = 3x(n-1) \rightarrow 1$$

find

$$x(n-1) = 3x(n-1-1)$$

$$\Rightarrow 3x(n-2) \rightarrow 2$$

Sub 2 in 1

$$x(n) = 3[3x(n-2)] \rightarrow 3$$

find

$$x(n-2) = 3x(n-2-1)$$

$$\Rightarrow 3x(n-3) \rightarrow 4$$

$$x(n) = 3^2[3x(n-3)]$$

here as observered..

$$x(n) = 3^k x(n-k) \rightarrow 3^{n-1} x(\cancel{n}-\cancel{n}-1)$$

$$\Rightarrow 3^{n-1} \cdot 4$$

$n - k = 1 \quad \Rightarrow n - 1 = k$

ignore constant $O(3^n)$

(c) $x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$

$n = 2^K$

Using Master's Theorem

$x(n) = 1x(n/2) + n$

$T(n) = aT(n/b) + f(n) \rightarrow$ Form

So $a = 1$
$b = 2$
$f(n) = n$

Step 1: $\log_2 1 \Rightarrow \log_a b$

$f(n) = n^K + \log_n P$

where in question no log value

$P = 1$ $K = 1$ as $n^1 \rightarrow n$

Case 1: $\log_2 1 > K$ $n^K \log_a b$

Case 2: $\log_2 1 = K$

Case 3: $\log_2 1 < K$

$0 < 1 \rightarrow$ Case 3

Check $P \geq 0$ $n^K \log_n^P$

$P \leq 0$ n^K

$1 \geq 0$ $n^1 \log_n^1 \Rightarrow O(n)$

(d) $x(n) = x(n/3) + 1$ for $n > 1$ $\quad x(1) = 1$

~~Master Theorem :-~~

~~$a = 1$~~

~~$b = 3$~~ $\Big\}$ ~~There is No Log~~

~~$f(n) = 1$~~

Step 1 : $\log_a b = \log_1 3 \rightarrow$

Tree Method

$x(n) = x(n/3) + 1 \longrightarrow$ 1

find $x(n/3) = x\left(\frac{n/3}{3}\right) + 1$

$= x(n/3^2) + 1 \longrightarrow$ 2

1. $x(n) = \left(x(n/3^2) + 1\right) + 1$

$= x(n/3^2) + 2$

find $x(n/3^2) = x\left(\frac{n/3^2}{3}\right) + 1$

$\Rightarrow x(n/3^3) + 1$

2. $x(n) = \left(x(n/3^3) + 1\right) + 2$

$\Rightarrow x(n/3^3) + 3$

Observed series

$$x(n) = x\left(\frac{n}{3^i}\right) + i$$

$x(1) = 1$

$$\frac{n}{3^i} = 1$$

$$n = 3^i$$

apply log

$\log_2 n = \log 3^i$

$\log_2 n = i \log_2 3$

$$\frac{\log_2 n}{\log_2 3} = i$$

$$\underline{\log_3 n = i}$$

$$x(n) = x\left(\frac{n}{3^{\log_3 n}}\right) + \log_3 n$$

$n = 3k$

$$x(3k) = x\left(\frac{3k}{3^{\log_3 n}}\right) + \log_3 n$$

$$= x\left(\frac{3k}{n}\right) + \log_3 n$$

$$\Rightarrow x\left(\frac{3k}{3k}\right) + \log_3 n$$

$$\rightarrow x(1) + \log_3 n$$

$x(3k) \rightarrow \log_3 n + 1$

$x(n) = \log_3 n + 1$

$\underline{O(\log n)}$

2) $T(n) = T(n/2) + 1$ when $n = 2k$.

Tree Method

$T(n) = T(n/2) + 1$

Find $T(n/2) = T\left(\frac{n/2}{2}\right) + 1$

$\Rightarrow T(n/2^2) + 1$

(1) $T(n) = T(n/2^2) + 1 + 1 \Rightarrow 2$

$= T(n/2^2) + 2$

Find $T(n/2^2) = T\left(\frac{n/2^2}{2}\right) + 1$

$= T(n/2^3) + 1$

(2) $T(n) = T(n/2^3) + 3$

Find $(T(n/2^3) = T(n/2^4) + 1$

(3) $T(n) = T(n/2^4) + 4$

Observed Series

$T(n) = T\left(\frac{n}{2^i}\right) + i$

$\frac{n}{2^i} = n = 2^i$

where $i = \log_2 n$

$$T(n) = T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n$$

$$T(n) = T\left(\frac{n}{n}\right) + \log_2 n. \qquad a^{\log_a b} = b$$

$$T(2k) \Rightarrow T\left(\frac{\cancel{2}k}{\cancel{2}k}\right) + \log_2 2k$$

$$T(2k) \Rightarrow 1 + \log_2 k$$

$$T(2k) = 1 + \log_2 k$$

$$O(\log k) \text{ (or) } O(\log n)$$

(ii) $T(n) = T(n/3) + T(2n/3) + cn.$

$$T(n) = T(n/3) + T(2n/3) + cn$$

Find $T(n/3) \Rightarrow T(n/3^2) + T\left(\frac{2n}{3^2}\right) + c\frac{n}{3^2}$

$$T(2n/3) = T\left(\frac{2n}{3^2}\right) + T\left(\frac{4n}{3^2}\right) + \frac{c2n}{3^2}$$

$$T(n) = T(n/3^2) + T\left(\frac{2n}{3^2}\right) + T\left(\frac{2n}{3^2}\right) + \left(\frac{4n}{3^2}\right) + \frac{cn}{3^2} + \frac{c \cdot 2n}{3^2}$$

$$T\left(\frac{n}{3^2}\right) = T\left(\frac{n}{3^3}\right) + T\left(\frac{2n}{3^3}\right)$$

$$T\left(\frac{2n}{3^2}\right) = T\left(\frac{2n}{3^3}\right) + T\left(\frac{4n}{3^3}\right)$$

$$T\left(\frac{4n}{3^2}\right) = T\left(\frac{4n}{3^3}\right) + T\left(\frac{8n}{3^3}\right)$$

$$T(n) = T\left(\frac{n}{3^3}\right) + T\left(\frac{2n}{3^3}\right) + T\left(\frac{2n}{3^3}\right) + T\left(\frac{4n}{3^2}\right)$$
$$+ T\left(\frac{2n}{3^3}\right) + T\left(\frac{4n}{3^3}\right) + T\left(\frac{4n}{3^3}\right) + T\left(\frac{8n}{3^3}\right)$$

3) Min1 (A[0 ..... n-1])

If n = 1 return A[0]

ELSE Temp = MIN1[A(0 ..... n-2]]

if temp <= A[n-1] return Temp

Else

Return A[n-1]

(a) Base Case [n = 1]

if the Size of the array is 1, the algorithm returns the only Element in the array

(b) Recursive [n - 1]

if the Size of the array is greater than 1, the algorithm recursively computes the minimum array of the Subarray A[0 ..... n-2]

(c) Comparsion :

Computing the minimum of the Subarray Compare the Mininnum value with the last Element of the array A[n-1]

(b) Recurrence Relation

$T(n) = T(n-1) + c$ $\quad c = n$

$T(n) = T(n-1) + c$

$T(n) = T(n-1) \rightarrow 1$

$T(n-1) = T(n-1-1)$

$= T(n-2) + 2c \rightarrow 2$

Sub 2 in 1

$T(n) = T(n-2) \rightarrow 3$

find $T(n-2) = T(n-2-1) \rightarrow T(n-3)$
$\rightarrow 4$

sub 4 in 3

$T(n) = T(n-3) + 3c$

by the series.

$T(n-2), T(n-3)$ $\quad T(1) = 1$

$T(n) = T(n-k) + kc$

$n-k = 0$, $n = k$ $\quad$ $n-k = 1$, $n-1 = k$

$T(n) = T(1) + (n-1)c$

$1 + (n-1)c \rightarrow O(n)$

4) (i) $2n^2+5$ & $g(n) = 7n$ $\quad \Omega(g(n))$

$f(n) = 2n^2+5 \quad g(n) = 7n$

$f(n) \geq g(n) \cdot c$

$2n^2+5 \geq 7n$

$2n^2+5 = 7n$

$n=1 \quad 2(1)^2+5 = 7(1) \rightarrow \boxed{7 = 7}$

$n=2 \quad 2(2)^2+5 = 7(2)$

$8+5 = 14$

$\boxed{13 = 14}$

$n=4 \quad 2(16)+5 = 7\times 4$

$32+5 = 28$

$\boxed{37 = 28}$

$n=3$

$2(9)+5) = 7\times 3$

$18+5 = 21$

$\boxed{23 = 21}$

as with $f(n)$ is growing faster than $g(n)$

$f(n) = \Omega(n^2)$

$g(n) = \Omega(n)$

at large value of ($n=3$) so on $f(n)$ is growing faster than $g(n)$.

$\Omega(7n)$.