

1. What is a null hypothesis (H_0) and why is it important in hypothesis testing?

Null Hypothesis (H_0):

The null hypothesis is a formal statement that there is no effect, no difference, or no relationship between variables in a population. It represents the status quo or default assumption that any observed difference in sample data is due to random chance.

Examples:

- H_0 : The average score of students is equal to 70.
- H_0 : There is no difference in mean salaries between two groups.
- H_0 : A new drug has no effect compared to the old drug.

Why is the Null Hypothesis Important?

1. Provides a Baseline for Testing

H_0 serves as a starting point against which the alternative hypothesis (H_1) is tested.

2. Enables Statistical Decision-Making

Hypothesis testing evaluates whether there is enough evidence to reject H_0 using sample data and a significance level (α).

3. Controls Errors

It helps manage Type I error (rejecting a true null hypothesis) by setting a confidence threshold.

4. Ensures Objectivity

Decisions are based on probability and evidence rather than assumptions or opinions.

5. Foundation for Statistical Tests

Test statistics (z-test, t-test, chi-square, ANOVA, etc.) are all designed to assess the validity of H_0 .

2. What does the significance level (α) represent in hypothesis testing?

The significance level (α) in hypothesis testing represents the maximum probability of making a Type I error—that is, rejecting the null hypothesis (H_0) when it is actually true.

Key Meaning of Significance Level (α)

- α is the risk you are willing to take of concluding that an effect or difference exists when it really does not.
- It acts as a decision threshold for judging whether the sample evidence is strong enough to reject H_0 .

How α Is Used in Testing

1. Set α before analysis (to avoid bias).
2. Calculate the p-value from sample data.
3. Decision rule:
 - If $p\text{-value} \leq \alpha \rightarrow$ Reject H_0
 - If $p\text{-value} > \alpha \rightarrow$ Fail to reject H_0

3. Differentiate between Type I and Type II errors.

In hypothesis testing, **Type I and Type II errors** describe the two kinds of mistakes we can make when making decisions about the null hypothesis.

Type I Error vs Type II Error

Aspect	Type I Error	Type II Error
Definition	Rejecting a true null hypothesis	Failing to reject a false null hypothesis
Also called	False positive	False negative
Probability	α (alpha)	β (beta)
Result	Concluding there is an effect when none exists	Missing a real effect
Example	Saying a drug works when it actually doesn't	Saying a drug doesn't work when it actually does

4. Explain the difference between a one-tailed and two-tailed test. Give an example of each.

The difference between one-tailed and two-tailed tests lies in the direction of the alternative hypothesis and how the rejection region is defined.

One-Tailed Test

Definition

A one-tailed test checks for an effect in only one specific direction (either greater than or less than).

Hypotheses

- H_0 : Parameter = value
- H_1 : Parameter > value or Parameter < value

Key Features

- Entire significance level (α) is placed in one tail of the distribution
- Used when only one direction of change is of interest

Example

A teacher believes a new teaching method increases average marks.

- $H_0: \mu = 70$
- $H_1: \mu > 70$

This is a right-tailed test.

Two-Tailed Test

Definition

A two-tailed test checks for an effect in both directions (either increase or decrease).

Hypotheses

- H_0 : Parameter = value
- H_1 : Parameter \neq value

Key Features

- Significance level (α) is split equally between both tails

- Used when any difference from the hypothesized value matters

Example

A manufacturer wants to check whether the average weight of a product differs from 500 g.

- $H_0: \mu = 500$ g
- $H_1: \mu \neq 500$ g

This is a two-tailed test.

- A company claims that the average time to resolve a customer complaint is 10 minutes. A random sample of 9 complaints gives an average time of 12 minutes and a standard deviation of 3 minutes. At $\alpha = 0.05$, test the claim.**

1. State the hypotheses

The company claims the average time is 10 minutes, so this goes in the null hypothesis.

- $H_0: \mu = 10$ minutes
- $H_1: \mu \neq 10$ minutes (two-tailed test)

2. Choose the test

- Sample size $n = 9$ (small sample)
- Population standard deviation is unknown
Use a one-sample t-test

3. Given data

- Sample mean, $\bar{x} = 12$
- Sample standard deviation, $s = 3$
- Sample size, $n = 9$
- Significance level, $\alpha = 0.05$

4. Calculate the test statistic

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{12 - 10}{3/\sqrt{9}} = \frac{2}{1} = 2$$

5. Critical value

- Degrees of freedom: $df = n - 1 = 8$
- Two-tailed test at $\alpha = 0.05$

From the t-table:

$$t_{critical} = \pm 2.306$$

6. Decision rule

- If $|t| > 2.306 \rightarrow$ Reject H_0
- If $|t| \leq 2.306 \rightarrow$ Fail to reject H_0

Here:

$$|t| = 2 < 2.306$$

7. Conclusion

Fail to reject the null hypothesis (H_0).

Final Interpretation

At the 5% significance level, there is not enough statistical evidence to conclude that the average complaint resolution time is different from 10 minutes.

6. When should you use a Z-test instead of a t-test?

Use a Z-test when:

1. Population standard deviation (σ) is known
 - o This is the most important condition.
 - o Example: Industrial processes where σ is already established.
2. Sample size is large ($n \geq 30$)
 - o By the Central Limit Theorem, the sampling distribution of the mean is approximately normal.
3. Population is normally distributed (*or approximately normal*)
 - o Especially important for smaller samples.
4. Data is quantitative and random
 - o Observations must be independent.

Use a t-test when (contrast):

- Population standard deviation is unknown
- Sample size is small ($n < 30$)
- Sample standard deviation (s) is used instead of σ

7. The productivity of 6 employees was measured before and after a training program.

1. State the hypotheses

Let $d = \text{After} - \text{Before}$

- $H_0: \mu_d = 0$ (training did not improve productivity)
- $H_1: \mu_d > 0$ (training improved productivity)

Right-tailed test

2. Compute the differences

Employee	Before	After	$d = \text{After} - \text{Before}$
1	50	55	5
2	60	65	5
3	58	59	1

Employee	Before	After	$d = \text{After} - \text{Before}$
4	55	58	3
5	62	63	1
6	56	59	3

3. Calculate mean and standard deviation of differences

- $\bar{d} = \frac{5+5+1+3+1+3}{6} = \frac{18}{6} = 3$

Deviations from mean:

(2, 2, -2, 0, -2, 0)

- Sum of squares = $4 + 4 + 4 + 0 + 4 + 0 = 16$

$$s_d = \sqrt{\frac{16}{6-1}} = \sqrt{3.2} \approx 1.789$$

4. Test statistic

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{3}{1.789/\sqrt{6}} \approx \frac{3}{0.730} \approx 4.11$$

5. Critical value

- $n = 6 \Rightarrow df = 5$
- $\alpha = 0.05$ (right-tailed)

From t-table:

$$t_{critical} = 2.015$$

6. Decision

$$t_{calculated} = 4.11 > t_{critical} = 2.015$$

Reject H_0

7. Conclusion

At the 5% significance level, there is sufficient evidence to conclude that the training program improved employee productivity.

8. A company wants to test if product preference is independent of gender

This is a Chi-square test of independence, since we are testing whether product preference depends on gender.

1. State the hypotheses

- H_0 : Product preference is independent of gender

- H_1 : Product preference is not independent of gender

2. Observed frequencies (O)

Gender	Product A	Product B	Row Total
Male	30	20	50
Female	10	40	50
Column Total	40	60	100

3. Expected frequencies (E)

$$E = \frac{(\text{Row total})(\text{Column total})}{\text{Grand total}}$$

- Male–A: $\frac{50 \times 40}{100} = 20$
- Male–B: $\frac{50 \times 60}{100} = 30$
- Female–A: $\frac{50 \times 40}{100} = 20$
- Female–B: $\frac{50 \times 60}{100} = 30$

4. Compute Chi-square statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Cell	O	E	$(O-E)^2/E$
Male–A	30	20	5
Male–B	20	30	3.33
Female–A	10	20	5
Female–B	40	30	3.33

$$\chi^2 = 5 + 3.33 + 5 + 3.33 = 16.66$$

5. Critical value

- Degrees of freedom:

$$df = (r - 1)(c - 1) = (2 - 1)(2 - 1) = 1$$

- $\alpha = 0.05$
- From Chi-square table:

$$\chi^2_{critical} = 3.841$$

6. Decision

$$\chi^2_{calculated} = 16.66 > 3.841$$

Reject H₀

7. Conclusion

At the 5% significance level, there is strong evidence that product preference is dependent on gender.