

Question 1: A die is rolled. What is the probability of getting: (a) An even number (b) A number greater than 4.

A standard die has 6 outcomes: {1, 2, 3, 4, 5, 6}

(a) Probability of getting an even number

Even numbers on a die: {2, 4, 6}

Number of favorable outcomes = 3

$$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$$

(b) Probability of getting a number greater than 4

Numbers greater than 4: {5, 6}

Number of favorable outcomes = 2

$$P(\text{number} > 4) = \frac{2}{6} = \frac{1}{3}$$

Question 2: In a class of 50 students: 20 like Mathematics (M) 15 like Science (S) 5 like both subjects What is the probability that a student chosen at random likes Mathematics or Science?

Given:

Total students = 50

$|M| = 20$

$|S| = 15$

$|M \cap S| = 5$

First, find the number of students who like Mathematics or Science using the formula:

$$\begin{aligned} |M \cup S| &= |M| + |S| - |M \cap S| \\ |M \cup S| &= 20 + 15 - 5 = 30 \end{aligned}$$

Now, calculate the probability:

$$P(M \text{ or } S) = \frac{|M \cup S|}{\text{Total students}} = \frac{30}{50} = \frac{3}{5}$$

Question 3: A bag has 3 red and 2 blue balls. If one ball is drawn randomly and is red, what is the probability that the next ball is also red (without replacement)?

Given:

Red balls = 3

Blue balls = 2

Total balls = 5

One ball is drawn and it is red (without replacement).

After drawing one red ball:

- Remaining red balls = 2
- Remaining total balls = 4

So, the probability that the next ball is also red is:

$$P(\text{next ball is red}) = \frac{2}{4} = \frac{1}{2}$$

Question 4: The population of a school is divided into 60% boys and 40% girls. If you want equal representation of both genders in the sample, which method should you use: Simple Random Sampling or Stratified Sampling? Why?

We should use Stratified Sampling.

Reason:

The population is divided into two clear groups (strata):

- Boys = 60%
- Girls = 40%

To get equal representation of both genders in the sample, the population should first be divided into these strata (boys and girls), and then samples should be selected from each group in the required proportion (or equally, as specified).

Simple Random Sampling may not guarantee equal representation of boys and girls, especially if the sample size is small.

Use Stratified Sampling, because it ensures proper and balanced representation of both boys and girls in the sample.

Question 5: The average height of 1000 students = 160 cm. A sample of 100 students shows an average height = 158 cm. Find the sampling error.

Given:

Population mean height = 160 cm

Sample mean height = 158 cm

Sampling Error Formula:

$$\begin{aligned}\text{Sampling Error} &= \text{Sample Mean} - \text{Population Mean} \\ &= 158 - 160 = -2 \text{ cm}\end{aligned}$$

Interpretation:

The negative sign indicates that the sample average is 2 cm less than the population average.

Question 6: The population mean salary is ₹50,000 with $\sigma = ₹5,000$. If we take a sample of 100 employees, what is the standard error of the mean (SEM)?

Given:

Population standard deviation (σ) = ₹5,000

Sample size (n) = 100

Formula for Standard Error of the Mean (SEM):

$$SEM = \frac{\sigma}{\sqrt{n}}$$

Calculation:

$$SEM = \frac{5000}{\sqrt{100}} = \frac{5000}{10} = 500$$

The Standard Error of the Mean (SEM) is ₹500.

Question 7: In a group of 100 students: 40 like Cricket (C) 30 like Football (F) 10 like both Cricket and Football Find the probability that a student likes at least one sport.

Given:

Total students = 100

$|C| = 40$

$|F| = 30$

$|C \cap F| = 10$

Step 1: Find students who like at least one sport

Use the formula:

$$\begin{aligned} |C \cup F| &= |C| + |F| - |C \cap F| \\ |C \cup F| &= 40 + 30 - 10 = 60 \end{aligned}$$

Step 2: Find the probability

$$P(\text{at least one sport}) = \frac{60}{100} = \frac{3}{5}$$

Question 8: From a deck of 52 cards, two cards are drawn without replacement. What is the probability that both are Aces?

A standard deck has 52 cards, of which 4 are Aces.

Two cards are drawn without replacement.

Step 1: Probability the first card is an Ace

$$P(\text{1st Ace}) = \frac{4}{52}$$

Step 2: Probability the second card is an Ace (after one Ace is drawn)

Remaining Aces = 3

Remaining cards = 51

$$P(\text{2nd Ace}) = \frac{3}{51}$$

Step 3: Multiply the probabilities

$$P(\text{both Aces}) = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$$

Question 9: A factory produces bulbs with 2% defective rate. If 5 bulbs are chosen at random, what is the probability that all are non-defective?

Given:

Defective rate = 2%

So, probability a bulb is non-defective = $1 - 0.02 = 0.98$

If 5 bulbs are chosen at random (assuming independence):

Probability that all 5 bulbs are non-defective:

$$P = (0.98)^5$$
$$P \approx 0.9039$$

Question 10: Differentiate between discrete and continuous random variables with examples.

Difference between Discrete and Continuous Random Variables

Basis	Discrete Random Variable	Continuous Random Variable
Definition	Takes countable values	Takes uncountable values within a range
Possible Values	Finite or countably infinite (0, 1, 2, 3, ...)	Any value in an interval (e.g., 1.2, 1.25, 1.257, ...)
Measurement	Obtained by counting	Obtained by measuring
Probability	Probability is assigned to exact values	Probability is assigned to a range of values
Probability Distribution	Probability Mass Function (PMF)	Probability Density Function (PDF)
$P(X = x)$	May be greater than 0	Always 0 for exact values

Examples

Discrete Random Variables:

- Number of students in a class
- Number of defective items in a batch
- Number of heads when tossing 3 coins

Continuous Random Variables:

- Height of a person
- Weight of an object
- Time taken to complete a task