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Assignment (Relations)

1. Determine whether the relation represented by zero-one matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$ is an equivalence relation.

Sol. 1:- Let $A = \{1, 2, 3, 4\}$

	1	2	3	4
1	1	0	1	0
2	0	1	0	1
3	1	0	1	0
4	0	1	0	1

Then given relation is
$R = \{(1,1), (2,2), (3,3), (4,4), (1,3), (2,4), (3,1), (4,2)\}$

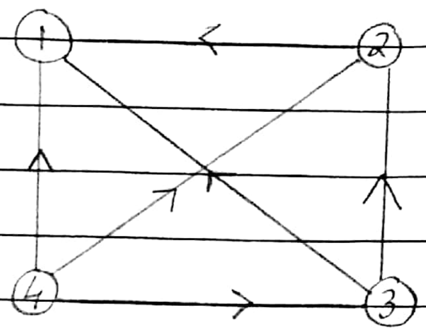
R is reflexive as $(x,x) \in R, \forall x \in A$.
R is symmetric as $(x,y) \in R \Rightarrow (y,x) \in R$
R is transitive as $(x,y) \in R$ ~~and~~ $(y,z) \in R$
$\Rightarrow (x,z) \in R$

So R is an equivalence relation.

2. Let $x = \{1,2,3,4\}$, $R = \{\langle x,y \rangle \mid x > y\}$. Draw the graph of R and also give its matrix.

Sol. 2:- $R = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

Graph of R



Matrix of R

	1	2	3	4
1	0	0	0	0
2	1	0	0	0
3	1	1	0	0
4	1	1	1	0

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

3. Let R be a relation on a set $A = \{1, 2, 3\}$ defined by:
$R = \{(1,1), (1,2), (2,3)\}$. Find the reflexive closure of R and symmetric closure of R. and transitive closure of R.

Sol. 3 $A = \{1, 2, 3\}$

$R = \{(1,1), (1,2), (2,3)\}$

$R^{-1} = \{(1,1), (2,1), (3,2)\}$

$R \cup R^{-1} = \{(1,1), (1,2), (2,1), (2,3), (3,2)\}$

Reflexive closure of R :-

$\{(1,1), (2,2), (3,3), (1,2), (2,3)\}$

Symmetric closure of R :-

$R \cup R^{-1} = \{(1,1), (1,2), (2,1), (2,3), (3,2)\}$

For transitive closure of R :-

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$ where M is matrix of R.

$$M^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M^3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M^+ = M + M^2 + M^3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{R^+ = \{(1,1), (1,2), (1,3), (2,3)\}}$$

4. Define different types of closures of R

⇒ Closures :- Let R be a relation in a set A. R may not satisfy particular property like reflexivity, symmetry or transitivity. The new relation, obtained after adding least number of new pairs so that R satisfies particular property, is called closure of R.

Types of Closure:-

① Reflexive Closure
② Symmetric Closure
③ Transitive Closure

① Reflexive closure :- Let R be a relation on A. A reflexive closure of R is the smallest reflexive relation that contains R.

② Symmetric closure :- Let R be a relation on A which is not symmetric.
∴ there exists $(a,b) \in R$ but $(b,a) \notin R$
Now $(b,a) \in R^{-1}$
∴ to make R symmetric, we add all pairs of R^{-1}.
∴ $R \cup R^{-1}$ is symmetric closure of R.

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(3) Transitive closure :- Let A be a set and R be a relation on A. The transitive closure of R, denoted by R^+, is the smallest relation which contains R as a subset and which is transitive.

Let A be a set and R be a relation on A.
The relation $R^+ =$
$R \cup R^2 \cup R^3 \ldots$ in A is called the transitive ~~clo~~ closure of R in A.