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Qus what is Boolean Algebra? Describe the method used for minimization of Boolean function with example. And explain the canonical forms for Boolean functions.

Ans METHODS FOR MINIMIZATION OF BOOLEAN ALGEBRA.

* Definition of Boolean Algebra:-

Boolean Algebra is a system of algebra which is based on only two values 0 and 1. Binary logic deals with variables that can take only two discreate values. The two values taken by variables can be true or false. Yes or No, 1 or 0.
Thus Boolean algebra is defined as "set of elements, set of postulates and set of operations to be performed on the elements.

set of elements :- Collection of ~~having~~ objects having a common property.

The operation performed in Boolean Algerbra are called as logical operations. Logical operations are:- AND, OR ; NOT.

*.1 Method used for minimization of Booleans function.

1. By using the laws or rules of Boolean Algebra.

2. By using Karnaugh Map (K-Map).

1. Minimization of Boolean function using Laws or rules of Boolean Algebra:

The Boolean function can be minimized by using the rules or laws of Boolean Algebra. This can be explain with the help of following example:

eg. 1. $xy + x'z + yz$.

$= xy + x'z + yz(x+x')$ (Multiply yz with $(x+x')$)

$= xy + x'z + xyz + x'yz$

$= xy + xyz + x'z + x'yz$.

$= xy(1+z) + x'z(1+y)$ $\{1+x = 1$ as per identity rule

$= xy + x'z$.

$\therefore$ Minimization of Boolean expression of $xy + x'z + yz$ will be: $xy + x'z$.

eg 2. $x'y + x$

$= x + x'y$

$= (x+x')\cdot(x+y)$ [By using Distributive law]

$= 1\cdot(x+y) = x+y$ $\{x+x'=1$ As per Complementry laws$\}$

eg 3. $(x+y)\cdot(x+y')$

$= x\cdot x + xy' + yx + yy'$ $\{x\cdot x = x, y\cdot y' = 0$. As per Complentry rule)

$= xx + xy' + xy + yy'$ and indempotent rule $\}$

$= x + xy' + xy + 0$ $\{y'+y = 1\}$ $\{x+x = x\}$

$= x + x(y'+y) + 0$

$= x + x$

$= x$.

Complement of a Boolean function

The Complement of a function f denoted f'. is obtained by interchanging 0's for 1's and 1's for 0's in the truth table that defines a function. Algebrically, the complement of a function may be derived using **De Margan's Law**.

Method are used to find Complement of a function.

1. Firstly, Complement expression representing a boolean function is written.

2. Next, laws of Boolean Algebric are used to simplify the Expression.

For example

$$f = (x+y+z)\cdot(xy+z)$$

$$f' = [(x+y+z)\cdot(xy+z)]'$$

$$= [(x+y+z)' + (xy+z)']$$

$$= ((x'\cdot y'\cdot z') + (x'+y')\cdot z')$$

$$\Rightarrow [x'y'z' + (x'+y')\cdot z']$$

find the Complement of the function.

$$f = x'y'z' + x'yz + xyz$$

$$f' = [(x'yz' + x'yz + xyz)]'$$

$$= [(x'y'z')'\cdot(x'yz)'\cdot(xyz)']$$

$$\Rightarrow ((x')' + y' + (z')')\cdot((x')' + y' + z')\cdot(x'+y'+z')$$

$$\Rightarrow (x+y'+z)\cdot(x+y'+z')\cdot(x'+y'+z').$$

Canonical forms for Boolean Expression

"A Boolean expression with n variable are said to be in canonical form if each terms of Boolean expression has exactly n variables in direct or indirect form i.e. complement form.

Canonical form of Boolean function

1. S-O-P form (Minterm)

P-O-S form (Maxterm)

Minterm : Minterm can be defined as product of all the variables or literals with or without bar within the logic system. A binary variable may appear either in its normal forms or in its complement form.

eg. (a) $xy + xy' + \bar{x}y$

~~Lets take~~

(A) **Sum of Products (S-O-P) :**

A Boolean expression that consist of sum of product of various literals or variables is called S-O-P form. Each literal appear only once in direct or indirect form of each minter of SOP form.

eg ~~$f(x,y,z) = x \cdot y \cdot z + x' \cdot y'$~~

$f(x,y,z) = x \cdot y \cdot z + x' \cdot y \cdot z + x \cdot y' z + x \cdot y \cdot z'$.

Example 1:- Express the Boolean function $F = A + B$ in sum of minterms.

$F = A + B$.

first step to represent each term in the form of minterms.

$\Rightarrow A \cdot (B + B') + B \cdot (A + A')$ $\{ A + A' = 1, B + B' = 1$ complementary law $\}$

$= AB + AB' + BA + BA'$

$= AB + AB' + AB + A'B$.

$= AB + AB + AB' + A'B$.

= ⌊ Remove Duplicate term using idempotent Law.

$\because AB + AB = AB$.

$= AB + AB' + A'B$

$\therefore$ Minterm $= AB,\ AB',\ A'B$ and

SOP Form :- $AB + AB' + A'B$.

Example 2: find the sum of minterms of $xy + z$.

$\Rightarrow xy(z + z') + z(x + x')$

$= xyz + xyz' + xz + x'z$

$= xyz + xyz' + xz(y + y') + x'z(y + y')$

$= xyz + xyz' + xyz + xy'z + x'yz + x'y'z$

$= \underbrace{xyz + xyz}_{} + xyz' + xy'z + x'yz + x'y'z$

$= xyz + xyz' + xy'z + x'yz + x'y'z$.

shorthand Notation to Express a function as sum of minter.

$f = \Sigma(7, 6, 3, 5, 1)$

or

$f = m_7 + m_6 + m_3 + m_5 + m_1$

$= m_1 + m_3 + m_5 + m_6 + m_7$

$f = x'y'z + x'yz + xy'z + xyz' + xyz$

Minterm in shorthand notation can be expressed using truth table:-

INPUT		OUTPUT	
x	y	z	m
0	0	0	$m_0 = x'y'z'$
0	0	1	$m_1 = x'y'z$
0	1	0	$m_2 = x'yz'$
0	1	1	$m_3 = x'yz$
1	0	0	$m_4 = xy'z'$
1	0	1	$m_5 = xy'z$
1	1	0	$m_6 = xyz'$
1	1	1	$m_7 = xyz$

Maxterm :- It is defined as sum of all the variable or elements or literals with or without bar within the logic system.

eg. $f(x,y) = (x'+y) \cdot (x'+y') \cdot (x+y)$.

(B) **Product of Sum (POS) form.**

A Boolean Expression that consist of product of sums of various literals or variables is called POS form and each literal only once. indirect or direct form in each maxterm.

Example to convert A·B into product of maxterms.

$A \cdot B$

$= A + (BB')\, B + (AA')$

$= (A+B) \cdot (A+B')\,(B+A)(B+A')$

$= (A+B) \cdot (A+B') \cdot (A+B)(A'+B)$

$= \underbrace{(A+B)\cdot(A+B)}\cdot(A+B')\cdot(A'+B)$

Remove duplicate using idempotent law

$= (A+B)\cdot(A+B')\cdot(A'+B)$

Maxterm $= (A+B), (A+B'), (A'+B)$ and

P.O.S for $= (A+B)\cdot(A+B')\cdot(A'+B)$

Exampl 1: find POS form of $x+y'z$ expression.

$f = x+y'z$

$= (x+y')\cdot(x+z)$

$= (x+y'+(zz'))\cdot(x+z)(yy')$

$= (x+y'+z)\cdot(x+y'+z')\cdot(x+y+z)(x+y'+z)$

$= \underline{(x+y'+z)}\cdot\underline{(x+y'+z)}\cdot(x+y'+z')(x+y+z)$

$= (x+y'+z)\cdot(x+y'+z')\cdot(x+y+z)$ pos for

shorthand rotation for $x+y'z$ expression can be find out using truth table for maxterm:

Inputs		Outputs	
x	y	z	m
0	0	0	$M_0 = x+y+z$
0	0	1	$M_1 = x+y+z'$
0	1	0	$M_2 = x+y'+z$
0	1	1	$M_3 = x+y'+z'$
1	0	0	$M_4 = x'y+z$
1	0	1	$M_5 = x'+y+z'$
1	1	0	$M_6 = x'+y'+z$
1	1	1	$M_7 = x'+y'+z'$