Volatility Voyage

Assignment 5

Summary and Answers

1. Which distribution best models the returns?

The Kolmogorov–Smirnov (KS) test indicates that the **Student's t-distribution** provides the best fit for the returns, with a p-value of **0.9591**. This is the only distribution for which we fail to reject the null hypothesis that the returns follow the specified distribution.

The Cauchy distribution is the next best fit (p-value = 0.5439), but the Student's t-distribution is clearly superior.

2. What are the estimated parameters?

The estimated parameters for the Student's t-distribution are:

- Degrees of Freedom (ν): 2.7897

- Location (μ): -0.000499

- Scale (σ): 0.008489

3. What are the test results and p-values?

Kolmogorov–Smirnov Test Results:

Rank	Distribution	KS Statistic	p-value
1	Student's t	0.0315	0.9591
2	Cauchy	0.0501	0.5439
3	Weibull	0.0915	0.0289
4	Log-normal	0.0922	0.0271
5	Normal	0.0974	0.0165

Additional Normality Tests:

- Anderson–Darling (Normal): Test statistic = 3.9392. All critical values exceeded, hence normality is rejected.

- Jarque–Bera: JB statistic = 164.08 with p-value < 0.0001, hence normality is rejected.

- Skewness = 0.7274

- Kurtosis = 6.7011

4. What do the confidence interval outliers indicate?

Outliers lying beyond the 95% confidence band of the fitted distribution indicate unusual return behavior. These deviations may arise due to high-volatility events or market anomalies. Such observations are particularly useful for stress testing and risk management analysis.

Conclusion

The returns are best modeled by a **Student's t-distribution** with estimated parameters:

$$\nu \approx 2.79, \quad \mu \approx -0.0005, \quad \sigma \approx 0.0085.$$

Goodness-of-fit tests strongly reject normality and favor the t-distribution, which successfully captures the heavy tails and excess kurtosis observed in the data.