

Volatility Voyage

GARCH, EGARCH + inclusion of user's risk aversion

What is GARCH?

GARCH stands for **Generalized Autoregressive Conditional Heteroskedasticity**. That's a mouthful, but it's easier than it sounds. At its core, GARCH is a way to **model and forecast the volatility (risk)** of a time series -like a stock's return - over time. Volatility in markets doesn't stay constant. Sometimes it's calm, sometimes extremely noisy. What GARCH does is use **past patterns of market shocks and volatility** to predict how volatile things will be tomorrow or next week.

Let's take a simple example. Say you're looking at the 1-minute returns of a stock. On some days, the market is calm -the returns fluctuate gently. But on budget day or when big global news hits, the returns bounce up and down sharply. You might notice that if today was very volatile, tomorrow is likely to be volatile too. GARCH formalizes this idea. It says: *"Today's volatility depends on yesterday's squared return (shock) and yesterday's volatility."*

So, if the market had a big unexpected move yesterday (say, a surprise rate hike), then tomorrow's expected volatility increases. GARCH models this using just **three main parameters**:

1. A base level of volatility
2. A measure of how much new shocks increase volatility
3. A measure of how persistent that volatility is over time

The GARCH(1,1) model is the simplest and most widely used form.

2. What is EGARCH and Why Is It Different?

EGARCH stands for **Exponential GARCH**. It's an improvement over the standard GARCH model in one important way: it can handle **asymmetry** in how the market reacts to different kinds of news.

Here's the intuition: Imagine two days –on Day 1, a stock drops 5%, and on Day 2, it rises 5%. Standard GARCH treats both as equally “risky” events, because both had large returns (positive or negative). But in real markets, **bad news (negative returns)** usually causes **more fear** and increases volatility more than good news of the same size. This is called the **leverage effect**, and GARCH doesn't capture this difference –but EGARCH does.

EGARCH does this by modelling volatility on a **log scale** and including a term that allows positive and negative shocks to affect volatility differently. That means if the market suddenly drops, EGARCH will forecast a **bigger spike in volatility** than if it had risen by the same amount. This makes EGARCH more realistic, especially in equity markets where fear spreads faster than optimism.

3. What is Asymmetry in Volatility?

Asymmetry simply means that **not all shocks are treated equally**. In the context of financial volatility, it refers to the fact that **negative returns (losses)** tend to cause a **greater increase in volatility** than positive returns (gains) of the same size.

Why does this happen? There are psychological and structural reasons:

- Investors fear losses more than they enjoy gains (called **loss aversion**).
- In equity markets, falling prices often lead to **margin calls, forced selling**, and panic, making the next moves even more volatile.
- In contrast, a big price rise may just lead to a little more buying – not panic.

The EGARCH model is designed to pick up this **directional sensitivity**, which GARCH cannot.

4. We will use rolling window throughout. Why?

Financial markets change constantly. A model trained on pre-COVID market data would likely do a bad job forecasting volatility during COVID. To keep our model **adaptive**, we use a **rolling window**.

Think of it like this: Instead of training your GARCH model on **all data from 2020**, you train it only on the most recent **6 months or 1 year** of data. Every time you move forward by one day, the oldest data drops out of the window, and the newest data gets added in. This way, the model always reflects the **latest market behavior** without being weighed down by outdated patterns.

For example, say you're forecasting volatility on 1st Jan 2024. You train the model using data from **July 2023 to Dec 2023**. When you forecast for 2nd Jan 2024, you shift the window forward by one day and use data from **July 2nd to Jan 1st**, and so on.

This method helps you:

- Adapt to changing volatility regimes
- Avoid overfitting to old, irrelevant data
- Stay realistic in a live trading environment

Rolling estimation is especially powerful when forecasting volatility in high-frequency data like minute-wise stock returns – where patterns change fast.

In GARCH(1,1):

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

- ω : Constant volatility (base level)
- α : Effect of new shocks (how yesterday's squared return influences today's volatility)
- β : Persistence of past volatility (how yesterday's volatility carries forward)

Example:

If $\omega = 0.0001$, $\alpha = 0.1$, and $\beta = 0.85$, then

$$\sigma_t^2 = 0.0001 + 0.1 \cdot \varepsilon_{t-1}^2 + 0.85 \cdot \sigma_{t-1}^2$$

Here, yesterday's shock and volatility both affect today's volatility. This captures *volatility clustering* effectively.

Stationarity condition:

persistence = $\alpha + \beta < 1$ (we get alpha and beta by directly estimating once you fit the model)

Persistence refers to how long the impact of a volatility shock (i.e., a sudden large return movement) lasts over time.

Interpretation of Persistence

- High persistence (close to 1):
Volatility shocks decay very slowly. This means if the market is volatile today, it will likely stay volatile for a while.
- Low persistence (< 0.8):
Volatility shocks die out quickly. The market settles down fast after sudden jumps.
- Persistence ≥ 1 :
The model is non-stationary — volatility could grow infinitely over time. Not desirable.

As a trader or risk modeler:

- **High persistence** → Volatility clustering → risk stays elevated
- **Low persistence** → Quick mean reversion → might mean opportunities to re-enter markets faster

Variance equation for EGARCH:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha |\varepsilon_{t-1} / \sigma_{t-1}| + \gamma (\varepsilon_{t-1} / \sigma_{t-1})$$

Explanation of terms:

- $\log(\sigma_t^2)$: Log of variance — ensures positivity of volatility
- ω : Constant
- β : Persistence (same as GARCH, but now applied to log-volatility)
- $\alpha|\varepsilon_{t-1} / \sigma_{t-1}|$: Shock size effect — large shocks increase volatility, regardless of direction
- $\gamma(\varepsilon_{t-1} / \sigma_{t-1})$: Asymmetry term — if γ is negative, negative returns increase volatility more than positive ones