Dutch-Bangla Bank -Prothom Alo Gonit Olympiad 2017

National Olympiad, 15 February 2017

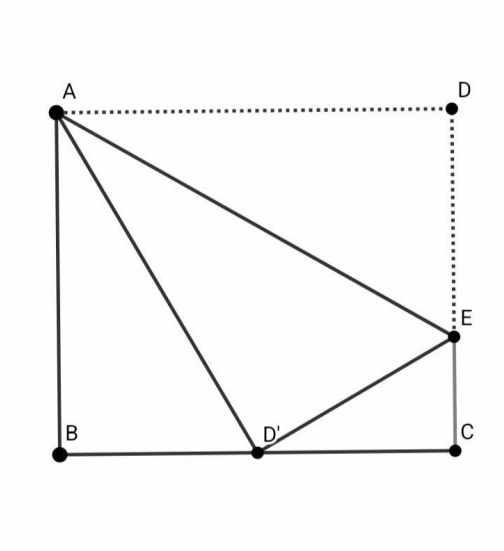
Bangladesh Mathematical Olympiad Committee

Junior Category, Time: 3 hours

1. **Evaluate:**
$$\frac{2+4+6+\cdots+34}{3+6+9+\cdots+51}=?$$

2. **What is the minimum possible product of three different numbers of the set {-8,-6,-4,0,3,5,7}?**

3. **As shown in the Figure 1, a rectangular piece of paper is folded along AE such that D becomes coincident with D' on BC and BD'=CD'. If AD=$4\sqrt{3}$, then what is EC?**



4. **a) Can two consecutive numbers n and n-1 both be devisible by 3?**

   b) Determine the smallest integer $n > 1$ such that $n^2(n-1)$ is devisible by 1971. [Note: $1971 = 3^3 \times 73$]

5. **a) What are the possible values for the sum of the digits of the multiples of 18 between 100 and 999?**

   b) Show that among any 18 consecutive 3-digit numbers that there is at least one number that is divisible by the sum of its digits.

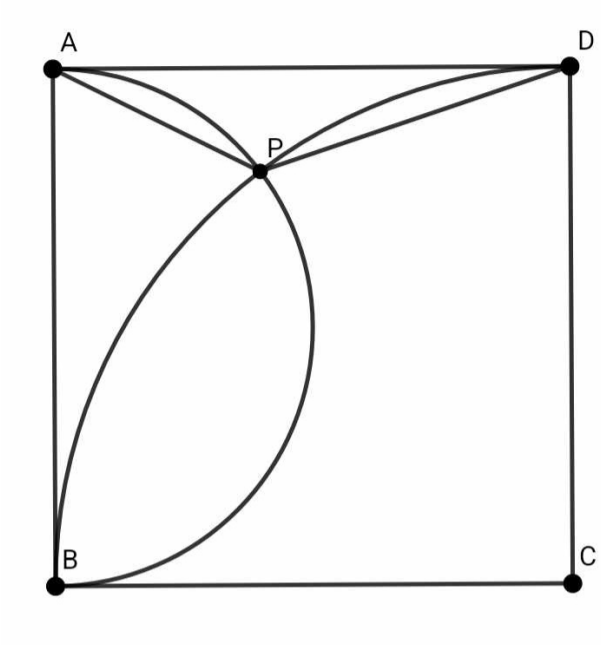
6. **Ishfaq has randomly chosen 4 numbers. Prove that it is possible to choose two of those numbers such that their differences is devisible by 3.**

7. **In a school, there are 5 classrooms. Each student in a classroom knows exactly one student from each of the other 4 classrooms. Prove that the number of students in each classroom is exactly same.**

   (Assume if student A knows student B, then student B also knows student A)

8. **In Triangle ABC, the perpendicular bisectors of AB and AC meet at O. Line AO intersects segment BC at D. if $OD = BD = \frac{BC}{3}$, find the angles of triangle ABC.**

9. **In figure 2, ABCD is a square. Circle with diameter AB and circle with center C and radius BC meet inside the square at P. Prove that $DP = \sqrt{2}AP$**



10. **Whenever Avik gets a sequence, he multiplies every two distinct terms of that sequence, and then sums up these products to get the 'Hocus-pocus' sum of the sequence. For example, the 'Hocus-pocus' sum for the sequence a,b,c,d is ab+ac+ad+bc+bd+cd. If Avik gets a sequence of 100 terms, where each term is either 2 or -1, what is the minimum 'Hocus-pocus' sum of that sequence?**