

Mathematical Calculations

- The way we are used to writing expressions is known as infix notation.
- What does $3 + 2 * 6$ equal? Ans: 15
What does $3 * 2 + 6$ equal? Ans: 12
- The precedence of operators affects the order of operations.
- A mathematical expression cannot simply be evaluated left to right.
- Postfix expression does not require any precedence rules.

Operator	Precedence	Associativity
()	1	L to R ✓
^	2	R to L ✓
*, /	3	L to R ✓
+, -	4	L to R ✓

$$(2+3)*6+4^2+8$$

$$2^3 = 2^{(3^2)}$$

$$= 2^9$$

$$= 512$$

Algorithm: Convert Infix to Postfix Expression

- Scan the infix expression from left to right.
- If the scanned character is an operand, output it.
- Else [scanned character is an operator]
 - if stack is empty or contains a left parenthesis '(' on top, push the incoming operator onto the stack.
 - if the incoming symbol is '(', push it onto the stack.
 - if the incoming symbol is ')', pop the stack and print the operators until left parenthesis is found.
 - if the incoming symbol has higher precedence than the top of the stack, push it onto the stack.
 - if the incoming symbol has lower precedence than the top of the stack, pop and print the top. Then test the incoming operator against the new top of the stack.
 - if the incoming symbol has equal precedence with the top of the stack, use associativity rule.
 - if associativity is L to R, then pop and print the top. Then push the incoming operator.
 - if associativity is R to L, then push the incoming operator.
- At the end of the expression, pop and print all operators of stack.

$$(8+2)*5$$

$$\text{Postfix: } 482+5*+$$

$$\text{stack: } +8**$$

$$\text{stack: } +8**+$$

$$\text{stack: } +8**+*$$

Simple Example 1

- Convert the following expressions from infix to postfix:

$$3 + 2 * 4$$

Infix Expression: $3 + 2 * 4$

PostFix Expression: $3 2 4 * +$

Operator Stack: $+ *$

Infix to Postfix: Practice Question 1

- Convert the following expressions from infix to postfix:

a) $2^3 \wedge 4 + 5 * 7$

Postfix: $2 3 4 \wedge \wedge 5 7 * +$

Stack

\wedge

$\wedge \wedge$

\wedge

$+$

$+ *$

$+$

Infix to Postfix: Practice Question 1

- Convert the following expressions from infix to postfix:

a) $2^3 \wedge 4 + 5 * 7$

Ans: $2\ 3\ 4\ \wedge\ 5\ 7\ *\ +$

b) $9 + 2 - 5 * 6 / 3 + 2^3 / 4$

Ans: $9\ 2\ +\ 5\ 6\ *\ 3\ /\ -\ 2\ 3\ \wedge\ 4\ /\ +$ ✓

c) $1 + 3^2 + (5 * 6 - 4) * 7$

Ans: $1\ 3\ 2\ \wedge\ +\ 5\ 6\ *\ 4\ -\ 7\ *\ +$ ✓

stack = operand

Application of Stacks:

Evaluation of Postfix Expression

for each character in postfix expression do

- if it is an operand then push it onto the stack
- else if it is an operator (say, op) then
 - ◆ pop the stack for the right hand operand (say, b)
 - ◆ pop the stack for the left hand operand (say, a)
 - ◆ apply the operator to the two operands (result = a op b)
 - ◆ push the result onto the stack
- when the expression has been exhausted the result is the top (and only element) of the stack

stack: $2\ \cancel{3}\ \cancel{4}\ 12$

$\cancel{3}\ \cancel{4}$

14

$\begin{matrix} 12 \\ 3 \times 4 \end{matrix}$

$\begin{matrix} 14 \\ 2 + 12 \end{matrix}$

result = 14

*post: 2 3 4 * + = ?*

↑ ↑ ↑ ↑ ↑

Practice Question 2

- What does the following postfix expression evaluate to?

6 3 2 + *

Postfix Expression: 6 3 2 + *

Stack: 6 3 2

- What does the following postfix expression evaluate to?

2 3 4 ^ ^ 5 7 * +

stack = 2 3 4

= 2 81

= 2 57

4

2 81 35

2 81 + 35

2 3 4 + - 5 7 * +