Data Structures for Disjoint Sets

Application:

Connected Components

Minimum Spanning Tree

Dr. Md. Abul Kashem Mia, Professor, CSE Dept and Pro-Vice Chancellor, UIU

Disjoint Sets

- Some applications require maintaining a collection of disjoint sets.

- A Disjoint Set S is a collection of sets $S_1, \ldots\ldots S_n$
  where $\forall_{i \neq j} S_i \cap S_j = \phi$

- Each set has a representative which is a member of the set (usually the minimum if the elements are comparable)

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Disjoint Set Operations

- Make-Set(x) – Creates a new set S_x where x is it's only element (and therefore it is the representative of the set).
    $O(1)$ time.

- Union(x, y) – Replaces S_x, S_y by $S_x \cup S_y$.
    One of the elements of $S_x \cup S_y$ becomes the representative of the new set.
    $O(\log n)$ time.

- Find(x) – Returns the representative of the set containing x
    $O(\log n)$ time.

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Analyzing Operations

- We usually analyze a sequence of m operations, of which n of them are Make_Set operations, and m is the total of Make_Set, Find, and Union operations.

- Each union operations decreases the number of sets in the data structure, so there can not be more than n-1 Union operations.

Applications

- Equivalence Relations (e.g Connected Components)

- Minimum Spanning Trees

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