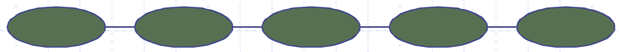
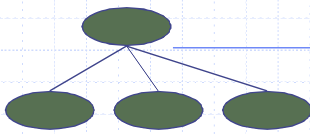
Graphs

sequence/linear (1 to 1)

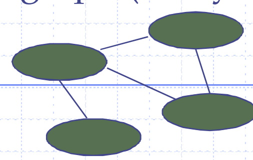


first ith last

hierarchical
(1 to many)

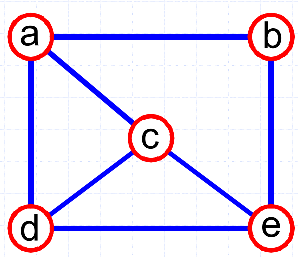


graph (many to many)



What is a Graph?

- A graph is a pair (V, E), where
  - V is a set of nodes, called vertices
  - E is a collection of pairs of vertices, called edges
- V(G) and E(G) represent the sets of vertices and edges of G, respectively
- Example:



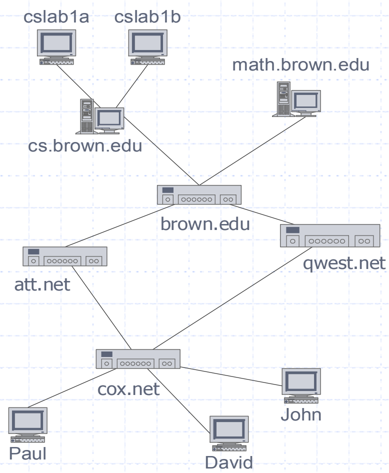
V= {a, b, c, d, e}

E= {(a,b), (a,c), (a,d), (b,e), (c,d), (c,e), (d,e)}

- **A tree is a special type of graph!**

Applications

- Electronic circuits
  - Printed circuit board
  - Integrated circuit
- Transportation networks
  - Highway network
  - Flight network
- Computer networks
  - Local area network
  - Internet
  - Web
- Databases
  - Entity-relationship diagram



What can we do with graphs?

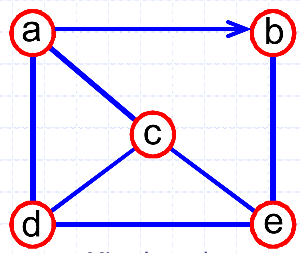
- Find a *path* from one place to another
- Find the *shortest path* from one place to another
- Determine connectivity
- Find the "weakest link" (min cut)
  - check amount of redundancy in case of failures
- Find the amount of flow that will go through them

Edge and Graph Types

- Directed edge
  - ordered pair of vertices (u,v)
  - first vertex u is the origin
  - second vertex v is the destination
- Undirected edge
  - unordered pair of vertices (u,v)
- Directed graph (Digraph)
  - all the edges are directed
  - e.g., route network
- Undirected graph
  - all the edges are undirected
  - e.g., flight network
- Mixed graph
  - some edges are undirected and some edges are directed
  - e.g., a graph modeling a city map

Directed edge

Undirected edge



Mixed graph

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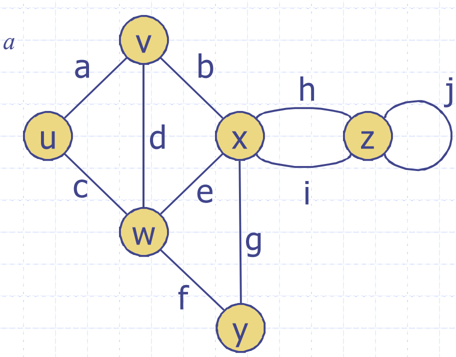
Terminology

- End vertices (or endpoints) of an edge
  - u and v are the *endpoints* of a
- Edges incident to a vertex
  - a, d, and b are *incident* to v
- Adjacent vertices
  - u and v are *adjacent*
- Degree of a vertex
  - x has *degree* 5
- Parallel edges
  - h and i are *parallel edges*
- Self-loop
  - j is a *self-loop*



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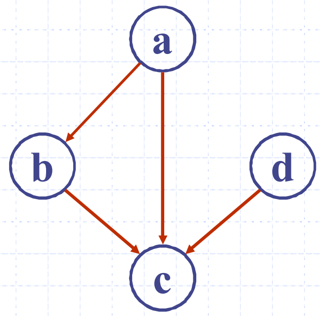
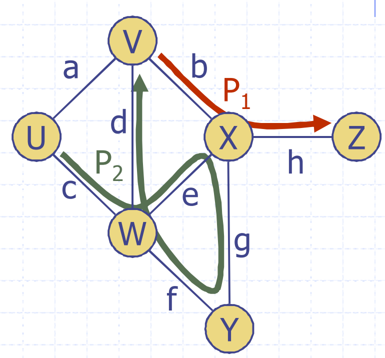
Terminology (cont.)

- Outgoing edges of a vertex
  - (a, b) and (a, c) are outgoing edges of vertex a
- Incoming edges of a vertex
  - (b, c), (d, c) and (a, c) are incoming edges of vertex c
- In-degree of a vertex
  - c has *in-degree* 3
  - b has *in-degree* 1
- Out-degree of a vertex
  - a has *out-degree* 2
  - b has *out-degree* 1



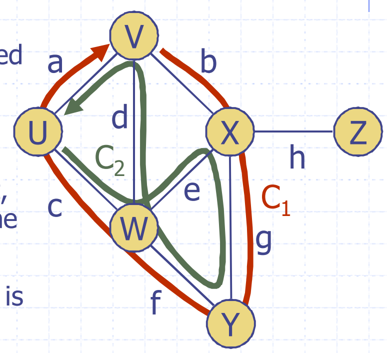
Terminology (cont.)

- Path
  - sequence of alternating vertices and edges
  - begins with a vertex
  - ends with a vertex
  - each edge is preceded and followed by its endpoints
- Simple path
  - path such that all its vertices and edges are distinct
- Examples
  - P_1=(V, b, X, h, Z) is a simple path
  - P_2=(U, c, W, e, X, g, Y, f, W, d, V) is a path that is not simple



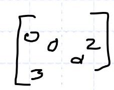
Terminology (cont.)

- Cycle
  - A cycle is a path whose start and end vertices are the same
  - each edge is preceded and followed by its endpoints
- Simple cycle
  - A cycle is simple if each edge is distinct and each vertex is distinct, except for the first and the last one
- Examples
  - C_1=(V, b, X, g, Y, f, W, c, U, a, V) is a simple cycle
  - C_2=(U,c,W,e,X,g,Y,f,W,d,V,a,U) is a cycle that is not simple



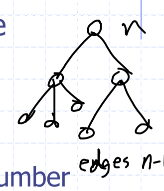
matrix

Sparse graph

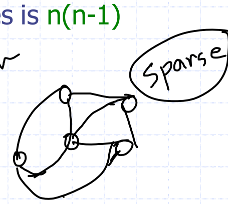


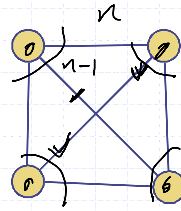
Terminology (cont.)

- *Dense* graph: |E| ≈ $|V|^2$; *Sparse* graph: |E| ≈ |V|
- A *weighted graph* associates weights with either the edges or the vertices
- A complete graph is a graph that has the maximum number of edges
  - for undirected graph with n vertices, the maximum number of edges is n(n-1)/2
  - for directed graph with n vertices, the maximum number of edges is n(n-1)



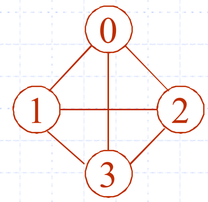
Planar graph



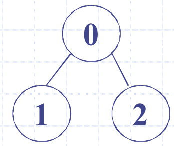


$\frac{n(n-1)}{2} = O(n^2)$

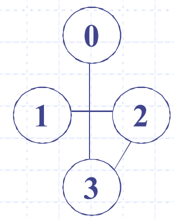
Terminology (cont.)



- A subgraph of G is a graph G′ such that
  - V(G′) is a subset of V(G) [V(G′) $\subseteq$ V(G)] and
  - E(G′) is a subset of E(G) [E(G′) $\subseteq$ E(G)]
- A spanning subgraph G′ of G is a subgraph of G that contains all the vertices of G, that is
  - V(G′) is equal to V(G) [V(G′) = V(G)] and
  - E(G′) is a subset of E(G) [E(G′) $\subseteq$ E(G)]
- A forest is a graph without cycles.
- A (free) tree is a connected forest, that is, a connected graph without cycles.
- A spanning tree of a graph G is a spanning subgraph that is a (free) tree.



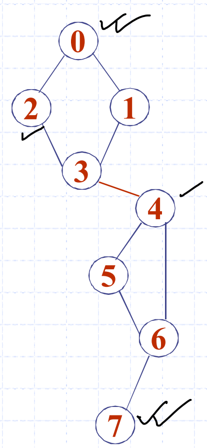
A subgraph

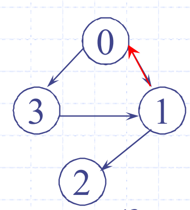


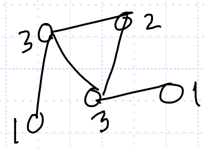
A spanning subgraph (tree)

Terminology (cont.)

- In a graph G, two vertices, v_0 and v_1, are connected if there is a path in G from v_0 to v_1
- A graph is connected if, for every pair of distinct vertices v_i and v_j, there is a path from v_i to v_j
- A connected component of an undirected graph is a maximal connected subgraph.
- A tree is a graph that is connected and acyclic.
- A directed graph is strongly connected if there is a directed path from v_i to v_j and also from v_j to v_i.
- A strongly connected component is a maximal subgraph that is strongly connected.







$1+3+2+3+1 = 10$

$10 = 2*5$

Properties

Property 1
For an undirected graph
$$\Sigma_v \deg(v) = 2m$$
Proof: each edge is counted twice

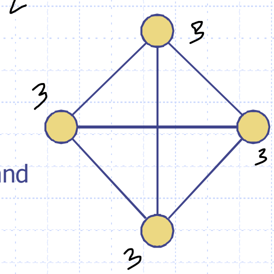
Property 2
For a directed graph
$$\Sigma_v \text{indeg}(v) = \Sigma_v \text{outdeg}(v) = m$$
Proof: each for out-degreeedge is counted once for in-degree and once

Property 3
If G is a simple undirected graph, then $m \le n(n-1)/2$, and if G is a simple directed graph, then $m \le n(n-1)$.
Proof: each vertex has degree at most $(n-1)$. Then use Property 1 and Property 2.

Notations:

n	number of vertices
m	number of edges
$\deg(v)$	degree of vertex v

$12 = 2*6$



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