Solve each of the following sets of simultaneous congruence:

(a) $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$

(b) $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$

(c) $x \equiv 5 \pmod 6$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$.

Sol^n (a):

Given that,

$x \equiv 1 \pmod 3$ .... (i)

$x \equiv 2 \pmod 5$ .... (ii)

$x \equiv 3 \pmod 7$ .... (iii)

using quotient reminder theorem we get,

$x \equiv 1 \pmod 3 \approx x = 3a+1$ .... (iv)

$x \equiv 2 \pmod 5 \approx x = 5a+2$ .... (v)

$x \equiv 3 \pmod 7 \approx x = 7a+3$ .... (vi)

from (iv) and (ii) we get,

$3a+1 \equiv 2 \pmod 5$

$\Rightarrow 3a = 1 \pmod 5$

The inverse of $3 \bmod 5$ is 2 so, $a \equiv 2 \pmod 5 \Rightarrow a = 5b+2$

Then $x = 3(5b+2)+1 = 15b+7$.

Now from eqn (iii).

$15b+7 = 3 \pmod 7$

since, $15 \equiv 1 \pmod 7$ and $7 \equiv 0 \pmod 7$ then the eqn simplifies to

$b = 3 \pmod 7 \Rightarrow b = 7c+3$

then, $x = 15(7c+3)+7 = 105c+52$

$x \equiv 52 \pmod{15} \Rightarrow 52$ (Ans.)

Soln (b):

Given that, $x \equiv 5 \pmod{11}$ .... (i)

$x \equiv 14 \pmod{29}$ .... (ii)

$x \equiv 15 \pmod{31}$ .... (iii)

combining the first two congruence,

$x = 11a + 5$. Substitute into the second ingruence

$11a + 5 \equiv 14 \pmod{29}$

$\Rightarrow 11a = 9 \pmod{29}$

The inverse of 11 module 29. This means we want $11x = 1 \bmod 29$

$\therefore 11 \times 8 = 1 \bmod 29$

So the inverse is 8. multiply both sides: $a = 9 \times 8 = 72 \equiv 14 \pmod{29}$

$a = 29b + 14$

Substitute back: $x = 11(29b + 14) + 5 = 319b + 159$

so $x \equiv 159 \pmod{319}$

combine with third congruence we have, $319b + 159 = 15 \pmod{31}$

$\therefore 319 \equiv 9 \pmod{31}$ and $159 \equiv 4 \pmod{31}$

So, $9b + 4 \equiv 15 \pmod{31} \Rightarrow 9b \equiv 11 \pmod{31}$

Finding the inverse of 9 modulo 31:

$9 \times 7 = 63 = 1 \pmod{31}$

So the inverse is 7. multiply both sides:

$b = 11 \times 7 = 77 \equiv 15 \pmod{31} \Rightarrow b = 31c + 15$

Substitute back. $x = 319(31c + 15) + 159 = 9889c + 4944$

$\therefore x = 4944 \pmod{9889} \Rightarrow 4944$

Soln: (c) : Given,

$x \equiv 5 \pmod 6$ ... ... (i)

$x \equiv 4 \pmod{11}$ ... ... (ii)

$x \equiv 3 \pmod{17}$ ... ... (iii)

Combining the first two ingruences, we get.

$6a+5 = 4 \pmod{11}$.

Substituting into the second congruence we get.

$6a+5 = 4 \pmod{11}$

$6a \equiv -1 \equiv 10 \pmod{11}$

The inverse of 6 modulo 11 is 2. ~~since 6 modulo 11 is 2:~~

since $6 \times 2 = 12 \equiv 1 \pmod{11}$:

$a = 2 \times 10 = 20 \equiv 9 \pmod{11} \Rightarrow a = 11b+9.$

Then $x = 6(11b+9)+5 = 66b+59$

Now using the third congruence

$66b + 59 \equiv 3 \pmod{17}$

$\Rightarrow 66 \equiv 15 \pmod{17}$, $59 \equiv 8 \pmod{17}$

So, $15b+8 \equiv 3 \pmod{17} \Rightarrow 15b = -5 \equiv 12 \pmod{17}$

The inverse of 15 mod 7 is 8 so, $15 \times 8 = 120 \equiv 1 \pmod{17}$

$b = 8 \times 12 = 96 \equiv 11 \pmod 7 \Rightarrow 17c+11$

Then $x = 66(17c+11)+59$

$= 1122c + 785$

$\therefore x \equiv 785 \pmod{1122} \Rightarrow 785$

(Ans.)