North South University

Department of Electrical and Computer Engineering

Senior Design Project Report - Summer, 2022

Implementation and Fidelity comparison of Quantum Error Correction techniques using Quantum Repetition Code

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Declaration

We hereby declare that this project is our original work. We didn't submit it elsewhere. Everything in this project has been appropriately cited.

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Approval

In this senior design project report on “Implementation and Fidelity comparison of Quantum Error Correction techniques using Quantum Repetition Code.” by Name: Farzana Rahman (ID: 1712173642), Tanvirul Hasan (ID: 1812949642), and Kaniz Fatema Roksana (ID: 1522057042) have been approved for partial fulfillment of the requirement of the Degree of Bachelor of Science in Computer Science and Engineering and accepted as satisfactory.

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Acknowledgment

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Abstract:

Quantum Error Correction (QEC), quantum information is protected against errors caused by decoherence and noise in Quantum Computers. Through the use of qubit redundancy, QEC focuses on error-correcting codes to increase the reliability of quantum systems. In this project, we adopt a practical methodology and employ Quantum Repetition Code as a scalable approach which is one of the easiest ways to prevent and correct classical channel bit-flip errors. Instead of using a physical qubit $| \psi\rangle = \alpha | 0\rangle + \beta | 1\rangle$, we encode a logical qubit using three physical qubits as $| \psi\rangle = \alpha | 000\rangle + \beta | 111\rangle$. We used this method on 3, 5 quantum error correcting codes to compare their fidelities to reduce bit flip errors at the best possible state.

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Chapter 1: Introduction

A quantum bit is represented by a linear combination of two orthogonal vectors $|0| >= [1\ 0]^\wedge(T)\ and\ |1| >= [0\ \ 1]^\wedge(T)$ and $B[quantum] = \alpha\,|0> + \beta\,|1>$, where α is the probability of $|0>$ and β is the probability of $|1>$, and $\alpha^2 + \beta^2 = 1$. Quantum bits or qubits allow these particles to exist in more than one state either 0 or 1 at the same time. Quantum computing requires information in qubits to encode. We can see that quantum states are very delicate. So, quantum error correction will be necessary to build reliable quantum computers. The last few decades have seen great advances in finding physical systems that behave as qubits, with better quality qubits being developed all the time [1][12]. In the current era of quantum computing, we ask to use physical qubits despite their imperfections. we necessarily find ways to build logical qubits from physical qubits. This can be done by the process of quantum error correction. Where logical qubits are encoded in a large number of physical qubits. The theory of quantum error correcting codes has some close ties and some striking differences from the theory of classical error correcting codes. Quantum computers have great potential. To realize that potential, we need some sort of protection from noise. Quantum error correction is essential to achieve the goal that can reduce the effects of noise on stored quantum information and so on. Quantum states of superposition (which stores quantum information) are extremely fragile. Quantum error correction is more tricky than classical error correction. In the field of quantum computation, what is attainable in theory is very far off from what can be implemented. Complex quantum computation is impossible without the ability to recover it from errors. Quantum error correction of a qubit encoded in mesh states of an oscillator. Quantum bits are more vigorous to noise when they are encoded nonlocally. In an encoding, errors affecting the underlying physical system can then be detected and corrected before they corrupt the encoded information.

Thermodynamic irreversibility is a great enemy of quantum computing, since it tends to nonlocal qubits, therefore introducing unwanted noise into the quantum computation. Effective techniques for quantum error correction were first developed in 1995 by Peter Shor. Probably, there will be more improvements in error correction methods as various physical realizations of quantum computers are developed. Quantum error correction was developed in analogy with classical error-correcting codes by encoding information. Using repetition code is one of them. The same basic idea underlies all QEC (Quantum Error Correction) codes, which is that qubit states are represented in collective states of several (noisy) qubits [4] that have specific symmetries. Errors can be found and fixed by examining these symmetries using so-called syndrome measurements. Keep in mind that errors in the quantum situation take on more intricate shapes than the straightforward bit flips. As a result, the repetition code alone is insufficient to adequately safeguard against quantum errors, and in the vast majority of cases, more advanced encoding techniques are needed. However, quantum versions of the repeating code are frequently a suitable place to start for QEC.

In this paper, we will look at a particular example of error correction: the repetition code. It gives out a simple guide to all the basic concepts in any quantum error-correcting code. We will also distinguish how it can be run on current prototype devices.

Chapter 2: Related Work

We have done quantum error correction by using repetition code. In the process of doing the project, we found a lot of other related works. Cornell University implements repetition codes of at most 15 qubits on the 16 qubits [3]. The results show strong evidence that the logical error rate decays exponentially with code distance, as the way it is expected and required for the development of fault-tolerant quantum computers. Some works on n-qubit repetition codes and simple and concatenated Calderbank Shor-Steane codes are used for quantum error correction against correlated noise [5]. By measuring the fidelity after an individual logical qubit in a quantum register has undergone error correction, we may characterize the performance of different codes which is shown in some articles. Also, there are journals which proved well of how benchmarking near-term devices with quantum error correction can be made as more advanced quantum computing hardware is being developed. To execute tests based on the repetition code and analyze the results, they utilized the quantum repetition code and graph decoder classes. Data from a 43-qubit code running on IBM's Rochester device is shown as an illustration [6][12].

Also, for optimizing the encoding circuit, a journal article of National Science Review has shown in an experimental exploration of five-qubit quantum error-correcting code with superconducting qubits how they use an array of superconducting qubits in the experiment to realize the [5, 1, 3] code for a number of common logical states, including the magic state, which is a crucial component for realizing non-Clifford gates. The prepared encoded states have a high fidelity of 98.6(1) % in the coding space and an average fidelity of 57.1(3) %.

The use of mixed ancilla qubits [8] to solve QEC for $(2t + 1)$ qubits in which t represents number of bit flip errors have also been shown. Here, they examined bit-flip error correction using three-, five-, seven-, and nine-qubit repetition codes.

Chapter 3: Project Plan

Planning of our project:

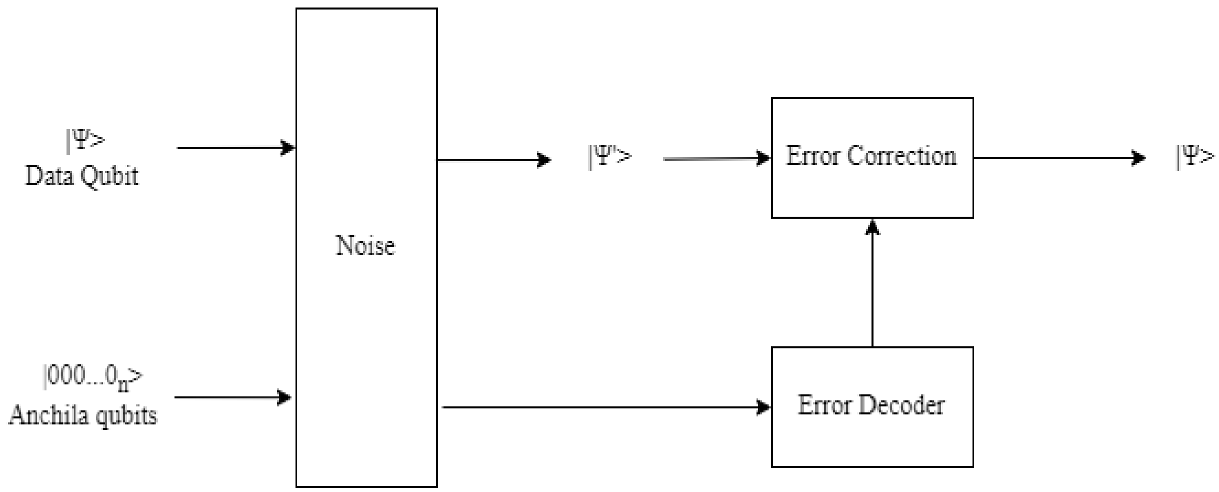

Figure: Flowchart of project plan to detect and reduce the noise of QEC in a Quantum Channel.

Chapter 4: Study Area

For doing this project we had to learn some key points on this topic. First of all, we had to know about quantum computing. A fast-developing technology called quantum computing uses the principles of quantum physics to solve issues that are too complicated for conventional computers. We are all aware that quantum computing is a cutting-edge technology that uses a special kind of computer that is 158 million times faster than the most advanced supercomputer currently in existence. Quantum computing is the application of quantum theory to computer science. The behavior of energy and matter at the atomic and subatomic scales is explained by quantum theory. Subatomic particles, such as electrons or photons, are used in quantum computing. A technology that scientists had only begun to conceive thirty years ago is now made by IBM Quantum and is accessible to thousands of developers. Then we learned about the errors of quantum computers. Quantum errors are more complex than a classical computer's errors. Classical errors consist only of bit flips, where bits might change from 1 to 0, but quantum bits can enter superposition, taking on combinations of 0 and 1 with an associated angle called a phase, which can also cause error [8]. Errors in our approach result from the qubits' information becoming randomized as a result of noise. The shorter the algorithm can run before it makes a mistake and produces an erroneous or even meaningless result, the more noise it can withstand.

Is it possible to fix quantum errors? Yes, and there are different ways: repetition code is one of them. Note that not all single qubit mistakes can be fixed by the proposed encoding. Only single bit-flip faults are correctable with this method. We needed to understand quantum logic gates to perform quantum error correction.

Qubits, as opposed to bits, are used by quantum computers. A qubit can exist in a "superposition" of 0 and 1, in contrast to conventional bits, which can only be 0 or 1. Quantum computers are incredibly powerful due to their capacity to live in several states simultaneously [18,19]. But until anyone can perform a quantum calculation on a qubit, it is meaningless. Additionally, a set of fundamental operations known as quantum logic gates are used to accomplish these quantum computations. The most prevalent quantum gates work with one- or two-qubit spaces. Thus, 2 x 2 or 4 x 4 matrices with orthonormal rows can be used to model quantum gates as matrices. Quantum gates come in many different varieties [2]. Such as:

Pauli X gate: The X-gate is represented by the Pauli-X matrix: To see the effect a gate has on a qubit; we simply multiply the qubit's state vector by the gate. We can see that the X-gate [9] switches the amplitudes of the states $|0\rangle$ and $|1\rangle$:

$$X\,|0> = \,|1> \qquad \text{.........Eq-01}$$

$$X\,|1> = \,|0> \qquad \text{.........Eq-02}$$

The Pauli-X gate is a single-qubit rotation through π radians around the x-axis.

Pauli Y gate: The Pauli-Y gate is a single-qubit rotation through π radians around the y-axis [9].

$$Y = \sigma_y = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \text{.........Eq-03}$$

$$Y = -i|0\rangle\langle 1| + i|1\rangle\langle 0| \qquad \text{.........Eq-04}$$

Hadamard Gate: The Hadamard gate (H-gate) is a fundamental quantum gate. This gate operates on a single qubit. It allows us to move away from the poles of the Bloch sphere and create a superposition of $|0\rangle$ and $|1\rangle$. It turns-

$$|0\rangle = (|0\rangle + |1\rangle)/2 \qquad \ldots\ldots\ldots \text{Eq-05}$$

$$|1\rangle = (|0\rangle + |1\rangle)/2 \qquad \ldots\ldots\ldots \text{Eq-06}$$

$$H|0\rangle = |+\rangle, H|1\rangle = |-\rangle \qquad \ldots\ldots\ldots \text{Eq-07}$$

Toffoli Gate: The Toffoli gate is a three-qubit operation that changes the state of a target qubit conditioned on the state of two control qubits. It is used for universal reversible classical computation. It also forms a universal set of qubit gates in quantum computation together with a Hadamard gate.

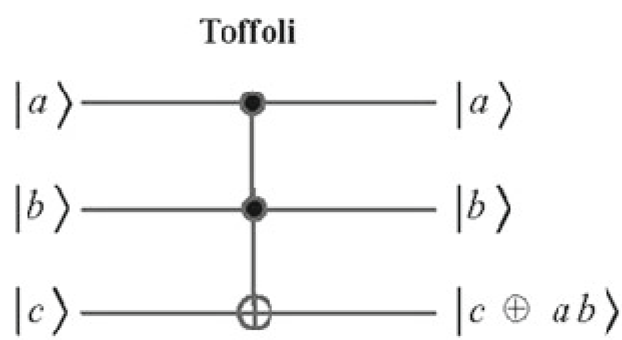

Figure: Toffoli gate

This has two control bits a and b and one target bit c. On output the control bits are unchanged and the target bit is flipped if both control bits are 1, so, $c \rightarrow c \oplus ab$.

CNOT Gate: In basic states the CNOT gate leaves the control qubit unchanged and performs a Pauli-X gate on the target qubit when the control qubit [9] is in state |1⟩. It leaves the target qubit unchanged when the control qubit is in state |0⟩.

x	y	x'	y'
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

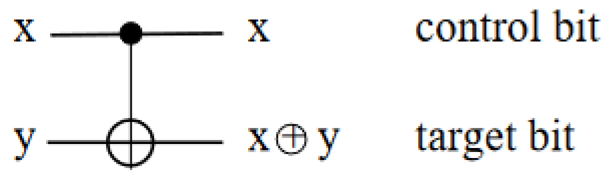

Figure: CNOT gate

It is very easy to say that CNOT is reversible. since, it is unitary. If we act twice, we get back the original input.

Quantum Teleportation: Quantum teleportation is a quantum information protocol by which the unknown quantum state of one particle can be transferred from one to another distant particle, using a pair of entangled particles, a projective measurement, and exchange of two bits of classical information. It's a technique to transfer quantum information from source to destination by employing entanglement. In quantum teleportation, the entanglement in the Bell stat is used to transport an arbitrary quantum state $|\psi\rangle$ between two distant observers A and B. The quantum teleportation system employs three qubits. Where qubit 1 is an arbitrary state to be teleported, qubits 2 and 3 are in Bell state.

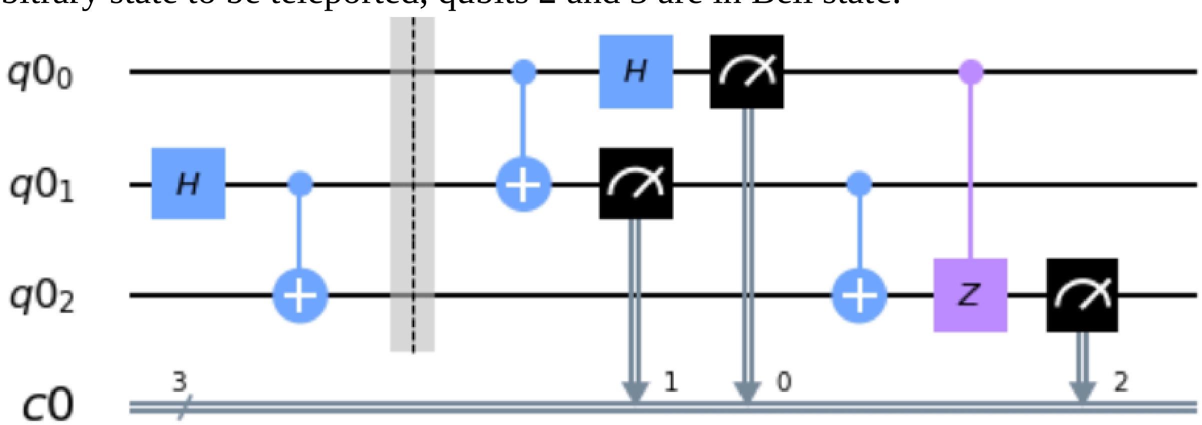

Figure: The "classical" circuit for quantum teleportation

Superdense Coding: Superdense coding is a form of secure quantum communication and a quantum communications protocol that allows a user to send 2 classical bits by sending only 1 qubit. In superdense coding, a sender (Alice) can send a message consisting of two classical bits by using one quantum bit to the receiver (Bob). The input to the circuit is one of a pair of qubits entangled in the Bell basis state. The other qubit from the pair is sent unchanged to Bob. After processing the upper qubit in one of four ways, it is sent to Bob, who measures the two qubits and yields two classical bits. The result is that Bob receives two classical bits, B_1 and B_2 that match those that Alice sent. But only a single qubit conveyed those two bits of information.

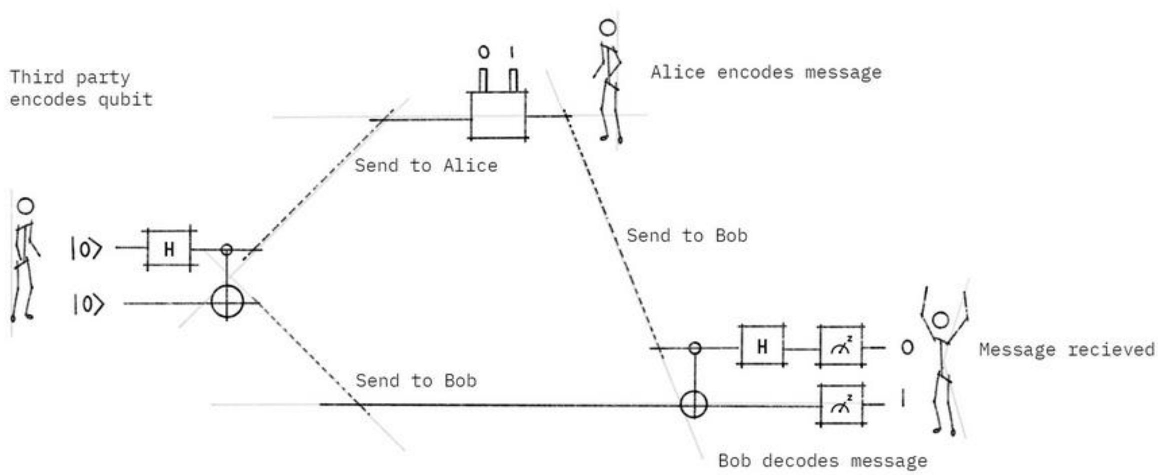

Figure: Superdense coding for sending 2 qubits at a time.

Density matrix: In quantum mechanics, any density matrix or density operator is Hermitian. A density matrix is a matrix that describes the quantum state of a physical system. It allows for the calculation of the probabilities of the outcomes of any measurement performed upon this system by using the Born rule. Below the given equation which denotes a multi particle state we have to calculate the reduced density matrix where we traced out the third particle-

$$|\psi\rangle = 1/\sqrt{3}\text{–}\sqrt{}(|\uparrow\uparrow\downarrow\rangle + i|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle) \quad \text{……..Eq-08}$$

Fidelity: In quantum information theory, fidelity is a measure of the closeness of two quantum states. Fidelity expresses the probability that one state will pass a test to identify as the other. Fidelity is the measure of the distance between two quantum states. Where fidelity equals 1, means that two states are equal. In the case of a density matrix, fidelity represents the overlap with a reference pure state [3]. Fidelity equal to 1, means that the square of the density matrix is equal to the density matrix itself and equivalent to the pure reference state. If the input state is ψ and the output density matrix is ρ, then the fidelity of the output is given by

$F(|\psi\rangle, \rho) = \langle\psi|\rho|\psi\rangle$ the probability of measuring $|\psi\rangle$ in a measurement that distinguishes this state from the orthogonal states.

$$F(\rho, \sigma) = |\langle\Psi\rho|\Psi\sigma\rangle|2 \quad \text{…. Eq-08}$$

Quantum Channel: A quantum computer needs to send an output state to another party, or two parties need to establish quantum entanglement or they need to secure keys via quantum communication and there a quantum channel is inevitably involved. It can be used to realize classical information transmission or to deliver quantum information, such as quantum entanglement.

$$E_U U\varphi U^+ = \int U\varphi U^+ dU = \frac{\mathrm{I}}{d} \quad \text{…….. Eq-09}$$

Quantum Repetition code: The repetition code used in quantum computing is a simple error-correcting code. Due to the ease of measuring and repeating classical bits, we are already aware that the repetition code functions in a classical channel. The repetition code in quantum computing simply repeats the message numerous times to transmit a message over a noisy channel that might distort the transmission and information in some locations. After some repetition, a lot of no's will appear similar to largely no with a few sporadic yeses. The encoding of our message makes it tolerant of minor errors. The message must be decoded by the recipient.

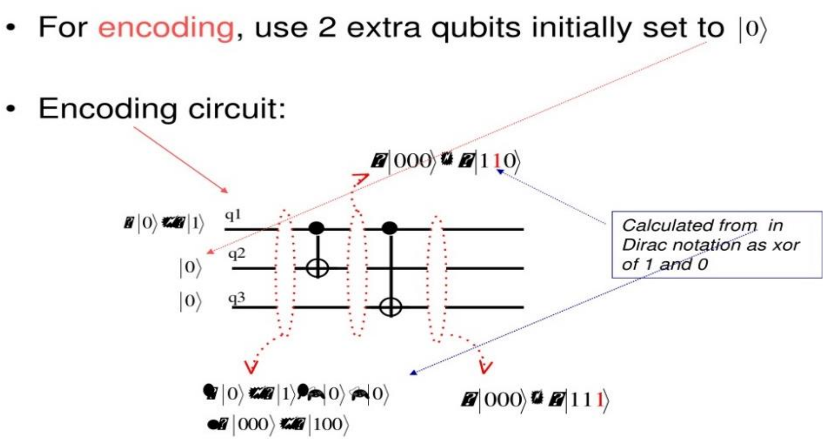

Figure: Encoder for (3,1) repetition code

Bit flip code: Due to the ease of measuring and repeating classical bits, the repetition code functions on a classical channel. This method is ineffective in a quantum channel because the no-cloning theorem prohibits the repetition of a single qubit more than three times. An alternative approach must be utilized to get around this, such as the three-qubit bit flip code first put forth by Asher Peres in 1985. This method, which compares favorably to the repetition code in terms of performance, makes use of entanglement and syndrome measurements. As an illustration, suppose the main qubits state was 0. The auxiliary qubits will be given CNOT gates, which will leave them untouched because the main qubits' state was 0. The primary qubits' state is then reversed to 1 by a bit flip that follows. The ancillary qubits are subjected to CNOT gates following the bit flip, which will cause them to change their states to 1 since the primary qubit's state is 1. The primary qubit is then given a Toffoli gate, which will flip its state because the auxiliary qubits are in the state of 1. This corrects the problem by flipping the primary qubit's state to zero.

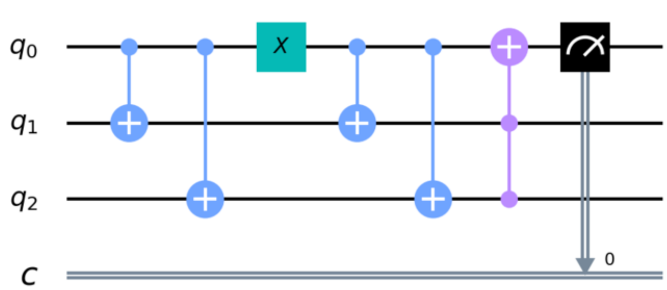

Figure: Circuit diagram of bit flip code with simulated error using a NOT gate

Bit Flip Channel: Assume, the probability of bit flip is given by p. Then the Krauss operators of bit flip channel for single qubit is given by

$$\{\sqrt{(p)}X, \sqrt{}(1 - p) I\} \qquad \text{..........Eq-10}$$

No Repetition Code: Since, we are using bipartite states, the Krauss Operators are $E1 = pXA \otimes XB, E2 = (1 - p)IA \otimes IB, E3 = \sqrt{p(1 - p)XA} \otimes IB,$ $E4 = \sqrt{}p(1 - p)IA \otimes XB$, and initial density matrix was given by $\rho 0 = |\varphi+\rangle AB\langle \varphi + |AB$. The final density matrix is given by $\rho AB = \sum_{i=1}^{4} Eip0Ei^{\dagger}$

This fidelity becomes:

$$F = \sqrt{\langle \varphi + |AB\rho AB|\varphi +\rangle AB} = \sqrt{p2} + (1 - p2) \text{ ....Eq-11}$$

Ancilla Qubits: Best methods trade extra ancilla qubits for error rate: Ancilla qubits create complex ancilla states to substitute for most gates on the data. Errors on ancillas are less serious, since bad ancillas can be discarded safely [23]. Extreme case in these qubits is creating all states using error-detecting codes, ensuring a low basic error rate but very high overheads (e.g., 10 or more physical qubits per logical qubit) [24].

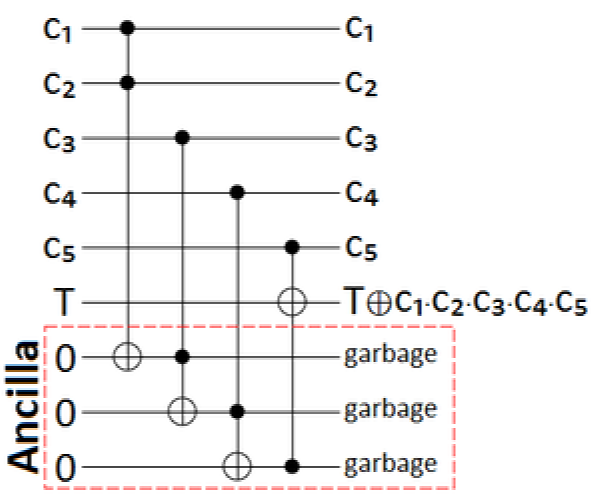

Figure: Ancilla Qubits

Chapter 5: Methodology

Methodology details will explain at this point-

3 Qubit:

Alice qubit, $|\psi >= \alpha|0> +\beta|1>$ ……. Eq-12

Apply CNOT (cx) gate where Alice qubit is controlled qubit and two ancilla qubits are different target qubits in two cases,

$$|\psi >= \alpha|000> +\beta|111> \qquad \text{……. Eq-13}$$

Now passing this state through a channel. It can be a bit flip error in any bit of the state. Here we take it as the second ancilla qubit (third qubit).

After the bit flip in the third qubit, the state will be,

$$|\psi >= \alpha|001> +\beta|110> \qquad \text{……. Eq-14}$$

Now again we use the CNOT gate. Here we add the Toffoli gates with CNOT gates for decoding the state.

Applying CNOT gate we get,

$$|\psi >= \alpha|100> +\beta|011> \qquad \text{……. Eq-15}$$

Then applying Toffoli gates we get,

$|\psi >= \alpha|1> +\beta|0>$; Final state (See the below Table-01 of 3-Bit Code: Bit Flip Error)

Here we corrected the error of the third qubit of the total three qubits. Using this model, we can correct the other qubits too.

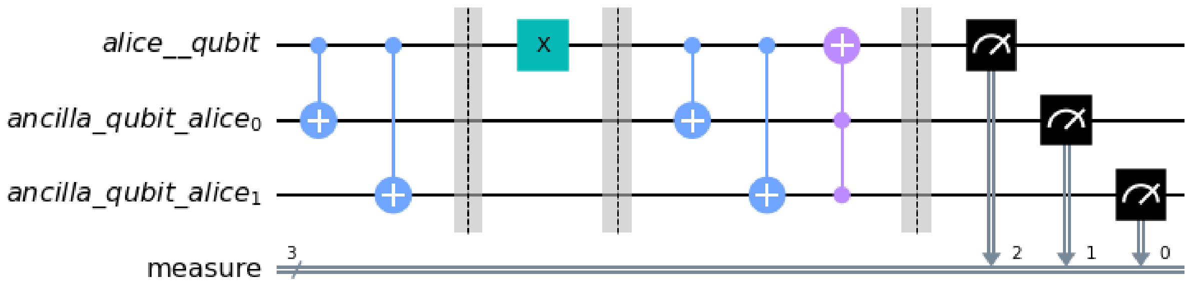

Figure: 3 Qubit Circuit

After the diagnosed and corrected quantum channel 8 possible results are found in 3-Bit Code: Bit Flip Error (Based on third qubit)-

State	Probability
$\alpha\|000 > +\beta\|111 >$	$(1-p)3$
$\alpha\|001 > +\beta\|110 >$	$p(1-p)2$
$\alpha\|010 > +\beta\|101 >$	$p(1-p)2$
$\alpha\|100 > +\beta\|011 >$	$p(1-p)2$
$\alpha\|011 > +\beta\|100 >$	$p2(1-p)$
$\alpha\|101 > +\beta\|010 >$	$p2(1-p)$
$\alpha\|110 > +\beta\|001 >$	$p2(1-p)$
$\alpha\|111 > +\beta\|000 >$	$p3$

Table: 01

5 Qubit:

Five qubit repetition code can correct up to 2 qubit bit-flip errors for encoding a single qubit.

Primarily, Alice qubit, $|\psi> = \alpha|0> + \beta|1>$

Applying CNOT (cx) gate where Alice qubit is controlled qubit and 4 ancilla qubits are different target qubits in two cases,

$$|\psi> = \alpha|00000> + \beta|11111> \quad \text{.......Eq-16}$$

Now passing this state through a channel. It can be a bit flip error in any bit of the state. Here we take it as the Alice qubit (first qubit) and ancilla qubit (second qubit).

After the bit flip in the first and second qubit, the state will be,

$$|\psi> = \alpha|11000> + \beta|00111> \quad \text{.......Eq-17}$$

Now again we use the CNOT gate. Here we add the Toffoli gates with CNOT gates for decoding the state.

After applying CNOT gate we get,

$$|\psi> = \alpha|00011> + \beta|11100> \quad \text{....... Eq-18}$$

Then applying Toffoli gates we get,

$$|\psi> = \alpha|11111> + \beta|00000> \quad \text{....... Eq-19}$$

$|\psi> = \alpha|1> + \beta|0>$; Final state (See the below Table-02 of 5-Bit Code: Bit Flip Error)

Here we corrected the error of the first and second qubit of the total five qubits. Using this model, we can correct the other qubits too.

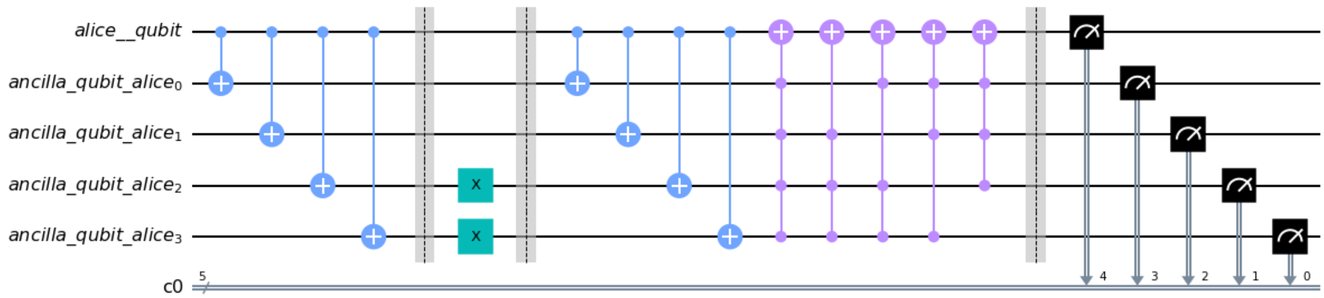

Figure: 5 Qubit Circuit

After the diagnosed and corrected quantum channel 8 possible results are found in 5-Bit Code: Bit Flip Error (Based on first and second qubit)-

State	Probability
α|00000>+β|11111>	(1-p)5
α|11000>+β|00111>	p(1-p)4
α|00110>+β|11001>	p(1-p)4
α|00011>+β|11100>	p(1-p)4
α|11100>+β|00011>	p4(1-p)
α|10011>+β|01100>	p4(1-p)
α|00111>+β|11000>	p4(1-p)
α|11111>+β|00000>	p5

Table-02

Chapter 6: Experimental Setup

6.1: Technology Stack

In this project, we used Qiskit (An open-source software development kit (SDK) for working with quantum computers at the level of circuits and algorithms). It provides support for developing and modifying quantum programs, as well as for executing them on local computer simulators or on prototype quantum devices through IBM Quantum Experience.

Chapter 7: Result & Analysis

Here, we have measured qubit fidelity. By the result of it, we show how good the repetition code is. We discuss it below:

7.1: 3 Qubit

In the 3-qubit quantum repetition code, we get better results than no repetition code. We have successfully removed some noise from the channel and made a better communication channel. In this process, we can solve 1 qubit error from 3 qubit codes. Below we have shown the mathematical equation of fidelity check of 3-Bit error correcting code (Bit-Flip Error):

$$\text{Failing probability} = \text{Pu} = 3\text{p}2(1-\text{p}) + \text{p}3$$

$$= 3\text{p}2 - 2\text{p}3 = \text{O}(\text{p}2)$$

$$\text{Fidelity} = \text{success probability} = 1 - \text{Pu} = 1 - 3\text{p}2$$

$$\text{Without error correction Pu} = \text{O}(\text{p})$$

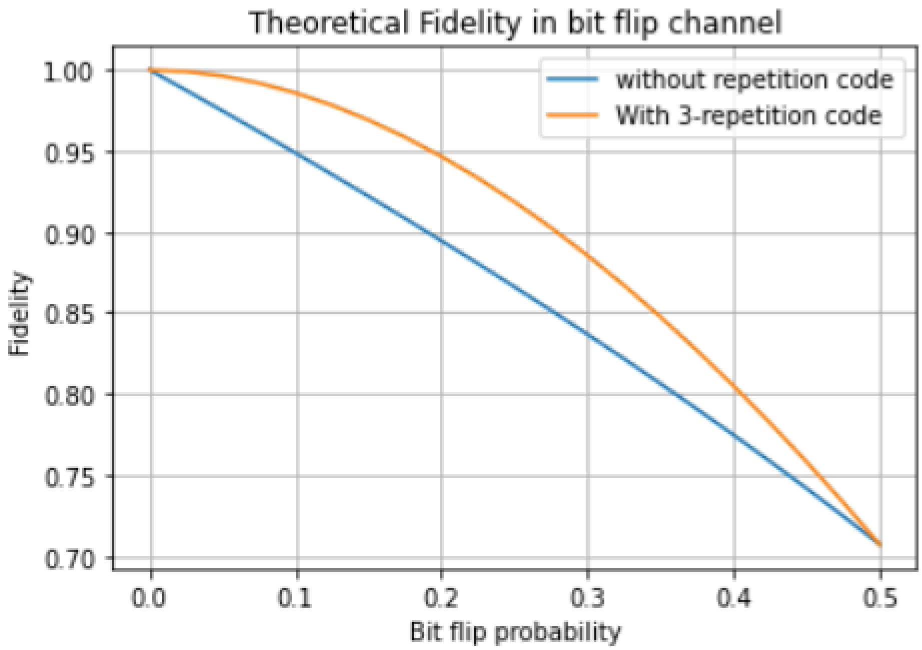

Figure: Fidelity Comparison between 3-qubit quantum repetition code and no repetition code.

Also, we can see that Bit flip probability having the smaller value than Fidelity in the figure above that It can be held both as bit flip and phase flip error. The probability of happening both of them is 1. So, for only Bit flip error, it can be 0.5 (highest).

So, the equation will be-

p<=0.5

7.2: 5 Qubit

In the 5-qubit quantum repetition code, We have successfully removed some more noise from the channel and made a better communication channel.

In this process, we can solve 2 qubit errors from 5 qubit codes. Below we have shown the mathematical equation of fidelity check of 5-Bit error correcting code (Bit-Flip Error):

$$\text{Failing probability} = \text{Pu} = 5\text{p}2(1-\text{p}) + \text{p}3$$

$$= 5\text{p}2 - 4\text{p}3 = \text{O}(\text{p}4)$$

$$\text{Fidelity} = \text{success probability} = 1 - \text{Pu} = 1 - 5\text{p}2$$

$$\text{Without error correction Pu} = \text{O}(\text{p})$$

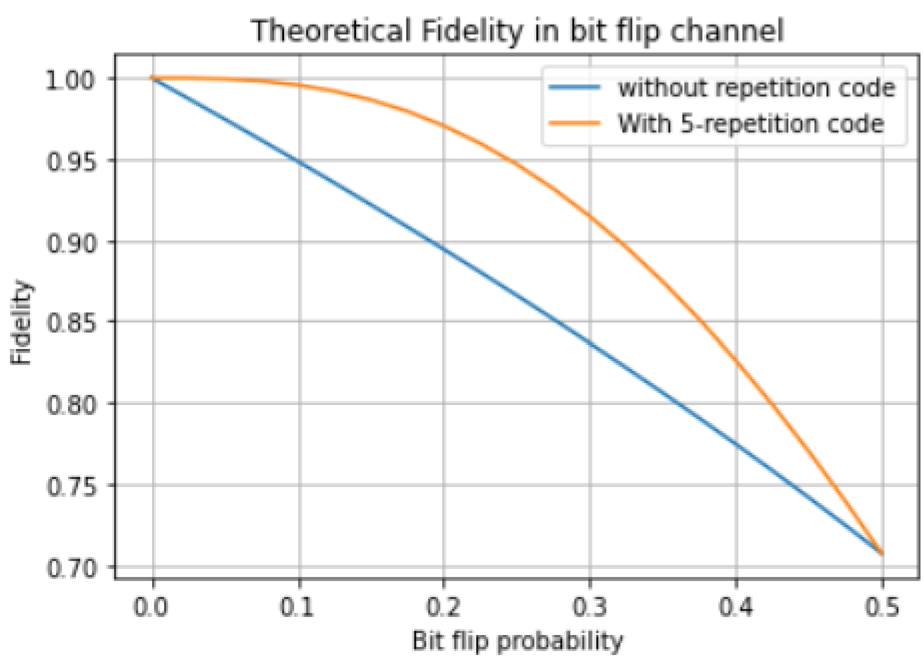

Figure: Fidelity Comparison between 5-qubit quantum repetition code and no repetition code.

7.3: Comparison Between 3 Qubit, 5 Qubit, and No Repetition Code

In 3-qubit and 5-qubit quantum repetition code, we removed some noise and channel barriers. But 5-qubit gives us better output. In the below diagram, we can see the real scenario through a graph:

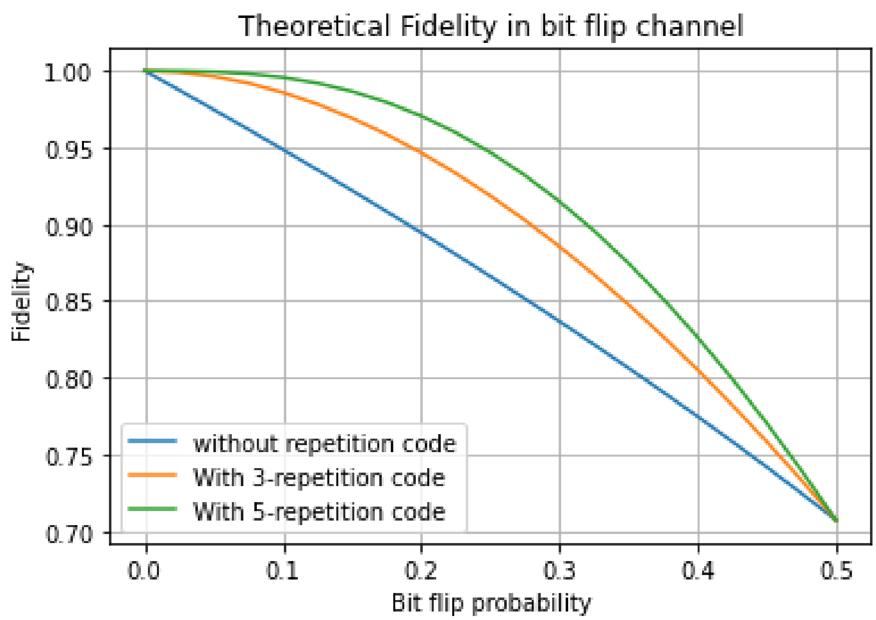

Figure: Fidelity Comparison between 3, 5-qubit quantum repetition codes and no repetition code.

7.4: Discussion

The development of error-corrected logical qubits that perform better than straightforward physical qubits and truly benefit from error correction is a crucial step toward fault-tolerant quantum computing. Three stages [18, 19] are necessary to accomplish this objective: (1) Realizing operations on encoded qubits and error-correction cycles, (2) Realizing operations on encoded qubits, and (3) Increasing the number of supplementary qubits and enhancing operation fidelity to achieve fault tolerance. By identifying the fundamental components of the fully operational five-qubit error-correcting code, our experiment completes step (1). Step 2 is partially accomplished by our work because we do logical operations and check error detection, but we are unable to fully remedy errors because we can only evaluate stabilizers destructively. The implementation of non-destructive error detection and correction [14,15,16] as well as logical operations on multiple logical qubits for the five-qubit code are directions for future investigation. Our research also has potential in the near future for quantum computing's error reduction [21].

Chapter 8: Project Impact

In this section we will discuss some useful aspects of the future:

8.1 Communication Impact:

It will sort out the bit flip communication channel error [19][21]. For this, we can pass out the state more accurately, which will decrease the noise of the channel. Communication channels will be smoother.

8.2 Impact on the research community:

There are many opportunities to do research in this field. We just solved the theoretical part using qiskit code. Using this, future researchers do better research on how many bits of repetition code will be better for bit flip error-correcting. In the last few years, the field's researchers have made major strides in quantum error correction, but there is still much work to be done. Today, researchers are collaborating with the larger quantum community to quickly implement intelligent quantum computing. We see the advancement in this discipline as a continuous way ahead as part of our development strategy, where we try to extract value from the noisy quantum hardware of today as IBM scientists and the larger research community develop scalable Quantum Error Correction (QEC) technologies. Their main goal is to develop quantum error correction technologies that will make it possible to build fault-tolerant quantum computers or quantum computers that can identify and fix problems as they happen [25]. In the fall of 2014, Google hired a quantum computing research team from the University of California, Santa Barbara. This was the beginning of their breakthrough. A system of superconducting quantum circuits that the UCSB researchers had previously developed performed accurately enough to allow for error correction. The researchers—many of whom are already working for Google—used that earlier success as a springboard to create a system that can fix faults that inevitably occur during quantum computing activities [27]. The journal Nature published a full article about their study on March 4, 2015.

Chapter 9: Conclusion and Future Work

9.1 Conclusion

Among those fidelity comparisons we now can say that the 5-qubit error correcting repetition code gives the best fidelity measurement to ensure prevention and correct errors.

9.2 Future Work

Our future work is to solve the 7-qubit quantum error correcting code problem. We did the construction, measurement [13] and fidelity check but we didn’t get accurate results for it with comparison to 3,5 qubits. The Decoding part of 7 qubits QEC will be sorted out very soon.

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Appendix

Project GitHub Link:

[TanvirulHasan/Quantum_Repetition_Code (github.com)]