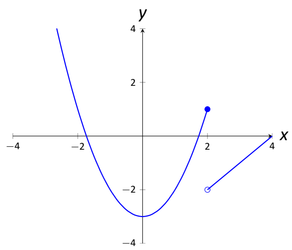
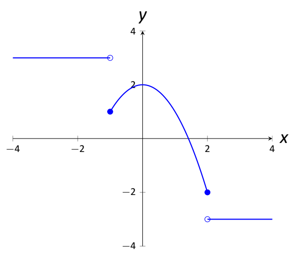
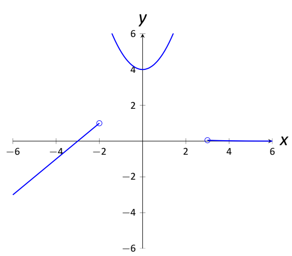
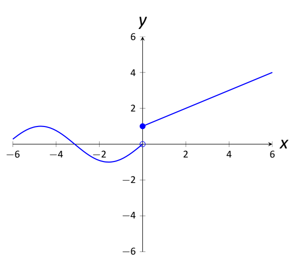
A: Instructor Solutions To All Problems

Chapter 1

Exercises 1.0

1. $(-\infty, \infty)$
2. $[-7, \infty)$
3. $(-\infty, -1] \cup [7, \infty)$
4. $(-\infty, \infty)$
5. $(-\infty, 2) \cup (2, \infty)$
6. $(-\infty, \infty)$
7. $(-\infty, \infty)$
8. $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
9. $(-\infty, 0) \cup (0, \infty)$
10. $(-\infty, \infty)$
11. 
12. 
13. 
14. 
15. (a) 14
    (b) 11
    (c) $3a^2 - 2a + 6$
    (d) $3(x+h)^2 - 2(x+h) + 6$
    (e) $\dfrac{h(3h + 6x - 2)}{h}$
16. (a) $\sqrt{2}$
    (b) undefined
    (c) $\sqrt{t-2}$
    (d) $\sqrt{x+h-2}$
    (e) $\dfrac{\sqrt{x+h-2} - \sqrt{x-2}}{h} = \dfrac{h}{h(\sqrt{x+h-2} + \sqrt{x-2})}$
17. (a) -1
    (b) $\dfrac{1}{9}$
    (c) $\dfrac{1}{t+3}$
    (d) $\dfrac{1}{x+h}$
    (e) $\dfrac{\frac{1}{x+h} - \frac{1}{x}}{h} = -\dfrac{h}{hx(x+h)}$

Exercises 1.1

1. Answers will vary.
2. An indeterminate form.
3. F
4. The function may approach different values from the left and right, the function may grow without bound, or the function might oscillate.
5. Answers will vary.
6. -1
7. -5
8. Limit does not exist
9. 2
10. 1.5
11. Limit does not exist.
12. Limit does not exist.
13. 7
14. 1
15. Limit does not exist.
16.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-7
-0.01	-7
0.01	-7
0.1	-7

The limit seems to be exactly 7.

17.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	9
-0.01	9
0.01	9
0.1	9

The limit seems to be exactly 9.

18.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	4.9
-0.01	4.99
0.01	5.01
0.1	5.1

The limit is approx. 5.

19.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.114943
-0.01	-0.111483
0.01	-0.110742
0.1	-0.107527

The limit is approx. -0.11.

20.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	29.4
-0.01	29.04
0.01	28.96
0.1	28.6

The limit is approx. 29.

21.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	0.202027
-0.01	0.2002
0.01	0.1998
0.1	0.198026

The limit is approx. 0.2.

22.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.998334
-0.01	-0.999983
0.01	-0.999983
0.1	-0.998334

The limit is approx. -1.

23.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.0499583
-0.01	-0.00499996
0.01	0.00499996
0.1	0.0499583

The limit is approx. 0.

24.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	0.251582
-0.01	0.250156
0.01	0.249844
0.1	0.248457

The limit is approx. 0.25.

Exercises 1.2

1. ε should be given first, and the restriction $|x-a|<\delta$ implies $|f(x)-K|<\varepsilon$, not the other way around.
2. The y–tolerance.
3. T
4. T
5. $\delta \leq 0.45$
6. $\delta \leq 0.71$
7. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-5|<\delta$, $|f(x)-(-2)|<\varepsilon$.
Scratch-Work: Consider $|f(x)-(-2)|<\varepsilon$:
$$\begin{aligned}|f(x)+2| &< \varepsilon\\ |(3-x)+2| &< \varepsilon\\ |5-x| &< \varepsilon\\ -\varepsilon < 5-x &< \varepsilon\\ -\varepsilon < x-5 &< \varepsilon.\end{aligned}$$
This implies we can let $\delta = \varepsilon$.
Proof: Given $\varepsilon > 0$, choose $\delta = \varepsilon$.
$$\begin{aligned}|x-5| &< \delta\\ -\delta < x-5 &< \delta\\ -\varepsilon < x-5 &< \varepsilon\\ -\varepsilon < (x-3)-2 &< \varepsilon\\ -\varepsilon < (-x+3)-(-2) &< \varepsilon\\ |3-x-(-2)| &< \varepsilon.\end{aligned}$$
Thus $\lim\limits_{x\to 5} 3-x = -2$.
8. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-5|<\delta$, $|f(x)-8|<\varepsilon$.
Scratch-Work: Consider $|f(x)-8|<\varepsilon$, keeping in mind we want to make a statement about $|x-5|$:
$$\begin{aligned}|f(x)-8| &< \varepsilon\\ |4x-12-8| &< \varepsilon\\ |4x-20| &< \varepsilon\\ 4|x-5| &< \varepsilon\\ |x-5| &< \frac{\varepsilon}{4}\end{aligned}$$
suggesting $\delta = \frac{\varepsilon}{4}$.
Proof: Given $\varepsilon > 0$, let $\delta = \dfrac{\varepsilon}{4}$. Then:
$$\begin{aligned}|x-5| &< \delta\\ |x-5| &< \frac{\varepsilon}{4}\\ 4|x-5| &< \frac{\varepsilon}{4}\cdot 4\\ |4x-20| &< \varepsilon\\ |4x-12-8| &< \varepsilon\end{aligned}$$
Thus $\lim\limits_{x\to 5} 4x-12 = 8$.
9. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-3|<\delta$, $|f(x)-(-1)|<\varepsilon$.
Scratch-Work: Consider $|f(x)-(-1)|<\varepsilon$, keeping in mind we want to make a statement about $|x-3|$:
$$\begin{aligned}|f(x)-(-1)| &< \varepsilon\\ |5-2x+1| &< \varepsilon\\ |-2x+6| &< \varepsilon\\ 2|x-3| &< \varepsilon\\ |x-3| &< \frac{\varepsilon}{2}\end{aligned}$$
suggesting $\delta = \frac{\varepsilon}{2}$.
Proof: Given $\varepsilon > 0$, let $\delta = \dfrac{\varepsilon}{2}$. Then:
$$\begin{aligned}|x-3| &< \delta\\ |x-3| &< \frac{\varepsilon}{2}\\ 2|x-3| &< \frac{\varepsilon}{2}\cdot 2\\ |-2x+6| &< \varepsilon\\ |5-2x+1| &< \varepsilon\end{aligned}$$
Thus $\lim\limits_{x\to 3} 5-2x = -1$.
10. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-3|<\delta$, $|f(x)-6|<\varepsilon$.
Scratch-Work: Consider $|f(x)-6|<\varepsilon$, keeping in mind we want to make a statement about $|x-3|$:
$$\begin{aligned}|f(x)-6| &< \varepsilon\\ |x^2-3-6| &< \varepsilon\\ |x^2-9| &< \varepsilon\\ |x-3|\cdot|x+3| &< \varepsilon\\ |x-3| &< \varepsilon/|x+3|\end{aligned}$$

Since x is near 3, we can safely assume that, for instance, $2 < x < 4$. Thus

$$\begin{aligned} 2+3 < x+3 < 4+3 \\ 5 < x+3 < 7 \\ \frac{1}{7} < \frac{1}{x+3} < \frac{1}{5} \\ \frac{\varepsilon}{7} < \frac{\varepsilon}{x+3} < \frac{\varepsilon}{5} \end{aligned}$$

This suggests $\delta = \frac{\varepsilon}{7}$.
Proof: Given $\varepsilon > 0$, let $\delta = \frac{\varepsilon}{7}$.

$$\begin{aligned} |x-3| < \delta \\ |x-3| < \frac{\varepsilon}{7} \\ |x-3| < \frac{\varepsilon}{x+3} \\ |x-3|\cdot|x+3| < \frac{\varepsilon}{x+3}\cdot|x+3| \end{aligned}$$

Assuming x is near 3, $x+3$ is positive and we can drop the absolute value signs on the right.

$$\begin{aligned} |x-3|\cdot|x+3| < \frac{\varepsilon}{x+3}\cdot(x+3) \\ |x^2-9| < \varepsilon \\ |(x^2-3)-6| < \varepsilon. \end{aligned}$$

Thus, $\lim\limits_{x\to 3} x^2-3 = 6$.

11. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-4| < \delta$, $|f(x)-15| < \varepsilon$.
Scratch-Work: Consider $|f(x)-15| < \varepsilon$, keeping in mind we want to make a statement about $|x-4|$:

$$\begin{aligned} |f(x)-15| < \varepsilon \\ |x^2+x-5-15| < \varepsilon \\ |x^2+x-20| < \varepsilon \\ |x-4|\cdot|x+5| < \varepsilon \\ |x-4| < \varepsilon/|x+5| \end{aligned}$$

Since x is near 4, we can safely assume that, for instance, $3 < x < 5$. Thus

$$\begin{aligned} 3+5 < x+5 < 5+5 \\ 8 < x+5 < 10 \\ \frac{1}{10} < \frac{1}{x+5} < \frac{1}{8} \\ \frac{\varepsilon}{10} < \frac{\varepsilon}{x+5} < \frac{\varepsilon}{8} \end{aligned}$$

suggesting $\delta = \frac{\varepsilon}{10}$.
Proof: Given $\varepsilon > 0$, let $\delta = \frac{\varepsilon}{10}$. Then:

$$\begin{aligned} |x-4| < \delta \\ |x-4| < \frac{\varepsilon}{10} \\ |x-4| < \frac{\varepsilon}{x+5} \\ |x-4|\cdot|x+5| < \frac{\varepsilon}{x+5}\cdot|x+5| \end{aligned}$$

Assuming x is near 4, $x+5$ is positive and we can drop the absolute value signs on the right.

$$\begin{aligned} |x-4|\cdot|x+5| < \frac{\varepsilon}{x+5}\cdot(x+5) \\ |x^2+x-20| < \varepsilon \\ |(x^2+x-5)-15| < \varepsilon. \end{aligned}$$

Thus, $\lim\limits_{x\to 4} x^2+x-5 = 15$.

12. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-2| < \delta$, $|f(x)-7| < \varepsilon$.
Scratch-Work: Consider $|f(x)-7| < \varepsilon$, keeping in mind we want to make a statement about $|x-2|$:

$$\begin{aligned} |f(x)-7| < \varepsilon \\ |x^3-1-7| < \varepsilon \\ |x^3-8| < \varepsilon \\ |x-2|\cdot|x^2+2x+4| < \varepsilon \\ |x-3| < \varepsilon/|x^2+2x+4| \end{aligned}$$

Since x is near 2, we can safely assume that, for instance, $1 < x < 3$. Thus

$$\begin{aligned} 1^2+2\cdot1+4 < x^2+2x+4 < 3^2+2\cdot3+4 \\ 7 < x^2+2x+4 < 19 \\ \frac{1}{19} < \frac{1}{x^2+2x+4} < \frac{1}{7} \\ \frac{\varepsilon}{19} < \frac{\varepsilon}{x^2+2x+4} < \frac{\varepsilon}{7}, \end{aligned}$$

suggesting $\delta = \frac{\varepsilon}{19}$.
Proof: Given $\varepsilon > 0$, let $\delta = \frac{\varepsilon}{19}$.

$$\begin{aligned} |x-2| < \delta \\ |x-2| < \frac{\varepsilon}{19} \\ |x-2| < \frac{\varepsilon}{x^2+2x+4} \\ |x-2|\cdot|x^2+2x+4| < \frac{\varepsilon}{x^2+2x+4}\cdot|x^2+2x+4| \end{aligned}$$

Assuming x is near 2, x^2+2x+4 is positive and we can drop the absolute value signs on the right.

$$\begin{aligned} |x-2|\cdot|x^2+2x+4| < \frac{\varepsilon}{x^2+2x+4}\cdot(x^2+2x+4) \\ |x^3-8| < \varepsilon \\ |(x^3-1)-7| < \varepsilon, \end{aligned}$$

which is what we wanted to prove.

13. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-2| < \delta$, $|f(x)-5| < \varepsilon$. However, since $f(x) = 5$, a constant function, the latter inequality is simply $|5-5| < \varepsilon$, which is always true. Thus we can choose any δ we like; we arbitrarily choose $\delta = \varepsilon$.

14. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-0| < \delta$, $|f(x)-0| < \varepsilon$. In simpler terms, we want to show that when $|x| < \delta$, $|\sin x| < \varepsilon$.
Set $\delta = \varepsilon$. We start with assuming that $|x| < \delta$. Using the hint, we have that $|\sin x| < |x| < \delta = \varepsilon$. Hence if $|x| < \delta$, we know immediately that $|\sin x| < \varepsilon$.

Exercises 1.3

1. Answers will vary.

2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. As x is near 1, both f and g are near 0, but f is approximately twice the size of g. (I.e., $f(x) \approx 2g(x)$.)
6. 9
7. 6
8. 0
9. Limit does not exist.
10. 3
11. Not possible to know.
12. 3
13. -45
14. 1
15. -1
16. 0
17. π
18. 7
19. $-0.000000015 \approx 0$
20. $1/2$
21. Limit does not exist
22. 64
23. 2
24. 0
25. $\frac{\pi^2+3\pi+5}{5\pi^2-2\pi-3} \approx 0.6064$
26. $\frac{3\pi+1}{1-\pi}$
27. -8
28. -1
29. 10
30. -2
31. $-3/2$
32. $-7/8$
33. $1/3$
34. $1/4$
35. $-1/9$
36. 1
37. -8
38. 0
39. 0
40. 1
41. 9
42. 0
43. 3
44. $5/8$
45. 1
46. $\pi/180$
47. $4/3$
48. $5/7$
49. We find $\lim_{x\to 0} \dfrac{\cos^2 x - 1}{x(\cos x + 1)} = \lim_{x\to 0} \dfrac{\sin^2 x}{x(\cos x + 1)} = \lim_{x\to 0} \dfrac{\sin x}{x} \lim_{x\to 0} \dfrac{\sin x}{\cos x + 1} = 0.$

Exercises 1.4

1. The function approaches different values from the left and right; the function grows without bound; the function oscillates.
2. F
3. F
4. T
5. (a) 2
   (b) 2
   (c) 2
   (d) 1
   (e) As f is not defined for $x < 0$, this limit is not defined.
   (f) 1
6. (a) 1
   (b) 2
   (c) Does not exist.
   (d) 2
   (e) 0
   (f) As f is not defined for $x > 2$, this limit is not defined.
7. (a) 2
   (b) 0
   (c) Does not exist.
   (d) 1
8. (a) 2
   (b) 2
   (c) 2
   (d) 2
9. (a) 4
   (b) -4
   (c) Does not exist.
   (d) 0
10. (a) 2
    (b) 2
    (c) 2
    (d) 0
    (e) 2
    (f) 2
    (g) 2
    (h) Not defined
11. (a) $a-1$
    (b) a
    (c) Does not exist.
    (d) a
12. (a) 2
    (b) -4
    (c) Does not exist.
    (d) 2
13. (a) -1

(b) 1
(c) Does not exist.
(d) 1

14. (a) 0
(b) 0
(c) 0
(d) 0
(e) 2
(f) 2
(g) 2
(h) 2

15. (a) -1
(b) 0
(c) Does not exist.
(d) 0

16. (a) $1 - \cos^2 a = \sin^2 a$
(b) $\sin^2 a$
(c) $\sin^2 a$
(d) $\sin^2 a$

17. (a) 2
(b) 0
(c) Does not exist
(d) 1

18. (a) 4
(b) 4
(c) 4
(d) 3

19. (a) c
(b) c
(c) c
(d) c

20. (a) -1
(b) 1
(c) Does not exist
(d) 0

21. Answers will vary.
22. Answers will vary.
23. Answers will vary.
24. Answers will vary.
25. $-3/5$
26. $2/3$
27. $\frac{1}{2\sqrt{3}}$
28. 2
29. -1.63

Exercises 1.5

1. F
2. T
3. F
4. T
5. T
6. Answers will vary.
7. Answers will vary.
8. The limit of f as x approaches 7 does not exist, hence f is not continuous. (Note: f could be defined at 7!)

9. (a) ∞
(b) ∞

10. (a) $-\infty$
(b) ∞
(c) Limit does not exist
(d) ∞
(e) ∞
(f) ∞

11. (a) 1
(b) 0
(c) $1/2$
(d) $1/2$

12. (a) Limit does not exist
(b) Limit does not exist

13. (a) Limit does not exist
(b) Limit does not exist

14. (a) 10
(b) ∞

15. Tables will vary.

(a)

x	$f(x)$
2.9	-15.1224
2.99	-159.12
2.999	-1599.12

It seems $\lim_{x\to 3^-} f(x) = -\infty$.

(b)

x	$f(x)$
3.1	16.8824
3.01	160.88
3.001	1600.88

It seems $\lim_{x\to 3^+} f(x) = \infty$.

(c) It seems $\lim_{x\to 3} f(x)$ does not exist.

16. Tables will vary.

(a)

x	$f(x)$
2.9	-335.64
2.99	-30350.6

It seems $\lim_{x\to 3^-} f(x) = -\infty$.

(b)

x	$f(x)$
3.1	-265.61
3.01	-29650.6

It seems $\lim_{x\to 3^+} f(x) = -\infty$.

(c) It seems $\lim_{x\to 3} f(x) = -\infty$.

17. Tables will vary.

(a)

x	$f(x)$
2.9	132.857
2.99	12124.4

It seems $\lim_{x\to 3^-} f(x) = \infty$.

(b)

x	$f(x)$
3.1	108.039
3.01	11876.4

It seems $\lim_{x\to 3^+} f(x) = \infty$.

(c) It seems $\lim_{x\to 3} f(x) = \infty$.

18. Tables will vary.

(a)

x	$f(x)$
2.9	-0.632
2.99	-0.6032
2.999	-0.60032

It seems $\lim_{x\to 3^-} f(x) = -0.6$.

(b)

x	$f(x)$
3.1	-0.5686
3.01	-0.5968
3.001	-0.59968

It seems $\lim_{x\to 3^+} f(x) = -0.6$.

(c) It seems $\lim_{x\to 3} f(x) = -0.6$.

19. Horizontal asymptote at $y = 2$; vertical asymptotes at $x = -5, 4$.
20. Horizontal asymptote at $y = -3/5$; vertical asymptote at $x = 3$.
21. Horizontal asymptote at $y = 0$; vertical asymptotes at $x = -1, 0$.
22. No horizontal asymptote; vertical asymptote at $x = 1$.
23. No horizontal or vertical asymptotes.
24. Horizontal asymptote at $y = -1$; no vertical asymptotes
25. ∞
26. $-\infty$
27. $-\infty$
28. ∞
29. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-1| < \delta$, $|f(x) - 3| < \varepsilon$.
Scratch-Work: Consider $|f(x) - 3| < \varepsilon$, keeping in mind we want to make a statement about $|x-1|$:
$$\begin{aligned} |f(x)-3| &< \varepsilon \\ |5x-2-3| &< \varepsilon \\ |5x-5| &< \varepsilon \\ 5\,|x-1| &< \varepsilon \\ |x-1| &< \frac{\varepsilon}{5} \end{aligned}$$
suggesting $\delta = \frac{\varepsilon}{5}$.
Proof: Given $\varepsilon > 0$, let $\delta = \dfrac{\varepsilon}{5}$. Then:
$$\begin{aligned} |x-1| &< \delta \\ |x-1| &< \frac{\varepsilon}{5} \\ 5\,|x-1| &< \frac{\varepsilon}{5}\cdot 5 \\ |5x-5| &< \varepsilon \\ |5x-2-3| &< \varepsilon \end{aligned}$$
Thus $\lim\limits_{x\to 1} 5x - 2 = 3$.
30. (a) 2
(b) -3
(c) -3
(d) $1/3$
31. Yes. The only "questionable" place is at $x = 3$, but the left and right limits agree.
32. 1

Exercises 1.6

1. Answers will vary.
2. Answers will vary.
3. A root of a function f is a value c such that $f(c) = 0$.
4. Consider the function $h(x) = g(x) - f(x)$, and use the Bisection Method to find a root of h.
5. F
6. T
7. T
8. F
9. F
10. T
11. No; $\lim\limits_{x\to 1} f(x) = 2$, while $f(1) = 1$.
12. No; $\lim\limits_{x\to 1} f(x)$ does not exist.
13. No; $f(1)$ does not exist.
14. Yes
15. Yes
16. Yes
17. (a) No; $\lim\limits_{x\to -2} f(x) \neq f(-2)$
(b) Yes
(c) No; $f(2)$ is not defined.
18. (a) Yes
(b) Yes
19. (a) Yes
(b) No; the left and right hand limits at 1 are not equal.
20. (a) Yes
(b) Yes
21. (a) Yes
(b) No. $\lim_{x\to 8} f(x) = 16/5 \neq f(8) = 5$.
22. $(-\infty, \infty)$
23. $(-\infty, -2] \cup [2, \infty)$
24. $[-1, 1]$
25. $(-\infty, -\sqrt{6}] \cup [\sqrt{6}, \infty)$
26. $(-1, 1)$
27. $(-\infty, \infty)$
28. $(-\infty, \infty)$
29. $(0, \infty)$
30. $(-\infty, \infty)$
31. $(-\infty, 0]$
32. $(-\infty, \infty)$
33. $(-\infty, -4) \cup (-4, 2) \cup (2, 5) \cup (5, \infty)$
34. $(-\infty, 2) \cup (2, \infty)$
35. Yes, by the Intermediate Value Theorem.
36. Yes, by the Intermediate Value Theorem. In fact, we can be more specific and state such a value c exists in $(0, 2)$, not just in $(-3, 7)$.
37. We cannot say; the Intermediate Value Theorem only applies to function values between -10 and 10; as 11 is outside this range, we do not know.
38. We cannot say; the Intermediate Value Theorem only applies to continuous functions. As we do know know if h is continuous, we cannot say.
39. $a = \frac{1}{3}$
40. $a = -1$ and $\frac{4}{3}$
41. $a = 1$ and $b = -1$
42. $a = -1$
43. Answers will vary.

44. Answers will vary.

45. Answers will vary.

46. Answers will vary.

47. Use the Bisection Method with an appropriate interval.

48. Use the Bisection Method with an appropriate interval.

49. Use the Bisection Method with an appropriate interval.

50. Use the Bisection Method with an appropriate interval.

51. (a) 20
(b) 25
(c) Limit does not exist
(d) 25

52.

x	$f(x)$
-0.81	-2.34129
-0.801	-2.33413
-0.79	-2.32542
-0.799	-2.33254

The top two lines give an approximation of the limit from the left: -2.33. The bottom two lines give an approximation from the right: -2.33 as well.

53. Answers will vary.

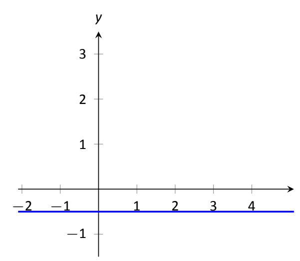
Chapter 2

Exercises 2.0

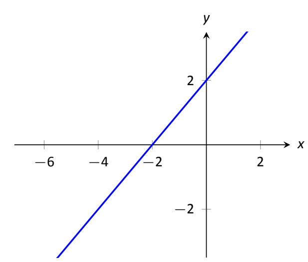
1. $80x^{12}y^{17}$
2. $\dfrac{a}{16b^7}$
3. $\dfrac{-x^3}{16y^{22}z^{35}}$
4. $x^2y^4z^5\sqrt[4]{z} = x^2y^4z^{21/4}$
5. $\frac{5(x-1)}{3x^{\frac{1}{3}}}$
6. $\frac{-5x+4}{2x^{\frac{1}{2}}(x+4)^2}$
7. $6x(3x^2+2)^3(x^2-5)^2(7x^2-18)$
8. (a) 8 (b) 44 (c) x^2-6x+8 (d) x^2+2x-4
9. (a) $-\frac{1}{3}$ (b) undefined (c) $\dfrac{1}{\sqrt{x-2}-5}$ (d) $\sqrt{\dfrac{1}{x-5}-2}$
10. (a) Possible solution: $f(x) = \frac{5}{x}$ and $g(x) = x+4$
(b) Possible solution: $f(x) = |x|$ and $g(x) = 4-x^2$
(c) Possible solution: $f(x) = \sqrt{x-5}$ and $g(x) = (x+2)^2$
11. (a) Possible solution: $f(x) = \sqrt[3]{x}$, $g(x) = x^2$, and $h(x) = 2x+1$
(b) Possible solution: $f(x) = 2x+1$, $g(x) = \sqrt[3]{x}$, and $h(x) = x^2$

Exercises 2.1

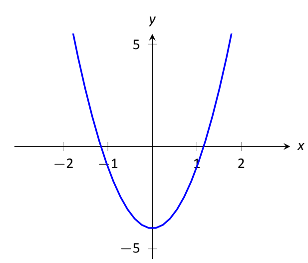
1. T
2. T
3. Answers will vary.
4. Answers will vary.
5. Answers will vary.
6. (a) $f'(x) = 0$, (b) $y = 6$
7. (a) $f'(x) = 2$, (b) $y = 2x$
8. (a) $f'(x) = -3$, (b) $y = 4-3x$
9. (a) $g'(x) = 2x$, (b) $y = -4x-4$
10. (a) $h'(x) = 2-2x$ (b) $y = 1$
11. (a) $f''(x) = 6x-1$, (b) $y = -7x+1$
12. (a) $g'(x) = \frac{1}{2\sqrt{x+3}}$, (b) $y = \frac{x}{4}+\frac{7}{4}$
13. (a) $r'(x) = \frac{-1}{x^2}$, (b) $y = -\frac{x}{4}-1$
14. (a) $h'(x) = -\dfrac{3}{2x\sqrt{x}}$, (b) $y = -\dfrac{3x}{16}+\dfrac{9}{4}$
15. (a) $f'(x) = \dfrac{-1}{(s-2)^2}$, (b) $y = -x+4$
16. $f(x) = \sqrt{x}$, $c = 16$.
17. $f(x) = x^4$, $c = 3$
18. $f(x) = \dfrac{1}{x}$, $c = 2$
19. $f(x) = \cos x$, $c = -\pi$.
20. $y = 8.1(x-3)+16$
21. $y = .248x + 1.006$
22. $y = -0.099(x-9)+1$
23. $y = 7.77(x-2)+e^2$, or $y = 7.77(x-2)+7.39$.
24. $y = -0.05x+1$
25. (a) Approximations will vary; they should match (c) closely.
(b) $f'(x) = 2x$
(c) At $(-1, 0)$, slope is -2. At $(0, -1)$, slope is 0. At $(2, 3)$, slope is 4.
26. (a) Approximations will vary; they should match (c) closely.
(b) $f'(x) = -1/(x+1)^2$
(c) At $(0, 1)$, slope is -1. At $(1, 0.5)$, slope is $-1/4$.
27.

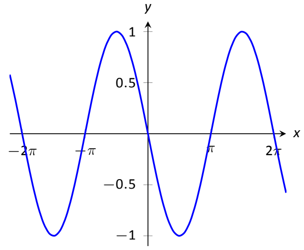


28.



29.



30. 

31. (a) Approximately on $(-1.5, 1.5)$.
    (b) Approximately on $(-\infty, -1.5) \cup (1.5, \infty)$.
    (c) Approximately at $x = \pm 1.5$.
    (d) On $(-\infty, -1) \cup (0, 1)$.
    (e) On $(-1, 0) \cup (1, \infty)$.
    (f) At $x = \pm 1$.

32. Approximately 24.

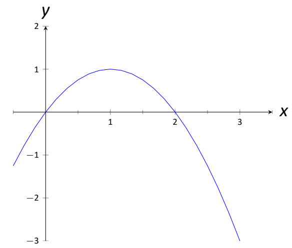
33. Approximately 0.54.

34. (a) $(-\infty, \infty)$
    (b) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
    (c) $(-\infty, 5]$
    (d) $[-5, 5]$

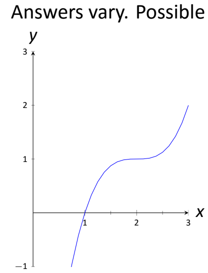
35. (a) 1
    (b) 3
    (c) Does not exist
    (d) $(-\infty, -3) \cup (3, \infty)$

Exercises 2.2

1. Velocity
2. Answers will vary.
3. Linear functions.
4. 12
5. -17
6. 102
7. $f(10.1)$ is likely most accurate, as accuracy is lost the farther from $x = 10$ we go.
8. -4
9. 6
10. decibels per person
11. ft/s^2
12. ft/h
13. (a) thousands of dollars per car
    (b) It is likely that $P(0) < 0$. That is, negative profit for not producing any cars.
14. (a) degrees Fahrenheit per hour
    (b) It is likely that $T'(8) > 0$ since at 8 in the morning, the temperature is likely rising.
    (c) It is very likely that $T(8) > 0$, as at 8 in the morning on July 4, we would expect the temperature to be well above 0.
15. $f(x) = g'(x)$
16. $g(x) = f'(x)$
17. Either $g(x) = f'(x)$ or $f(x) = g'(x)$ is acceptable. The actual answer is $g(x) = f'(x)$, but is very hard to show that $f(x) \neq g'(x)$ given the level of detail given in the graph.
18. $g(x) = f'(x)$
19. $f(6) = 1, f'(6) = -\frac{3}{4}$
20. Answers vary. Possible solution



21. Answers vary. Possible solution



22. $f'(x) = 10x$
23. $f'(x) = 3x^2 - 12x + 12$
24. $f'(\pi) \approx 0$.
25. $f'(9) \approx 0.1667$.

Exercises 2.3

1. Power Rule.
2. $1/x$
3. One answer is $f(x) = 10e^x$.
4. One answer is $f(x) = 10$.
5. $f(x)$, $g(x)$, $h(x)$, and $m(x)$
6. Answers will vary.
7. One possible answer is $f(x) = 17x - 205$.
8. Answers will vary.
9. $f'(x)$ is a velocity function, and $f''(x)$ is acceleration.
10. lbs/ft^2.
11. $f'(x) = 14x - 5$
12. $g'(x) = 42x^2 + 14x + 11$
13. $m'(t) = 45t^4 - \frac{3}{8}t^2 + 3$
14. $f'(\theta) = 9\cos\theta - 10\sin\theta$
15. $f'(r) = 6e^r$
16. $g'(t) = 40t^3 + \sin t + 7\cos t$
17. $f'(x) = \frac{2}{x} - 1$
18. $p'(s) = s^3 + s^2 + s + 1$
19. $h'(t) = e^t - \cos t + \sin t$
20. $f'(x) = \frac{2}{x}$
21. $f'(t) = 0$
22. $g'(t) = 18t + 6$
23. $g'(x) = 24x^2 - 120x + 150$
24. $f'(x) = -3x^2 + 6x - 3$
25. $f'(x) = 18x - 12$

26. $h'(x) = 3x^2 - 2$

27. $f'(x) = \frac{3}{2}\sqrt{x} - \frac{1}{2x\sqrt{x}}$

28. $g'(\theta) = -\sin\theta$

29.

30. $\frac{d}{dx}(c) = \lim_{h\to 0} \frac{c-c}{h} = \lim_{h\to 0} 0 = 0$

31. $n = -3, 2$

32. a is f, b is f', c is f''

33. d is f, c is f', b is f'', and a is f'''

34. $f'(x) = 6x^5$ $f''(x) = 30x^4$ $f'''(x) = 120x^3$ $f^{(4)}(x) = 360x^2$

35. $g'(x) = -2\sin x$ $g''(x) = -2\cos x$ $g'''(x) = 2\sin x$ $g^{(4)}(x) = 2\cos x$

36. $h'(t) = 2t - e^t$ $h''(t) = 2 - e^t$ $h'''(t) = -e^t$ $h^{(4)}(t) = -e^t$

37. $p'(\theta) = 4\theta^3 - 3\theta^2$ $p''(\theta) = 12\theta^2 - 6\theta$ $p'''(\theta) = 24\theta - 6$ $p^{(4)}(\theta) = 24$

38. $f'(\theta) = \cos\theta + \sin\theta$ $f''(\theta) = -\sin\theta + \cos\theta$ $f'''(\theta) = -\cos\theta - \sin\theta$ $f^{(4)}(\theta) = \sin\theta - \cos\theta$

39. $f'(x) = f''(x) = f'''(x) = f^{(4)}(x) = 0$

40. (a) $v(t) = 4t^3 - 8t$, $a(t) = 12t^2 - 8$
    (b) $a(1.5) = 19$ ft/s^2
    (c) $t = 0$ sec and $t = \sqrt{\frac{3}{2}}$ sec

41. (a) $v(t) = 5e^x - 5$, $a(t) = 5e^x$
    (b) $a(2) = 5e^2$ ft/s^2
    (c) $v(t) = 0$ at $t = 0$ sec, $a(0) = 5$ in/s^2

42. Tangent line: $y = 2(x-1)$

43. Tangent line: $y = t + 4$

44. Tangent line: $y = x - 1$

45. Tangent line: $y = 4$

46. Tangent line: $y = \sqrt{2}(x - \frac{\pi}{4}) - \sqrt{2}$

47. Tangent line: $y = 2x + 3$

48. The tangent line to $f(x) = e^x$ at $x = 0$ is $y = x + 1$; thus $e^{0.1} \approx y(0.1) = 1.1$.

49. The tangent line to $f(x) = x^4$ at $x = 3$ is $y = 108(x-3) + 81$; thus $(3.01)^4 \approx y(3.01) = 108(.01) + 81 = 82.08$.

Exercises 2.4

1. F
2. F
3. T
4. Quotient Rule
5. F
6. Answers will vary.
7.

$$\begin{aligned}\frac{d}{dx}(\cot x) &= \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)\\ &= \frac{\sin x(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2}\\ &= \frac{-[(\sin x)^2 + (\cos x)^2]}{(\sin x)^2}\\ &= \frac{-1}{(\sin x)^2} = -\csc^2 x\end{aligned}$$

8.

$$\begin{aligned}\frac{d}{dx}(\sec x) &= \frac{d}{dx}\left(\frac{1}{\cos x}\right)\\ &= \frac{\cos x \cdot 0 - 1\cdot(-\sin x)}{(\cos x)^2}\\ &= \frac{\sin x}{(\cos x)^2} = \sec x \tan x\end{aligned}$$

9.

$$\begin{aligned}\frac{d}{dx}(\csc x) &= \frac{d}{dx}\left(\frac{1}{\sin x}\right)\\ &= \frac{\sin x \cdot 0 - 1\cdot(\cos x)}{(\sin x)^2}\\ &= \frac{-\cos x}{(\sin x)^2} = -\csc x \cot x\end{aligned}$$

10. (a) $f'(x) = (x^2 + 3x) + x(2x + 3)$
    (b) $f'(x) = 3x^2 + 6x$
    (c) They are equal.

11. (a) $g'(x) = 4x(5x^3) + 2x^2(15x^2)$
    (b) $g'(x) = 50x^4$
    (c) They are equal.

12. (a) $h'(s) = 2(s+4) + (2s-1)(1)$
    (b) $h'(s) = 4s + 7$
    (c) They are equal.

13. (a) $f'(x) = 2x(3 - x^3) + (x^2 + 5)(-3x^2)$
    (b) $f'(x) = -5x^4 - 15x^2 + 6x$
    (c) They are equal.

14. (a) $f'(x) = \frac{x(2x) - (x^2+3)1}{x^2}$
    (b) $f'(x) = 1 - \frac{3}{x^2}$
    (c) They are equal.

15. (a) $g'(x) = \frac{2x^2(3x^2-4x) - (x^3-2x^2)(4x)}{4x^4}$
    (b) $g'(x) = 1/2$
    (c) They are equal.

16. (a) $h'(s) = \frac{4s^3(0) - 3(12s^2)}{16s^6}$
    (b) $h'(s) = -9/4s^{-4}$
    (c) They are equal.

17. (a) $f'(t) = \frac{(t+1)(2t) - (t^2-1)(1)}{(t+1)^2}$
    (b) $f(t) = t - 1$ when $t \neq -1$, so $f'(t) = 1$.
    (c) They are equal.

18. $f'(x) = \sin x + x\cos x$

19. $f'(t) = \frac{-2}{t^3}(\csc t - 4) + \frac{1}{t^2}(-\csc t \cot t)$

20. $H'(y) = (y^5 - 2y^3)(14y + 1) + (5y^4 - 6y^2)(7y^2 + y - 8)$

21. $F'(y) = \frac{8}{3}y^{5/3} + 15y^{2/3} = \frac{\sqrt[3]{y^2}(8y+45)}{3}$

22. $g'(x) = \frac{-12}{(x-5)^2}$

23. $y' = \frac{4-x}{2\sqrt{x}(x+4)^2}$

24. $g'(x) = \frac{\sqrt{x}+8}{2(\sqrt{x}+4)^2}$

25. $g'(t) = \frac{(\cos t - 2t^2)(5t^4) - (t^5)(-\sin t - 4t)}{(\cos t - 2t^2)^2}$

26. $h'(x) = -\csc^2 x - e^x$

27. $h'(t) = 14t + 6$

28. (a) $f'(x) = \frac{(x+2)(4x^3+6x^2)-(x^4+2x^3)(1)}{(x+2)^2}$

(b) $f(x) = x^3$ when $x \neq -2$, so $f'(x) = 3x^2$.

(c) They are equal.

29. $f'(x) = -\dfrac{1}{x^2} + \dfrac{5}{2x^3\sqrt{x}} = \dfrac{-2x\sqrt{x}+5}{2x^3\sqrt{x}}$

30. $y' = 2x - 5 + \dfrac{10}{x^2} = \dfrac{2x^3 - 5x^2 + 10}{x^2}$

31. $g'(x) = -\frac{1+2x+3x^2}{(1+x+x^2+x^3)^2}$

32. $p'(x) = -\dfrac{1}{x^2} - \dfrac{2}{x^3} - \dfrac{3}{x^4} = -\dfrac{x^2+2x+3}{x^4}$

33. $f'(x) = 7$

34. $f'(t) = 5t^4(\sec t + e^t) + t^5(\sec t \tan t + e^t)$

35. $f'(x) = \frac{\sin^2(x)+\cos^2(x)+3\cos(x)}{(\cos(x)+3)^2}$

36. $g'(x) = 0$

37. $g'(t) = 12t^2e^t + 4t^3e^t - \cos^2 t + \sin^2 t$

38. $f'(y) = y(2y^3 - 5y - 1)(12y) + y(6y^2 - 5)(6y^2 + 7) + 1(2y^3 - 5y - 1)(6y^2 + 7) = 72y^5 - 64y^3 - 18y^2 - 70y - 7$

39. $F'(x) = (8x - 1)(x^2 + 4x + 7)(3x^2) + (8x - 1)(2x + 4)(x^3 - 5) + (8)(x^2 + 4x + 7)(x^3 - 5)$

40. $f'(x) = \frac{(t^2\cos t+2)(2t\sin t+t^2\cos t)-(t^2\sin t+3)(2t\cos t-t^2\sin t)}{(t^2\cos t+2)^2}$

41. $f'(x) = 2xe^x \tan x = x^2e^x \tan x + x^2e^x \sec^2 x$

42. $g'(x) = 2\sin x \sec x + 2x\cos x \sec x + 2x\sin x \sec x \tan x = 2\tan x + 2x + 2x\tan^2 x = 2\tan x + 2x\sec^2 x$

43. $y = 2x + 2$

44. $y = -(x - \frac{3\pi}{2}) - \frac{3\pi}{2} = -x$

45. $y = 4$

46. $y = -9x + 1$

47. $x = 3/2$

48. $x = 0$

49. $f'(x)$ is never 0.

50. $x = -2, 0$

51. $f''(x) = 2\cos x - x\sin x$

52. $f^{(4)}(x) = -4\cos x + x\sin x$

53. $f''(x) = \cot^2 x \csc x + \csc^3 x$

54. $f^{(8)} = 0$

55. 1

56. -3

57. -4

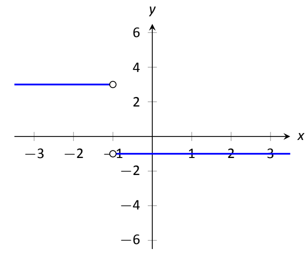
58. 11

59. $-\dfrac{1}{25}$

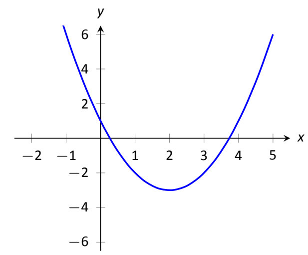
60. $\dfrac{1}{4}$

61. (a) $-\frac{7}{2}$ (b) $\frac{1}{8}$ (c) $-\frac{9}{2}$ (d) $\frac{15}{2}$

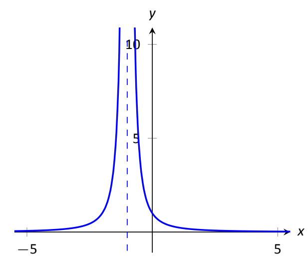
62.



63.



64.



Exercises 2.5

1. T

2. F

3. F

4. T

5. T

6. T

7. $f'(x) = 10(4x^3 - x)^9 \cdot (12x^2 - 1) = (120x^2 - 10)(4x^3 - x)^9$

8. $f'(t) = 15(3t - 2)^4$

9. $g'(\theta) = 3(\sin\theta + \cos\theta)^2(\cos\theta - \sin\theta)$

10. $h'(t) = (6t + 1)e^{3t^2+t-1}$

11. $f'(x) = 4\left(x + \frac{1}{x}\right)^3\left(1 - \frac{1}{x^2}\right)$

12. $p'(x) = 12\left(x^2 - \dfrac{1}{x^2}\right)^5\left(x + \dfrac{1}{x^3}\right)$

13. $f'(x) = -3\sin(3x)$

14. $g'(x) = 5\sec^2(5x)$

15. $h'(t) = 8\sin^3(2t)\cos(2t)$

16. $p'(t) = -3\cos^2(t^2 + 3t + 1)\sin(t^2 + 3t + 1)(2t + 3)$

17. $g'(x) = 2(\tan x \sec^2 x - x\sec^2(x^2))$

18. $w'(x) = 3x^2e^{x^3}(\sec e^{x^3})(\tan e^{x^3})$

19. $f'(x) = -\tan x$

20. $f'(x) = 2/x$

21. $f'(x) = 2/x$

22. $g'(t) = 0$

23. $r'(x) = \dfrac{-6(x-1)}{x^3\sqrt{4x-3}}$

24. $f'(x) = \dfrac{12x(2x^3-1)(3x^2-5)^3(x^2+5x-2)}{(2x^3-1)^4}$

25. $h'(x) = 200(2x+1)^9[(2x+1)10+1]^9$

26. $f'(t) = \dfrac{-t^4}{(2t+1)(t+1)}$

27. $F'(x) = 2(2x+1)(2x+3)^2(24x^2+26x+3)$

28. $f'(x) = 5x^2\cos(5x) + 2x\sin(5x)$

29. $g'(t) = 5\cos(t^2+3t)\cos(5t-7) - (2t+3)\sin(t^2+3t)\sin(5t-7)$

30. $g'(t) = 10t\cos(\frac{1}{t})e^{5t^2} + \frac{1}{t^2}\sin(\frac{1}{t})e^{5t^2}$

31. $a'(t) = 7t^2e^{\tan(t^2)}(2t^2\sec^2(t^2)+3)$

32. $y' = \dfrac{-\cos x\sin x\cos(\cos^2 x)}{\sqrt{\sin(\cos^2 x)}}$

33. $k'(x) = -\sin(x\sin x^3)(3x^3\cos x^3 + \sin x^3)$

34. 15

35. 90

36. (a) 6 (b) 1 (c) -4 (d) 1.5

37. (a) 12 (b) 2.5 (c) 9 (d) 35

38. Tangent line: $y = 0$
Normal line: $x = 0$

39. Tangent line: $y = 15(t-1)+1$
Normal line: $y = -1/15(t-1)+1$

40. Tangent line: $y = -3(\theta - \pi/2)+1$
Normal line: $y = 1/3(\theta - \pi/2)+1$

41. Tangent line: $y = -5e(t+1)+e$
Normal line: $y = 1/(5e)(t+1)+e$

42. In both cases the derivative is the same: $1/x$.

43. In both cases the derivative is the same: k/x.

44. Let $g(x) = -x$. Then
   (a) $f \circ g = f$, so $f'(-x) = f' \circ g(x) = -f' \circ g(x)g'(x) = -(f \circ g)'(x) = -f'(x)$
   (b) $f \circ g = -f$, so $f'(-x) = f' \circ g(x) = -f' \circ g(x)g'(x) = -(f \circ g)'(x) = f'(x)$

45. Let $h(x) = x^{-1}$. Then $\dfrac{d}{dx}\dfrac{f(x)}{g(x)} = \dfrac{d}{dx}[f(x)\cdot h(g(x))] = \dfrac{d}{dx}[f(x)]\cdot h(g(x)) + f(x)\cdot \dfrac{d}{dx}[h(g(x))] = f'(x)\cdot h(g(x)) + f(x)\cdot h'(g(x))\cdot g'(x) = f'(x)[g(x)]^{-1} - f(x)[g(x)]^{-2}g'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

46. $[f(g(x))]'' = [f'(g(x))g'(x)]' = [f'(g(x))]'g'(x) + f'(g(x))g''(x) = f''(g(x))g'(x)g'(x) + f'(g(x))g''(x) = f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$

47. (a) $^\circ$ F/mph
   (b) The sign would be negative; when the wind is blowing at 10 mph, any increase in wind speed will make it feel colder, i.e., a lower number on the Fahrenheit scale.

48. $2xe^x\cot x + x^2e^x\cot x - x^2e^x\csc^2 x$

Exercises 2.6

1. Answers will vary.
2. The Chain Rule.
3. T
4. T
5. $\frac{dy}{dx} = \frac{-4x^3}{2y+1}$
6. $\frac{dy}{dx} = -\frac{y^{3/5}}{x^{3/5}}$
7. $\frac{dy}{dx} = \sin(x)\sec(y)$
8. $\frac{dy}{dx} = \frac{y}{x}$
9. $\frac{dy}{dx} = \frac{y}{x}$
10. $-\frac{2\sin(y)\cos(y)}{x}$
11. $-\frac{x}{y^2}$
12. $\frac{1}{2y+2}$
13. $\frac{x^2+2xy^2-y}{2x^2y-x+y^2}$
14. $\frac{-\cos(x)(x+\cos(y))+\sin(x)+y}{\sin(y)(\sin(x)+y)+x+\cos(y)}$
15. $-\frac{x}{y}$
16. $-\frac{2x+y}{2y+x}$
17. $\dfrac{e^x(x+1)}{e^y(y+1)}$
18. $\dfrac{3x^2y\cos x^3 - \sin y^3}{3xy^2\sin y^3 - \sin x^3}$
19. $\dfrac{y - 4xy\sqrt{xy}}{2x^2\sqrt{xy} - x}$
20. In each, $\frac{dy}{dx} = -\frac{y}{x}$.
21. (a) $y = 0$
    (b) $y = -1.859(x-0.1)+0.281$
22. (a) $x = 1$
    (b) $y = -\frac{3\sqrt{3}}{8}(x - \sqrt{.6}) + \sqrt{.8} \approx -0.65(x-0.775)+0.894$
    (c) $y = 1$
23. (a) $y = 4$
    (b) $y = 0.93(x-2) + \sqrt[4]{108}$
24. (a) $y = -1/3x + 1$
    (b) $y = 3\sqrt{3}/4$
25. (a) $y = -\frac{1}{\sqrt{3}}(x - \frac{7}{2}) + \frac{6+3\sqrt{3}}{2}$
    (b) $y = \sqrt{3}(x - \frac{4+3\sqrt{3}}{2}) + \frac{3}{2}$
26. (a) $y = -\dfrac{3x}{4} - \dfrac{3}{2}$
    (b) $y = \dfrac{7}{2}x - \dfrac{3}{2}$
27. $\frac{d^2y}{dx^2} = \frac{(2y+1)(-12x^2)+4x^3\left(2\frac{-4x^3}{2y+1}\right)}{(2y+1)^2}$
28. $\frac{d^2y}{dx^2} = \frac{3}{5}\frac{y^{3/5}}{x^{8/5}} + \frac{3}{5}\frac{1}{yx^{6/5}}$
29. $\frac{d^2y}{dx^2} = \frac{\cos x\cos y + \sin^2 x\tan y}{\cos^2 y}$
30. $\frac{d^2y}{dx^2} = 0$

Chapter 3

Exercises 3.1

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.

5. F
6. A: none B: abs. max and rel. max C: rel. min D: none E: none F: rel. min G: none
7. A: abs. min B: none C: abs. max D: none E: none
8. Answers will vary.
9. $f'(0) = 0$
10. $f'(0) = 0\, f'(2) = 0$
11. $f'(\pi/2) = 0\, f'(3\pi/2) = 0$
12. $f'(0) = 0\, f'(3.2) = 0\, f'(4)$ is undefined
13. $f'(0) = 0$
14. $f'(0)$ is not defined
15. $f'(2)$ is not defined $f'(6) = 0$
16. min: $(-0.5, 3.75)$
    max: $(2, 10)$
17. min: $(5, -134.5)$
    max: $(0, 3)$
18. min: $(\pi/4, 3\sqrt{2}/2)$
    max: $(\pi/2, 3)$
19. min: $(0, 0)$ and $(\pm 2, 0)$
    max: $(\pm 2\sqrt{2/3}, 16\sqrt{3}/9)$
20. min: $(\sqrt{3}, 2\sqrt{3})$
    max: $(5, 28/5)$
21. min: $(0, 0)$
    max: $(5, 5/6)$
22. min: $(\pi, -e^{\pi})$
    max: $(\pi/4, \frac{\sqrt{2}e^{\pi/4}}{2})$
23. min: $(0, 0)$ and $(\pi, 0)$
    max: $(3\pi/4, \frac{\sqrt{2}e^{3\pi/4}}{2})$
24. min: $(1, 0)$
    max: $(e, 1/e)$
25. min: $(2, 2^{2/3} - 2)$
    max: $(8/27, 4/27)$
26. $f'(x) = 3(x-4)^2$, so $f'(4) = 0$. But $f(x) > 7$ for $x > 4$ and $f(x) < 7$ for $x < 4$.
27. (a) $x^3 - x$, x^3, and $x^3 + x$ have 2, 1, and 0 critical numbers respectively. Because the derivative is a quadratic with at most 2 roots, a cubic cannot have 3 or more critical numbers.
    (b) A cubic can only have 2 or 0 extreme values.
28. min: $(0, 0)$ and $(1, 0)$
    max: $(\frac{a}{a+b}, \frac{a^a b^b}{(a+b)^{a+b}})$
29. $\frac{dy}{dx} = \frac{y(y-2x)}{x(x-2y)}$
30. $y = -\frac{4}{5}(x-1) + 2$
31. $3x^2 + 1$

Exercises 3.2

1. Answers will vary.
2. Answers will vary.
3. Any c in $[-1, 1]$ is valid.
4. Rolle's Thm. does not apply.
5. $c = -1/2$
6. $c = -1/2$
7. Rolle's Thm. does not apply.
8. $c = \pi/2$
9. Rolle's Thm. does not apply.
10. Rolle's Theorem does not apply.
11. $c = 0$
12. $c = 5/2$
13. $c = 3/\sqrt{2}$
14. $c = 19/4$
15. The Mean Value Theorem does not apply.
16. $c = 4/\ln 5$
17. $c = \pm \sec^{-1}(2/\sqrt{\pi})$
18. $c = -2/3$
19. $c = \frac{5 \pm 7\sqrt{7}}{6}$
20. With c given by the Mean Value Theorem, $f(4) = f(1) + f'(c)(4-1) = 10 + 3f'(c) \geq 16$.
21. No. Otherwise, with c given by the Mean Value Theorem, $\dfrac{4 - -1}{2 - 0} = f'(c) \leq 2$, a contradiction.
22. $f(-1) < 0 < f(0)$, so it has at least one root. $f' = 2 + 3x^2 + 20x^4 \geq 2$, so more than one root would contradict Rolle's Theorem.
23. If f has more than 3 real roots, then Rolle's Theorem implies f' is a quadratic with more than 2 real roots.
24. (a) is Rolle's Theorem. For (b), applying Rolle's Theorem to roots 1 and 2 and roots 2 and 3 shows that f' has two roots, and we can then apply (a).
25. $2pc + q = f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{pb^2+qb+r-pa^2-qa-r}{b-a} = \frac{p(b^2-a^2)+q(b-a)}{b-a} = p(b+a) + q$ implies that $c = \frac{a+b}{2}$.
26. Max value of 19 at $x = -2$ and $x = 5$; min value of 6.75 at $x = 1.5$.
27. They are the odd, integer valued multiples of $\pi/2$ (such as $0, \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$, etc.)
28. They are the odd, integer valued multiples of $\pi/2$ (such as $0, \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$, etc.)

Exercises 3.3

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. Increasing
6. decreasing on $(-3, -1) \cup (1, 3)$,
   increasing on $(-\infty, -3) \cup (-1, 1) \cup (3, \infty)$;
   local maxima when $x = -3, 1$,
   local minima when $x = -1, 3$.
7. decreasing on $(0, \frac{\pi}{6}) \cup (\frac{\pi}{2}, \frac{5\pi 6}{)} \cup (\frac{3\pi}{2}, 2\pi)$,
   increasing on $(\frac{\pi}{6}, \frac{\pi}{2}) \cup (\frac{5\pi}{6}, \frac{3\pi}{2})$;
   local maxima when $x = \frac{\pi}{2}, \frac{3\pi}{2}$,
   local minima when $x = \frac{\pi}{6}, \frac{5\pi}{6}$.
8. decreasing on $(-\infty, -2) \cup (2, \infty)$,
   increasing on $(-2, 2)$;
   local maxima when $x = 2$,
   local minima when $x = -2$.

9. decreasing on $(-1, 1)$,
increasing on $(-\infty, -1) \cup (1, \infty)$;
local maxima when $x = -1$,
local minima when $x = 1$.
10. Graph and verify.
11. Graph and verify.
12. Graph and verify.
13. Graph and verify.
14. Graph and verify.
15. Graph and verify.
16. Graph and verify.
17. Graph and verify.
18. domain: $(-\infty, \infty)$
c.p. at $c = -1$;
decreasing on $(-\infty, -1)$;
increasing on $(-1, \infty)$;
rel. min at $x = -1$.
19. domain=$(-\infty, \infty)$
c.p. at $c = -2, 0$;
increasing on $(-\infty, -2) \cup (0, \infty)$;
decreasing on $(-2, 0)$;
rel. min at $x = 0$;
rel. max at $x = -2$.
20. domain=$(-\infty, \infty)$
c.p. at $c = \frac{1}{6}(-1 \pm \sqrt{7})$;
decreasing on $(\frac{1}{6}(-1 - \sqrt{7}), \frac{1}{6}(-1 + \sqrt{7})))$;
increasing on $(-\infty, \frac{1}{6}(-1 - \sqrt{7})) \cup (\frac{1}{6}(-1 + \sqrt{7}), \infty)$;
rel. min at $x = \frac{1}{6}(-1 + \sqrt{7})$;
rel. max at $x = \frac{1}{6}(-1 - \sqrt{7})$.
21. domain=$(-\infty, \infty)$
c.p. at $c = 1$;
increasing on $(-\infty, \infty)$;
22. domain=$(-\infty, \infty)$
c.p. at $c = 1$;
decreasing on $(1, \infty)$
increasing on $(-\infty, 1)$;
rel. max at $x = 1$.
23. domain=$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
c.p. at $c = 0$;
decreasing on $(-\infty, -1) \cup (-1, 0)$;
increasing on $(0, 1) \cup (1, \infty)$;
rel. min at $x = 0$;
24. domain=$(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$
no c.p.;
decreasing on entire domain, $(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$
25. domain=$(-\infty, 0) \cup (0, \infty)$;
c.p. at $c = 2, 6$;
decreasing on $(-\infty, 0) \cup (0, 2) \cup (6, \infty)$;
increasing on $(2, 6)$;
rel. min at $x = 2$; rel. max at $x = 6$.
26. domain=$(-\infty, \infty)$
c.p. at $c = -3\pi/4, -\pi/4, \pi/4, 3\pi/4$;
decreasing on $(-3\pi/4, -\pi/4) \cup (\pi/4, 3\pi/4)$;
increasing on $(-\pi, -3\pi/4) \cup (-\pi/4, \pi/4) \cup (3\pi/4, \pi)$;
rel. min at $x = -\pi/4, 3\pi/4$;
rel. max at $x = -3\pi/4, \pi/4$.
27. domain = $(-\infty, \infty)$;
c.p. at $c = -1, 1$;
decreasing on $(-1, 1)$;
increasing on $(-\infty, -1) \cup (1, \infty)$;
rel. min at $x = 1$;
rel. max at $x = -1$
28. domain=$(-\infty, \infty)$;
c.p. at $c = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$;
decreasing on $(0, \frac{\pi}{3}) \cup (\frac{5\pi}{3}, \frac{7\pi}{3})$;
increasing on $(\frac{\pi}{3}, \frac{5\pi}{3}) \cup (\frac{7\pi}{3}, 3\pi)$;
rel. min at $x = \frac{\pi}{3}, \frac{7\pi}{3}$;
rel. max at $x = \frac{5\pi}{3}$
29. domain=$(-\infty, \infty)$;
c.p. at $c = \frac{\pi}{2}, \frac{3\pi}{2}$;
decreasing on $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$;
increasing on $(\frac{\pi}{2}, \frac{3\pi}{2})$;
rel. min at $x = \frac{\pi}{2}$;
rel. max at $x = \frac{3\pi}{2}$
30. domain=$[3, \infty)$;
no c.p.;
increasing on $(3, \infty)$;
31. domain=$(-\infty, \infty)$;
c.p. at $c = -1, 0, 1$;
decreasing on $(-\infty, 0)$;
increasing on $(0, \infty)$;
rel. min at $x = 0$
32. $c = 1/2$
33. $c = \pm \cos^{-1}(2/\pi)$

Exercises 3.4

1. Answers will vary.
2. Answers will vary.
3. Yes; Answers will vary.
4. No.
5. concave up on $(-2, 2)$;
concave down on $(-\infty, -2) \cup (2, \infty)$;
inflection points when $x = \pm 2$
6. concave up on $(-\infty, -1) \cup (1, \infty)$;
concave down on $(-1, 1)$;
inflection points when $x = \pm 1$
7. concave up on $(-\infty, -1) \cup (1, \infty)$;
concave down on $(-1, 1)$;
inflection points when $x = \pm 1$
8. Graph and verify.
9. Graph and verify.
10. Graph and verify.
11. Graph and verify.
12. Graph and verify.
13. Graph and verify.
14. Graph and verify.
15. Graph and verify.
16. Graph and verify.
17. Graph and verify.

18. Graph and verify.

19. (a) Possible points of inflection: none
    (b) concave up on $(-\infty, \infty)$
    (c) min: $x = 1$
    (d) f' has no maximal or minimal value.

20. (a) Possible points of inflection: none
    (b) concave down on $(-\infty, \infty)$
    (c) max: $x = -5/2$
    (d) f' has no maximal or minimal value

21. (a) Possible points of inflection: $x = 0$
    (b) concave down on $(-\infty, 0)$, concave up on $(0, \infty)$
    (c) max: $x = -1/\sqrt{3}$, min: $x = 1/\sqrt{3}$
    (d) f' has a minimal value at $x = 0$

22. (a) Possible points of inflection: $x = 1/2$
    (b) concave down on $(-\infty, 1/2)$, concave up on $(1/2, \infty)$
    (c) No relative extrema
    (d) f' has a minimal value at $x = 1/2$

23. (a) Possible points of inflection: $x = -2/3, 0$
    (b) concave down on $(-2/3, 0)$, concave up on $(-\infty, -2/3) \cup (0, \infty)$
    (c) min: $x = 1$
    (d) f' has a relative min at: $x = 0$, relative max at: $x = -2/3$

24. (a) Possible points of inflection: $x = (1/3)(2 \pm \sqrt{7})$
    (b) concave up on $((1/3)(2 - \sqrt{7}), (1/3)(2 + \sqrt{7}))$, concave down on $(-\infty, (1/3)(2 - \sqrt{7})) \cup ((1/3)(2 + \sqrt{7}), \infty)$
    (c) max: $x = -1, 2$, min: $x = 1$
    (d) f' has a relative max at: $x = (1/3)(2 + \sqrt{7})$, relative min at: $x = (1/3)(2 - \sqrt{7})$

25. (a) Possible points of inflection: $x = 1$
    (b) concave up on $(-\infty, \infty)$
    (c) min: $x = 1$
    (d) f' has no relative extrema

26. (a) Possible points of inflection: $x = \pm 1/\sqrt{3}$
    (b) concave down on $(-1/\sqrt{3}, 1/\sqrt{3})$, concave up on $(-\infty, -1/\sqrt{3}) \cup (1/\sqrt{3}, \infty)$
    (c) max: $x = 0$
    (d) f' has a relative max at $x = -1/\sqrt{3}$, relative min at $x = 1/\sqrt{3}$

27. (a) Possible points of inflection: $x = 0, \pm 1$
    (b) concave down on $(-\infty, -1) \cup (0, 1)$, concave up on $(-1, 0) \cup (1, \infty)$
    (c) critical values: $x = -1, 1$, no max/min
    (d) f' has a relative max at $x = 0$

28. (a) Possible points of inflection: $x = -\pi/4, 3\pi/4$
    (b) concave down on $(-\pi/4, 3\pi/4)$, concave up on $(-\pi, -\pi/4) \cup (3\pi/4, \pi)$
    (c) max: $x = \pi/4$, min: $x = -3\pi/4$
    (d) f' has a relative min at $x = 3\pi/4$, relative max at $x = -\pi/4$

29. (a) Possible points of inflection: $x = -2 \pm \sqrt{2}$
    (b) concave down on $(-2 - \sqrt{2}, -2 + \sqrt{2})$, concave up on $(-\infty, -2 - \sqrt{2}) \cup (-2 + \sqrt{2}, \infty)$
    (c) max: $x = -2$, min: $x = 0$
    (d) f' has a relative max at $x = -2 - \sqrt{2}$, relative min at $x = -2 + \sqrt{2}$

30. (a) Possible points of inflection: $x = 1/e^{3/2}$
    (b) concave down on $(0, 1/e^{3/2})$, concave up on $(1/e^{3/2}, \infty)$
    (c) min: $x = 1/\sqrt{e}$
    (d) f' has a relative min at $x = 1/\sqrt{e^3} = e^{-3/2}$

31. (a) Possible points of inflection: $x = \pm 1/\sqrt{2}$
    (b) concave down on $(-1/\sqrt{2}, 1/\sqrt{2})$, concave up on $(-\infty, -1/\sqrt{2}) \cup (1/\sqrt{2}, \infty)$
    (c) max: $x = 0$
    (d) f' has a relative max at $x = -1/\sqrt{2}$, a relative min at $x = 1/\sqrt{2}$

32. (a) Possible points of inflection: none
    (b) concave up on $(-3, \infty)$
    (c) min: $x = -2$
    (d) f' has no relative extrema

33. (a) Possible points of inflection: $x = \pi/6, 5\pi/6, 3\pi/2$
    (b) concave down on $(0, \pi/6) \cup (5\pi/6, 2\pi)$, concave up on $(\pi/6, 5\pi/6)$
    (c) max: $x = 3\pi/2$, min: $x = 3\pi/2$
    (d) f' has a relative max at $x = 5\pi/6$, f' has a relative min at $x = \pi/6$

Exercises 3.5

1. Answers will vary.
2. T
3. T
4. T
5. concave up on $(-\infty, -1) \cup (1, \infty)$
   concave down on $(-1, 1)$
   inflection points when $x = \pm 1$
   increasing on $(-2, 0) \cup (2, \infty)$
   decreasing on $(-\infty, -2) \cup (0, 2)$
   relative maximum when $x = 0$
   relative minima when $x = \pm 2$
6. concave up on $(-2, 0) \cup (2, \infty)$
   concave down on $(-\infty, -2) \cup (0, 2)$
   inflection points when $x = 0, \pm 2$
   increasing on $(-\infty, -2.3) \cup (-1, 1) \cup (2.3, \infty)$
   decreasing on $(-2.3, -1) \cup (1, 2.3)$
   relative maximum when $x = -2.3, 1$
   relative minima when $x = -1, 2.3$
7. various possibilities
8. A good sketch will include the x and y intercepts and draw the appropriate line.
9. A good sketch will include the x and y intercepts..
10. Use technology to verify sketch.
11. Use technology to verify sketch.
12. Use technology to verify sketch.
13. Use technology to verify sketch.
14. Use technology to verify sketch.
15. Use technology to verify sketch.
16. Use technology to verify sketch.
17. Use technology to verify sketch.
18. Use technology to verify sketch.

19. Use technology to verify sketch.
20. Use technology to verify sketch.
21. Use technology to verify sketch.
22. Use technology to verify sketch.
23. Use technology to verify sketch.
24. Use technology to verify sketch.
25. Use technology to verify sketch.
26. Use technology to verify sketch.
27. Use technology to verify sketch.
28. Use technology to verify sketch.
29. Use technology to verify sketch.
30. Use technology to verify sketch.
31. Use technology to verify sketch.
32. Use technology to verify sketch.
33. Use technology to verify sketch.
34. Use technology to verify sketch.
35. Critical point: $x = 0$ Points of inflection: $\pm b/\sqrt{3}$
36. Critical points: $x = \frac{n\pi/2-b}{a}$, where n is an odd integer Points of inflection: $(n\pi - b)/a$, where n is an integer.
37. Critical point: $x = (a+b)/2$ Points of inflection: none
38. $\dfrac{dy}{dx} = -x/y$, so the function is increasing in second and fourth quadrants, decreasing in the first and third quadrants.
    $\dfrac{d^2y}{dx^2} = -1/y^3$, which is positive when $y < 0$ and is negative when $y > 0$. Hence the function is concave down in the first and second quadrants and concave up in the third and fourth quadrants.

Chapter 4

Exercises 4.1

1. T
2. F
3. (a) $5/(2\pi) \approx 0.796$cm/s
   (b) $1/(4\pi) \approx 0.0796$ cm/s
   (c) $1/(40\pi) \approx 0.00796$ cm/s
4. (a) $5/(2\pi) \approx 0.796$cm/s
   (b) $1/(40\pi) \approx 0.00796$ cm/s
   (c) $1/(4000\pi) \approx 0.0000796$ cm/s
5. 63.14mph
6. (a) 64.44 mph
   (b) 78.89 mph
7. Due to the height of the plane, the gun does not have to rotate very fast.
   (a) 0.0573 rad/s
   (b) 0.0725 rad/s
   (c) In the limit, rate goes to 0.0733 rad/s
8. Due to the height of the plane, the gun does not have to rotate very fast.
   (a) 0.073 rad/s
   (b) 3.66 rad/s (about 1/2 revolution/sec)
   (c) In the limit, rate goes to 7.33 rad/s (more than 1 revolution/sec)
9. (a) 0.04 ft/s
   (b) 0.458 ft/s
   (c) 3.35 ft/s
   (d) Not defined; as the distance approaches 24, the rates approaches ∞.
10. (a) 30.59 ft/min
    (b) 36.1 ft/min
    (c) 301 ft/min
    (d) The boat no longer floats as usual, but is being pulled up by the winch (assuming it has the power to do so).
11. (a) 50.92 ft/min
    (b) 0.509 ft/min
    (c) 0.141 ft/min
    As the tank holds about 523.6ft^3, it will take about 52.36 minutes.
12. (a) 0.63 ft/sec
    (b) 1.6 ft/sec
    About 52 ft.
13. (a) The rope is 80ft long.
    (b) 1.71 ft/sec
    (c) 1.87 ft/sec
    (d) About 34 feet.
14. (a) The balloon is 105ft in the air.
    (b) The balloon is rising at a rate of 17.45ft/min. (Hint: convert all angles to radians.)
15. The cone is rising at a rate of 0.003ft/s.

Exercises 4.2

1. T
2. F
3. 2500; the two numbers are each 50.
4. The minimum sum is $2\sqrt{500}$; the two numbers are each $\sqrt{500}$.
5. There is no maximum sum; the fundamental equation has only 1 critical value that corresponds to a minimum.
6. The only critical point of the fundamental equation corresponds to a minimum; to find maximum, we check the endpoints.
   If one number is 300, the other number y satisfies $300y = 500$; $y = 5/3$. Thus the sum is $300 + 5/3$.
   The other endpoint, 0, is not feasible as we cannot solve $0 \cdot y = 500$ for y. In fact, if $0 < x < 5/3$, then $x \cdot y = 500$ forces $y > 300$, which is not a feasible solution.
   Hence the maximum sum is $301.\overline{6}$.
7. Area = 1/4, with sides of length $1/\sqrt{2}$.
8. Each pen should be $500/3 \approx 166.67$ feet by 125 feet.
9. The radius should be about 3.84cm and the height should be $2r = 7.67$cm. No, this is not the size of the standard can.
10. The radius should be about 3.2in and the height should be $2r = 6.4$in. As the #10 is not a perfect cylinder (with extra material to aid in stacking, etc.), the dimensions are close enough to assume that minimizing surface area was a consideration.
11. The height and width should be 18 and the length should be 36, giving a volume of $11,664$in^3.
12. $w = 4\sqrt{3}$, $h = 4\sqrt{6}$
13. $5 - 10/\sqrt{39} \approx 3.4$ miles should be run underground, giving a minimum cost of $374,899.96.

14. The power line should be run directly to the off shore facility, skipping any underground, giving a cost of about \$430,813.
15. The dog should run about 19 feet along the shore before starting to swim.
16. The dog should run about 13 feet along the shore before starting to swim.
17. The largest area is 2 formed by a square with sides of length $\sqrt{2}$.
18. The largest volume is 62.5 in^3 formed by cutting 2.5 in squares at each corner.
19. A length of 2 in and height of 2.5 will give a cost of 52¢.
20. A box that is 1 in wide, 2 in long and 4/3 in high will have a volume of 8/3 in^3.

Exercises 4.3

1. T
2. T
3. F
4. T
5. Answers will vary.
6. Use $y = x^2$; $dy = 2x \cdot dx$ with $x = 2$ and $dx = 0.05$. Thus $dy = .2$; knowing $2^2 = 4$, we have $2.05^2 \approx 4.2$.
7. Use $y = x^2$; $dy = 2x \cdot dx$ with $x = 6$ and $dx = -0.07$. Thus $dy = -0.84$; knowing $6^2 = 36$, we have $5.93^2 \approx 35.16$.
8. Use $y = x^3$; $dy = 3x^2 \cdot dx$ with $x = 5$ and $dx = 0.1$. Thus $dy = 7.5$; knowing $5^3 = 125$, we have $5.1^3 \approx 132.5$.
9. Use $y = x^3$; $dy = 3x^2 \cdot dx$ with $x = 7$ and $dx = -0.2$. Thus $dy = -29.4$; knowing $7^3 = 343$, we have $6.8^3 \approx 313.6$.
10. Use $y = \sqrt{x}$; $dy = 1/(2\sqrt{x}) \cdot dx$ with $x = 16$ and $dx = 0.5$. Thus $dy = .0625$; knowing $\sqrt{16} = 4$, we have $\sqrt{16.5} \approx 4.0625$.
11. Use $y = \sqrt{x}$; $dy = 1/(2\sqrt{x}) \cdot dx$ with $x = 25$ and $dx = -1$. Thus $dy = -0.1$; knowing $\sqrt{25} = 5$, we have $\sqrt{24} \approx 4.9$.
12. Use $y = \sqrt[3]{x}$; $dy = 1/(3\sqrt[3]{x^2}) \cdot dx$ with $x = 64$ and $dx = -1$. Thus $dy = -1/48 \approx 0.0208$; we could use $-1/48 \approx -1/50 = -0.02$; knowing $\sqrt[3]{64} = 4$, we have $\sqrt[3]{63} \approx 3.98$.
13. Use $y = \sqrt[3]{x}$; $dy = 1/(3\sqrt[3]{x^2}) \cdot dx$ with $x = 8$ and $dx = 0.5$. Thus $dy = 1/24 \approx 1/25 = 0.04$; knowing $\sqrt[3]{8} = 2$, we have $\sqrt[3]{8.5} \approx 2.04$.
14. Use $y = \sin x$; $dy = \cos x \cdot dx$ with $x = \pi$ and $dx \approx -0.14$. Thus $dy = 0.14$; knowing $\sin \pi = 0$, we have $\sin 3 \approx 0.14$.
15. Use $y = \cos x$; $dy = -\sin x \cdot dx$ with $x = \pi/2 \approx 1.57$ and $dx \approx -0.07$. Thus $dy = 0.07$; knowing $\cos \pi/2 = 0$, we have $\cos 1.5 \approx 0.07$.
16. Use $y = e^x$; $dy = e^x \cdot dx$ with $x = 0$ and $dx = 0.1$. Thus $dy = 0.1$; knowing $e^0 = 1$, we have $e^{0.1} \approx 1.1$.
17. $dy = (2x + 3)dx$
18. $dy = (7x^6 - 5x^4)dx$
19. $dy = \frac{-2}{4x^3}dx$
20. $dy = 2(2x + \sin x)(2 + \cos x)dx$
21. $dy = \left(2xe^{3x} + 3x^2e^{3x}\right)dx$
22. $dy = \frac{-16}{x^5}dx$
23. $dy = \frac{2(\tan x+1) - 2x\sec^2 x}{(\tan x+1)^2}dx$
24. $dy = \frac{1}{x}dx$
25. $dy = (e^x \sin x + e^x \cos x)dx$
26. $dy = (-\sin(\sin x)\cos x)dx$
27. $dy = \frac{1}{(x+2)^2}dx$
28. $dy = ((\ln 3)3^x \ln x + \frac{3^x}{x})dx$
29. $dy = (\ln x)dx$
30. $dV = \pm 0.157$
31. (a) ± 12.8 feet
    (b) ± 32 feet
32. $\pm 15\pi/8 \approx \pm 5.89$in^2
33. ± 48in^2, or 1/3ft^2
34. (a) 297.8 feet
    (b) ± 62.3 ft
    (c) ± 20.9%
35. (a) 298.8 feet
    (b) ± 17.3 ft
    (c) ± 5.8%
36. (a) 298.9 feet
    (b) ± 8.67 ft
    (c) ± 2.9%
37. The isosceles triangle setup works the best with the smallest percent error.
38. 1%

Exercises 4.4

1. F
2. F
3. $x_0 = 1.5$, $x_1 = 1.5709148$, $x_2 = 1.5707963$, $x_3 = 1.5707963$, $x_4 = 1.5707963$, $x_5 = 1.5707963$
4. $x_0 = 1$, $x_1 = -0.55740772$, $x_2 = 0.065936452$, $x_3 = -0.000095721919$, $x_4 = 2.9235662 * 10^{-13}$, $x_5 = 0$
5. $x_0 = 0$, $x_1 = 2$, $x_2 = 1.2$, $x_3 = 1.0117647$, $x_4 = 1.0000458$, $x_5 = 1$
6. $x_0 = 1.5$, $x_1 = 1.4166667$, $x_2 = 1.4142157$, $x_3 = 1.4142136$, $x_4 = 1.4142136$, $x_5 = 1.4142136$
7. $x_0 = 2$, $x_1 = 0.6137056389$, $x_2 = 0.9133412072$, $x_3 = 0.9961317034$, $x_4 = 0.9999925085$, $x_5 = 1$
8. roots are: $x = -5.156$, $x = -0.369$ and $x = 0.525$
9. roots are: $x = -3.714$, $x = -0.857$, $x = 1$ and $x = 1.571$
10. roots are: $x = -1.013$, $x = 0.988$, and $x = 1.393$
11. roots are: $x = -2.165$, $x = 0$, $x = 0.525$ and $x = 1.813$
12. $x = \pm 0.824$,
13. $x = -0.637$, $x = 1.410$
14. $x = \pm 0.743$
15. $x = \pm 4.493$, $x = 0$
16. The approximations alternate between $x = 1$ and $x = 2$.
17. The approximations alternate between $x = 1$, $x = 2$ and $x = 3$.
18. $f(x) = x^2 - 16.5$ and $x_0 = 4$ yield $x_1 = \frac{65}{16} = 4.0625$ and $x_2 = \frac{8449}{2080} \approx 4.0620192$.
19. $f(x) = x^2 - 24$ and $x_0 = 5$ yield $x_1 = \frac{49}{10} = 4.9$ and $x_2 = \frac{4801/980}{\approx}4.898980$.
20. $f(x) = x^3 - 63$ and $x_0 = 4$ yield $x_1 = \frac{191}{48} \approx 3.97916667$ and $x_2 \approx 3.9790572$.

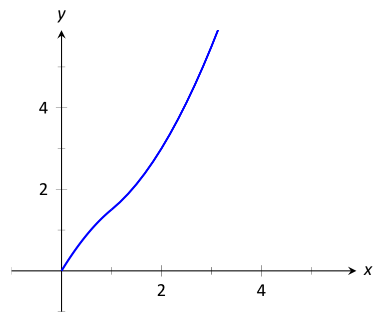
21. $f(x) = x^3 - 8.5$ and $x_0 = 2$ yield $x_1 = \frac{49}{24} \approx 2.0416667$ and $x_2 \approx 2.0408279$.

22. (a) $x_n \to -\infty$
 (b) x_1 is undefined
 (c) $x_n \to 2$
 (d) x_1 is undefined
 (e) $x_n \to 6$

Chapter 5

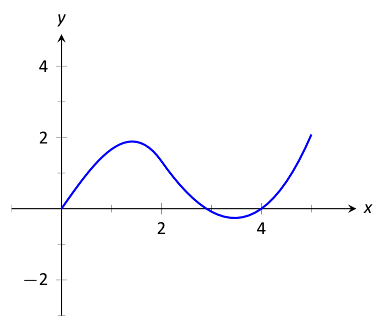
Exercises 5.1

1. Answers will vary.
2. "an"
3. Answers will vary.
4. opposite; opposite
5. Answers will vary.
6. velocity
7. velocity
8. $3/4x^4 + C$
9. $1/9x^9 + C$
10. $10/3x^3 - 2x + C$
11. $t + C$
12. $s + C$
13. $-1/(3t) + C$
14. $-3/(t) + C$
15. $2\sqrt{x} + C$
16. $\tan\theta + C$
17. $-\cos\theta + C$
18. $\sec x - \csc x + C$
19. $5e^{\theta} + C$
20. $3^t/\ln 3 + C$
21. $\frac{5^t}{2\ln 5} + C$
22. $4/3t^3 + 6t^2 + 9t + C$
23. $t^6/6 + t^4/4 - 3t^2 + C$
24. $x^6/6 + C$
25. $e^{\pi}x + C$
26. $ax + C$
27. $-x^{-3} + C$
28. $\frac{4}{3}x^3 + \frac{7}{2}x^{-2} + C$
29. $\frac{2}{9}x^{9/2} + C$
30. $\frac{2}{7}x^{7/2} - \frac{14}{3}x^{3/2} + C$
31. (a) $x > 0$
 (b) $1/x$
 (c) $x < 0$
 (d) $1/x$
 (e) $\ln|x| + C$. Explanations will vary.
32. $-\cos x + 3$
33. $5e^x + 5$
34. $x^4 - x^3 + 7$
35. $\tan x + 4$
36. $7^x/\ln 7 + 1 - 49/\ln 7$
37. $5/2x^2 + 7x + 3$
38. $\frac{7x^3}{6} - \frac{9x}{2} + \frac{40}{3}$
39. $5e^x - 2x$
40. $\theta - \sin(\theta) - \pi + 4$
41. $\frac{2x^4\ln^2(2) + 2^x + x\ln 2(\ln 32 - 1) + \ln^2(2)\cos(x) - 1 - \ln^2(2)}{\ln^2(2)}$
42. $3x - 2$
43. $x^{-2} + 1$
44. $2\sqrt{x} - 4$
45. $s(t) = 2t^{3/2}$.
46. $s(t) = 241.67 - 16t^2$ ft, so $s(t) = 0$ at $t = 3.89$sec.
47.



Other antiderivatives are vertical shifts of this one.

48.



Other antiderivatives are vertical shifts of this one.

49. No answer provided.
50. $dy = (2xe^x\cos x + x^2e^x\cos x - x^2e^x\sin x)dx$

Exercises 5.2

1. Answers will vary.
2. Answers will vary.
3. 0
4. $\int 0^2(2x + 3)\,dx$
5. (a) 3
 (b) 4
 (c) 3
 (d) 0
 (e) -4
 (f) 9
6. (a) -4

(b) -5
(c) -3
(d) 1
(e) -2
(f) 10

7. (a) 4
(b) 2
(c) 4
(d) 2
(e) 1
(f) 2

8. (a) $-1/2$
(b) 0
(c) $3/2$
(d) $3/2$
(e) $9/2$
(f) $15/2$

9. (a) π
(b) π
(c) 2π
(d) 10π

10. (a) -59
(b) -48
(c) -27
(d) -33
(e) 70
(f) 91

11. (a) $4/\pi$
(b) $-4/\pi$
(c) 0
(d) $2/\pi$
(e) $4/\pi$
(f) $8/\pi$

12. (a) 4
(b) 4
(c) -4
(d) -2
(e) 6
(f) 2

13. (a) $40/3$
(b) $26/3$
(c) $8/3$
(d) $38/3$

14. (a) 2ft/s
(b) 2ft
(c) 1.5ft

15. (a) 3ft/s
(b) 9.5ft
(c) 9.5ft

16. (a) 64ft/s
(b) 64ft
(c) $t = 2$
(d) $t = 2 + \sqrt{7} \approx 4.65$ seconds

17. (a) 96ft/s
(b) 6 seconds
(c) 6 seconds
(d) Never; the maximum height is 208ft.

18. 2
19. 5
20. 16
21. Answers can vary; one solution is $a = -2$, $b = 7$
22. 24
23. -7
24. -7
25. Answers can vary; one solution is $a = -11$, $b = 18$
26. $1/4x^4 - 2/3x^3 + 7/2x^2 - 9x + C$
27. $-\cos x - \sin x + \tan x + C$
28. $3/4t^{4/3} - 1/t + 2^t/\ln 2 + C$
29. $\ln|x| + \csc x + C$

Exercises 5.3

1. limits
2. 14
3. Rectangles.
4. T
5. $2^2 + 3^2 + 4^2 = 29$
6. $-6 - 2 + 2 + 6 + 10 = 10$
7. $0 - 1 + 0 + 1 + 0 = 0$
8. $1 + 1/2 + 1/3 + 1/4 + 1/5 = 137/60$
9. $-1 + 2 - 3 + 4 - 5 + 6 = 3$
10. $1/2 + 1/6 + 1/12 + 1/20 = 4/5$
11. $1 + 1 + 1 + 1 + 1 + 1 = 6$
12. Answers may vary; $\sum_{i=1}^{5} 3i$
13. Answers may vary; $\sum_{i=0}^{8} (i^2 - 1)$
14. Answers may vary; $\sum_{i=1}^{4} \frac{i}{i+1}$
15. Answers may vary; $\sum_{i=0}^{4} (-1)^i e^i$
16. 325
17. 1045
18. 28,650
19. -8525
20. 2050
21. 5050
22. 2870
23. 155
24. $91, 225$
25. 24
26. $11, 700$
27. 19
28. $59/8$

29. $\pi/3 + \pi/(2\sqrt{3}) \approx 1.954$

30. 8.16986

31. 0.388584

32. $496/315 \approx 1.5746$

33. (a) Exact expressions will vary; $\frac{(1+n)^2}{4n^2}$.
    (b) $121/400$, $10201/40000$, $1002001/4000000$
    (c) $1/4$

34. (a) Exact expressions will vary; $2 + 4/n^2$.
    (b) $51/25$, $5001/2500$, $500001/250000$
    (c) 2

35. (a) 8.
    (b) 8, 8, 8
    (c) 8

36. (a) Exact expressions will vary; $20/3 - 96/(3n) + 64/(3n^2)$.
    (b) $92/25$, $3968/625$, $103667/15625$
    (c) $20/3$

37. (a) Exact expressions will vary; $100 - 200/n$.
    (b) 80, 98, $499/5$
    (c) 100

38. (a) Exact expressions will vary; $-1/12(1 - 1/n^2)$.
    (b) $-33/400$, $-3333/40000$, $-333333/4000000$
    (c) $-1/12$

39. (a) Exact expressions will vary; 80.5.
    (b) 72.25
    (c) 62.5

40. (a) $(5\text{ s})((0+6+14+23+30+36)\text{ mph}) = 545\frac{\text{mi s}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times 5280\text{ ft}1\text{ mi} = 799\text{ ft}$
    (b) $(5\text{ s})((6+14+23+30+36+40)\text{ mph}) = 585\frac{\text{mi s}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times 5280\text{ ft}1\text{ mi} = 858\text{ ft}$

41.
$$\begin{aligned}\int_a^b k\cdot f(x)\,dx &= \lim_{n\to\infty}\sum_{i=1}^n k\cdot f(c_i)\Delta x \quad \text{T36.2}\\ &= \lim_{n\to\infty} k\cdot\sum_{i=1}^n k\cdot f(c_i)\Delta x \quad \text{T35.3}\\ &= k\cdot\lim_{n\to\infty}\sum_{i=1}^n k\cdot f(c_i)\Delta x \quad \text{T1.4}\\ &= k\int_a^b f(x)\,dx \quad \text{T36.2}\end{aligned}$$

42. Let f and M be as given.
$$\begin{aligned}\int_a^b f(x)\,dx &= \lim_{n\to\infty}\sum_{i=1}^n f(c_i)\Delta x \quad \text{T36.2}\\ &\leq \lim_{n\to\infty}\sum_{i=1}^n M\Delta x\\ &= \int_a^b M\,dx \quad \text{T36.2}\\ &= M(b-a)\end{aligned}$$

43. $F(x) = 5\tan x + 4$

44. $F(x) = 7\ln|x| + 14$

45. $G(t) = 4/6t^6 - 5/4t^4 + 8t + 9$

46. $G(t) = \sin t - \cos t - 78$

47. $F(x) = 2\sqrt{x} - \pi$

Exercises 5.4

1. Answers will vary.
2. 0
3. T
4. Answers will vary.
5. 20
6. $28/3$
7. 0
8. 1
9. 1
10. 1
11. $(5 - 1/5)/\ln 5$
12. $23/2$
13. -4
14. $e^3 - e$
15. $16/3$
16. 4
17. $45/4$
18. $\ln 2$
19. $1/2$
20. $3/8$
21. $1/2$
22. $1/3$
23. $1/4$
24. $1/101$
25. 8
26. 15
27. 0
28. $2 - 2/\sqrt{3}$
29. 2
30. $\frac{7}{2}$
31. Explanations will vary. A sketch will help.
32. $c = 2/\sqrt{3}$
33. $c = \pm 2/\sqrt{3}$
34. $c = \ln(e-1) \approx 0.54$
35. $c = 64/9 \approx 7.1$
36. $2/\pi$
37. $2/pi$
38. 2
39. $16/3$
40. 16
41. $1/(e-1)$
42. (a) -300ft; (b) 312.5ft
43. (a) 400ft; (b) 850ft

44. (a) -1ft; (b) 3ft

45. (a) $128/5$ft; (b) same

46. -64ft/s

47. 50ft/s

48. 2ft/s

49. 0ft/s

50. $F'(x) = (3x^2+1)\frac{1}{x^3+x}$

51. $F'(x) = 3x^{11}$

52. $F'(x) = 2x(x^2+2) - (x+2)$

53. $F'(x) = e^x \sin(e^x) - 1/x \sin(\ln x)$

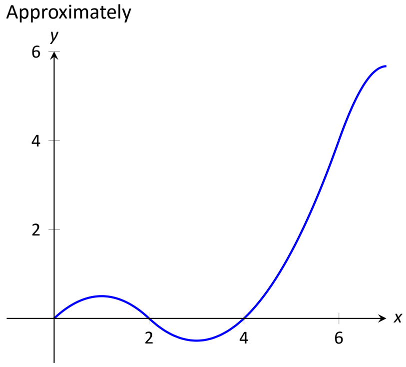
54. (a)

x	0	1	2	3	4	5	6
g(x)	0	.5	0	-.5	0	1.5	4

(b) $g(7) \approx 5.7$

(c) min at $x = 3$; max at $x = 7$

(d) Approximately



Exercises 5.5

1. Chain Rule.

2. T

3. $\frac{1}{8}(x^3-5)^8 + C$

4. $\frac{1}{4}(x^2-5x+7)^4 + C$

5. $\frac{1}{18}\left(x^2+1\right)^9 + C$

6. $\frac{1}{3}(3x^2+7x-1)^6 + C$

7. $\frac{1}{2}\ln|2x+7| + C$

8. $\sqrt{2x+3} + C$

9. $\frac{2}{3}(x+3)^{3/2} - 6(x+3)^{1/2} + C = \frac{2}{3}(x-6)\sqrt{x+3} + C$

10. $\frac{2}{21}x^{3/2}\left(3x^2-7\right) + C$

11. $2e^{\sqrt{x}} + C$

12. $\frac{2\sqrt{x^5+1}}{5} + C$

13. $-\frac{1}{2x^2} - \frac{1}{x} + C$

14. $\frac{\ln^2(x)}{2} + C$

15. $\frac{\sin^3(x)}{3} + C$

16. $-\frac{1}{6}\sin(3-6x) + C$

17. $-\tan(4-x) + C$

18. $\frac{1}{2}\ln|\sec(2x)+\tan(2x)| + C$

19. $\frac{\tan^3(x)}{3} + C$

20. $\frac{\sin(x^2)}{2} + C$

21. The key is to rewrite $\cot x$ as $\cos x/\sin x$, and let $u = \sin x$.

22. The key is to multiply $\csc x$ by 1 in the form $(\csc x + \cot x)/(\csc x + \cot x)$.

23. $\frac{1}{3}e^{3x-1} + C$

24. $\frac{e^{x^3}}{3} + C$

25. $\frac{1}{2}e^{(x-1)^2} + C$

26. $x - e^{-x} + C$

27. $\frac{e^{-3x}}{3} - e^{-x} + C$

28. $\frac{1}{2}\ln^2(x) + C$

29. $\frac{\left(\ln x\right)^3}{3} + C$

30. $\frac{1}{6}\ln^2\left(x^3\right) + C$

31. $\frac{1}{2}\ln\left(\ln\left(x^2\right)\right) + C$

32. $-\frac{1}{3(x^3+3)} + C$

33. $\frac{1}{45}(5x^3+5x^2+2)^9 + C$

34. $-\sqrt{1-x^2} + C$

35. $-\frac{1}{3}\cot\left(x^3+1\right) + C$

36. $-\frac{2}{3}\cos^{\frac{3}{2}}(x) + C$

37. $\ln|x-5| + C$

38. $\frac{7}{3}\ln|3x+2| + C$

39. $\ln\left|x^2+7x+3\right| + C$

40. $3\ln\left|3x^2+9x+7\right| + C$

41. $3\sqrt{x^2-2x-6} + C$

42. $\sqrt{x^2-6x+8} + C$

43. $2\sin\sqrt{x} + C$

44. $\frac{1}{2}\sec^2\theta + C$ or $\frac{1}{2}\tan^2\theta + C$

45. $-\ln 2$

46. $352/15$

47. $2/3$

48. $1/5$

49. $(1-e)/2$

50. $e-1$

51. 0

52. $\ln\left(\frac{4}{1+e}\right)$

53. $\frac{15}{392}$

54. $\frac{2}{3}$

55. $\frac{1}{12}$

Chapter 6

Exercises 6.1

1. T

2. T

3. Answers will vary.

4. $4\pi + \pi^2 \approx 22.436$

5. $16/3$
6. π
7. π
8. $1/2$
9. $2\sqrt{2}$
10. 4.5
11. $4/3$
12. $2 - \pi/2$
13. 8
14. $1/6$
15. $37/12$
16. On regions such as $[\pi/6, 5\pi/6]$, the area is $3\sqrt{3}/2$. On regions such as $[-\pi/2, \pi/6]$, the area is $3\sqrt{3}/4$.
17. 1
18. $5/3$
19. $9/2$
20. $9/4$
21. $1/12(9 - 2\sqrt{2}) \approx 0.514$
22. 1
23. 5
24. 4
25. 133/20

Exercises 6.2

1. T
2. Answers will vary.
3. Recall that "dx" does not just "sit there;" it is multiplied by $A(x)$ and represents the thickness of a small slice of the solid. Therefore dx has units of in, giving $A(x)\ dx$ the units of in^3.
4. $48\pi\sqrt{3}/5$ units^3
5. $175\pi/3$ units^3
6. $\pi/6$ units^3
7. $\dfrac{768\pi}{7}$
8. $9\pi/2$ units^3
9. $35\pi/3$ units^3
10. $2\pi/15$ units^3
11. $\dfrac{96\pi}{5}$
12. (a) $\pi/2$
    (b) $5\pi/6$
    (c) $4\pi/5$
    (d) $8\pi/15$
13. (a) $512\pi/15$
    (b) $256\pi/5$
    (c) $832\pi/15$
    (d) $128\pi/3$
14. (a) $4\pi/3$
    (b) $2\pi/3$
    (c) $4\pi/3$
    (d) $\pi/3$
15. (a) $104\pi/15$
    (b) $64\pi/15$
    (c) $32\pi/5$
16. (a) 8π
    (b) 8π
    (c) $16\pi/3$
    (d) $8\pi/3$
17. (a) $\frac{\pi^2}{8} + \frac{\pi}{4}$
    (b) $\frac{3\pi^2}{8} + \frac{\pi}{4} - \pi\sqrt{2}$
    (c) $\frac{\pi^2}{8} + \frac{\pi}{4} + \pi\sqrt{2}$
18. Placing the tip of the cone at the origin such that the x-axis runs through the center of the circular base, we have $A(x) = \pi x^2/4$. Thus the volume is $250\pi/3$ units^3.
19. The cross–sections of this cone are the same as the cone in Exercise 18. Thus they have the same volume of $250\pi/3$ units^3.
20. Orient the cone such that the tip is at the origin and the x-axis is perpendicular to the base. The cross–sections of this cone are right, isosceles triangles with side length $2x/5$; thus the cross–sectional areas are $A(x) = 2x^2/25$, giving a volume of $80/3$ units^3.
21. Orient the solid so that the x-axis is parallel to long side of the base. All cross–sections are trapezoids (at the far left, the trapezoid is a square; at the far right, the trapezoid has a top length of 0, making it a triangle). The area of the trapezoid at x is $A(x) = 1/2(-1/2x + 5 + 5)(5) = -5/4x + 25$. The volume is 187.5 units^3.

Exercises 6.3

1. T
2. F
3. F
4. T
5. $9\pi/2$ units^3
6. $70\pi/3$ units^3
7. $\dfrac{96\pi}{5}$
8. $2\pi/15$ units^3
9. $48\pi\sqrt{3}/5$ units^3
10. $350\pi/3$ units^3
11. $\dfrac{768\pi}{7}$
12. $\pi/6$ units^3
13. (a) $4\pi/5$
    (b) $8\pi/15$
    (c) $\pi/2$
    (d) $5\pi/6$
14. (a) $128\pi/3$
    (b) $128\pi/3$
    (c) $512\pi/15$
    (d) $256\pi/5$
15. (a) $4\pi/3$
    (b) $\pi/3$
    (c) $4\pi/3$

(d) $2\pi/3$

16. (a) $16\pi/3$
    (b) $8\pi/3$
    (c) 8π

17. (a) $16\pi/3$
    (b) $8\pi/3$
    (c) 8π
    (d) 8π

18. (a) Disk: $\pi\int_0^1 \left[1^2-(\sqrt[4]{y})^2\right]\,dy=\frac{\pi}{3}$
Shell: $2\pi\int_0^1 x\cdot x^4\,dx=\frac{\pi}{3}$
(b) Disk: $\pi\int_0^1 (x^4)^2\,dx=\frac{\pi}{9}$
Shell: $2\pi\int_0^1 y(1-\sqrt[4]{y})\,dy=\frac{\pi}{9}$.

19. (a) Disk: $\pi\int_1^2(\sqrt[3]{y-1})^2\,dy=\frac{3\pi}{5}$
Shell: $2\pi\int_0^1 x(2-(x^3+1))\,dx=\frac{3\pi}{5}$
(b) Disk: $\pi\int_0^1\left[2-(x^3+1)\right]^2\,dx=\frac{9\pi}{14}$
Shell: $2\pi\int_1^2(2-y)\sqrt[3]{y-1}\,dy=\frac{9\pi}{14}$.

20. (a) Disk: $\pi\int_{-2}^1\left[(-4x+8)^2-(4x^2)^2\right]\,dx=\frac{1152\pi}{5}$
Shell:
$2\pi\int_0^4 y\sqrt{y}\,dy+2\pi\int_4^{16} y\left[\left(2-\frac{y}{4}\right)+\frac{\sqrt{y}}{2}\right]\,dy=\frac{128\pi}{5}+\frac{1024\pi}{5}$
(b) Disk: $\pi\int_0^4\left[1+\frac{\sqrt{y}}{2}\right]^2-\left[1-\frac{\sqrt{y}}{2}\right]^2\,dy+$
$\pi\int_4^{16}\left[1+\frac{\sqrt{y}}{2}\right]^2-\left[1-\left(2-\frac{y}{4}\right)\right]^2\,dy=\frac{32\pi}{3}+\frac{130\pi}{3}$
Shell: $2\pi\int_{-2}^1(1-x)\left[(-4x+8)-4x^2\right]\,dx=54\pi$
(c) Disk: $\pi\int_{-2}^1\left[(16-4x^2)^2-(16-(-4x+8))^2\right]\,dx=\frac{1728\pi}{5}$
Shell:
$2\pi\int_0^4(16-y)\left[\sqrt{y}\right]\,dy+2\pi\int_4^{16}(16-y)\left[\left(2-\frac{y}{4}\right)+\frac{\sqrt{y}}{2}\right]\,dy=\frac{2176\pi}{15}+\frac{3008\pi}{15}$.

Exercises 6.4

1. In SI units, it is one Joule, i.e., one Newton–meter, or kg·m/s^2·m. In Imperial Units, it is ft–lb.
2. The same.
3. Smaller.
4. (a) 500 ft–lb
   (b) $100-50\sqrt{2}\approx 29.29$ ft–lb
5. (a) 2450 J
   (b) 1568 J
6. (a) $\frac{1}{2}\cdot d\cdot l^2$ ft–lb
   (b) 75 %
   (c) $\ell(1-\sqrt{2}/2)\approx 0.2929\ell$
7. 735 J
8. (a) 756 ft–lb
   (b) 60,000 ft–lb
   (c) Yes, for the cable accounts for about 1% of the total work.
9. 11,100 ft–lb
10. 575 ft–lb
11. 125 ft–lb
12. 0.05 J
13. 12.5 ft–lb
14. 5/3 ft–lb
15. 0.2625 = 21/80 J
16. $f\cdot d/2$ J
17. 45 ft–lb
18. 5 ft–lb
19. 953, 284 J
20. (a) 52,929.6 ft–lb
    (b) 18,525.3 ft–lb
    (c) When 3.83 ft of water have been pumped from the tank, leaving about 2.17 ft in the tank.
21. 192,767 ft–lb. Note that the tank is oriented horizontally. Let the origin be the center of one of the circular ends of the tank. Since the radius is 3.75 ft, the fluid is being pumped to $y=4.75$; thus the distance the gas travels is $h(y)=4.75-y$. A differential element of water is a rectangle, with length 20 and width $2\sqrt{3.75^2-y^2}$. Thus the force required to move that slab of gas is $F(y)=40\cdot 45.93\cdot\sqrt{3.75^2-y^2}dy$. Total work is $\int_{-3.75}^{3.75}40\cdot 45.93\cdot(4.75-y)\sqrt{3.75^2-y^2}\,dy$. This can be evaluated without actual integration; split the integral into $\int_{-3.75}^{3.75}40\cdot 45.93\cdot(4.75)\sqrt{3.75^2-y^2}\,dy+\int_{-3.75}^{3.75}40\cdot 45.93\cdot(-y)\sqrt{3.75^2-y^2}\,dy$. The first integral can be evaluated as measuring half the area of a circle; the latter integral can be shown to be 0 without much difficulty. (Use substitution and realize the bounds are both 0.)
22. 212,135 ft–lb
23. (a) approx. 577,000 J
    (b) approx. 399,000 J
    (c) approx. 110,000 J (By volume, half of the water is between the base of the cone and a height of 3.9685 m. If one rounds this to 4 m, the work is approx 104,000 J.)
24. 187,214 ft–lb
25. 617,400 J
26. 4,917,150 J