Exercises 0.0 (solutions)

Terms and Concepts

06 01 ex 01 1. Substitution "undoes" what derivative rule?

06 01 ex 02 2. T/F: One can sometimes use algebra to rewrite the integrand of an integral to make it easier to evaluate.

Problems

06 01 exset 01 **In Exercises 3 – 49, evaluate the indefinite integral.**

06 01 ex 03 3. $\int 3x^2\left(x^3-5\right)^7\,dx$

06 01 ex 04 4. $\int (2x-5)\left(x^2-5x+7\right)^3\,dx$

06 01 ex 05 5. $\int x\left(x^2+1\right)^8\,dx$

06 01 ex 06 6. $\int (12x+14)\left(3x^2+7x-1\right)^5\,dx$

06 01 ex 11 7. $\int \frac{1}{2x+7}dx$

06 01 ex 12 8. $\int \frac{1}{\sqrt{2x+3}}dx$

06 01 ex 13 9. $\int \frac{x}{\sqrt{x+3}}dx$

06 01 ex 17 10. $\int \frac{x^3-x}{\sqrt{x}}dx$

06 01 ex 18 11. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

06 01 ex 19 12. $\int \frac{x^4}{\sqrt{x^5+1}}dx$

06 01 ex 20 13. $\int \frac{\frac{1}{x}+1}{x^2}dx$

06 01 ex 21 14. $\int \frac{\ln(x)}{x}dx$

06 01 ex 08 15. $\int \sin^2(x)\cos(x)dx$

06 01 ex 09 16. $\int \cos(3-6x)dx$

06 01 ex 10 17. $\int \sec^2(4-x)dx$

06 01 ex 16 18. $\int \sec(2x)dx$

06 01 ex 22 19. $\int \tan^2(x)\sec^2(x)dx$

06 01 ex 25 20. $\int x\cos\left(x^2\right)\,dx$

06 01 ex 82 21. $\int \cot x\,dx$. Do not just refer to Theorem 2 for the answer; justify it through Substitution.

06 01 ex 83 22. $\int \csc x\,dx$. Do not just refer to Theorem 2 for the answer; justify it through Substitution.

06 01 ex 29 23. $\int e^{3x-1}dx$

06 01 ex 30 24. $\int e^{x^3}x^2dx$

06 01 ex 31 25. $\int e^{x^2-2x+1}(x-1)dx$

06 01 ex 32 26. $\int \frac{e^x+1}{e^x}dx$

06 01 ex 33 27. $\int \frac{e^x-e^{-x}}{e^{2x}}dx$

06 01 ex 36 28. $\int \frac{\ln x}{x}dx$

06 01 ex 37 29. $\int \frac{\left(\ln x\right)^2}{x}dx$

06 01 ex 38 30. $\int \frac{\ln\left(x^3\right)}{x}dx$

06 01 ex 39 31. $\int \frac{1}{x\ln\left(x^2\right)}dx$

06 01 ex 14 32. $\int \frac{x^2}{\left(x^3+3\right)^2}dx$

06 01 ex 07 33. $\int \left(3x^2+2x\right)\left(5x^3+5x^2+2\right)^8\,dx$

06 01 ex 15 34. $\int \frac{x}{\sqrt{1-x^2}}dx$

06 01 ex 26 35. $\int x^2\csc^2\left(x^3+1\right)\,dx$

06 01 ex 28 36. $\int \sin(x)\sqrt{\cos(x)}dx$

06 01 ex 23 37. $\int \frac{1}{x-5}dx$

06 01 ex 24 38. $\int \frac{7}{3x+2}dx$

06 01 ex 47 39. $\int \frac{2x+7}{x^2+7x+3}dx$

06 01 ex 48 40. $\int \frac{9(2x+3)}{3x^2+9x+7}dx$

06 01 ex 72 41. $\int \frac{3x-3}{\sqrt{x^2-2x-6}}dx$

06 01 ex 73 42. $\int \frac{x-3}{\sqrt{x^2-6x+8}}dx$

06 01 ex 84 43. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

06 01 ex 85 44. $\int \sec^2\theta\tan\theta\,d\theta$

06 01 exset 08 **In Exercises 45 – 55, evaluate the definite integral.**

06 01 ex 74 45. $\int_1^3 \frac{1}{x-5}dx$

06 01 ex 75 46. $\int_2^6 x\sqrt{x-2}dx$

06 01 ex 76 47. $\int_{-\pi/2}^{\pi/2} \sin^2 x\cos x\,dx$

06 01 ex 77 48. $\int_0^1 2x(1-x^2)^4\,dx$

06 01 ex 78 49. $\int_{-2}^{-1} (x+1)e^{x^2+2x+1}\,dx$

06 01 ex 86 50. $\int_0^{\pi/4} e^{\tan x}\sec^2 x\,dx$

06 01 ex 87 51. $\int_{-1}^{1} \frac{x}{1+x^2}\,dx$

06 01 ex 88 52. $\int_1^{\ln 3} \frac{e^x}{1+e^x}\,dx$

06 01 ex 89 53. $\int_0^1 \frac{2x^2+1}{(2x^3+3x+2)^3}\,dx$

06 01 ex 90 54. $\int_{-1}^{2} \frac{x}{\sqrt{x+2}}\,dx$

06 01 ex 91 55. $\int_0^{\frac{\pi}{4}} \cos^5(2x)\sin(2x)\,dx$

06 01 exset 02 **Exercises 56 – 91 were removed from 6.1 and are free for the taking.**

06 01 ex 27 56. $\int \tan^2(x)dx$

06 01 ex 34 57. $\int 3^{3x}dx$

06 01 ex 35 58. $\int 4^{2x}dx$

06 01 ex 40 59. $\int \frac{x^2+3x+1}{x}dx$

06 01 ex 41 60. $\int \frac{x^3+x^2+x+1}{x}dx$

06 01 ex 42 61. $\int \frac{x^3-1}{x+1}dx$

06 01 ex 43 62. $\int \frac{x^2+2x-5}{x-3}dx$

06 01 ex 44 63. $\int \frac{3x^2-5x+7}{x+1}dx$

06 01 ex 45 64. $\int \frac{x^2+2x+1}{x^3+3x^2+3x}dx$

06 01 ex 50 65. $\int \frac{7}{x^2+7}dx$

06 01 ex 51 66. $\int \frac{3}{\sqrt{9-x^2}}dx$

06 01 ex 52 67. $\int \frac{14}{\sqrt{5-x^2}}dx$

06 01 ex 53 68. $\int \frac{2}{x\sqrt{x^2-9}}dx$

06 01 ex 54 69. $\int \frac{5}{\sqrt{x^4-16x^2}}dx$

06 01 ex 55 70. $\int \frac{x}{\sqrt{1-x^4}}dx$

06 01 ex 60 71. $\int \frac{1}{x^2-2x+8}dx$

06 01 ex 61 72. $\int \frac{2}{\sqrt{-x^2+6x+7}}dx$

06 01 ex 62 73. $\int \frac{3}{\sqrt{-x^2+8x+9}}dx$

06 01 ex 63 74. $\int \frac{5}{x^2+6x+34}dx$

06 01 ex 46 75. $\int \frac{3x^3+4x^2+2x-22}{x^2+3x+5}dx$

06 01 ex 49 76. $\int \frac{-x^3+14x^2-46x-7}{x^2-7x+1}dx$

06 01 ex 56 77. $\int \frac{x}{x^4+81}dx$

06 01 ex 57 78. $\int \frac{2}{4x^2+1}dx$

06 01 ex 58 79. $\int \frac{1}{x\sqrt{4x^2-1}}dx$

06 01 ex 59 80. $\int \frac{1}{\sqrt{16-9x^2}}dx$

06 01 ex 64 81. $\int \frac{3x-2}{x^2-2x+10}dx$

06 01 ex 65 82. $\int \frac{7-2x}{x^2+12x+61}dx$

06 01 ex 66 83. $\int \frac{x^2+5x-2}{x^2-10x+32}dx$

06 01 ex 67 84. $\int \frac{x^3}{x^2+9}dx$

06 01 ex 68 85. $\int \frac{x^3-x}{x^2+4x+9}dx$

06 01 ex 69 86. $\int \frac{\sin(x)}{\cos^2(x)+1}dx$

06 01 ex 70 87. $\int \frac{\cos(x)}{\sin^2(x)+1}dx$

06 01 ex 71 88. $\int \frac{\cos(x)}{1-\sin^2(x)}dx$

06 01 ex 79 89. $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$

06 01 ex 80 90. $\int_{2}^{4} \frac{1}{x^2-6x+10}\,dx$

06 01 ex 81 91. $\int_{1}^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$

A: Solutions To Selected Problems

06 01 ex 01 1. Chain Rule.

06 01 ex 02 2. T

06 01 ex 03 3. $\frac{1}{8}(x^3-5)^8+C$

06 01 ex 04 4. $\frac{1}{4}(x^2-5x+7)^4+C$

06 01 ex 05 5. $\frac{1}{18}\left(x^2+1\right)^9+C$

06 01 ex 06 6. $\frac{1}{3}(3x^2+7x-1)^6+C$

06 01 ex 11 7. $\frac{1}{2}\ln|2x+7|+C$

06 01 ex 12 8. $\sqrt{2x+3}+C$

06 01 ex 13 9. $\frac{2}{3}(x+3)^{3/2}-6(x+3)^{1/2}+C=\frac{2}{3}(x-6)\sqrt{x+3}+C$

06 01 ex 17 10. $\frac{2}{21}x^{3/2}\left(3x^2-7\right)+C$

06 01 ex 18 11. $2e^{\sqrt{x}}+C$

06 01 ex 19 12. $\frac{2\sqrt{x^5+1}}{5}+C$

06 01 ex 20 13. $-\frac{1}{2x^2}-\frac{1}{x}+C$

06 01 ex 21 14. $\frac{\ln^2(x)}{2}+C$

06 01 ex 08 15. $\frac{\sin^3(x)}{3}+C$

06 01 ex 09 16. $-\frac{1}{6}\sin(3-6x)+C$

06 01 ex 10 17. $-\tan(4-x)+C$

06 01 ex 16 18. $\frac{1}{2}\ln|\sec(2x)+\tan(2x)|+C$

06 01 ex 22 19. $\frac{\tan^3(x)}{3}+C$

06 01 ex 25 20. $\frac{\sin\left(x^2\right)}{2}+C$

06 01 ex 82 21. The key is to rewrite $\cot x$ as $\cos x/\sin x$, and let $u=\sin x$.

06 01 ex 83 22. The key is to multiply $\csc x$ by 1 in the form $(\csc x+\cot x)/(\csc x+\cot x)$.

06 01 ex 29 23. $\frac{1}{3}e^{3x-1}+C$

06 01 ex 30 24. $\frac{e^{x^3}}{3}+C$

06 01 ex 31 25. $\frac{1}{2}e^{(x-1)^2}+C$

06 01 ex 32 26. $x-e^{-x}+C$

06 01 ex 33 27. $\frac{e^{-3x}}{3}-e^{-x}+C$

06 01 ex 36 28. $\frac{1}{2}\ln^2(x)+C$

06 01 ex 37 29. $\frac{\left(\ln x\right)^3}{3}+C$

06 01 ex 38 30. $\frac{1}{6}\ln^2\left(x^3\right)+C$

06 01 ex 39 31. $\frac{1}{2}\ln\left(\ln\left(x^2\right)\right)+C$

06 01 ex 14 32. $-\frac{1}{3\left(x^3+3\right)}+C$

06 01 ex 07 33. $\frac{1}{45}(5x^3+5x^2+2)^9+C$

06 01 ex 15 34. $-\sqrt{1-x^2}+C$

06 01 ex 26 35. $-\frac{1}{3}\cot\left(x^3+1\right)+C$

06 01 ex 28 36. $-\frac{2}{3}\cos^{\frac{3}{2}}(x)+C$

06 01 ex 23 37. $\ln|x-5|+C$

06 01 ex 24 38. $\frac{7}{3}\ln|3x+2|+C$

06 01 ex 47 39. $\ln\left|x^2+7x+3\right|+C$

06 01 ex 48 40. $3\ln\left|3x^2+9x+7\right|+C$

06 01 ex 72 41. $3\sqrt{x^2-2x-6}+C$

06 01 ex 73 42. $\sqrt{x^2-6x+8}+C$

06 01 ex 84 43. $2\sin\sqrt{x}+C$

06 01 ex 85 44. $\frac{1}{2}\sec^2\theta+C$ or $\frac{1}{2}\tan^2\theta+C$

06 01 ex 74 45. $-\ln 2$

06 01 ex 75 46. $352/15$

06 01 ex 76 47. $2/3$

06 01 ex 77 48. $1/5$

06 01 ex 78 49. $(1-e)/2$

06 01 ex 86 50. $e-1$

06 01 ex 87 51. 0

06 01 ex 88 52. $\ln\left(\frac{4}{1+e}\right)$

06 01 ex 89 53. $\frac{15}{392}$

06 01 ex 90 54. $\frac{2}{3}$

06 01 ex 91 55. $\frac{1}{12}$

06 01 ex 27 56. $\tan(x)-x+C$

06 01 ex 34 57. $\frac{27^x}{\ln 27}+C$

06 01 ex 35 58. $\frac{16^x}{\ln(16)}+C$

06 01 ex 41 59. $\frac{x^2}{2}+3x+\ln|x|+C$

06 01 ex 42 60. $\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x|+C$

06 01 ex 43 61. $\frac{x^3}{3}-\frac{x^2}{2}+x-2\ln|x+1|+C$

06 01 ex 44 62. $\frac{1}{2}\left(x^2+10x+20\ln|x-3|\right)+C$

06 01 ex 45 63. $\frac{3}{2}x^2-8x+15\ln|x+1|+C$

06 01 ex 50 64. $\frac{1}{3}\ln\left|x^2+3x+3\right|+\frac{\ln|x|}{3}+C$

06 01 ex 51 65. $\sqrt{7}\tan^{-1}\left(\frac{x}{\sqrt{7}}\right)+C$

06 01 ex 52 66. $3\sin^{-1}\left(\frac{x}{3}\right)+C$

06 01 ex 53 67. $14\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)+C$

06 01 ex 54 68. $\frac{2}{3}\sec^{-1}(|x|/3)+C$

06 01 ex 55 69. $\frac{5}{4}\sec^{-1}(|x|/4)+C$

06 01 ex 60 70. $\frac{1}{2}\sin^{-1}\left(x^2\right)+C$

06 01 ex 61 71. $\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{7}}\right)}{\sqrt{7}}+C$

06 01 ex 62 72. $2\sin^{-1}\left(\frac{x-3}{4}\right)+C$

06 01 ex 63 73. $3\sin^{-1}\left(\frac{x-4}{5}\right)+C$

06 01 ex 46 74. $\tan^{-1}\left(\frac{x+3}{5}\right)+C$

06 01 ex 46 75. $\frac{3x^2}{2}+\ln\left|x^2+3x+5\right|-5x+C$

06 01 ex 49 76. $-\frac{x^2}{2} + 2\ln\left|x^2 - 7x + 1\right| + 7x + C$

06 01 ex 56 77. $\frac{1}{18}\tan^{-1}\left(\frac{x^2}{9}\right) + C$

06 01 ex 57 78. $\tan^{-1}(2x) + C$

06 01 ex 58 79. $\sec^{-1}(|2x|) + C$

06 01 ex 59 80. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$

06 01 ex 64 81. $\frac{3}{2}\ln\left|x^2 - 2x + 10\right| + \frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right) + C$

06 01 ex 65 82. $\frac{19}{5}\tan^{-1}\left(\frac{x+6}{5}\right) - \ln\left|x^2 + 12x + 61\right| + C$

06 01 ex 66 83. $\frac{15}{2}\ln\left|x^2 - 10x + 32\right| + x + \frac{41\tan^{-1}\left(\frac{x-5}{\sqrt{7}}\right)}{\sqrt{7}} + C$

06 01 ex 67 84. $\frac{x^2}{2} - \frac{9}{2}\ln\left|x^2 + 9\right| + C$

06 01 ex 68 85. $\frac{x^2}{2} + 3\ln\left|x^2 + 4x + 9\right| - 4x + \frac{24\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)}{\sqrt{5}} + C$

06 01 ex 69 86. $-\tan^{-1}(\cos(x)) + C$

06 01 ex 70 87. $\tan^{-1}(\sin(x)) + C$

06 01 ex 71 88. $\ln|\sec x + \tan x| + C$ (integrand simplifies to $\sec x$)

06 01 ex 79 89. $\pi/2$

06 01 ex 80 90. $\pi/2$

06 01 ex 81 91. $\pi/6$