6.3 Additional Volume problems

Choose the appropriate method (disk, washer, shell) to find the volume of the solid generated by revolving the bounded region about the given axes. Set-up the integral, but do NOT integrate.

1. $y = x^4, y = 0, x = 1$ about

   y-axis - Solutions: V = $2\pi \int_0^1 x \cdot x^4 \, dx = 2\pi \int_0^1 x^5 \, dx$ or $V = \pi \int_0^1 \left[1^2 - (\sqrt[4]{y})^2\right] \, dy$

   x-axis - Solutions: V = $2\pi \int_0^1 y\left(1 - \sqrt[4]{y}\right) \, dx$ or $V = \pi \int_0^1 (x^4)^2 \, dy$

2. $y = x^3 + 1, x = 0, y = 2$ about

   y-axis - Solutions: V = $2\pi \int_0^1 x\left(2 - (x^3 + 1)\right) \, dx$ or $V = \pi \int_1^2 \left(\sqrt[3]{y-1}\right)^2 \, dy$

   $y = 2$ - Solutions: V = $2\pi \int_1^2 (2 - y)\sqrt[3]{y-1} \, dy$ or $V = \pi \int_0^1 \left[2 - (x^3 + 1)\right]^2 \, dx$

3. $y = 4x^2, 4x + y = 8$ about

   x-axis - Solution: $V = \pi \int_{-2}^1 \left[(-4x + 8)^2 - (4x^2)^2\right] \, dx$

   $x = 1$ - Solution: V = $2\pi \int_{-2}^1 (1 - x)\left[(-4x + 8) - 4x^2\right] \, dx$

   $y = 16$ - Solution: $V = \pi \int_{-2}^1 \left[(16 - 4x^2)^2 - \left(16 - (-4x + 8)\right)^2\right] \, dx$