Chapter 2: Derivatives

Section 2.5 The Chain Rule

All page numbers refer to original APEX text page numbers.

p. 97
After Chain Rule box We can think of this as taking the derivative of the outer function evaluated at the inner function times the derivative of the inner function. To help...Example 59.

pp. 98-99
Sentence right before Generalized Power Rule box add bracketed text:
"Example 60....$f(x) = x^n$ [and $y = f(g(x))$], then..."

Generalized Power Rule change restriction on n to any real number.

For problem before Example 61 add step to show CR details:
$y' = 20(3x^2 - 5x + 7 + \sin x)^{19} \cdot \dfrac{d}{dx}(3x^2 - 5x + 7 + \sin x)$

Similarly in Example 61 insert additional steps in the solutions.
For #1: insert $\cos(2x) \cdot \frac{d}{dx}(2x) =$
For #2: insert $= \frac{1}{4x^3-2x^2} \cdot \frac{d}{dx}(4x^3 - 2x^2)$
For #3: insert $= e^{x^2} \cdot \frac{d}{dx}(x^2)$

Example 62 solution in last line of text insert bracketed word: "...Thus the equation of the ...is [approximately]"

In line: $\dfrac{d}{dx}$ (ln(anything)) = ... use $\frac{d}{dx}$ instead of $'$ to indicate derivative.

p. 100
In Example 63 insert additional steps in the solutions.
For #1: insert $= x^5 \cdot \dfrac{d}{dx}(\sin 2x^3) + \dfrac{d}{dx}(x^5) \cdot \sin 2x^3 = x^5 \cdot [\cos 2x^3 \cdot \dfrac{d}{dx}(2x^3)] + 5x^4 \cdot \sin 2x^3$

For #2: insert $= \dfrac{e^{-x^2} \cdot \frac{d}{dx}(5x^3) - 5x^3 \frac{d}{dx} e^{-x^2}}{(e^{-x^2})^2} = \dfrac{e^{-x^2} \cdot 15x^2 - 5x^3 \cdot e^{-x^2} \cdot \frac{d}{dx}(-x^2)}{(e^{-x^2})^2} = ...$

pp. 101-102

Example 64 in paragraph below y': "The function is frankly...several ~~simple~~ small steps and be..."

Change Example 65 to

$$f(x) = \frac{x\cos(x^{-2}) - \sin^2(e^{4x})}{\ln x^2}$$

Tim: I think this solution will fit in the solution will fit in the space allowed with only one line in the numerator. Let me know if it doesn't.

$$= \frac{(\ln x^2)[-x(\sin x^{-2})(-2x^{-3}) + 1 \cdot (\cos(x^{-2})) - 2\sin e^{4x} \cos e^{4x} \cdot (4e^{4x})] - \frac{1}{x^2}(2x) \cdot [x\cos(x^{-2}) - \sin^2(e^{4x})]}{(\ln x^2)^2}$$

In paragraph below this derivative: "The reader is highly... ~~(I.e., the Quotient... term, etc.)~~ This example..."

pp. 102-103

Move example 66 through Theorem 20 to Calculus II somewhere - I think Ricard has this part.

I changed my mind about the use of "cancel" on this p. 103.
In paragraph: "Here the "fractional"... terms ~~cancel~~ divide out, leaving."
In paragraph: "It is important to realize that we are not ~~canceling~~ dividing these terms..."

Exercises

Cut a^x-type problems. This means # 19, 20, 22-25, 36b (I think I got that's all of them)

Add:
After current #11:
$p(x) = \left(x^2 - \frac{1}{x^2}\right)^6$

Answer: $p'(x) = 12\left(x^2 - \frac{1}{x^2}\right)^5\left(x + \frac{1}{x^3}\right)$

After current #15:
$g(x) = \tan^2 x - \tan(x^2)$

Answer: $g'(x) = 2(\tan x \sec^2 x - x \sec^2(x^2))$

$w(x) = \sec(e^{x^3})$

Answer: $w'(x) = 3x^2 e^{x^3}(\sec e^{x^3})(\tan e^{x^3})$

After current #21:
$r(x) = \frac{\sqrt{4x-3}}{x^2}$

Answer: $r'(x) = \frac{-6(x-1)}{x^3\sqrt{4x-3}}$

$f(x) = \frac{(3x^2-5)^4}{(2x^3-1)^2}$

Answer: $f'(x) = \frac{12x(2x^3-1)(3x^2-5)^3(x^2+5x-2)}{(2x^3-1)^4}$

$h(x) = [(2x+1)^{10}+1]^{10}$

Answer: $h'(x) = 200(2x+1)^9[(2x+1)10+1]^9$

$f(t) = \left[\left(1+\frac{1}{t}\right)^{-1}+1\right]^{-1}$

Answer: $\frac{-t^4}{(2t+1)(t+1)}$

$F(x) = 2x(2x+1)^2(2x+3)^3$

Answer: $F'(x) = 2(2x+1)(2x+3)^2(24x^2+26x+3)$

After current #28:

$a(t) = 7t^3 e^{\tan t^2}$

Answer: $a'(t) = 7t^2 e^{\tan(t^2)}(2t^2\sec^2(t^2)+3)$

$y = \sqrt{\sin(\cos^2 x)}$

Answer: $y' = \dfrac{-\cos x \sin x \cos(\cos^2 x)}{\sqrt{\sin(\cos^2 x)}}$

$k(x) = \cos(x \sin x^3)$

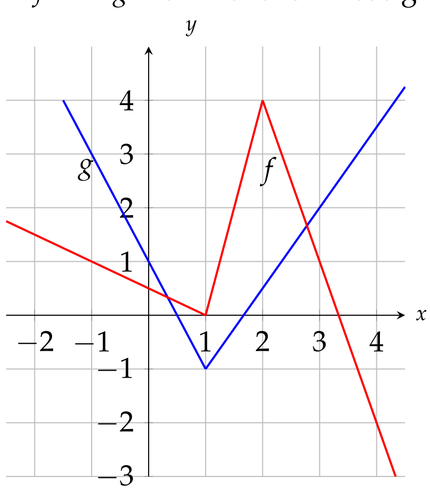
Answer: $k'(x) = -\sin(x\sin x^3)(3x^3\cos x^3 + \sin x^3)$

If $k(x) = f(g(x)$ with $f(2) = -4, g(2) = 2, f'(2) = 3,$ and $g'(2) = 5$. Find $k'(2)$.

Suppose $r(x) = f(g(h(x)))$, where $h(1) = 2, g(2) = 3, h'(1) = 3, g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.

Answer: 15

If f and g are functions whose graphs are shown, evaluate the expressions.



You can write this as 4 separately numbered problems, instead of 4 parts of one problem, if it works better that way.
(a) $(f \circ g)'(-1)$ (b) $(g \circ f)'(0)$ (c) $(g \circ g)'(-1)$ (d) $(f \circ f)'(4)$

Answers: (a) $(f \circ g)'(-1) = 6$ (b) $(g \circ f)'(0) = 1$ (c) $(g \circ g)'(-1) = -4$ (d) $(f \circ f)'(4) = 1.5$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	5	4	5
4	0	7	1	$\frac{1}{2}$
6	6	4	6	3

Use the given table of values for f, g, f', andg' to find
(a) $(f \circ g)'(6)$
(b) $(g \circ f)'(1)$
(b) $(g \circ g)'(6)$
(b) $(f \circ f)'(1)$

Answers: (a) $(f \circ g)'(6) = 12$ (b) $(g \circ f)'(1) = 2.5$ (c) $(g \circ g)'(6) = 9$ (d) $(f \circ f)'(1) = 35$

After current #34:
Use the Chain Rule to prove the following:
(a) The derivative of an even function is an odd function.
(b) The derivative of an odd function is an even function.

Use the Chain Rule and Product Rule to give an alternative proof of the Quotient Rule. (Hint: write $f(x)/g(x)$ as $f(x) \cdot [g(x)]^-1)$.

Use the Chain Rule to express the second derivative of $f(g(x))$ in terms of first and second derivatives of f and g.