Chapter 1

The Graphical Behavior of Functions

1.1 Extreme Values

Write the definition of **absolute minimum**.

Write the definition of **absolute maximum**.

Collectively, the absolute minimum and absolute maximum are called ____________________.

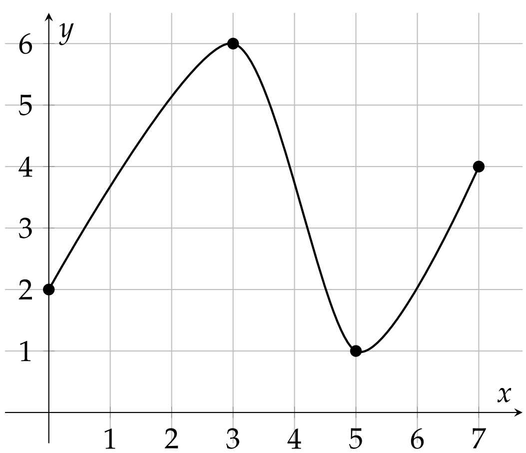
Write the **Extreme Value Theorem**.

The Extreme Value Theorem guarantees that a continuous function on a closed interval will have a maximum and a minimum value.

Write the definition of **relative minimum**.

Write the definition of **relative maximum**.

What is another name for a relative minimum? ____________________



On the graph to the left:

What is the absolute minimum value of f? ____________________

What is the absolute minimum value of f? ____________________

Is (3,6) a relative maximum? ____________________

Is (5,1) a relative minimum? ____________________

Is (7,4) a relative maximum? ____________________

Match the following statements with the correct graph below.

On each graph CLEARLY label where the relative max, relative min, absolute max, and absolute min are, if applicable.

Note: a point may be both a relative and absolute max or both a relative and absolute min!

a) Graph has no absolute or relative extrema.

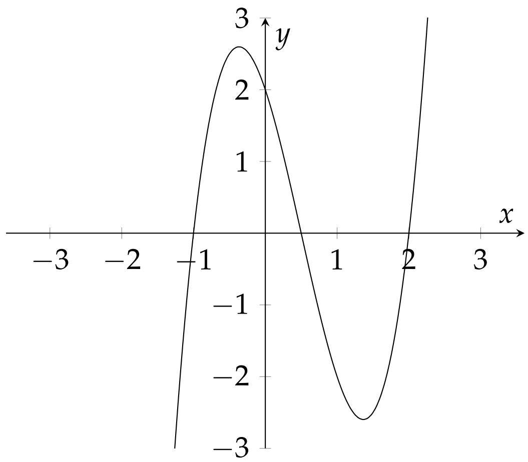
b) Graph has relative max and min, but no absolute extrema.

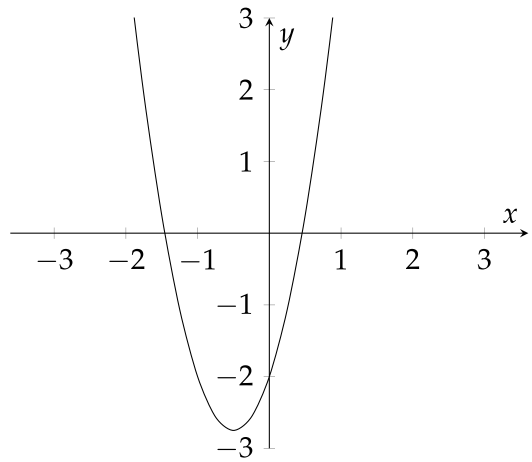
c) Graph has a relative max, relative min, absolute max, and absolute min.

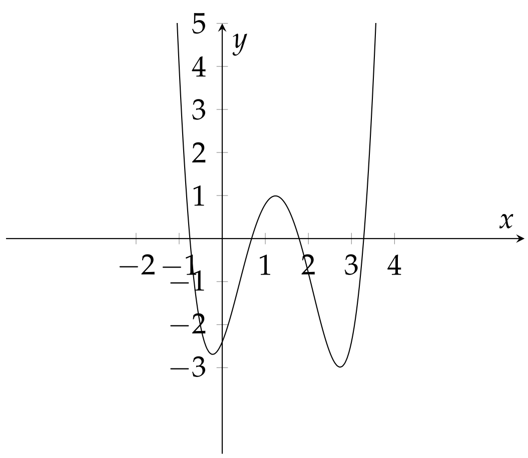
d) Graph has a relative min, absolute min, and absolute max.

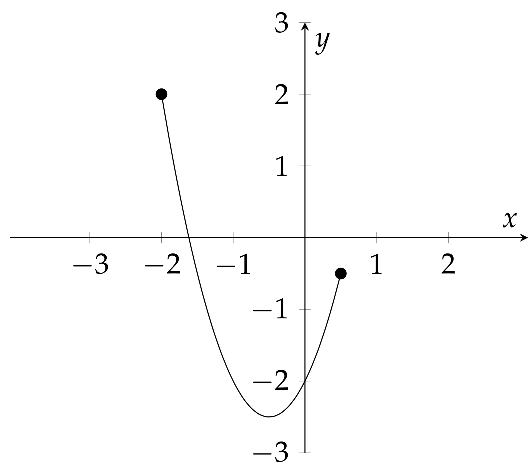
e) Graph has a relative and absolute min.

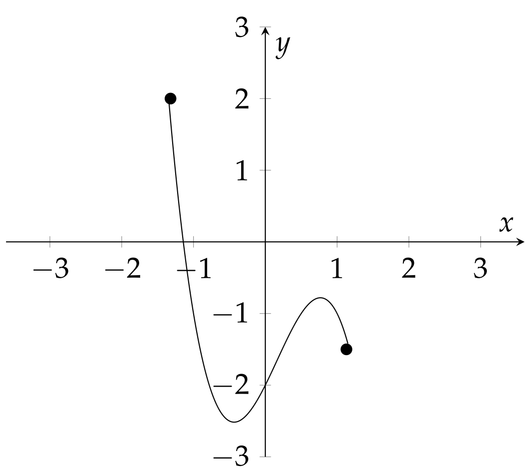
f) Graph has a relative min, relative max, and absolute max.

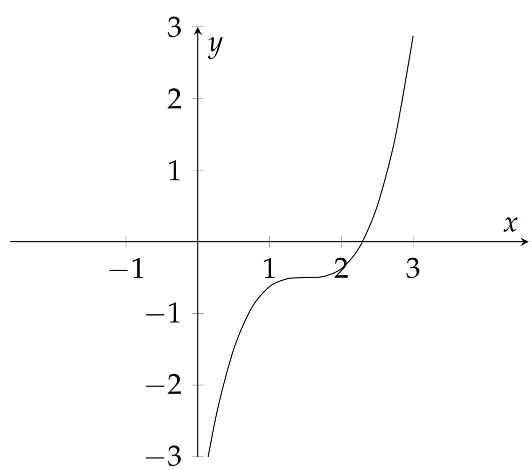












Write the definition of **critical number**.

Find the critical numbers of $f(x) = x^3 - 6x^2 - 15x + 7$

Use **Key Idea 4** to find the extreme values of $f(x) = 5 + 54x - 2x^3$ on the interval $[0, 4]$. Clearly show your work for each of the 4 steps given in Key Idea 4.

1.2 The Mean Value Theorem

Write the **Mean Value Theorem of Differentiation (MVT)**.

Let f be a function that satisfies the following conditions.

1.

2.

Then

Write **Rolle's Theorem**. Let f be a function that satisfies the following conditions.

1.

2.

3.

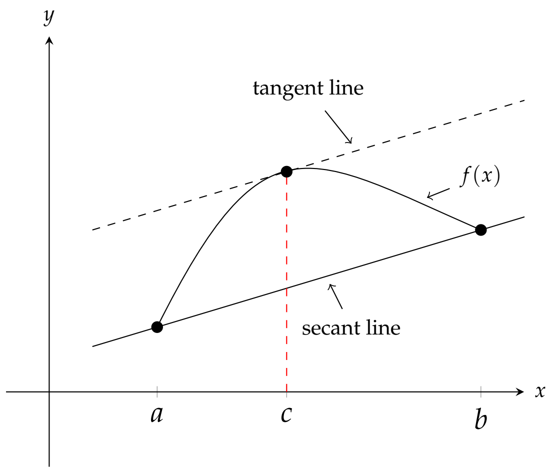
Then

Compare **MVT** and **Rolle's Theorem**.

What is the same for each theorem?

How do the theorems differ?

What is the conclusion of the MVT if $f(a) = f(b)$?



In your own words, use the graph above to explain the Mean Value Theorem.

I made a pretty graph so I want to use it!!! Is the above question good enough?
Should we be asking questions about the videos?

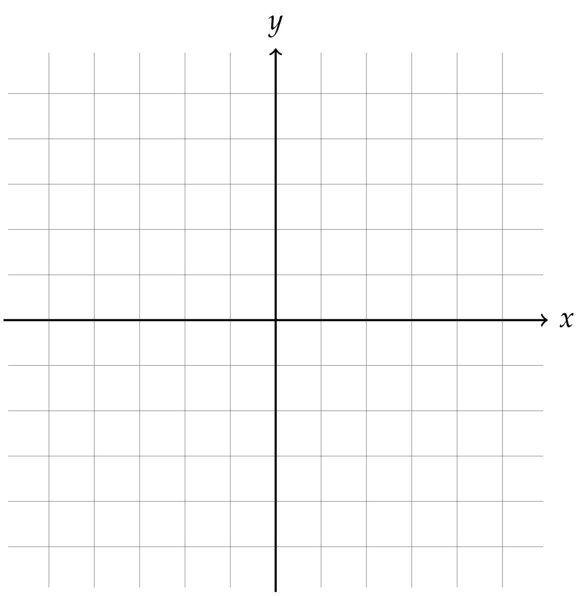
1.3 Increasing and Decreasing Functions

Write the definition of **increasing function**.

Write the definition of **decreasing function**.

Write the **Test for Increasing/Decreasing Functions**.

Graph the functions $f(x) = (x-1)^2 + 2$ and $f'(x) = 2(x-1) = 2x-2$ on the axes below.



1. On what interval is $f(x)$ increasing? ____

2. On what interval is $f'(x) > 0$? ____

3. On what interval is $f(x)$ decreasing? ____

4. On what interval is $f'(x) < 0$? ____

5. Do your results from 1-4 support or conflict with the Test for Increasing/Decreasing Functions? Explain.

State the **First Derivative Test**.

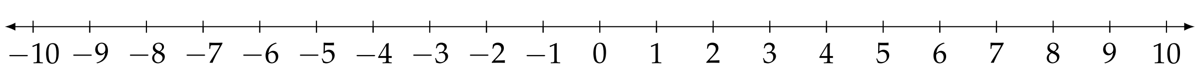
Look at your graph of $f(x) = (x-2)^2 + 2$ and $f'(x) = 2(x-1) = 2x - 2$ on the previous page.

1. At what x value does $f'(x)$ change signs?____________________
2. Does the function $f(x)$ have a relative maximum or minimum at this value?____________________

Complete the following steps to find the intervals on which $f(x) = x\sqrt{6-x}$ is increasing or decreasing.

1. Find the critical numbers of f.
   (a) To do this, determine where $f'(x) = 0$ or is undefined.
   (b) Show your work to find $f'(x) = \dfrac{-3x+12}{2\sqrt{6-x}}$.

   (c) The critical number(s) of f are ____________________
2. Split the domain of f, which is $(-\infty, 6]$, into intervals using the critical numbers of f.



3. Determine the sign of f' in each of the intervals above.
   NOTE: You should have OPEN intervals in each case.
   f will not be increasing or decreasing when f' is 0 or at an endpoint of the domain.

Interval	**Sign of f'**	**Behavior of f**

4. f is increasing on ____________________ and f is decreasing on ____________________.

5. Use the First Derivative Test and the information above to determine where f has a relative maximum or a relative minimum if applicable.

1.4 Concavity and the Second Derivative

State the definition of **Concave Up and Concave Down**.

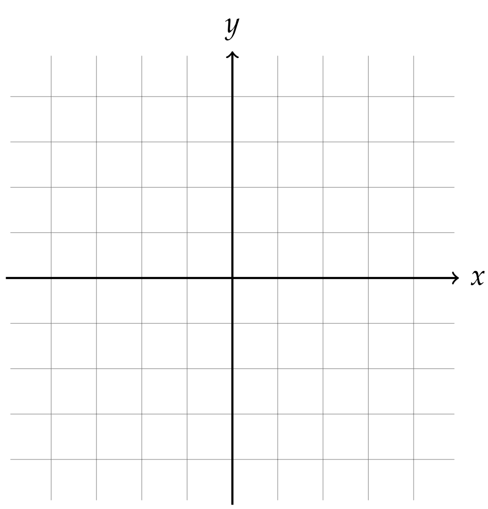
Geometrically, what does it mean when a function is

1. concave up?____
2. concave down?____

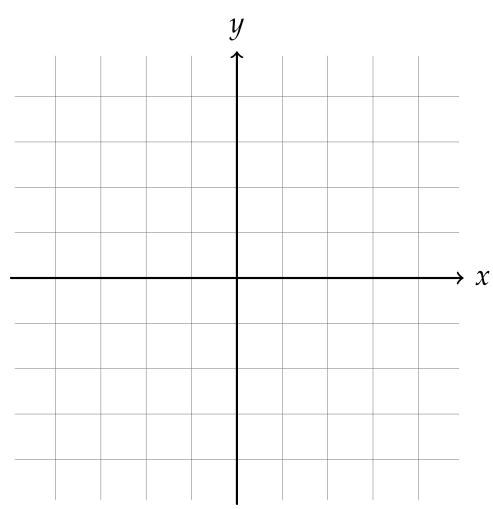
State the **Test for Concavity**.

State the definition of **point of inflection**.

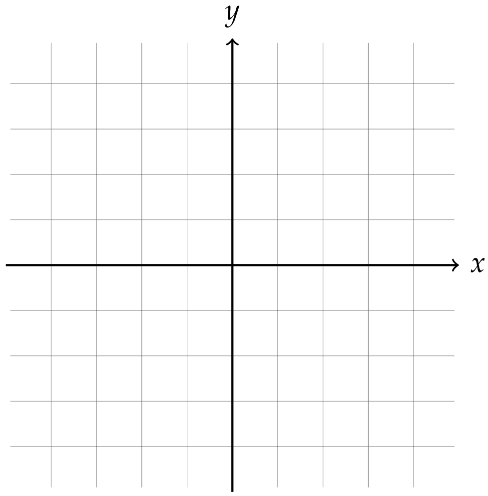
Draw a curve that satisfies the following requirements.



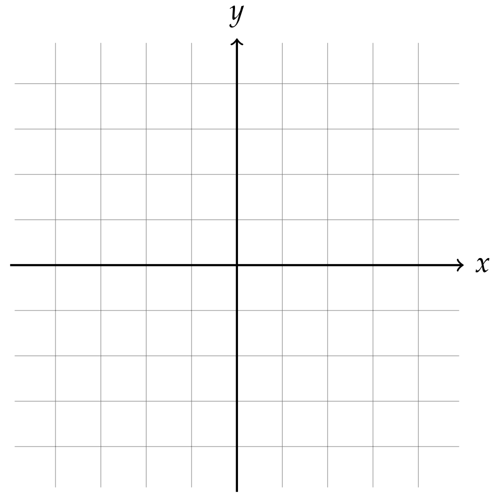
Concave upward and decreasing



Concave upward and increasing



Concave downward and decreasing



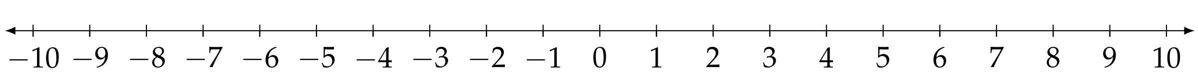
Concave downward and increasing

Complete the following steps to determine the intervals on which $f(x) = \dfrac{x}{x^2+1}$ is concave up and concave down.

1. Determine where $f''(x)$ is 0 or undefined.

   Show your work to find $f''(x) = \dfrac{2x(x^2-3)}{(x^2+1)^3}$.

2. Use the values from 1. to split the domain of $f(x)$, which is $(-\infty, \infty)$.



3. Determine the sign of $f''(x)$ in each of the intervals.

Interval	**Sign of f''**	**Behavior of f**

4. f is concave up on ________________ and f is concave down on ________________.

5. Determine if $f(x)$ has any inflection points and if applicable find them.

State the **Second Derivative Test**.

Should we have an example here?
Nothing in book about test failing when $f''(x) = 0$

1.5 Curve Sketching

Read through the curve sketching guidelines given in **Key Idea 7** and **Examples 1-3**. Complete the following problem.

We will complete the steps to graph $\mathbf{f(x) = \frac{x^2 - 1}{x^2 - 9}}$.

1. Find the domain of $f(x)$.

2. Find any vertical asymptotes.

   (a) $f(x) = \frac{x^2 - 1}{x^2 - 9}$ has vertical asymptotes at $x = 3$ and $x = -3$.

   (b) Show the appropriate limits to verify the vertical asymptotes. You should show four limits.

3. Find the x and y intercepts of f and any symmetry.

   (a) Find the x-intercept.

   (b) Find the y-intercept.

   (c) Determine if there is any symmetry.
   Remember: $f(x)$ is even and has y-axis symmetry if $f(-x) = f(x)$.
   $f(x)$ is odd and has origin symmetry if $f(-x) = -f(x)$.

4. Determine the limits $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$.
   Does $f(x)$ have any horizontal asymptotes? If so, clearly state the horizontal asymptote.

   ____

5. Find the critical values of $f(x)$.

   (a) Show your work to find $f'(x) = \dfrac{-16x}{(x^2-9)^2}$.

   (b) The critical values of $f(x)$ are ________________________.

6. Find the possible points of inflection of $f(x)$.

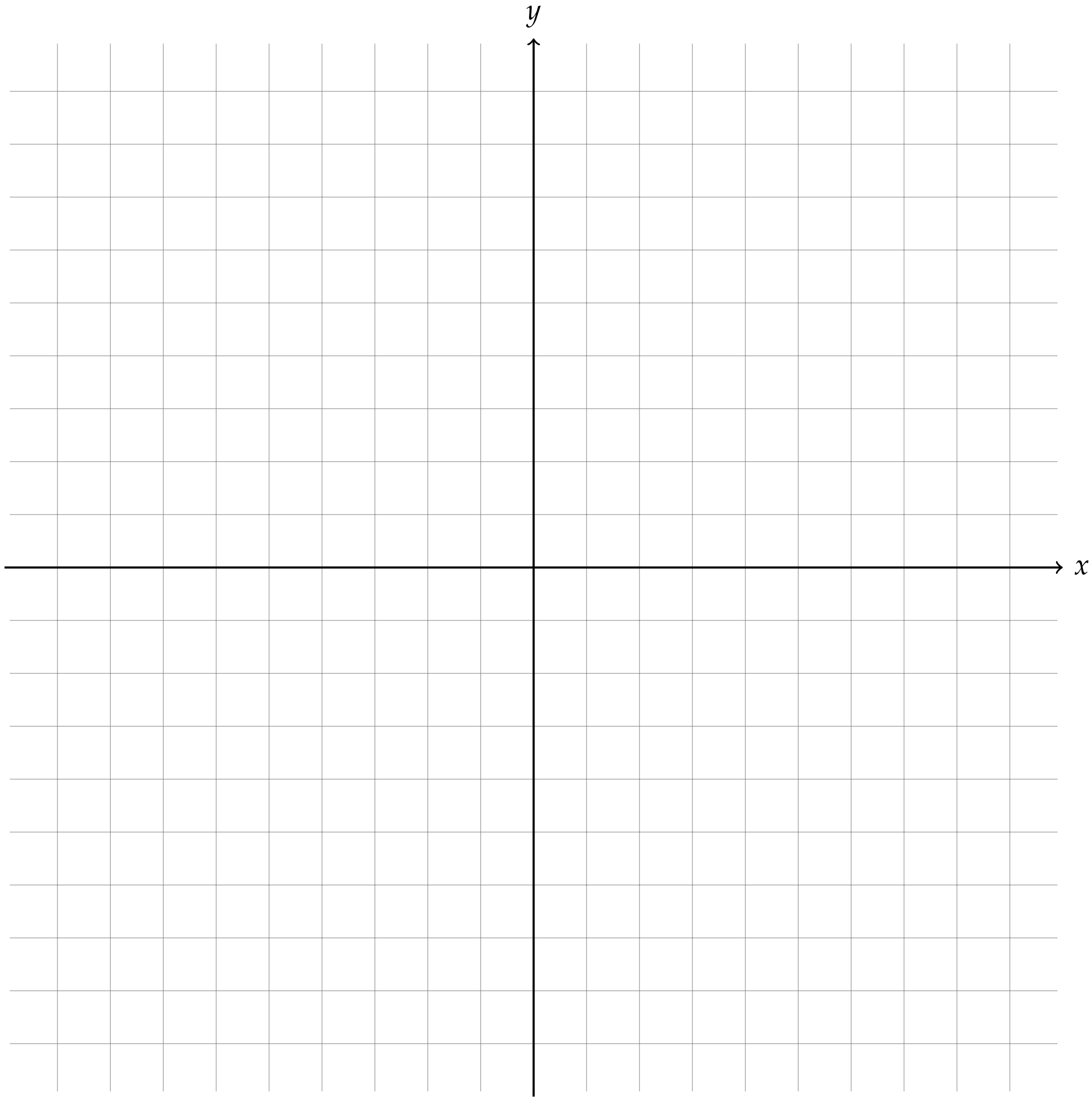
   (a) Show your work to find $f''(x) = \dfrac{48x^2+144}{(x^2-9)^3}$.

   (b) Possible x value(s) for points of inflection are ________________________.

7. Determine where $f(x)$ is increasing, decreasing, concave up and concave down. Complete the chart as in Examples 1-3.

x		-3		0		3	
f'							
f''							
f		undef		$\frac{1}{9}$		undef	

8. Sketch the graph.


y
x