

6.3 Additional Volume problems

Choose the appropriate method (disk, washer, shell) to find the volume of the solid generated by revolving the bounded region about the given axes. Set-up the integral, but do NOT integrate.

1. $y = x^4, y = 0, x = 1$ about

$$y\text{-axis - Solutions: } V = 2\pi \int_0^1 x \cdot x^4 dx = 2\pi \int_0^1 x^5 dx \text{ or } V = \pi \int_0^1 [1^2 - (\sqrt[4]{y})^2] dy$$

$$x\text{-axis - Solutions: } V = 2\pi \int_0^1 y(1 - \sqrt[4]{y}) dy \text{ or } V = \pi \int_0^1 (x^4)^2 dy$$

2. $y = x^3 + 1, x = 0, y = 2$ about

$$y\text{-axis - Solutions: } V = 2\pi \int_0^1 x(2 - (x^3 + 1)) dx \text{ or } V = \pi \int_1^2 (\sqrt[3]{y-1})^2 dy$$

$$y = 2 \text{ - Solutions: } V = 2\pi \int_1^2 (2 - y)\sqrt[3]{y-1} dy \text{ or } V = \pi \int_0^1 [2 - (x^3 + 1)]^2 dx$$

3. $y = 4x^2, 4x + y = 8$ about

$$x\text{-axis - Solution: } V = \pi \int_{-2}^1 [(-4x + 8)^2 - (4x^2)^2] dx$$

$$x = 1 \text{ - Solution: } V = 2\pi \int_{-2}^1 (1 - x)[(-4x + 8) - 4x^2] dx$$

$$y = 16 \text{ - Solution: } V = \pi \int_{-2}^1 [(16 - 4x^2)^2 - (16 - (-4x + 8))^2] dx$$