Machine Learning Homework 9

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Problem 1. *Prove that*

$$\phi : R^{|S|} \to R^{|S|}, \forall s \in S, \phi(v(s)) = v'(s)$$

is a contraction mapping with respect to l_∞-norm

Proof. We would like to show that Bellman Operator is a γ contraction mapping with respect to l_∞-norm.

$$\begin{aligned}
|\phi(v'(s)) - \phi(v(s))| &= \left|\max_a[R(s,a) + \gamma \sum P(s'|s,a)v'(s')] - \max_a[R(s,a) + \gamma \sum P(s'|s,a)v(s')]\right| \\
&\leq \left|R(s,a) + \gamma \sum_{s'} P(s'|s,a)v'(s') - R(s,a) + \gamma \sum_{s'} P(s'|s,a)v(s')\right| \\
&= \gamma \left|\sum_{s'} P(s'|s,a)\left[v'(s') - v(s')\right]\right| \\
&\leq \gamma \max_{s'} |v'(s') - v(s')| \sum_{s'} P(s'|s,a) \\
&= \gamma \max_{s'} |v'(s') - v(s')| \\
&= \gamma \|v' - v\|_\infty
\end{aligned}$$

Therefore,

$$\|\phi(v') - \phi(v)\|_\infty \leq \gamma \|v' - v\|_\infty$$

where $\gamma \in [0, 1)$. Thus, $\phi(\cdot)$ is a contraction mapping with respect to l_∞-norm.

□

Problem 2. *Show that*

$$v_{\pi'}(s) \geq v_\pi(s)$$

Proof.

$$\begin{aligned}
v_{\pi'}(s) &= \max_a \left[R(s,a) + \gamma \sum P(s'|s,a)v(s')\right] \\
&\geq R(s,\pi(s)) + \gamma \sum P(s'|s,\pi(s))v(s') \\
&= v_\pi(s)
\end{aligned}$$

□