Machine Learning Homework 7

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Problem 1. *For any functional f of classifier h, prove that*

$$E_{h\sim Q}[f(h)] \le \ln E_{h\sim P}[\exp\{f(h)\}] + D(Q||P)$$

Proof.

$$\begin{aligned}
E_{h\sim Q}[f(h)] &= E_{h\sim Q}[\ln e^{f(h)}] \\
&= E_{h\sim Q} \ln e^{f(h)} \frac{dP}{dQ}] + D(Q||P) \\
&\le \ln E_{h\sim Q}[e^{f(h)} \frac{dP}{dQ}] + D(Q||P) \\
&= \ln E_{h\sim P}[\exp\{f(h)\}] + D(Q||P)
\end{aligned}$$

□

Problem 2. *Denote that $f(h) = n[\mathrm{err}_D(h) - \mathrm{err}_S(h)]^2$, prove that*

$$\mathbb{P}\left[E_{h\sim P} \exp\{f(h)\} \ge \frac{3}{\delta}\right] \le \delta$$

Proof. According to chernoff bound,

$$\mathbb{P}\left[|\mathrm{err}_D(h) - \mathrm{err}_S(h)| \ge \epsilon\right] \le 2e^{-2n\epsilon^2}$$

Therefore,

$$\mathbb{P}[e^{f(h)} \ge t] = \mathbb{P}\left[|\mathrm{err}_D(h) - \mathrm{err}_S(h)| \ge \sqrt{\frac{\ln t}{n}}\right] \le \frac{2}{t^2}$$

$$\begin{aligned}
\therefore E_{h\sim P} \exp\{f(h)\} &= \int_0^{+\infty} \mathbb{P}[e^{f(h)} \ge t] dt \\
&\le \int_0^1 1 dt + \int_1^{+\infty} \frac{2}{t^2} dt \\
&= 3
\end{aligned}$$

By Markov's inequality, the result holds. □