Machine Learning Homework 3

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Problem 1. *Prove the following inequality:* $\sum_{k=0}^{d} C_n^k \leq (\frac{en}{d})^d$

Proof.

$$\begin{aligned}
\sum_{k=0}^{d} C_n^k &= \sum_{k=0}^{d} \frac{n...(n-k+1)}{k!} \\
&\leq \sum_{k=0}^{d} \frac{n^k}{k!} \\
&= \sum_{k=0}^{d} \frac{d^k}{k!} \frac{n^k}{d^k} \\
&\leq \left(\frac{n}{d}\right)^d \sum_{k=0}^{d} \frac{d^k}{k!} \\
&\leq \left(\frac{n}{d}\right)^d \sum_{k=0}^{+\infty} \frac{d^k}{k!} \\
&= \left(\frac{ne}{d}\right)^d
\end{aligned}$$

□