TUNKU ABDUL RAHMAN UNIVERSITY OF MANAGEMENT AND TECHNOLOGY

FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY

ACADEMIC YEAR 2023/2024

JANUARY EXAMINATION

AAMS1164 PRE-CALCULUS

WEDNESDAY, 17 JANUARY 2024 TIME: 9.00 AM – 11.00 AM (2 HOURS)

DIPLOMA IN COMPUTER SCIENCE

Instructions to Candidates:

Answer **ALL** questions. All questions carry equal marks.

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Question 1

a) (i) Rationalise the denominator of $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$. (2 marks)

(ii) Solve the following equations.

(1) $5^{2x} + 5^3 = 30(5^x)$ (3 marks)

(2) $\log_3(2x - 1) = 1 - \log_3(x - 1)$ (4 marks)

b) Given $f(x) = \frac{3-x}{x+2}$ and $g(x) = \sqrt{2x - 3}$, find

(i) the domains of $f(x)$ and $g(x)$, (2 marks)

(ii) the composite function $(f \circ g)(x)$, (2 marks)

(iii) the inverse function $f^{-1}(x)$ if it exists. (3 marks)

c) Expand $(1 + 3x)^{\frac{1}{2}}$ in ascending power of x up to the term in x^3 by using the binomial expansion. (5 marks)

d) Lily saves RM 500 at the end of every quarter into a saving account paying 5% per annum compounded quarterly.

(i) Find the accumulated amount in 15 years. (3 marks)

(ii) Find the total interest earned over 15 years. (1 mark)

[Total: 25 marks]

Question 2

a) Let $f(x) = \frac{x+2}{(x-2)(x+4)}$.

(i) Express $f(x)$ in terms of partial fractions. (5 marks)

(ii) Find the intercepts and asymptotes of $f(x)$, if any. (4 marks)

(iii) Sketch the graph of $f(x)$. (5 marks)

b) Given that $(x + 1)$ is the factor of the polynomial $P(x) = 6x^3 + 7x^2 + ax - 2$.

(i) Find the value of a. (2 marks)

(ii) Use synthetic division or otherwise, factorise $P(x)$ completely. (4 marks)

c) Use long division to divide $3x^4 - 5x^3 + 10x - 4$ by $x^2 + x + 2$. Find the quotient and the remainder. (5 marks)

[Total: 25 marks]

Question 3

a) Let $A = \begin{pmatrix} 2 & 4 & 1 \\ -1 & 1 & -1 \\ 1 & 4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & -4 & 5 \\ 1 & 1 & -1 \\ 5 & 4 & -6 \end{pmatrix}$.

(i) Find AB. (3 marks)

(ii) Find A^{-1} if exist. (2 marks)

(iii) Hence solve the following system of equations:

$$2x + 4y + z = 1$$
$$-x + y - z = 8$$
$$x + 4y = 7$$

(6 marks)

b) Given that the forces $\mathbf{F_1} = 9\mathbf{i} - 12\mathbf{j}$, $\mathbf{F_2} = -5\mathbf{i} + 11\mathbf{j}$ and $\mathbf{F_3} = 7\mathbf{i} - \mathbf{j}$ are acting on a point P, find

(i) the resultant force and its magnitude, (3 marks)

(ii) the additional force required in order for the forces to be equilibrium. (1 mark)

Question 3 (Continued)

c) Given the complex numbers $z_1 = 5 + 3i$ and $z_2 = -5 + 8i$.

(i) Express z_1 and z_2 in trigonometric form. (4 marks)

(ii) Hence use De Moivre's Theorem to find the values of z_2^4 and $z_1 z_2$. Show your answers in the form of $a + bi$, where a and b are real numbers. (6 marks)

[Total: 25 marks]

Question 4

a) Convert the polar equation $r = 3\sin\theta$ into rectangular form. (3 marks)

b) By completing the square, show that the equation $16y^2 - 9x^2 - 32y - 72x = 272$ represents an equation of a hyperbola. Hence find its centre and equation of asymptotes. (9 marks)

c) Express $15\sin x + 8\cos x$ in the form of $R\sin(x + \alpha)$, where α is an acute angle. Hence solve the equation $15\sin x + 8\cos x = 12$, giving all solutions between $0°$ and $360°$. (8 marks)

d) Solve the equation $2\,\mathrm{cosec}\,x + 3\sin x = 7$ for $0° \leq x \leq 360°$. (5 marks)

[Total: 25 marks]

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Formulae

Logarithms: $\log_a xy = \log_a x + \log_a y$ $\qquad$ $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

Quadratic Equation: $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If α and β are the roots, $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

Identities: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\qquad$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

Progression

AP: $T_n = a + (n-1)d$, $S_n = \dfrac{n}{2}[2a + (n-1)d] = \dfrac{n}{2}[a + l]$ where $l = a + (n-1)d$.

GP: $T_n = ar^{n-1}$, $\quad |r| < 1, S_n = \dfrac{a(1-r^n)}{(1-r)}$, $\quad |r| > 1,\ S_n = \dfrac{a(r^n - 1)}{(r-1)}$, $\quad S_\infty = \dfrac{a}{1-r}$

Binomial Expansion

$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + \ldots + {}^nC_{n-1} a^1 b^{n-1} + {}^nC_n a^0 b^n$ where ${}_nC_r = \dfrac{n!}{(n-r)!r!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{1 \cdot 2}x^2 + \dfrac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^3 + \ldots$ provided $|x| < 1$.

Trigonometry

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\tan^2\theta + 1 = \sec^2\theta$ $\qquad$ $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin P + \sin Q = 2\sin\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\sin P - \sin Q = 2\cos\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\cos P + \cos Q = 2\cos\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\cos P - \cos Q = -2\sin\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

If $t = \tan\frac{1}{2}A$, $\sin A = \dfrac{2t}{1+t^2}$

$\cos A = \dfrac{1-t^2}{1+t^2}$