TUNKU ABDUL RAHMAN UNIVERSITY OF MANAGEMENT AND TECHNOLOGY

FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY

ACADEMIC YEAR 2023/2024

JANUARY EXAMINATION

AAMS1143 INTRODUCTORY CALCULUS

FRIDAY, 12 JANUARY 2024   TIME: 3.00 PM – 5.00 PM (2 HOURS)

DIPLOMA IN COMPUTER SCIENCE

Instructions to Candidates:

Answer **ALL** questions. All questions carry equal marks.

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Question 1

a)   Find the values of the following limits.

(i) $\displaystyle\lim_{x\to 9} \frac{2x^2-23x+45}{x^2-10x+9}$ (3 marks)

(ii) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{49x^6-5x^4}}{11x^3}$ (3 marks)

(iii) $\displaystyle\lim_{x\to -6} \frac{x+6}{\sqrt{x^2-11}-5}$ (5 marks)

b)   Function f is defined as follows:

$$f(x) = \begin{cases} 3x-6, & x < 7 \\ x^2-5x, & x = 7 \\ \frac{2x^2-13x-7}{x-7}, & x > 7 \end{cases}$$

(i)   Explain whether $\displaystyle\lim_{x\to 7} f(x)$ exists. (5 marks)

(ii)   Determine whether the function f is continuous at $x = 7$. (3 marks)

c)   Use the first principles, find the derivative of $y = \frac{2x}{x-1}$. (6 marks)

[Total: 25 marks]

Question 2

a)   Differentiate the following functions with respect to x.

(i) $y = (x^2 + 4)e^{5-6x}$ (4 marks)

(ii) $y = \left(\frac{1-x}{1+x}\right)^3$ (4 marks)

(iii) $y = \ln\sqrt{\frac{x^2-5}{4x+3}}$ (4 marks)

b)   Find $\frac{d^2y}{dx^2}$ for $y = \ln(1+x^5) + e^{2x}$. (6 marks)

c)   Use logarithmic differentiation to find the derivative of function $y = \frac{e^{x^3}(7-4x)^2}{\sqrt{1+6x}}$. (7 marks)

[Total: 25 marks]

This question paper consists of 4 questions on 4 printed pages.

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Question 3

a)   Given $x^3 + 3y^2 - 6x^2y^3 - 1 = 0$, find $\frac{dy}{dx}$ in terms of x and y. Hence determine the equation of the tangent line at the point $\left(1, \frac{1}{2}\right)$ to the curve $x^3 + 3y^2 - 6x^2y^3 - 1 = 0$.   (7 marks)

b)   The area of a cube is decreasing at the rate of 120 cm²/s. At the instant when the side length is 8 cm, find the rate of change of

   (i)   the side length of the cube;   (3 marks)

   (ii)   the volume of the cube.   (3 marks)

c)   Let the curve $y = -2x^3 + 3x^2 + 12x - 6$ for $-3 \le x \le 3$.

   (i)   Find the coordinates of the turning points and determine the nature of each of the turning points.   (7 marks)

   (ii)   Find the point of inflexion if there is any.   (2 marks)

   (iii) Sketch the curve.   (3 marks)

[Total: 25 marks]

Question 4

a)   Integrate the following with respect to x.

   (i) $\int \frac{(4x^2+3)^2}{x} \, dx$,   (3 marks)

   (ii) $\int xe^{x^2} + \frac{1}{e^x} \, dx$.   (3 marks)

b)   By substituting $u = 2 + 3x$, or otherwise, evaluate $\int_1^2 \frac{x}{2+3x} dx$.   (6 marks)

c)   Find the average value of $f(x) = \frac{1}{\sqrt{10-x}}$ on $[1, 6]$.   (4 marks)

d)   Sketch the region bounded by the curve $y = -x^2 - x + 2$, the x-axis and the lines $x = -2$ and $x = 2$. Hence, find the area of this region.   (9 marks)

[Total: 25 marks]

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Formulae

Differentiation:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

y	$\frac{dy}{dx}$
x^n	nx^{n-1}
$ln(x)$	$\frac{1}{x}$
$ln(ax + b)$	$\frac{a}{ax + b}$
e^{mx}	me^{mx}

Integration:

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{f'(x)}{f(x)}\,dx = ln\,f(x) + c$$

y	$\int y\,dx + c$
x^n	$\frac{1}{n+1}x^{n+1}$
$\frac{1}{x}$	$ln(x)$
$\frac{1}{ax + b}$	$\frac{1}{a}ln(ax + b)$
e^{mx}	$\frac{1}{m}e^{mx}$

This question paper consists of 4 questions on 4 printed pages.