

TUNKU ABDUL RAHMAN UNIVERSITY OF MANAGEMENT AND TECHNOLOGY

FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY

ACADEMIC YEAR 2023/2024

JANUARY EXAMINATION

**AAMS3244 STATISTICS II**

WEDNESDAY, 10 JANUARY 2024

TIME: 9.00 AM – 11.00 AM (2 HOURS)

DIPLOMA IN COMPUTER SCIENCE

**Instructions to Candidates:**

Answer **ALL** questions. All questions carry equal marks.

**AAMS3244 STATISTICS II****Question 1**

- a) Let  $X$  have the probability density function given by

$$f(x) = \begin{cases} k(x+5), & 2 \leq x \leq 4; \\ 0, & \text{elsewhere.} \end{cases}$$

- (i) Show that the constant  $k = \frac{1}{16}$ . (3 marks)
- (ii) Find  $P(X < 3)$ . (3 marks)
- b) 20% of people go to work by public transportation. Find the probability that in a random sample of 10 people,
- (i) exactly 4 people go to work by public transportation; (3 marks)
- (ii) less than 3 people go to work by public transportation. (3 marks)
- c) The lives of a certain type of motors produced by an electric company can be approximated closely to a normal distribution with an average life of 8 years and a standard deviation of 1.5 years.
- (i) If a motor is selected randomly from the production, what is the probability that it has a life less than 9 years? (3 marks)
- (ii) Find the probability that the life is between 8.5 years and 9 years. (3 marks)
- (iii) The company replaces all motors that fail under guarantee. If the company wishes to replace only 2% of the motor that fail under guarantee, what guarantee period should the company offer? (4 marks)
- d) A car service centre receives an average of 3 complaints per day. Find the probability that the service centre will receive no complaint on any given day. (3 marks)

[Total: 25 marks]

**Question 2**

- a) A random sample of 10 accountants in a city showed a mean monthly salary of RM8500 and a standard deviation of RM700. Construct a 95% confidence interval for the mean monthly salary of the population of accountants. (5 marks)
- b) A random sample of 100 savings accounts in a local bank showed that 70% of the accounts have savings values of at least RM10,000. Obtain a 98% confidence interval for the proportion of all saving accounts in the bank with saving values of at least RM10,000. (4 marks)

**AAMS3244 STATISTICS II****Question 2 (Continued)**

- c) A survey found that the average hotel room rate in Kuala Lumpur is RM 86 and the average room rate in Johor Bahru is RM 95. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviation of the populations are RM 5 and RM 10 respectively. At the 5% level of significance, can it be concluded that there is a significant difference in the rates? (8 marks)
- d) A local bank randomly selected 200 current account clients and found that 90 of them also had savings accounts with the bank. The bank wishes to have a target of at least 50% of their current and savings accounts at the same time. Test the hypothesis that the bank has achieved its target at the 1% level of significance. (8 marks)

[Total: 25 marks]

**Question 3**

- a) The cost of a research project is normally distributed with a mean of RM50,000 and a standard deviation of RM5,000. Find the probability that a random sample of 4 research projects has a mean cost of between RM49,000 and RM51,000. (6 marks)
- b) A departmental store recorded the following information on three models of refrigerators for the year 2020 and 2022.

Model	Year 2020		Year 2022	
	Price (RM)	Unit Sold	Price (RM)	Unit Sold
A	3700	25	4000	22
B	2500	31	2700	35
C	1000	56	1200	60

By using year 2020 as the base year, calculate

- (i) the simple price index of model C for the year 2022; (2 marks)
- (ii) the Laspeyres quantity index for the year 2022 and interpret; (4 marks)
- (iii) the Paasche price index for the year 2022 and interpret. (4 marks)
- c) A publishing company has 100 different book titles classified by type of book and cost as shown in the table. Conduct a relevant test to investigate the relationship between the type of book and cost at 5% significance level.

Type	Cost			Total
	RM50	RM100	RM150	
Fiction	18	13	15	46
Non-fiction	16	28	10	54
Total	34	41	25	100

(9 marks)

[Total: 25 marks]

**AAMS3244 STATISTICS II****Question 4**

- a) A music teacher recorded the numbers of hours her students spent in practising piano and the scores obtained in the piano examination. She believes that the more time her students spend in practising the piano, the higher the score they obtain.

Time (hours), $X$	60	80	150	130	160	180
Score, $Y$	95	100	130	135	141	135

$$\sum X = 760, \sum Y = 736, \sum XY = 97610, \sum X^2 = 107400, \sum Y^2 = 92256$$

- (i) Calculate the product moment correlation coefficient. (2 marks)
  - (ii) Calculate the Spearman's rank correlation coefficient. (4 marks)
  - (iii) Find the least squares regression line for the scores obtained on the number of hours spent practising. (5 marks)
  - (iv) Estimate the score obtained if a student spends 140 hours in practising the piano. Comment on the accuracy of the answer obtained. (2 marks)
- b) The following table shows the daily income (in RM) of a nasi lemak stallholder who sells nasi lemak from Monday to Friday over a 3-week period:

Week	Monday	Tuesday	Wednesday	Thursday	Friday
1	200	250	500	600	640
2	150	280	450	550	650
3	210	230	410	520	560

- (i) Calculate the 5-point moving average trend. (4 marks)
- (ii) By using an additive model, compute the average daily variations. (5 marks)
- (iii) Forecast the daily income for Monday of week 4. (3 marks)

[Total: 25 marks]

**AAMS3244 STATISTICS II****AAMS3244 Formulae****Discrete Uniform Probability Function**

$$f(x; k) = \frac{1}{k} \text{ where } x = x_1, x_2, \dots, x_k$$

**Geometric Probability Function**

$$g(x; p) = pq^{x-1} \text{ where } x = 1, 2, 3, \dots$$

**Binomial Probability Function**

$$P(X = x) = {}^nC_x p^x (1-p)^{n-x}; \quad x = 0, 1, \dots, n$$

**Poisson Probability Function**

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}; \quad \lambda > 0, \quad x = 0, 1, 2, \dots$$

**Continuous Uniform Probability Density Function**

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

**Exponential Probability Density Function**

$$f(x) = \begin{cases} \frac{1}{\mu} e^{-\frac{x}{\mu}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

**Sample Average**

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

**Sample Variance**

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

$$= \frac{1}{n-1} \left[ \sum_{i=1}^n X_i^2 - \frac{1}{n} \left( \sum_{i=1}^n X_i \right)^2 \right]$$

**Confidence Interval for One Population**

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

$$\bar{X} \pm t_{\frac{\alpha}{2}; n-1} \frac{S}{\sqrt{n}}$$

$$\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

**Confidence Interval for Two Populations**

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}; n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}$$

$$(\hat{p}_1 - \hat{p}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

**Confidence Interval for Paired Samples**

$$\bar{D} \pm t_{\frac{\alpha}{2}; n-1} \frac{S_D}{\sqrt{n}}$$

where

$$\bar{D} = \frac{\sum D}{n} \text{ and } S_D = \sqrt{\frac{\sum D^2 - \frac{(\sum D)^2}{n}}{n-1}}$$

**AAMS3244 STATISTICS II****Test Statistic for One Population**

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

$$Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

$$T = \frac{\bar{X} - \mu_0}{S / \sqrt{n}} \text{ with d.f.} = n - 1$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

**Test Statistic for Two Populations**

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \text{ with d.f.} = n_1 + n_2 - 2$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \text{ with } \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

**Test Statistic for Paired Samples**

$$T = \frac{\bar{D} - \mu_D}{S_D / \sqrt{n}} \text{ with d.f.} = n - 1$$

**Product Moment Correlation Coefficient**

$$r = \frac{n \sum XY - (\sum X)(\sum Y)}{\sqrt{[n \sum X^2 - (\sum X)^2][n \sum Y^2 - (\sum Y)^2]}}$$

**Spearman's Rank Correlation Coefficient**

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

**Simple Linear Regression**

$$\hat{Y} = \hat{a} + \hat{b}X$$

where

$$\hat{b} = \frac{n \sum XY - (\sum X)(\sum Y)}{n \sum X^2 - (\sum X)^2}$$

$$\hat{a} = \bar{Y} - \hat{b} \bar{X}$$

**Chi-Square Test Statistic**

$$\chi^2 = \sum_{k=1}^m \frac{(f_k - e_k)^2}{e_k}$$

$$\chi^2 = \sum_{k=1}^m \frac{(|f_k - e_k| - 0.5)^2}{e_k}$$

$$\chi^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{(f_{ij} - e_{ij})^2}{e_{ij}}$$

$$\chi^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{(|f_{ij} - e_{ij}| - 0.5)^2}{e_{ij}}$$

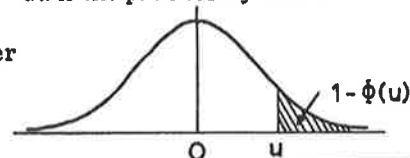
**Index Numbers**

	Price	Quantity
Laspeyres Index	$\frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$	$\frac{\sum p_0 q_1}{\sum p_0 q_0} \times 100$
Paasche Index	$\frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$	$\frac{\sum p_1 q_1}{\sum p_1 q_0} \times 100$

**AAMS3244 STATISTICS II****AREAS IN TAIL OF THE NORMAL DISTRIBUTION**

The function tabulated is  $1 - \Phi(u)$  where  $\Phi(u)$  is the cumulative distribution function of a standardised Normal variable  $u$ . Thus  $1 - \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-u^2/2} du$  is the probability that a

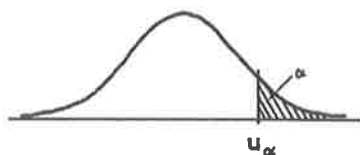
standardised Normal variable selected at random will be greater than a value of  $u \left( = \frac{x - \mu}{\sigma} \right)$



$\frac{(x - \mu)}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									
3.1	.00097									
3.2	.00069									
3.3	.00048									
3.4	.00034									
3.5	.00023									
3.6	.00016									
3.7	.00011									
3.8	.00007									
3.9	.00005									
4.0	.00003									

**PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION**

The table gives the  $100\alpha$  percentage points,  $u_\alpha$ , of a standardised Normal distribution where  $\alpha = \frac{1}{\sqrt{2\pi}} \int_{u_\alpha}^{\infty} e^{-u^2/2} du$ . Thus  $u_\alpha$  is the value of a standardised Normal variate which has probability  $\alpha$  of being exceeded.



$\alpha$	$u_\alpha$	$\alpha$	$u_\alpha$	$\alpha$	$u_\alpha$	$\alpha$	$u_\alpha$	$\alpha$	$u_\alpha$	$\alpha$	$u_\alpha$
.50	0.0000	.050	1.6449	.030	1.8808	.020	2.0537	.010	2.3263	.050	1.6449
.45	0.1257	.048	1.6646	.029	1.8957	.019	2.0749	.009	2.3656	.010	2.3263
.40	0.2533	.046	1.6849	.028	1.9110	.018	2.0969	.008	2.4089	.001	3.0902
.35	0.3853	.044	1.7060	.027	1.9268	.017	2.1201	.007	2.4573	.0001	3.7190
.30	0.5244	.042	1.7279	.026	1.9431	.016	2.1444	.006	2.5121	.00001	4.2649
.25	0.6745	.040	1.7507	.025	1.9600	.015	2.1701	.005	2.5758	.025	1.9600
.20	0.8416	.038	1.7744	.024	1.9774	.014	2.1973	.004	2.6521	.005	2.5758
.15	1.0364	.036	1.7991	.023	1.9954	.013	2.2262	.003	2.7478	.0005	3.2905
.10	1.2816	.034	1.8250	.022	2.0141	.012	2.2571	.002	2.8782	.00005	3.8906
.05	1.6449	.032	1.8522	.021	2.0335	.011	2.2904	.001	3.0902	.000005	4.4172



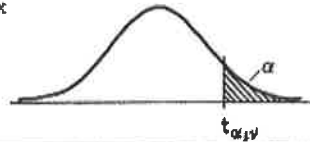
## PERCENTAGE POINTS OF THE t DISTRIBUTION

The table gives the value of  $t_{\alpha, \nu}$  — the  $100\alpha$  percentage point of the t distribution for  $\nu$  degrees of freedom.

The values of  $t$  are obtained by solution of the equation:-

$$\alpha = \frac{\Gamma\left\{\frac{1}{2}(\nu+1)\right\}}{\Gamma\left\{\frac{1}{2}\nu\right\}} \left\{\frac{\nu}{\pi}\right\}^{-1/2} \int_{t_{\alpha, \nu}}^{\infty} (1 + x^2/\nu)^{-(\nu+1)/2} dx$$

Note. The tabulation is for one tail only i.e. for positive values of  $t$ . For  $|t|$  the column headings for  $\alpha$  must be doubled.



$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
$\nu = 1$	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.898	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
$\infty$	1.282	1.645	1.960	2.326	2.576	3.090	3.291

This table is taken from Table III of Fisher & Yates: Statistical Tables for Biological, Agricultural and Medical Research, published by Oliver & Boyd Ltd., Edinburgh, and by permission of the authors and publishers and also from Table 12 of Biometrika Tables for Statisticians, Volume 1, by permission of the Biometrika Trustees.

