

TUNKU ABDUL RAHMAN UNIVERSITY OF MANAGEMENT AND TECHNOLOGY

FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY

ACADEMIC YEAR 2023/2024

JANUARY EXAMINATION

AAMS1143 INTRODUCTORY CALCULUS

FRIDAY, 12 JANUARY 2024

TIME: 3.00 PM – 5.00 PM (2 HOURS)

DIPLOMA IN COMPUTER SCIENCE

Instructions to Candidates:

Answer **ALL** questions. All questions carry equal marks.

AAMS1143 INTRODUCTORY CALCULUS**Question 1**

a) Find the values of the following limits.

(i) $\lim_{x \rightarrow 9} \frac{2x^2 - 23x + 45}{x^2 - 10x + 9}$ (3 marks)

(ii) $\lim_{x \rightarrow -\infty} \frac{\sqrt{49x^6 - 5x^4}}{11x^3}$ (3 marks)

(iii) $\lim_{x \rightarrow -6} \frac{x+6}{\sqrt{x^2 - 11} - 5}$ (5 marks)

b) Function f is defined as follows:

$$f(x) = \begin{cases} 3x - 6, & x < 7 \\ x^2 - 5x, & x = 7 \\ \frac{2x^2 - 13x - 7}{x - 7}, & x > 7 \end{cases}$$

(i) Explain whether $\lim_{x \rightarrow 7} f(x)$ exists. (5 marks)

(ii) Determine whether the function f is continuous at $x = 7$. (3 marks)

c) Use the first principles, find the derivative of $y = \frac{2x}{x-1}$. (6 marks)

[Total: 25 marks]

Question 2

a) Differentiate the following functions with respect to x .

(i) $y = (x^2 + 4)e^{5-6x}$ (4 marks)

(ii) $y = \left(\frac{1-x}{1+x}\right)^3$ (4 marks)

(iii) $y = \ln \sqrt{\frac{x^2-5}{4x+3}}$ (4 marks)

b) Find $\frac{d^2y}{dx^2}$ for $y = \ln(1 + x^5) + e^{2x}$. (6 marks)

c) Use logarithmic differentiation to find the derivative of function $y = \frac{e^{x^3}(7-4x)^2}{\sqrt{1+6x}}$. (7 marks)

[Total: 25 marks]

AAMS1143 INTRODUCTORY CALCULUS**Question 3**

- a) Given $x^3 + 3y^2 - 6x^2y^3 - 1 = 0$, find $\frac{dy}{dx}$ in terms of x and y . Hence determine the equation of the tangent line at the point $\left(1, \frac{1}{2}\right)$ to the curve $x^3 + 3y^2 - 6x^2y^3 - 1 = 0$. (7 marks)
- b) The area of a cube is decreasing at the rate of $120 \text{ cm}^2/\text{s}$. At the instant when the side length is 8 cm, find the rate of change of
- (i) the side length of the cube; (3 marks)
 - (ii) the volume of the cube. (3 marks)
- c) Let the curve $y = -2x^3 + 3x^2 + 12x - 6$ for $-3 \leq x \leq 3$.
- (i) Find the coordinates of the turning points and determine the nature of each of the turning points. (7 marks)
 - (ii) Find the point of inflexion if there is any. (2 marks)
 - (iii) Sketch the curve. (3 marks)

[Total: 25 marks]

Question 4

- a) Integrate the following with respect to x .
- (i) $\int \frac{(4x^2+3)^2}{x} dx$, (3 marks)
 - (ii) $\int xe^{x^2} + \frac{1}{e^x} dx$. (3 marks)
- b) By substituting $u = 2 + 3x$, or otherwise, evaluate $\int_1^2 \frac{x}{2+3x} dx$. (6 marks)
- c) Find the average value of $f(x) = \frac{1}{\sqrt{10-x}}$ on $[1, 6]$. (4 marks)
- d) Sketch the region bounded by the curve $y = -x^2 - x + 2$, the x -axis and the lines $x = -2$ and $x = 2$. Hence, find the area of this region. (9 marks)

[Total: 25 marks]

AAMS1143 INTRODUCTORY CALCULUS**Formulae**

Differentiation:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

y	$\frac{dy}{dx}$
x^n	nx^{n-1}
$\ln(x)$	$\frac{1}{x}$
$\ln(ax + b)$	$\frac{a}{ax + b}$
e^{mx}	me^{mx}

Integration:

$$\int u dv = uv - \int v du$$

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$$

y	$\int y dx + c$
x^n	$\frac{1}{n+1} x^{n+1}$
$\frac{1}{x}$	$\ln(x)$
$\frac{1}{ax + b}$	$\frac{1}{a} \ln(ax + b)$
e^{mx}	$\frac{1}{m} e^{mx}$