Here are **all 9 formulas** with detailed variable explanations in terms of **beauty trend analysis**:

1. Hawkes Process — Measuring Self-Sustaining Momentum in Comment Activity

$$\lambda_i(t) = \mu_i + \sum_{t_j<t} \alpha_{ij}\, \kappa_\phi(t - t_j)$$

- $\lambda_i(t)$: **Comment generation rate** for creator i at time t - measures viral momentum
- μ_i: **Baseline comment activity** - natural engagement without influence
- α_{ij}: **Influence strength** from creator j to i - cross-content impact
- $\kappa_\phi(t - t_j)$: **Temporal decay function** - recent comments have stronger viral effects

$$\mathcal{R}_{c,t} = \rho\big(\mathbf{A}_c(t)\big)$$

- $\mathcal{R}_{c,t}$: **Reproduction number** - measures if a beauty trend is self-sustaining (>1 = viral)
- $\rho(\cdot)$: **Spectral radius** - maximum eigenvalue indicating trend propagation strength
- $\mathbf{A}_c(t)$: **Excitation matrix** - cross-influence between different beauty creators/content

2. Topological Burst Index (TBI) — Detecting Novelty in Conversations Before Volume Spikes

$$\mathrm{TBI}_{c,t} = \frac{\sum_k \left(d_k - b_k\right)_t \;-\; \sum_k \left(d_k - b_k\right)_{t-1}}{\sum_k \left(d_k - b_k\right)_{t-1} + \epsilon}$$

- $\mathrm{TBI}_{c,t}$: **Structural novelty score** - detects new conversation patterns before volume spikes
- b_k, d_k: **Birth/death times** of discussion topics - when new beauty concepts emerge/disappear
- $(d_k - b_k)$: **Persistence lifetime** - how long beauty topics remain relevant in discussions
- ϵ: **Smoothing constant** - prevents division by zero in stability calculation

3. Fusion Score — Combining Fundamentals and Momentum Into a Robust Trend Indicator

$$\tilde{\alpha}_{c,t} = \frac{r_{c,t} - \mathbb{E}[r_{c,t} \mid \mathcal{F}_{t-1}]}{\sqrt{\mathrm{Var}(r_{c,t} \mid \mathcal{F}_{t-1})}}$$

- $\tilde{\alpha}_{c,t}$: **Risk-adjusted trend surprise** - measures unexpected beauty trend performance
- $r_{c,t}$: **Raw trend metric** (e.g., engagement rate) for category c at time t
- $\mathbb{E}[\cdot]$: **Expected value** - baseline trend expectation from historical data
- $\mathcal{F}_{t-1}$: **Information set** - all available data up to previous time period

$$\mathrm{FusionScore}_{c,t} = w_1 \cdot \tilde{\alpha}_{c,t} + w_2 \cdot \log(1 + \mathcal{R}_{c,t}) + w_3 \cdot \mathrm{TBI}_{c,t}$$

- $\mathrm{FusionScore}_{c,t}$: **Comprehensive trend strength** - unified beauty trend indicator
- w_1, w_2, w_3: **Weight coefficients** - balance between surprise, momentum, and novelty

4. TF-IDF-Based Trend Scoring — Quantifying Term Importance in Beauty Content

$$\mathrm{tfidf}_{t,d} = \mathrm{tf}_{t,d} \cdot \log\left(\frac{N}{\mathrm{df}_t}\right)$$

- $\mathrm{tfidf}_{t,d}$: **Term importance score** - identifies key beauty keywords in content
- $\mathrm{tf}_{t,d}$: **Term frequency** - how often beauty term t appears in document d
- df_t: **Document frequency** - number of beauty videos/discussions mentioning term t
- N: **Total documents** - complete beauty content corpus size

$$\mathrm{trend\_score}(t) = \sum_{d=1}^{N} \mathrm{tfidf}_{t,d}$$

- $\mathrm{trend\_score}(t)$: **Aggregate trend importance** - overall significance of beauty concept t

5. Market Gap Detection — Identifying Under-Served Product Segments

$$\mathrm{GapScore}_p = \mathrm{freq}(p) \cdot (1 - \mathrm{LoR}(p)) \cdot \mathrm{novelty}(p)$$

- GapScore_p: **Market opportunity score** - identifies profitable beauty product gaps
- $\text{freq}(p)$: **Mention frequency** - how often product concept p is discussed
- $\text{LoR}(p)$: **L'Oréal presence indicator** - flags if brand already serves this segment
- $\text{novelty}(p)$: **Innovation potential** - uniqueness of the product concept

$$\text{LoR}(p) = \begin{cases} 1 & \text{if } p \text{ contains L'Oréal brand terms} \\ 0 & \text{otherwise} \end{cases}$$

6. Ingredient Trend Momentum — Measuring Rise in Ingredient Popularity

$$\text{Momentum}_i(t) = \frac{\text{count}_i(t) - \text{count}_i(t - \Delta t)}{\text{count}_i(t - \Delta t) + \epsilon}$$

- $\text{Momentum}_i(t)$: **Growth rate** - measures rising popularity of beauty ingredient i
- $\text{count}_i(t)$: **Ingredient mentions** in current time period for ingredient i
- Δt: **Time comparison window** - typically 30-90 days for beauty trends
- ϵ: **Numerical stability** - prevents division errors

$$\text{TrendScore}_i = \text{Momentum}_i(t) \cdot \text{Frequency}_i$$

- TrendScore_i: **Comprehensive ingredient trend** - combines growth and popularity

7. Content Similarity Clustering — Grouping Related Beauty Concepts

$$\text{similarity}(d_i, d_j) = \frac{\mathbf{v}_i \cdot \mathbf{v}_j}{\|\mathbf{v}_i\| \cdot \|\mathbf{v}_j\|}$$

- $\text{similarity}(d_i, d_j)$: **Content relatedness** - measures similarity between beauty videos/content
- $\mathbf{v}_i, \mathbf{v}_j$: **TF-IDF vectors** - numerical representations of beauty content
- $\cdot$: **Dot product** - captures shared beauty terminology/concepts

$$\text{Cohesion}_C = \frac{2}{|C|(|C| - 1)} \sum_{i,j \in C, i<j} \text{similarity}(d_i, d_j)$$

- Cohesion_C: **Cluster tightness** - how related beauty concepts are within a trend group
- $|C|$: **Cluster size** - number of beauty videos/concepts in trend category

8. Semantic Innovation Index — Measuring Novelty in Product Concepts

$$\text{InnovationIndex}(p) = \frac{|\text{unique\_terms}(p)|}{|\text{total\_terms}(p)|} \cdot \left(1 - \frac{\text{common\_terms}(p)}{|\text{terms}(p)|}\right)$$

- $\text{InnovationIndex}(p)$: **Novelty score** - measures uniqueness of beauty product concept p
- $\text{unique\_terms}(p)$: **Distinctive keywords** - rare/innovative beauty terminology used
- $\text{common\_terms}(p)$: **Standard vocabulary** - overused beauty marketing terms
- $\text{terms}(p)$: **Total concept vocabulary** - all words describing the beauty product

9. Consumer Interest Heatmap — Spatial-Temporal Trend Analysis

$$\text{Interest}_{c,t} = \alpha \cdot \log(1 + \text{mentions}_{c,t}) + \beta \cdot \text{sentiment}_{c,t} + \gamma \cdot \text{engagement}_{c,t}$$

- $\text{Interest}_{c,t}$: **Consumer attention level** - overall interest in beauty category c at time t
- $\text{mentions}_{c,t}$: **Discussion volume** - raw comment/review count for beauty trend
- $\text{sentiment}_{c,t}$: **Emotional response** - positive/negative feelings toward beauty concept
- $\text{engagement}_{c,t}$: **Interaction intensity** - likes/shares per beauty content piece
- α, β, γ: **Weight parameters** - balance between volume, sentiment, and engagement