A PROJECT REPORT ON

USING BAYESIAN STATISTICS FOR A/B TESTING

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Tanya JAIN

Abstract

The aim of the project was to find a method to develop Bayesian A/B Tester and understand the mathematical and scientific reasons behind the working of the A/B analyzer. The Beta-Bernoulli model is a binary outcome model connected in the A/B testing setting, where the objective of derivation is understanding the likelihood that the test assembly performs superior to the control assembly. The Bayesian A/B analyzer utilizes a Beta distribution as the prior for the success probability.

Bayesian inference comprises of initially examining the prior belief and the likelihood of the effects that shall take place, then updating the earlier with recent information for a refreshed prior. For instance, for a determined conversion rate of 7%, it may seem practically likely for the change to take place for the benefit after the test by 7%. T if our transformation rate is 5%, we may state that it's sensibly likely that a here is also, a 93% chance for there to be no impact after the test, which is of higher likelihood. Moreover, it indicates that it is not possible for the conversion rates to shoot up more than 25%.

As the information begins coming in, the set beliefs are refreshed. In the event that the updated information indicates towards an increased conversion rate, the gauge of the impact shifts upwards from the prior. Constant gathering of information helps in validation of a result and moving further away from the prior. Posterior probability distribution is achieved as the end product of the treatment performed on the prior data.

Finally, the A/B tester is tested using Statistical significance based on Frequentist inference, an approach less preferred over Bayesian approach for A/B Testing due to lack in hypothesis generation.

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Chapter 1

Introduction

> Frequentist arguments are more counter-factual in nature, and resemble the type of logic that lawyers use in court. Most of us learn frequentist statistics in entry-level statistics courses. A t-test, where we ask, âĂIJIs this variation different from the control?âĂİ is a basic building block of this approach."
>
> — Leonid Pekelis (Data Scientist)

Fundamentally, utilizing a Frequentist strategy implies making forecasts on basic certainties of the trial utilizing just information from the present investigation.

1.1 A/B Testing

A/B testing (otherwise called split testing or pail testing) is a technique for contrasting two renditions of a page or application against each other to figure out which one performs better. AB testing is basically an investigation where at least two variations of a page appear to clients aimlessly, and measurable examination is utilized to figure out which variety performs better for a given change objective.

1.1.1 History

Like most fields, setting a date for the appearance of another strategy is troublesome in view of the persistent advancement of a subject. Where the distinction could be characterized is the point at which the change was produced using utilizing any accepted data from the populaces to a test performed on the specimens alone. This work was done in 1908 by William Sealy Gosset when he modified the Z-test to build Student's t-test.

Google engineers ran their initial A/B test in the year 2000 trying to figure out what the ideal number of results to show on its web index page would be. The principal test was unsuccessful due to glitches that came about because of moderate stacking times. Later A/B testing examination would be more higher in class, yet the establishment and fundamental standards, for the most part, remain, and in 2011, 11 years after Google's initial test, Google has run more than 7,000 diverse A/B tests.

1.1.2 Why do A/B Testing?

A/B testing permits people, groups, and organisations to roll out watchful improvements to their client encounters while gathering information on the outcomes. This enables them to develop theories and to learn better why certain components of their encounters affect client conduct. In another way, they can be demonstrated wrongâĂŤtheir sentiment about the best involvement for a given objective can be demonstrated wrong through an A/B test.
Something other than noting an erratic inquiry or settling a difference, AB testing can be utilised reliably to consistently enhance a given affair, enhancing a solitary objective like change rate after some time.
For example, a B2B innovation organisation might need to enhance their prospective customer quality and volume from crusade points of arrival. Keeping in mind the end goal to accomplish that objective, the group would attempt A/B testing changes to the feature, visual symbolism, frame fields, invitation to take action, and general design of the page. Testing one change at any given moment causes them pinpoint which changes affected their guests' conduct, and which ones did not. After some time, they can consolidate the impact of numerous triumphant changes from trials to exhibit the quantifiable change of the new experience over the old one.

1.2 Document Structure

The introductory chapter has laid a more in depth base on the objective of the paper. It introduces the readers to a topic in the field of Statistics and Probabilities which is not only favourable for programmers, but designers as well. The tools used in the development of the A/B tester and its application will be discussed in the upcoming chapter. Chapter 2 lays down the various tools that have aided in the successful development of the A/B tester. It covers the materials including the programming languages, their framework and their modules, as well as the methods in the utilisation of these materials. The well laid formatting and structure is a result of the utilities provided by the tool, LaTeX.

Chapter 2

Materials and Methods

> Make software that you want to use
> and that you would want to use often.
> As long as you are making something
> that you want to use, then your heart
> will be in it.
>
> Cabel Sasser, Sink or Swim, SXSW 2006

This chapter gives a full fledged description of the development environment setup used for coding the Python based Baysian A/B Tester. Since development started from scratch, a lot of research went in deciding which programming language, Software Development Kit, platform *etc.* to use. So here is a bottom up listing of the tools used along with the arguments which illustrated why they were chosen.

2.1 Programming Language

Python and R are the languages that were the most apt to carry out this Term Paper work. These two languages are famous for their wide range of capabilities in the User Groups communities, especially those for the Data Science analysts and Artificial Intelligence enthusiasts. These are some of my considerations while choosing the language:

1. **Apt for the Work**: They have the requirements needed for the completion of this project.

2. **Statistics and Probabilities**: It is great for analysis and calculations pertaining to the subject with numerous modules and functions defined to ease out the work with increased efficiency.

3. **Richness in the language**: A programming language is viewed as intense, when a

considerable measure can be accomplished with less number of lines of code. The develops of Python as well as R are rich in support and characteristic features.

4. **Lucid**: Rather than supports and semicolons for denoting the finish of code pieces and explanations, the greater part of the projects can be composed in a solitary line while making it to required to indent the code legitimately.

5. **Great Community Support**:

   (a) **Large engagement**: With many projects being carried out in these two languages, there is a wide number of people engaged in User Groups to provide instant help over various resources such as IRC channels.

   (b) **Constant Development**: These languages are a part of the Open Source Software culture and hence developers contributing to Open Source are there constantly improving the languages to eliminate any bugs that would exist and improve them with new features.

2.2 Python Framework

Software engineers are naturally sluggish which is the reason they make instruments which can be reused independent from anyone else and others as well. It is these devices that we call libraries, toolboxes, systems and so on, which is only proficient code for rudimentary undertakings like associating with server, drawing a catch on the screen and so forth. Python has an exceptionally rich arrangement of foreign Frameworks, i.e structures which have been made by individuals other than the center Python improvement group. For the current issue the accompanying were contemplated for the document sharing application nearby.

1. **Python SciPy library**:

   (a) It is a fundamental Python scientific library which comprises of the modules: Numpy, MatPlotlib

   (b) It is way more compact than the functions in the standard Python library

   (c) It is widely utilised in scientific calculations and data analysis, especially for research work

   (d) The functions and modules written in these libraries ease out the work of typing the regular scientific formulas on a redundant basis.

2.3 Theorems

2.3.1 Conditional Probability

Probability is understood as the likelihood for the validity or truthness in a matter. It is represented by a number in the vicinity of 0 and 1 (both inclusive) that speaks to a level of faith in a reality or expectation. The value 1 convicts a reality or a fact is valid, or that an expectation will work out. The value 0 assures that the certainity of the matter is false.

2.3.2 Bayes Theorem

Let there be two independent events A and B Hence, their complements will be represented as A^c and B^c respectively. where $P(A^c) = 1 - P(A)$ and $P(B^c) = 1 - P(B)$ We know,

$$P(AandB) = P(BandA)$$

According to conjoint probability,

$$P(AandB) = P(A)P(B|A)$$

and

$$P(BandA) = P(B)P(A|B)$$

Which on comparison, leads us to the following equation:

$$P(B)P(A|B) = P(A)P(B|A)$$

Hence, we get to the Bayes's Theorem:

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} \tag{2.1}$$

This equation is equivalent to saying that the probability of an event A to occur, given that the event B has already taken place, is the product of probability of event A to occur and the probabilty of event B to occur given that the event A has already taken place, divided by the probability of event B to occur.

Also, the the Bayes Theorem can also be wriiten as:

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)} \tag{2.2}$$

2.3.3 The Diachronic Interpretation

Let H and D be the events representing hypothesis and data respectively. Hence, the derived Bayesian equation (2.1) can be written as:

$$P(H|D) = \frac{P(H)P(D|H)}{P(D)} \tag{2.3}$$

We know that:

1. **Prior** or $P(H)$ is the probability of the hypothesis formed before the data is known. It is made via the background information, that is, the opinion and judgment conducted on various issues pertaining to the data.

2. **Posterior** or $P(H|D)$ is the probability of the hypothesis formed after the given set of data is known.

3. **Likelihood** or $P(D|H)$ is the probability of the data under the hypothesis.

4. **Normalizing constant** or $P(D)$ is the probability of data under any hypothesis.

Hence, our core of the Bayesian statistics can be interpreted as:

$$P(Posterior) = \frac{P(Prior)P(Likelihood)}{P(Normalizing - constant)} \tag{2.4}$$

For a set of hypotheses, it is necessary that the hypothesis are:

1. **Mutually exclusive**: Only one shall be true.

2. **Collectively exhaustive**: At one shall be true. No other possibilities can exist.

Using the law of total probabilities, the probability of data for two given hypothesis $H1$ and $H2$ are:

$$P(D) = P(H1)P(D|H1) + P(H2)P(D|H2) \tag{2.5}$$

2.4 Procedure

2.4.1 A/A Testing

A/A testing is the strategy of utilizing A/B testing to test two indistinguishable renditions of a page against each other. Ordinarily, this is done to watch that the apparatus being utilized to run the trial is measurably reasonable. In A/A test, the apparatus should report no distinction in transformations between the control and variety, if the test is executed accurately.
Why might you need to run a test where the variety and unique are indistinguishable?

Now and again, you might need to utilize this to screen the quantity of transformations on the page where you are running the A/A test to track the quantity of changes and decide the pattern change rate before starting an A/B or multivariate test.
In most different cases, the A/A test is a technique for twofold checking the viability and precision of the A/B testing programming. You should hope to check whether the product reports that there is a statistically significant (>95%) distinction between the control and variety. In the event that the product reports that there is a measurably huge distinction, that is an issue, and the product would need to be watched for accurate execution on the site or portable application.

2.4.2 Define the Prior

While starting off, there may or may not be data available in prior to have a prior beta distribution available for further calculations. The data could be some data retrieved through observations, known as the **Likelihood**, as well as some background information on the topic, regarded as the **Prior Probability**. These two variants in data may or may not be in liaison with each other, which isn't a matter of concern. The task is to model this prior. Suppose there is a background information regarding conversion rates for an action on a web page as $x\%$. Hence, the prior probability distribution will be formed with respect to the Beta function (B.1), as $Beta(\alpha, \beta)$ where

1. α is the number of users that added up to the conversion rates,
2. β is the number of users that didn't contribute to conversion rates, and
3. $\alpha + \beta$ is equivalent to the total number of users on whom the A/B testing has been analysed.

On plotting the various discrete data obtained overtime, it is noticed that the lower the sum of α and β, that is, number of users, the wider the Beta probability distribution is modeled.
We can calculate the **Posterior Distribution** as

$$Beta(\alpha_{posterior}, \beta_{posterior}) = Beta(\alpha_{likelihood} + \alpha_{prior}, \beta_{likelihood} + \beta_{prior}) \qquad (2.6)$$

2.4.3 Gather and Study the Website Data

Your investigation will frequently give knowledge into where you can start upgrading. It starts with high activity territories (where traffic is in excess) of your site or application, as that will enable you to accumulate information speedier. Search for pages with low change rates or high drop-off rates that can be progressed.

2.4.4 Observe User Behaviour

Study the web pages with high as well as low bounce rates via visitor behavior analysis tools such as Form Analysis, On-page Surveys, Heatmaps, and Visitor Recordings to find the loopholes which are preventing the conversion rates from increasing.

2.4.5 Distinguish and Examine Goals

Your transformation objectives are the measurements that you are utilising to decide if the variety is more effective than the first form. Objectives need not be specific, ranging from, clicking a button or a link to advertisements of items available for purchase on the same or a different eCommerce, and email driven subscription for information exchanges.

2.4.6 Produce Hypothesis

Once you've recognised an objective, you can start creating A/B testing thoughts and theories for why you think they will be superior to the present form. The rundown of thoughts shall then be organised by expected effect and inconvenience of usage.

2.4.7 Make Variations

Using your A/B testing programming, roll out the coveted improvements to a component of your site or portable application encounter. Most of the A/B testing tools and services offer editors with rich user-interface that enable the users to adapt changes indicated by the testing results. These alterations would then improve the user-experience and hence aid in transforming this data to increased conversion rates. Changes, talked about, could be as simple as modifying the position or hierarchy of elements on a website, or making a particular div more prominent than the other.
A/B Testing must be regarded as an essential task in setting up brands, especially eCommerce, as they can result in significant improvements in getting increased and returning hits by end users. The expense put into A/B testing shall be regarded as an investment. For instance, charges demanded by an A/B Tester for comparing the most suitable ads to be displayed on a website can be easily recovered in a few Google AdSense and Google AdWords clicks, which ironically would have increased after performing the A/B Testing.

2.4.8 Run Experiment

Kick off your analysis and sit tight for guests to take part! Now, guests to your site or application will be arbitrarily relegated to either the control or variety of your experience. Their cooperation with each experience is measured, tallied, and contrasted with decide how each performs.

2.4.9 Investigate Results

Once your examination is finished, it's a great opportunity to break down the outcomes. Your A/B testing programming will exhibit the information from the prior beliefs and likelihood to give out posterior probability. For both A and B variants, there will be a certain conversion rate derived overtime of constant collection of data. This when plotted using the Beta normal function, will pictorially represent the variance between the two hypothesis. Initially, the two Beta distributions might look the same. But, over time there will be a significant difference observed. This would exhibit the variant which is more likely to get the client more conversion rates.

2.4.10 Monte Carlo Simulations

Monte Carlo simulations are a great way to bridge the gab between the A/B testing by deterining the amount by which a variant, say B, is better than the other variant, say A. These simulations were carried out in the R function.

In this n number of trials are taken with both A and B variants having it's own prior. The samples of both variants are calculated as a beta functions of the posterior probability of the two variants (2.6). Then the normalised value of the two samples is generated on dividing the sum of the Beta functions of posterior probabilities of sample A and B by the n number of trials. This as a result gives the actual amount by which one variant (here, B) is greater than the other (here, A).

Hence, the Bayesian A/B testing has been successfully been carried out.

Chapter 3

Results and Discussion

> The combination of hard work and smart work is efficient work.
>
> Robert Half

Inspired by the keynote talk on Bayesian Statistics by Chris Stucchio, Data Scientist at Wingify, attended at the PyDelhi Conference 2017, a conference for Python programming User Groups; I was inspired to take it up as my topic for my term paper. Since Bayesian Statistics is regarded as an intense topic under Statistics and Probabilities, I had the responsibility to first read in depth about the basics under the topic. Understanding the mathematical notations used in the subject, weren't always a cake walk, but was a great learning experience to understand it further.
In the first week I started with understanding the Classical,Frequential and Bayesian Statistics on introductory level as a part of Data Science.I analysed why Bayesian statistics proves to be better than the former two methods. I learnt about the need and benefits to study it and know what makes it a trending topic in Data Science. My journey in understanding Bayesian statistics was aided by an online MOOC offered by University of California Santa Cruz and reading articles available on Wikipedia.
For week two, I searched in depth the applications of Bayesian statistics and the wide variety of fields they were used in. With some further research and reading documentations and tutorials, I learnt about Bayesian A/B testing. I chose it as my topic as it is an essential task for both a programmer and a designer.
In week three, I studied in depth about A/B testing and how Bayesian statistics can be incorporated in it for better and efficient results. I admired the benefits of Bayesian A/B testing and started working on understanding how to carry it out. I realised my need to practice Python programming, especially the SciPy Python library consisting of Numpy and Matplotlib modules. I took this up as an exercise simultaneously. I also delivered a talk on Exploring Numpy meanwhile in a PyDelhi meetup. I formulated the steps to be carried out during A/B testing which are well formulated in chapter 2. This initiated my

coding process.

Bayesian A/B testing is all about forming priors and using the hypothesis to collect data. It is observed that as more and more data is collected the posterior probability gets further away from the priors.

Statistical significance is the probability that the distinction in change rates between a given variety and the pattern is not because of an arbitrary shot. A consequence of an analysis is said to have factual centrality, or be measurably huge, on the off chance that it is likely not caused by chance for a given factual importance level.

Your measurable noteworthiness level mirrors your hazard resistance and certainty level. For instance, in the event that you run an A/B testing try different things with a hugeness level of 97% or the odds of 97:3, this implies on the off chance that you can be 97% sure that the watched comes about are genuine and not a mistake caused by irregularity. It additionally implies that there is a 3% chance that you could not be right.

Statistical significance is a method for numerically demonstrating that a specific measurement is dependable. When you settle on choices in view of the consequences of tests that you're running, you will need to ensure that a relationship really exists. Online web proprietors, advertisers, and promoters have as of late turned out to be keen on ensuring their a/b test tests (example, transformation rate a/b testing, advertisement duplicate changes, email title changes) get factual centrality before forming a hasty opinion.

Statistical significance is most for all intents and purposes utilized as a part of measurable speculation testing. For instance, you need to know regardless of whether changing the shade of a catch on your site from red to green will bring about more individuals tapping on it. This statistical significance helps to understand the significance of one hypothesis than other variant hypothesis.

Though, it does not tell the magnitude buy which one variant stands superior to the other. After a great deal of study in the various probability distributions, especially the Beta distribution to help define the prior and hence the Posterior probabilities, I realised my need to know about R programming which would ease out my way to perform Monte Carlo simulations. These simulations have helped to validate the findings derived in Bayesian A/B testing in terms of magnitude.

Talking about accomplishments, the targets and accomplishments on a weekly basis has been summarized in table 3.

Week	Target	Accomplishment
1	Get an overview on Bayesian Statistics and know other similar statistics' topics. Read about its pros, cons and applications.	Understanding the Classical,Frequential and Bayesian Statistics on introductory level. Examining why Bayesian statistics proves to be better than the former two methods. Learning about what makes it a trending topic in Data Science and hence its need.
2	Read practical real world applications of Bayesian Statistics in all industries inclusive of tech and find one to be the basis of my term paper.	Get acquainted with the basics of topics under Statistics and Probabilities. Chose Bayesian A/B testing as my main focus of term paper.
3	Study in depth about A/B testing and how Bayesian statistics can be incorporated in it for better and efficient results.	Read about benefits of Bayesian A/B testing and how to carry it out. Practised Python and the SciPy library.
4	See if any other topics are needed to be done for better performance of A/B tester.	Learnt the basics of R programming to carry out Monte Carlo simulations, that aid in validating results of A/B testing.

Table 3.1: Weekly log of accomplishments

Chapter 4

Proposing the Future

> The best thing about the future is that it comes one day at a time.
>
> Abraham Lincoln

Bayesian statistics has a long road ahead for development. There is an intensive scope for expansion both in terms of efficiency and features, even in Bayesian A/B testing. The enumerated list given below is an attempt to document the future scopes that bayesian statistic has:

1. **Immune to peeking**: Bayesian insights are helpful in test settings since you can stop a test at whatever point you please and the outcomes will in any case be substantial.

2. **Revenue is the goal**:The final objective is to max income and different business measurements, yet there is no better approach to do that in the domain of A/B testing than to find target realities about the world the business works in, as most ideal as essentially.

3. **Answers are given to questions of interest**: Bayesian methodologies are intended to give the response to what is the likelihood of Variant A being superior to the Control given a few information and our earlier learning about Variant A and the Control?

Appendices

Appendix A

301 and 302 redirect

A 301 divert implies that the page has for all time moved to another area. A 302 divert implies that the move is just brief. Web crawlers need to make sense of whether to keep the old page, or supplant it with the one found at the new area. On the off chance that the wrong sort of divert has been set up, web crawlers may wind up plainly befuddled, bringing about lost activity.

Why does this make a difference? On the off chance that you are moving a website page or a whole site to another area, for example on the off chance that you change your space name, you need guests to have the capacity to discover your webpage. A divert makes the client's program naturally forward from the old area to the better one. You may surmise that Google and the other web search tools would simply take after the sidetracks, yet that is the place things get entangled. At the point when a site moves, that can trigger the Google maturing delay. Typically the site drops out of the scan rankings for a while, some of the time even a year. We'll return to this later.

There aren't an excessive number of circumstances where a 302 is fitting. How regularly have you briefly moved a page? It's a great deal more typical to move pages for all time. By the by, it appears to be simpler to make 302 sidetracks than 301s. You can utilize Javascript or a meta tag to make a 302. Making a 301 divert requires extraordinary orders in your .htaccess document in the event that you utilize an Apache server. With Windows servers, making 301 takes much additional time and inconvenience. That is the reason there's an inclination for individuals to erroneously utilize 302 rather than 301.

Google perceives that many individuals utilize 302 when they truly mean 301. Luckily, Google isn't bound by any law to take individuals actually. For delivering the most ideal query items, Google can and should take a gander at 302s and make sense of if the website admin truly implies 302, or if it's common perplexity and they truly mean 301.

Regardless of whether Google really handles 302s appropriately is an open inquiry. In the event that a 302 is utilized rather than a 301, web search tools may keep on indexing the old URL, and carelessness the better and brighter one as a copy. Connection prominence may be separated between the two urls, harming look rankings. Web crawlers may make sense of how to deal with the 302, or they may not. Google representatives have said that they will regard a 302 as a 301 on the off chance that they think the website admin has made a blunder, yet why take a risk, and shouldn't something be said about other web crawlers?

At the point when for all time moving a site, or a page, best practice is to utilize a 301 divert. 302s in this circumstance appear to be erroneous. By saying "transitory move" a 302 advises web crawlers to keep the old space or page filed, yet it would be attractive for them to list the new area. In the past individuals have utilized 302 diverts with an end goal to dodge the Google maturing delay. This workaround may have worked sooner or later, yet it is not a present best practice.

On the off chance that worried about losing rankings due to a 301, the arrangement is not to change an area, and not to end up noticeably fiscally dependent on rankings. In this present reality, organizations abstain from changing their names since it can seem shady. Who can reprimand Google for utilizing a similar rationale: in case you're changing space names, you may be planning something sinister. How about we hold up a while and check whether you stay under control before we suggest you.

Appendix B

Continuous Distribution

B.1 Beta Distribution

1. This distribution takes randomly varied values between 0 and 1.

2. It is extensively used to model probabilities, such as the Bayesian inference.

3. For a random variable X,

$$
\begin{aligned}
X &\sim Beta(\alpha, \beta) \\
f(x|\alpha, \beta) &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} I_{(0<x<1)}(x) \\
E[X] &= \frac{\alpha}{\alpha+\beta} \\
Var[X] &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}
\end{aligned}
\tag{B.1}
$$

4. where $\Gamma()$ is the gamma distribution's function.

6. $\alpha > 0$ and $\beta > 0$

7. For a distribution, $X\ Beta(1,1)$, that is, $= \beta = 1$, a standard Uniform(0,1) distribution forms.

Bibliography

[1] Allen B. Downey, *Think Bayes - Bayesian Statistics Made Simple*. Green Tea Press, Version 1.0.9, 2012.

[2] Herbert Lee, *Coursera - Bayesian Statistics*, `https://www.coursera.org/learn/bayesian-statistics` University of California Santa Cruz.

[3] Python Software Foundation, *Python v2.7.2 documentation*, `http://docs.python.org/`.

[4] Hochman Consultants, *302 Redirect vs. 301 Redirect: Which is Better?*, `https://www.hochmanconsultants.com/301-vs-302-redirect/`

[5] Count Bayesie *Bayesian A/B Testing: A Hypothesis Test that Makes Sense* `https://www.countbayesie.com`