

Date: 17 April, 2022

Practical 15: To fit a Negative Binomial distribution to the given data of a blood bank which collects B-negative blood samples only.

Question: A blood bank collects B-negative blood samples only. The probability of getting B-negative blood is 'p' and is treated as success. It takes only one bottle of blood from one person and purchases 5 bottles per day. The failures of 400 days before getting 5th bottle of blood of this kind were recorded as follows:

Number of failures (not getting B-negative)	0	1	2	3	4	5	6	7
No. of days	131	131	79	37	14	5	2	1

Fit a negative binomial distribution.

Methods and Formula:

Negative Binomial Distribution: The negative binomial distribution is a discrete probability distribution that models the number of successes in a sequence of independent and identically distributed Bernoulli trials before a specified (non-random) number of failures (denoted r) occurs.

$$X \sim \text{NB}(r, p)$$

$$p(x) = P[X = x] = \binom{x+r-1}{r-1} p^r q^x, \quad q = 1-p, \quad x=0, 1, 2, \dots$$

$$\text{Mean} = \frac{rq}{p}$$

The recurrence relation for Negative binomial probabilities is $p(X=x) =$

$$p(x) = \frac{(r+x-1)}{x} p^r q^x, \quad x = 0, 1, 2, \dots$$

$$\text{Expected Frequencies} = N \cdot p(x)$$

Bar graph: A bar graph is a chart that plots data using rectangular bars or columns (called bins) that represent the total amount of observations in the data for that category.

Analysis:

To fit a Negative Binomial distribution to the given data of a blood bank which collects Bnegative blood samples only. The tables 15.1 have been prepared in MS Excel.

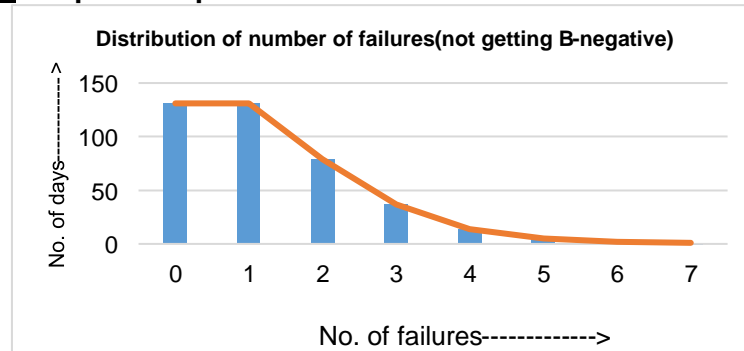
Table 15.1: Tabular form of Data for calculating Negative Binomial distribution of the data

Number of failures (not getting Bnegative)	No. of days	fx	f*x^2	(x+r)/(x+1)q	p(x)	exp. Frequency	exp. Frequency round off
0	131	0	0	1	0.32768	131.072	131
1	131	131	131	0.6	0.32768	131.072	131
2	79	158	316	0.4666667	0.196608	78.6432	79
3	37	111	333	0.4	0.0917504	36.70016	37
4	14	56	224	0.36	0.03670016	14.680064	15

5	5	25	125	0.33333333	0.013212058	5.284823	5
6	2	12	72	0.3142857	0.004404019	1.7616077	2
7	1	7	49	0.3	0.00138412	0.5536481	0
Total	400	500	1250		0.999418757		400

From Table 15.1, this represents the calculation of recurrence relation for probabilities of Negative binomial distribution of the given data of A blood bank collects B-negative blood samples only.

Fig 15.1: Graphical representation of distribution of number of failures



From Fig15.1, it shows the graphical representation of the distribution of number of failures of not getting B-negative blood sample. It shows the data is positively skewed.

CONCLUSION:

The above analysis forms a pattern which can be used to predict the number of failures i.e. number of days it will take for blood bank to collect as many bottles as required. As in the given data we can see that out of 400 days, 131 days have 0 failure (not getting B-negative blood sample) i.e. they have a good collection of B-negative blood.

EXCEL OPTION:

1. Enter data in column.
2. Then create table according to your need. (using all border option present in the HOME tab).
3. To create bar graph, select the data from the table then click on INSERT=>COLUMNS=> select the bar graph.

And then add graph title and suitable headings to the y-axis and x-axis.

Now, the bar-graph is created.

4. Calculate probabilities under Negative Binomial distribution

In excel, you can use any of the following three methods

Method 1: using pmf

Method 2: using recurrence relation for probabilities

$$p(X=x) = p(x) = \binom{r+x-1}{x} p^r q^x, \quad x = 0, 1, 2, \dots$$

Method 3: using Excel function for binomial Probabilities (NEGBINOMIAL.DIST())

