TUTORIAL → 4

1) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Sol $T(n) = aT\left(\frac{n}{2}\right) + f(n)$

$a \geq 1, \; b > 1$

on comparing,

$a = 3, \; b = 2, \; f(n) = n^2$

Now,

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n^2)$

2) $T(n) = 4T(n/2) + n^2$

Sol $a \geq 1, b = 2, f(n) = n^2$

$c = \log_2 4 = 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

$T(n) = T(n/2) + 2^n$

$a = 1, \; b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 \quad c = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

(4) $T(n) = 2^n T(n/2) + n^n$

Sol

$a = 2^n$

$b = 2, \quad f(n) = n^n$

$c = \log_b a = \log_2 2^n = n^2$

$n^c \Rightarrow n^n$

$\therefore f(n) = n^c$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

(5) $T(n) = 16\, T\left(\frac{n}{4}\right) + n$

Sol

$a = 16, \quad b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

(6) $T(n) = 2T(n/2) + n\log n$

Sol $a = 2, \quad b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $n\log n > n$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n\log n)$

) $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

Sol $a=2, b=2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Sol $a=2, b=4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

Since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

) $T(n) = 0.5T\left(\frac{n}{2}\right) + 1/n$

Sol $a=0.5, b=2$

Since, acc to Master's theorem $a \geq 1$, but here a is 0.5

so, we ~~not~~ cannot apply master theorem.

) $T(n) = 16T\left(\frac{n}{4}\right) + n!$

Sol $a=16, b=4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

Now, $n^c = n^2$

as $n! > n^2$

$\therefore T(n) = \Theta(n!)$

(11) $4T(n/2) + \log n$

Sol

$a=4,\ b=2,\ f(n)=\log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n$

Since, $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^c)$

$= \Theta(n^2)$

(12) $T(n) = sqrt(n)\, T(n/2) + \log n$

Sol

$a = \sqrt{n},\ b=2,$

$\therefore c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(f(n))$

$= \Theta(\log(n))$

(13) $T(n) = 3T(n/2) + n$

$a=3,\ b=2\ f(n)=n$

$c = \log_b a = \log_2 3 = 1.5848$

$\therefore n^c = n^{1.5849}$

$\Rightarrow f(n) < n^c$

$\therefore T(n) = \Theta(n^{1.5849})$

4) $T(n) = 3T(n/2) + sqrt(n)$

$a = 3,\ b = 3,\ c = \log_b a = \log_3 3 = 1$

$\therefore n^c = n^1 = n$

as $sqrt(n) < n$

$\therefore f(n) < n^c$

$\therefore T(n) = O(n)$

5) $T(n) = 4T(n/2) + cn$

$a = 4,\ b = 2$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$\therefore cn = n^2$ (for any constant)

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

6) $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\therefore n^{0.792} < n\log n$

$\therefore T(n) = \Theta(n\log n)$

7) $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3,\quad c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

as $n/2 < n$

$\therefore f(n) < n^c$

$TC = \Theta(n)$

(18) $T(n) = 6T\left(\frac{n}{2}\right) + n^2 \log n$

$a = 6, \; b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

as, $n^{1.6309} < n^2 \log n$

$\therefore T(n) = \Theta(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n/\log n$

Sol $a = 4, \; b = 2, \; f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \Theta(n^2)$

(20) $T(n) = 64\,T\left(\frac{n}{8}\right) - n^2 \log n$

Sol $a = 64, \; b = 8, \; c = \log_b a = \log_8 64 = \log_8$ [illegible]

$c = 2$

$n^c = n^2$

$\therefore n^2 \log n > n^2$

$\therefore T(n) = \Theta(n^2 \log n)$

iii) $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$\Rightarrow n^{1.7712} < n^2$

$\therefore T(n) = \Theta(n^2)$

iv) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1,\ b = 2,\ c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \Theta(n(2 - \cos n))$