Homework: Dynamic Programming

This document defines the **homework assignments** for the "Algortihms" course @ Software University. Please submit a single **zip** / **rar** / **7z** archive holding the solutions (source code) of all below described problems.

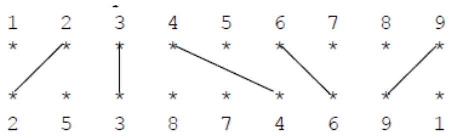
1. Connecting Cables

We are in a rectangular room. On opposite sides of the room there are sets of **n** cables (n < 1000). The cables are indexed from 1 to n.

On each side of the room there is a permutation of the cables, e.g. on one side we always have ordered {1, 2, 3, 4, 5} and on the other side we have some permutation {5, 1, 3, 4, 2}. We are trying to connect each cable from one side with the corresponding cable on the other side – connect 1 with 1, 2 with 2, etc. **Cables are straight and should not overlap!**

The task is to find the maximum number of pairs we can connect given the restrictions above.

Examples

Input	Output	Comments
`2 5 3 8 7 4 6 9 1`	`Maximum pairs connected: 5`	
`4 3 2 1`	`Maximum pairs connected: 1`	`Any other pair can be conntected as well.`
`1 2 3`	`Maximum pairs connected: 3`	

2. Minimum Edit Distance

We have two strings, **s1** and **s2**. The goal is to obtain **s2** from **s1** by applying the following operations:

- **replace(i, x)** – in **s1**, replaces the symbol at index **i** with the character **x**
- **insert(i, x)** – in **s1**, inserts the character **x** at index **i**
- **delete(i)** – from **s1**, removes the character at index **i**

We are only allowed to modify **s1**, **s2** stays unchanged at all times. Each of the three operations has a certain **cost** associated with it (positive integer number). **Note**: the cost of the **replace(i, x)** operation is 0 if it doesn't actually change the character.

The goal is to find the sequence of operations which will produce **s2** from **s1** with **minimal cost**.

Examples

Input	Output	Comments

`cost-replace = 3`
`cost-insert = 2`
`cost-delete = 1`
`s1 = abracadabra`
`s2 = mabragabra`
	`Minimum edit distance: 7`
	`INSERT(0, m)`
	`DELETE(3)`
	`DELETE(4)`
	`REPLACE(6, g)`
		`Indices refer to the original s1 string – DELETE(3) deletes the symbol at index 3 from abracadabra, not from the modified string mabracadabra after the INSERT(0, m) operation.`

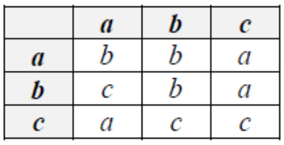
cost-replace = 5
cost-insert = 2
cost-delete = 1
s1 = nqma bira
s2 = ima bira
	Minimum edit distance: 4
	DELETE(0)
	DELETE(1)
	INSERT(1, i)
		We can obtain s2 with two operations – DELETE(0) + REPLACE(1, i), but the cost of the REPLACE operation is high, that's why the solution involves three operations, their total cost is smaller. The INSERT can be performed also at index 0 and index 2.

cost-replace = 3
cost-insert = 3
cost-delete = 3
s1 = equal
s2 = equal
	Minimum edit distance: 0	

cost-replace = 1
cost-insert = 1
cost-delete = 1
s1 = equal
s2 = different
	Minimum edit distance: 8
	INSERT(0,d)
	INSERT(1,i)
	INSERT(2,f)
	INSERT(3,f)
	REPLACE(1,r)
	REPLACE(2,e)
	REPLACE(3,n)
	REPLACE(4,t)	

3. * Symbol Multiplication

We have an **alphabet** of k symbols (a finite number) and a **multiplication table** showing the result of multiplying each two symbols of the alphabet. E.g., the alphabet is {a, b, c} and the multiplication table is:



	a	*b*	*c*
a	*b*	*b*	*a*
b	*c*	*b*	*a*
c	*a*	*c*	*c*

This shows that a*a = b, a*b = b, b*a = c, etc. As shown in the example, multiplication is **not commutative or associative** – a*b != b*a, therefore, the order of multiplication is essential.

We have a string **S** comprised of characters from the alphabet. The task is to find whether we can obtain the symbol **'a'** by inserting brackets in the string – all symbols in brackets are multiplied. If so, print the string with the brackets inserted. Print "No solution" otherwise. Assume 'a' will always be in the alphabet.

Examples

Input	Output	Comments
Alphabet = {a,b,c} Table = bba cba aac S = abc	((a*b)*c)	((a*b)*c) = (b*c) = a
Alphabet = {a,b,c} Table = bba cba aac S = bacacbcabbbcacab	(((b*a)*(c*a))*(((c*(b*c))*a)*((b*((b*b)*(c*a)))*(c*(a*b)))))	
Alphabet = {a,b}	No solution	No combination of

Table = bb bb S = abbbaaba		two symbols produces 'a' after multiplication.