

LWE encryption

First we generate a public key and a private key (Alice generates)

~~Alice~~ Bob

$$\boxed{A} \quad \boxed{s} + \boxed{e} = \boxed{t}$$

A is random

(s, e) small coefficients

public key $\leftarrow (A, t)$ {Concatenation

$s \rightarrow$ private key

u and v sent to Bob to recompute the message m

$$v = r \times t + e_2 + m$$

$$= r(As + e) + e_2 + m$$

$$= r(As) + \underbrace{e' + m}_{\text{small noise}}$$

$$us = (rA + e_1)s$$

$$= rAs + e's \approx v - m$$

so Alice can ~~decrypt~~ decrypt to obtain

$$v - us = m + e'$$

obtain the message by removing the noise somehow

\rightarrow For each bit of message you need to send large sizes of A, t

~~Bob~~ Alice

$$\begin{aligned} & \xrightarrow{\text{secret}} \boxed{r} \text{ random val} \\ & \quad \boxed{A} \quad \boxed{t} \\ & + \boxed{e_1} \quad \boxed{e_2} \\ & + \boxed{0} \quad \boxed{m} \\ & \leftarrow \boxed{v} \quad \boxed{v} \end{aligned}$$

Alice uses Bob's public key and does $R \times A, t$ then adds e_1 and e_2 and the message m she ~~pro~~ provides the encrypted text of u and v