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ASSIGNMENT-1

Main Numerical Proplem

p	q	r	p→q	~r	~r∨q	(p→q)∧(¬r∨q)]	(p→r)	
T	T	T	T	F	T	T	T	T
T	T	F	T	T	T	T	F	F
T	F	T	F	F	F	F	T	T
T	F	F	T	T	T	T	F	F
F	T	T	T	F	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	F	F	T	T
F	F	F	T	T	T	T	T	T

The Statement is contigency since the final expression contain mixture of Truth & False.

p : user is authenticated
q : user has a valid token
r : user can access confidential files
The subformula ~r∨q is logically equivalent to r → q. Treating ~r∨q as a security constraint enforces the policy "access implies token".
So if q is false, then ~r∨q becomes false that forces ~r to be true. If no token is present, the user can't have access to confidential files

Sub problem 1

$\sim(p \to q) \equiv p \wedge \sim q$

p	q	$p \to q$	$\sim(p \to q)$	$\sim q$	$p \wedge \sim q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

1. Hence using the above Truth Table we can say that the following prepositions are logically equival

2. $p \to q \equiv \neg p \vee q$

$\neg(p \to q) \equiv \neg(\neg p \vee q)$

Applying De-morgan law,

$\neg(\neg p \vee q) \equiv \neg(\neg p) \wedge \neg q$

Simplifying double negation

$\neg(\neg p) \equiv p$

So, we get :

$\neg(p \to q) \equiv p \wedge \neg q$

$\therefore$ Hence proved

Sub problem 2

Given = All smart student study regularly.

1. $P(x)$ = x is smart
$Q(x)$ = x studies regularly.

Given: $\cancel{\exists x \exists y [P(x) \land Q(x)]}$
$\forall x (smart(x) \rightarrow studies(x))$

2. Negation
$\neg \forall x (smart(x) \rightarrow studies(x))$
$\Rightarrow \exists x \neg (smart(x) \rightarrow studies(x))$

$smart(x) \rightarrow studies(x) = \neg smart(x) \lor studies(x)$
So,
$smart(x) \land \neg studies(x)$
$\exists x (smat(x) \land \neg studies(x))$
Meaning: There exist atleast one smart student who does not study regularly

Sub Problem 3

Let

$M(x, y)$: "x is a mentor of y"

$G(x)$: "x is a good Mentor"

Given:

1. $\forall x \forall y (M(x, y) \to G(x))$
2. $M(Amit, Reena)$

Goal : $G(Amit)$

1. $\forall x \forall y (M(x, y) \to G(x))$

$\forall y (M(Amit, y) \to G(Amit))$

$M(Amit, Reena) \to G(Amit)$. — (3)

$M(Amit, Reena)$ — Premise (given) — (4)

From (3) and (4) by Modus Ponens.

$G(Amit)$

- Universal instantiation : From $\forall x, \phi(x)$, you may infer $\phi(t)$ for any term t, we applied UI twice to specialize the general rule to Amit and Reena.
- Modus Ponens (MP): From $A \to B$ and A infer B.

We used MP on the implication we obtained and the fact $M(Amit, Reena)$

That completes the proof : Amit is a good Teacher

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Sub Problem-4

$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8\}$

$U = \{1 - 9\}$

(1) Find

- $A \cup B = \{1, 2, 3, 4, 6, 8\}$
- $A \cap B = \{2, 4\}$
- $A - B = \{1, 3\}$
- $A' = \{5, 6, 7, 8, 9\}$
- $B' = \{1, 3, 5, 7, 9\}$

(2) Verify : $(A \cup B)' = A' \cap B'$

LHS = $(A \cup B)'$

$= A \cup B = \{1, 2, 3, 4, 6, 8\}$

$= (A \cup B)' = \{5, 7, 9\}$

Hence LHS = RHS

∴ verified.

RHS = $A' \cap B'$

$A' = \{5, 6, 7, 8, 9\}$

$B' = \{1, 3, 5, 7, 9\}$

$A' \cap B' = \{5, 7, 9\}$

Sub problem 5

let $A = \{1,2,3\}$
$R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$

1. As matrix

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

2. For Reflexive
aRa
$(1,1) \in R$, $(2,2) \in R$, $(3,3) \in R$
Hence its a Reflexive reln.

• for symmetric:
aRb then bRa.
$(1,2) \in R$
then $(2,1)$ should also belong to R
$(2,1) \notin R$
Hence not symmetric

• For transitive: if aRb, bRc then aRc
$(1,1) \in R$
$(1,2) \in R$
$(1,2) \in R$.
Hence its a transitive Reln