Introduction:

There were four loading cases for soils following the Cam-Clay (CC) and Modified Cam-Clay model (MCC) discussed in the paper. In the case of pure strain controlled loading, the author states that violation of local uniqueness is unlikely in the realistic values of the parameters in table 1.

However, when the loading is mixed, then it becomes more likely to observe a numerical phenomenon called "snap-back" in our stress-strain plots even at realistic and common values of the parameters. Snap-back phenomenon goes against the conditions of local uniqueness that are necessary for the plastic flow. Basically,it violates the consistency conditions and thus the numerical solution becomes invalid. Then it becomes necessary for us to identify the values of stress ratio "eeta" at which this snap-back phenomenon appears. So that we can avoid those values and those numerical results This is called the **critical value of stress-ratio**.

The sections below will describe the critical stress-ratio for different loading conditions. For almost all the cases, the critical stress ratio would also depend on the combination of values of parameters in Table 1.

1- Expression for Critical stress ratio for arbitrary strain controlled loading:

$$\frac{\eta_d}{M} = \sqrt{\frac{1-\frac{2}{bM^2}-\sqrt{\left(1-\frac{2}{bM^2}\right)^2-1+\left(\frac{1}{\lambda/\kappa-1}\right)^2}}{1-\frac{1}{\lambda/\kappa-1}}}, \qquad b = \frac{2(1+\nu)}{9(1-2\nu)}.$$

Depends on:

Table 1: Parameters that affect the function 'f'

M	Critical state line
v	Poisson ratio
lambda	Virgin Consolidation Line slope
kappa	Swelling line slope

function **'h'** should always remain positive for the **local uniqueness** to hold.

$$h(\eta) = \left(1-\frac{1}{\lambda/\kappa-1}\right)\left(\frac{\eta}{M}\right)^4-2\left(1-\frac{2}{bM^2}\right)\left(\frac{\eta}{M}\right)^2+1+\frac{1}{\lambda/\kappa-1} \quad > \quad 0.$$

The function h is positive for certain values of lambda/kappa and eeta .

2- Mixed Loading Case 1 : Constant Mean Effective Stress

$$f(\eta) = b + \frac{(\lambda/\kappa - 1)2\eta}{M^2 + \eta^2}\left(\frac{1}{3} + \frac{2\eta}{M^2 - \eta^2}\right) \quad < \quad 0. \tag{36}$$

if

$$\eta > M$$

sign of ‘f’ depends on:

Table 2: Parameters on which 'f' depends

M	Critical state line
v	Poisson ratio
lambda	Virgin Consolidation Line slope
kappa	Swelling line slope
eeta	Current stress ratio

We plot the function f over a range of stress ratios for given material parameters and see where f=0. Additionally, there is also a limit on the over-consolidation ratio.

condition that $\eta < \eta_m$ then imposes, together with Eq. (32), the following restriction on the overconsolidation ratio:

$$R < R_m = 1 + (\eta_m/M)^2, \tag{37}$$

where R_m denotes the overconsolidation ratio R such that $\eta_p = \eta_m$ (that is, the maximum allowable overconsolidation ratio while not exceeding the snap-back threshold at yielding).

Fig. 8 shows (a) the normalized stress ratio η_m and (b) the corresponding overconsolidation ratio R_m as a function of the critical stress ratio M for some typical values of the Poisson's ratio and the slope ratio κ/λ. The results reveal that the snap-back threshold values can be very low, decreasing with the material constants M, ν and κ/λ; for the present parameter set R_m equals 5.55 (marked point in Fig. 8b). Moreover, it should be pointed out that the snap-back threshold exists in cases where the standard condition of local uniqueness is satisfied. A specific example is analysed in detail in the following in order to illustrate the model behaviour when η_m is exceeded.

3- Load Case 2: drained loading

This is the case that we are concerned with. This function should remain less than zero to avoid numerical error. The point “eeta” where the function becomes zero and greater, we will observe the “snap-back” effect according to the authors.

The function ‘f’ below must remain less than or equal to zero to avoid the snap-back phenomenon.

$$f(\eta) = \left[\frac{\lambda/\kappa}{3} + b\eta + \frac{(\lambda/\kappa - 1)2\eta}{M^2 - \eta^2}\right]\frac{1}{\alpha - \eta} + b + \frac{(\lambda/\kappa - 1)2\eta}{M^2 + \eta^2}\left(\frac{1}{3} + \frac{2\eta}{M^2 - \eta^2}\right) < 0. \tag{45}$$

Table 3: Parameters on which 'f' depends for drained triaxial compression loading

M	Critical state line (1-1.3)
v	Poisson ratio (0.2-0.4)
lambda	Virgin Consolidation Line slope (10^-1- 10^-2)
kappa	Swelling line slope (kappa less than lambda) (10^-3 - 10^-2)
eeta	Current stress ratio
alpha	Slope of linear effective stress path in p’ q plane

4- Case 3 : Undrained triaxial loading

$$f(\eta) = 4\Lambda\frac{\eta^2}{M^4 - \eta^4} + b\left(1 - 2\Lambda\frac{\eta^2}{M^2 + \eta^2}\right) < 0. \tag{50}$$

Table 4: Parameters on which 'f' depends for undrained loading

M	Critical state line
v	Poisson ratio
lambda	Virgin Consolidation Line slope
kappa	Swelling line slope
eeta	Current stress ratio