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"ASSIGNMENT "

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SECTION:

A

COURSE:

LINEAR ALGEBRA

SUBMITTED TO:

MAM ASMA SHAHEEN

QUESTION : 1

The set of all ordered pairs of real numbers, V, with the operations of component-wise addition and scalar multiplication are defined as :-

$$(a, b) + (c, d) = (a+c, b+d)$$

$$k(a, b) = (ka, kb)$$

To show that, we must verify that V satisfies the axioms of a vector space. These axioms are as follows :-

i) Close Under addition. For any two elements (a, b) and (c, d) in V, their sum $(a, b) + (c, d)$ must also be in V.

This condition is satisfied because the sum of any two ordered pairs of real numbers is itself in an ordered pair of real numbers.

$$(a, b) + (c, d) = (a+c, b+d) \in \mathbb{R}^2$$

ii) Compatibility of Scalar multiplication with field multiplication : For all Scalars k in $\mathbb{R}$ and all (a, b) in V, we must have.

$$k(a, b) = (ka, kb)$$

$$k(a, b) \neq k(ka, kb) \neq k(a, b)$$

e:g;

$$3(2, 5) = (6, 15) \neq (6, 5)$$

Since V deesnot satisfy all the axioms of a vector space, where V is not a V.space over $\mathbb{R}$.

Question No 2:-

For what value of k will the vectors $(1,-2,k)$ in $\mathbb{R}^3$ be a linear combination of the vector $(3,0,-2)$ & $(2,-1,-5)$?

Solution:-

$$\left[\begin{array}{cc:c} 3 & 2 & 1 \\ 0 & -1 & -2 \\ -2 & -5 & K \end{array}\right]$$

$$\begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} x_2 = \begin{bmatrix} 1 \\ -2 \\ K \end{bmatrix}$$

$$3x_1 + 2x_2 = 1 \rightarrow 1)$$
$$0x_1 - 1x_2 = -2 \rightarrow 2)$$
$$-2x_1 - 5x_2 = K \rightarrow 3)$$

From eq 2)

$$0x_1 - 1x_2 = -2$$
$$-1x_2 = -2$$
$$\boxed{x_2 = 2}$$

Put $x = 2$ in eq 1)

$$3x_1 + 2(2) = 1$$
$$3x_1 + 4 = 1$$
$$3x_1 = 1 - 4$$
$$3x_1 = -3$$
$$\boxed{x_1 = -1}$$

Putting values of x_1 & x_2 in eq 3)

$-2(-1) - 5(2) = k$

$2 - 10 = K$

$-8 = K$

$\boxed{k = -8}$

Check:

- Putting values of x_1, x_2 & k in eq 3)

$-2(-1) - 5(2) = -8$

$2 - 10 = -8$

$-8 = -8$

Question : 03

Let $v_1 = (0, 1, 2)$, $v_2 = (0, 2, 3)$ and $v_3 = (0, 3, 1)$

Consider $w = av_1 + bv_2 + cv_3$

$(0, y, z) = a(0, 1, 2) + b(0, 2, 3) + c(0, 3, 1)$

$(0, y, z) = (0, a, 2a) + (0, 2b, 3b) + (0, 3c, c)$

$(0, y, z) = (0, (a + 2b + 3c), (2a + 3b + c))$

$0 = 0$

$a + 2b + 3c = y \longrightarrow eq(1)$

$2a + 3b + c = z \longrightarrow eq(2)$

Multiplying eq 1 by 2

$2(a + 2b + 3c = y) \Rightarrow 2a + 4b + 6c = 2y \rightarrow eq(3)$

Now subtracting eq(2) and eq(3)

$$\begin{array}{r} \cancel{2a} + 4b + 6c = 2y \\ \pm\ \cancel{2a} \pm 3b \pm c = \pm z \\ \hline b + 5c = 2y - z \\ \hline \end{array}$$

$b = 2y - z - 5c$

Multiplying eq(1) by 3 and eq(2) by 2

$3(a + 2b + 3c = y) \Rightarrow 3a + 6b + 9c = 3y \rightarrow eq(4)$

$2(2a + 3b + c = z) \Rightarrow 4a + 6b + 2c = 2z \longrightarrow eq(5)$

Now subtracting eqn (4) and eqn (5)

$$\begin{array}{r} 3a + \cancel{6b} + 9c = 3y \\ \pm 4a \pm \cancel{6b} \pm 2c = \pm 2z \\ \hline -a \quad + 7c = 3y - 2z \\ \hline \end{array}$$

$\therefore -a + 7c = 3y - 2z$

$a = 2z - 3y + 7c$

Now putting values of a and b in eqn (2) to find c.

$$2(2z-3y+7c)+3(2y-z-5c)+c=z$$

$$4z-6y+14c+6y-3z-15c+c=z$$

$$z-z+c-c=0$$

$$0=0$$

The solution doesn't exist so W is not spanned by $(0,1,2),(0,2,3),(0,3,1)$.

Question no 4

Check whether the transformation is linear or not: $T(x_1 + x_2 + x_3) = (|x_1|, x_2 - x_3)$

Solution:

Given transformation is

$T(x_1 + x_2 + x_3) = (|x_1|, x_2 - x_3)$

Let $U_1 = (x_1, x_2, x_3)$, $U_2 = (y_1, y_2, y_3)$

let

$U_1 = (0, 0, 0)$

$T(0, 0, 0) = (0, 0)$

i) $T(U_1 + U_2) = T(U_1) + T(U_2)$

$T(U_1 + U_2) = T((x_1, x_2, x_3) + (y_1, y_2, y_3))$

$T(U_1 + U_2) = T(x_1 + y_1, x_2 + y_2, x_3 + y_3)$

$T(U_1 + U_2) = ((|x_1 + x_2|), (x_2 + y_2) - (x_3 + y_3))$

$T(U_1 + U_2) = (|x_1 + y_1|), x_2 + y_2 - x_3 - y_3)$ .... (1)

Now,

$T(U_1) + T(U_2) = T(x_1, x_2, x_3) + T(y_1, y_2, y_3)$

$T(U_1) + T(U_2) = (|x_1|, x_2 - x_3) + (|y_1|, y_2 - y_3)$

$T(U_1) + T(U_2) = (|x_1| + |y_1|, x_2 - x_3 + y_2 - y_3)$

$T(U_1) + T(U_2) = (|x_1| + |y_1|, x_2 + y_2 - x_3 - y_3)$ .... (2)

From (1) & (2)

$|x_1 + y_1| \leq |x_1| + |y_1|$

For example :

a) $U_1 = 5$, $U_2 = 2$

$|5+2| = |7| = 7$

$|5| + |2| = 5+2 = 7$

b) $U_1 = 5$, $U_2 = -2$

$|5-2| = |3| = 3$

$|5| + |-2| = 5+2 = 7$

Equality does not hold for all values of U_1 and U_2

$|U_1 + U_2| \neq |U_1| + |U_2|$ for some values of U_1 and U_2.

$T(U_1 + U_2) \neq T(U_1) + T(U_2)$

Hence,

T is not a linear transformation.

Question no 5 (Part A)

Find Basis and dimension of null space and column space where $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2 - x_3, x_2 + x_3, x_1 + x_2 - 2x_3)$. Also find transformation matrix

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$R_3 - R_1$

$$= \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$R_3 + R_2$

$$= \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 - x_3 = 0 \rightarrow (i)$$

$$x_2 + x_3 = 0 \rightarrow (ii)$$

$$x_2 = -x_3$$

Putting value of x_2 in eq (i)

$$x_1 + 2(-x_3) - x_3 = 0$$

$$x_1 - 2x_3 - x_3 = 0$$

$$x_1 - 3x_3 = 0$$

$x_1 = 3x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_3 \\ -x_3 \\ x_3 \end{bmatrix} \Rightarrow x_3 \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

Null space has basis $= \left\{ \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \right.$ so dim $= 1$

Now

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Here $C_3 = C_1 - C_2$ where C_1 and C_2 are linearly independent. So any vector can be written as linear combination of C_1. So b has basis

dim $= 2$

Transformation matrix.

$B = \{(1,0,0), (0,1,0), (0,0,1)\}$

$E = \{(1,0,0), (0,1,0), (0,0,1)\}$

$T(e_1) = T(1,0,0) = (1,0,1)$

$T(e_2) = T(0,1,0) = (2,1,1)$

$T(e_3) = T(0,0,1) = (-1,1,-2)$

$T(e_1) = (1,0,1) = a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1)$

$(1,0,1) = a_1, a_2, a_3$

$a_1 = 1, a_2 = 0, a_3 = 1$

$T(e_2) = (2,1,1) = a_1(1,0,1) + a_2(0,1,0) + a_3(0,0,1)$

$a_1 = 2, a_2 = 1, a_3 = 1$

$T(e_3) = (-1,1,-2) = a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1)$

$\neq a_1 = -1, a_2 = 1, a_3 = -2$

Question NO: 6

Diagonalize matrise.

$$A = \begin{bmatrix} 7 & -2 & 1 \\ -2 & 15 & -2 \\ 1 & -2 & 7 \end{bmatrix}$$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 7 & -2 & 1 \\ -2 & 15 & -2 \\ 1 & -2 & 7 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 7-\lambda & -2 & 1 \\ -2 & 15-\lambda & -2 \\ 1 & -2 & 7-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 7-\lambda & -2 & 1 \\ -2 & 15-\lambda & -2 \\ 1 & -2 & 7-\lambda \end{vmatrix} = 0$$

$$7-\lambda \begin{vmatrix} 15-\lambda & -2 \\ -2 & 7-\lambda \end{vmatrix} + 2 \begin{vmatrix} -2 & -2 \\ 1 & 7-\lambda \end{vmatrix} + 1 \begin{vmatrix} -2 & 15-\lambda \\ 1 & -2 \end{vmatrix} = 0$$

$$7-\lambda\left[(15-\lambda)(7-\lambda) - (-2)(-2)\right] + 2\left[(-2)(7-\lambda) - (-2)(1)\right] + 1\left[(-2)(-2) - 15 + \lambda\right) = 0$$

$$7-\lambda\left[105 - 15\lambda - 7\lambda + \lambda^2 - 4\right] + 2\left[-14 + 2\lambda + 2\right] + \{4 - 15 + \lambda\} = 0$$

$$7-\lambda\left[101 - 22\lambda + \lambda^2\right] + 2\left[-12 + 2\lambda\right] - 11 + \lambda = 0$$

$$707 - 154\lambda + 7\lambda^2 - 101\lambda + 22\lambda^2 - \lambda^3 - 24 + 4\lambda - 11 + \lambda = 0$$

$$-\lambda^3 + 29\lambda^2 - 250\lambda + 672 = 0$$

$$\boxed{\lambda^3 - 29\lambda^2 + 250\lambda - 672 = 0}$$

By synthetic division

$$\begin{array}{c|cccc} 6 & 1 & -29 & 250 & -672 \\ & \downarrow & 6 & -138 & 672 \\ \hline & 1 & -23 & 112 & \boxed{0} \end{array}$$

$(\lambda-6)(\lambda^2-23\lambda+112)=0$

$\boxed{\lambda=6}$; $\lambda^2-7\lambda-16\lambda+112=0$

$\lambda(\lambda-7)-16(\lambda-7)=0$

$\boxed{\lambda=16}$, $\boxed{\lambda=7}$

$$D=\begin{bmatrix} 6 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 16 \end{bmatrix}.$$

$\because$ For $\lambda=6$ putting

$$\begin{bmatrix} 7-6 & -2 & 1 \\ -2 & 15-6 & -2 \\ 1 & -2 & 7-6 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 9 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

By R_3-R_1

$$\sim\begin{bmatrix} 1 & -2 & 1 \\ -2 & 9 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

By R_2+2R_1

$$\sim\begin{bmatrix} 1 & -2 & 1 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\div$ by 5 in R_2

$$\sim\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

By R_1+2R_2

$$\sim\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

$x_1+x_3=0$

$x_2=0$

x_3 is free

$x_1 = -x_3$

$x_2 = 0$

x_3 is free

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ 0 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} x_3 .$$

for $\lambda = 7$.

$$\sim \begin{bmatrix} 7-7 & -2 & 1 \\ -2 & 15-7 & -2 \\ 1 & -2 & 7-7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & -2 & 1 \\ -2 & 8 & -2 \\ 1 & -2 & 0 \end{bmatrix}$$

$\div R_2$ by 2

$$\sim \begin{bmatrix} 0 & -2 & 1 \\ -1 & 4 & -1 \\ 1 & -2 & 0 \end{bmatrix}$$

By $R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & -2 & 0 \\ -1 & 4 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

By $R_2 + R_1$

$$\sim \begin{bmatrix} 1 & -2 & 0 \\ 0 & 2 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

By $R_1 + R_2$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

By $R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\div$ by 2 by R_2

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$x_1 - x_3 = 0$

$\boxed{x_1 = x_3}$

$x_2 = \frac{1}{2}$

x_3 is free

$$\begin{bmatrix} x_3 \\ \frac{1}{2} \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ 1 \end{bmatrix} x_3 .$$

for $\lambda = 16$.

$$\begin{bmatrix} 7-16 & -2 & 1 \\ -2 & 15-16 & -2 \\ 1 & -2 & 7-16 \end{bmatrix}$$

$$\begin{bmatrix} -9 & -2 & 1 \\ -2 & -1 & -2 \\ 1 & -2 & -9 \end{bmatrix}$$

By $R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & -2 & -9 \\ -2 & -1 & -2 \\ -9 & -2 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -9 \\ +2 & +1 & +2 \\ -9 & -2 & 1 \end{bmatrix}$$

$R_2 - 2R_1$

$$\sim \begin{bmatrix} 1 & -2 & -9 \\ 0 & 5 & 20 \\ -9 & -2 & 1 \end{bmatrix}$$

$\div$ by 5 by R_2

$$\sim \begin{bmatrix} 1 & -2 & -9 \\ 0 & 1 & 4 \\ -9 & -2 & 1 \end{bmatrix}$$

$R_3 + 9R_1$

$$\sim \begin{bmatrix} 1 & -2 & -9 \\ 0 & 1 & 4 \\ 0 & -20 & -80 \end{bmatrix} \quad \div \text{by} -20 \text{ by } R_3$$

$$\sim \begin{bmatrix} 1 & -2 & -9 \\ 0 & 1 & 4 \\ 0 & 1 & 4 \end{bmatrix}$$

By $R_3 - R_2$

$$\begin{bmatrix} 1 & -2 & -9 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

By $R_1 + 2R_2$.

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$x_1 - x_3 = 0$

$\boxed{x_1 = x_3}$

$x_2 = -4x_3$.

x_3 is free

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} x_3$$

$$P = \begin{bmatrix} -1 & 1 & 1 \\ 0 & \frac{1}{2} & -4 \\ 1 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 16 \end{bmatrix}$$

Check result:

$PD = AP$.

so

$$P \cdot D = \begin{bmatrix} -1 & 1 & 1 \\ 0 & \frac{1}{2} & -4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(6)+0+0 & 0+7+0 & 0+0+16 \\ 0+0+0 & 0+\frac{7}{2}+0 & 0+0-64 \\ 6+0+0 & 0+7+0 & 0+0+16 \end{bmatrix} = \begin{bmatrix} -6 & 7 & 16 \\ 0 & \frac{7}{2} & -64 \\ 6 & 7 & 16 \end{bmatrix}.$$

$$AP = \begin{bmatrix} 7 & -2 & 1 \\ -2 & 15 & -2 \\ 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 0 & \frac{1}{2} & -4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (7)(-1)+0+1 & (7)(1)+(-2)(\frac{1}{2})+(1)(1) & (7)(1)+(-2)(-4)+(1)(1) \\ (-2)(-1)+0+(-2)(1) & (-2)(1)+\frac{15}{2}+(-2)(1) & (-2)(1)+(15)(-4)+(-2)(1) \\ (1)(-1)+0+(7)(1) & (1)(1)+(-2)(\frac{1}{2})+(7)(1) & (1)(1)+(-2)(-4)+(7)(1) \end{bmatrix}$$

$$
\begin{bmatrix} -7+1 & 7-1+1 & 7+8+1 \\ +2-2 & -2+\frac{15}{2}-2 & -2-60-2 \\ -1+7 & 1-1+7 & 1+8+7 \end{bmatrix}
$$

$$
= \begin{bmatrix} -6 & 7 & 16 \\ 0 & 7/2 & -64 \\ 6 & 7 & 16 \end{bmatrix}
$$

$4+\frac{15}{2}$

Hence $AP = PD$ so answer is correct.