Logarithms

We shall use the following notations:

$$\begin{aligned}
\lg n &= \log_2 n && \text{(binary logarithm)} \; , \\
\ln n &= \log_e n && \text{(natural logarithm)} \; , \\
\lg^k n &= (\lg n)^k && \text{(exponentiation)} \; , \\
\lg \lg n &= \lg(\lg n) && \text{(composition)} \; .
\end{aligned}$$

An important notational convention we shall adopt is that *logarithm functions will apply only to the next term in the formula*, so that $\lg n + k$ will mean $(\lg n) + k$ and not $\lg(n + k)$. If we hold $b > 1$ constant, then for $n > 0$, the function $\log_b n$ is strictly increasing.

For all real $a > 0$, $b > 0$, $c > 0$, and n,

$$\begin{aligned}
a &= b^{\log_b a} \; , \\
\log_c(ab) &= \log_c a + \log_c b \; , \\
\log_b a^n &= n \log_b a \; , \\
\log_b a &= \frac{\log_c a}{\log_c b} \; , && (3.15) \\
\log_b(1/a) &= -\log_b a \; , \\
\log_b a &= \frac{1}{\log_a b} \; , \\
a^{\log_b c} &= c^{\log_b a} \; , && (3.16)
\end{aligned}$$

where, in each equation above, logarithm bases are not 1.