Assignment-01

80 marks. 2 time complexity , 3 sorting and 3 graph

Time Complexity-1 (5+5 mark)

1 **a.** **CO3** In the primary scholarship exam in Bangladesh, four lakh (n=4,00,000) students take part but only the top **50** students are given an award. **Write** the asymptotic time complexity to give the awards. Assume that each award is given in a constant time. **02**

b. **CO1** Write the asymptotic time complexity of the following function. **0**

```
1. def contains_duplicates(elements):
2.     for outer in range(len(elements)):
3.         for inner in range(len(elements)):
4.             if outer == inner:
5.                 continue
6.
7.             if elements[outer] == elements[inner]:
8.                 return True
9.
10.    return False
```

c. **CO3** **Express** the following running time $T(n)$ with an asymptotic bound. **04**

$$T(n) = 625T(\frac{n}{5}) + n^3$$

Any method is acceptable as long as you show calculations.

Time Complexity-2 (5+5 Mark)

Total Marks:10

a.**[CO7]Calculate** the time complexity of the following function

Finding_Worst_Case(n):

```
int i,j,k,m,multi,a,b,c
for( i = n; i >= 1; i = i / 7 ){
      for( j = 1; j <= n; j = j + 3 ) {
            for( k=1; k<=40 ; k=k+1){
                  multi=a*b
             }
            for( m=n ; m>=1 ; m=m-5 ){
                  multi=multi*c
            }
       }
 }
```

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b.**[CO7] Calculate** the time complexity of the following recurrence relation.

[Any method is acceptable as long as steps are shown]

$$T(n) = T(n/2) + T(n/4) + n$$

Sorting-1 (7+3 = 10 mark)

Renowned Progressive Rock band 'Porcupine Tree' released an album called 'Closure/Continuation' after about 13 years. Now as a Progressive Rock Music fan you are going to listen to the tracks of the album but in the order of their Youtube views (highest one at first, lowest one at last) instead of the order of the album tracklist.

You have chosen an Algorithm to order them as per your preference. If multiple tracks have the same views, you are going to listen to any of them the Algorithm puts first in the list after ordering. This algorithm also solves your concern about the issues regarding space that your device is facing. Porcupine Tree made the fans wait for 13 years for a new album and so you think you have the patience to wait as much time as the Algorithm may take to order the tracks.

The following table contains the list of the tracks and their Youtube views (in thousand) :

Track	Harridan	Of The New Day	Rats Return	Dignity	Herd Culling	Walk the Plank	Chimera's Wreck	Population Three
Views	15	8	11	112	33	39	88	41

a. CO2 **Specify** the name of the algorithm you have chosen and **simulate** the Algorithm to order the tracks of the album as per your preference. Show your workings in detail. **7**

b. CO5 Suppose you want to run the algorithm again on the ordered list. **Determine** the run-time complexity of the algorithm in this scenario. **3**

Sorting - 2 (3 + 4 + 3 = 10 mark)

Total Marks : 10

Jack loves to play with integers. He created a list of n integers where the even indices hold numbers in decreasing order and the odd indices hold numbers in increasing order. For example, this is a list of n=8 integers Jack made.

Index	0	1	2	3	4	5	6	7
Number	23	2	19	3	7	11	5	13

[Explanation:
The indices 1, 3, 5, and 7 have numbers 2, 3, 11, and 13 in increasing order.
The indices 0, 2, 4, and 6 have numbers 23, 19, 7 and 5 in decreasing order.]

You sorted the the list in quadratic time [$O(n^2)$]. To your utter surprise, Jack replies, "It could be sorted in linear time".

1) ***[CO2]*** **Show** the steps how you sorted the list using a suitable algorithm. Mention the name of the algorithm. — 3
2) ***[CO2]*** **Describe** how Jack sorted the list in linear time. Show the steps too.— 4
3) ***[CO3]*** Jack wants to add the number 15 to your sorted list from (1). He wants you to come up with an efficient idea about how he is going to find the accurate index position for 15 in the list.
   Describe how you are going to assist him in this. **Show** the steps too. — 3

Sorting - 03 (4+4+2 = 10 mark)

b. **CO2** In case of sorting an array in ascending order, *Bubble Sort* extends a sorted subarray at the rightmost side and, *Insertion Sort* extends a sorted subarray at the leftmost side. Your friend, Jimmy was told to find the first five largest and smallest numbers from a list of N distinct integers ($N > 10$). To solve the task, he modified the Bubble sort and Insertion sort algorithm for only 5 iterations and used the rightmost and leftmost 5 numbers as the 5 largest and smallest numbers respectively. Do you support his strategy? **Explain** with logical reasons.

c. **CO2** While sorting a list of 10 integers in ascending order using Quick Sort, after the first partition (using the first element as a pivot), the list looks like this : [13, 11, 19, 7, 23, 37, 29, 53, 59, 41]. Which element was the pivot before partitioning?
Explain your answer in brief.

d. **CO2** Consider the scenario in 2(c). **Show** how the list will look like after partitioning using **13** as the pivot.

Graph - 01 (7+3 = 10 mark)

You are the coach of the renowned football team "Real Madrid". Your team is behind one goal and there is still some time left to back in the game by scoring a goal. Benzema is your main striker and if you can pass the ball to him he will give the goal for sure. Currently the ball is in your goalkeeper's (Courtois) hand. So, your main target should be Passing the ball in minimum steps from Courtois to Benzema as the time left is very low.

Courtois can pass the ball to Rudiger or Alaba
Rudiger can pass the ball to Modric
Alaba can pass the ball to Modric, Hazard or Nacho
Modric can pass the ball to Benzema
Hazard can pass the ball to Kroos
Nacho can pass the ball to Vini
Kroos can pass the ball to Benzema
Vini can pass the ball to Benzema

Now answer the following questions.

a) Using a suitable algorithm, **find** the minimum number of passes your team will give to reach the ball from Courtois to Benzema. **Mention** the name of the algorithm, **Show** each step of the simulation properly to find out the minimum number of passes. **07**

b) **Find** out the players who are required for this minimum number of passes scenario. **03**

Graph - 02 (1+2+6+1 = 10 mark)

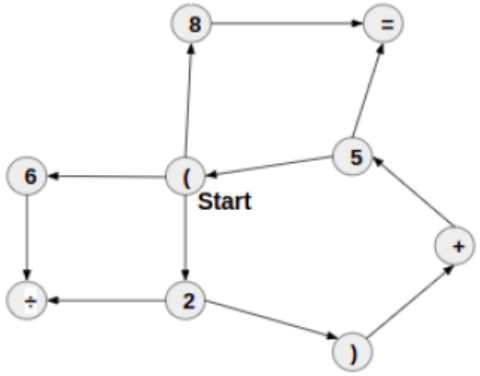
3. Bill has this weird characteristic of playing with different types of graphs. This time he comes up with an undirected simple graph of 10 nodes. Nodes are labeled from 3 to 12. Some of the edges are:
(5,6), (5,10), (6,11), (7,11). He created other edges following two rules.

a. All the nodes that are labeled with a number which is a multiple of 3 have edges among them.
b. All the nodes that are labeled with a number which is a multiple of 4 have edges among them.

a. CO3	**Draw** the graph to show all the edges.	**1**
b. CO3	Bill says, there are at least four triangles in the graph. Do you agree? **Support** your answer by showing the nodes which form these triangles.	**2**
c. CO2	**Simulate** a BFS algorithm on the graph to find the shortest distance from Node 3 to all others nodes.	**6**
d. CO3	Bill told you to keep on adding edges between the nodes according to your wish, keeping the graph simple (without adding multiple edges between any two nodes, self edges). **Compute** the number of more edges that you can add.	**1**

Graph -03 (1 + 5 + 4 = 10 Mark)

3. Consider the graph shown below:



Bill says he has found a valid mathematical equation while running DFS from the node denoted by '('. According to him, the equation is:

$$(6 \div 2) + 5 = 8$$

a. CO3	**Compute** the number of edges this DFS tree (of the equation) contains.	**1**
b. CO3	Is what Bill said right i.e is the equation achievable by running such a DFS? **Validate** it by showing steps. (Neighbor/Edge selection should be done according to the necessity of the verification process)	**5**
c. CO2	**Classify** the edges of the graph into tree edge, back edge and forward edge while running DFS from the node denoted by '('.	**4**