CS101 Algorithms and Data Structures
Fall 2023
Homework 7

Due date: 23:59, October 26th, 2023

1. Please write your solutions in English.

2. Submit your solutions to gradescope.com.

3. Set your FULL name to your Chinese name and your STUDENT ID correctly in Account Settings.

4. If you want to submit a handwritten version, scan it clearly. `CamScanner` is recommended.

5. When submitting, match your solutions to the problems correctly.

6. No late submission will be accepted.

7. Violations to any of the above may result in zero points.

1. (4 points) Multiple Choices

Each question has **one or more** correct answer(s). Select all the correct answer(s). For each question, you will get 0 points if you select one or more wrong answers, but you will get 1 point if you select a non-empty subset of the correct answers.

Write your answers in the following table.

(a)	(b)
AD	BC

(a) (2') Consider an AVL tree whose number of nodes is n and height is h, which of the following are true?

A. $n = O(2^h)$.

B. $n = O(\alpha^h)$, where $\alpha = \dfrac{1 + \sqrt{5}}{2}$.

C. h is always not greater than the height of a BST with n nodes.

D. $h = \Theta(\log n)$ in all cases.

> **Solution:**
>
> B. $n = \Omega(\alpha^h)$.
>
> C. The height of a BST with n nodes can be as small as $\lfloor \log_2 n \rfloor$.

(b) (2') Which of the following statements are true for an AVL tree? Here one balance correction means a single rotation (in left-left or right-right cases) or a double rotation (in left-right or right-left cases).

A. Inserting an item causes at most one node imbalanced before checking if any balance correction is needed.

B. At most one balance correction has to be performed after inserting an item.

C. Removing an item in leaf nodes causes at most one node imbalanced before checking if any balance correction is needed.

D. At most one balance correction has to be performed after removing an item.
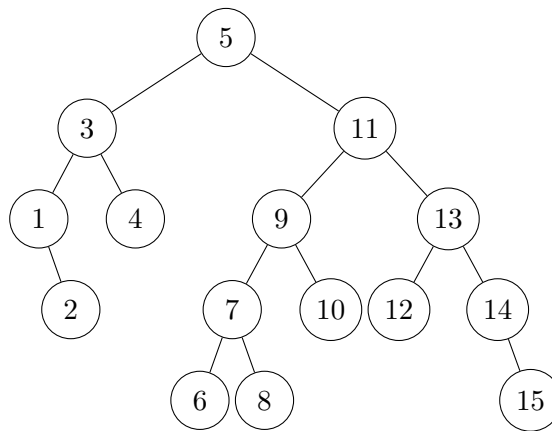
> **Solution:**
>
> B. Suppose x is the deepest imbalanced node. After the imbalance of x is corrected, the height of x's subtree always becomes the same as when the item is not inserted. Therefore other ancestors of x will not needed to be corrected.
>
> C. Suppose x is the deepest imbalanced node. The height of x's subtree always remain unchanged because the removed leaf node is not the deepest node of x's subtree. Therefore other ancestors of x will not be imbalanced.
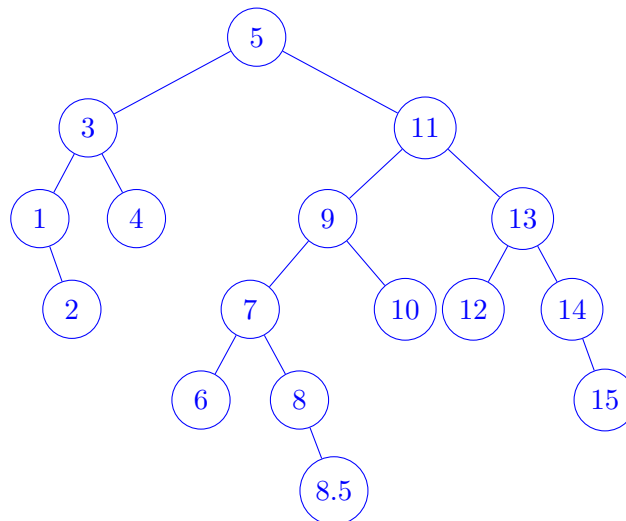
2. (8 points) AVL tree operations

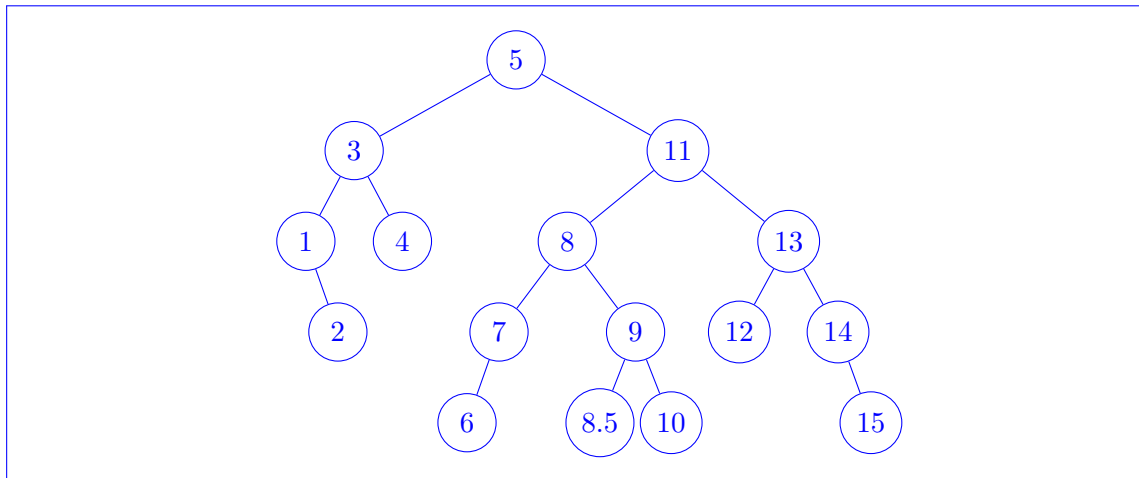Here is an AVL tree. Denote it as T.

(a) (2') Insert 8.5 into T. Draw the AVL tree before checking if any balance correction is needed.
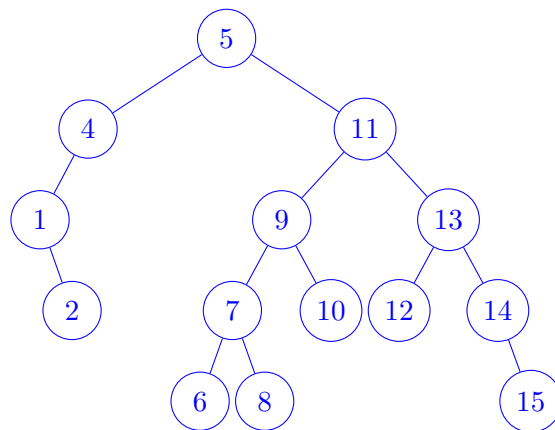
Solution:

(b) (2') Insert 8.5 into T. Draw the AVL tree after balance corrections.

Solution:

(c) (2') Remove 3 from T (**NOT from the previous answer!**). Draw the AVL tree after replacing and before checking if any balance correction is needed.

(d) (2') Remove 3 from T. Draw the AVL tree after balance corrections.