CS150A Database

Lu Sun
School of Information Science and Technology
ShanghaiTech University
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Today:
- Index Files
- B+ Tree Refinements

Readings:
- Database Management Systems (DBMS), Chapters 9&10

Acknowledgement: Joe Hellerstein@UCBerkeley's scourse notes

General characteristics of an index: An Outline

- Issues to consider in any index structure (not just B+-trees)
  - Query support: what class of queries does the index allow?
  - Choice of Search Key
    - Affects the queries for which we can use an index.
  - Data Entry Storage
    - Affects performance of the index
  - Variable-length key tricks
    - Affects performance of the index
  - Cost Model for Index vs Heap vs Sorted File

QUERY SUPPORT

Indexes: Basic Selection

- **Basic Selection:** <key> <op> <constant>
  - Equality selections (op is =)
  - Range selections (op is one of <, >, <=, >=, BETWEEN)
  - B+-trees provide both
  - Linear Hash indexes provide only equality (but are interesting!)

Indexes: Other Selections

- **More Exotic Selections:**
  - 2-d box (current map boundaries)
  - 2-d circle (“within 2 miles of Empire State Building”)
  - Common **n-dimensional indexes**: R-tree, KD-tree, etc.
    - Beware of the curse of dimensionality
  - Near-neighbor queries (“10 restaurants closest to Empire State Building”)
  - Regular expression matches, genome string matches, etc.
  - See Postgres’ GiST indexes for a flexible structure developed at Berkeley

For Today

- In the remainder of our discussion, we'll focus on traditional 1-d range search
  - And equality as a special case
  - As in B+-trees

Search Key and Ordering

- Can index on any ordered subset of columns. Order matters!
  - Determines the queries supported
- In an ordered index (e.g. B+-tree) the keys are ordered **lexicographically** by the search key columns:
  - Ordered by the 1st column
  - 2 items match on 1st column? Ordered by 2nd
  - Match on 1st and 2nd column? Ordered by 3rd
  - Etc.
- E.g. table to right ordered lexicographically by the search key <Age, Salary>

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400

Search Key and Ordering, Pt 2.

- Defn: A **composite search key** on columns (k_1, k_2, …, k_n) “matches” a query if:
  - The query is a *conjunction* of *m >= 0* equality clauses of the form:
    $k_1 = <val_1>$ AND $k_2 = <val_2>$ AND .. AND $k_m = <val_m>$
    and at most 1 additional *range* clause of the form:
    AND k_{m+1} op <val>, where op is one of {<, >}

- Why does this “match”? *Lookup and scan in lexicographic order*
  - Can do a lookup on equality conjuncts to find start-of-range
  - Can do a scan of contiguous data entries at leaves
    - satisfy the $m+1^{st}$ conjunct
    - or if there is no $m+1^{st}$ conjunct
      - scan the entire set of matches to the first m conjuncts

Search Key and Ordering, Pt 3

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>
- Legend

Green for rows we visit that are in the range

Red for rows we visit that are not in the range

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

Search Key and Ordering, Pt 4

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - Age = 31 & Salary = 400

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

Search Key and Ordering, Pt 5

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

Search Key and Ordering, Pt 6

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - Age = 55 & Salary > 200

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

Search Key and Ordering, Pt 6, cont

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - ✓ Age = 55 & Salary > 200

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

Search Key and Ordering, Pt. 7

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - ✓ Age = 55 & Salary > 200
    - Age > 31 & Salary = 400

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

Search Key and Ordering, Pt 8

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - ✓ Age = 55 & Salary > 200
    - ✗ Age > 31 & Salary = 400

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

✗ Not a lexicographic range. Either visits useless rows or has to “bounce through” the index.

Search Key and Ordering, Pt 9

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - ✓ Age = 55 & Salary > 200
    - ✗ Age > 31 & Salary = 400
    - Age = 31

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

✗ Not a lexicographic range. Either visits useless rows or has to "bounce through" the index.

Search Key and Ordering, Pt 10

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - ✓ Age = 55 & Salary > 200
    - ✗ Age > 31 & Salary = 400
    - ✓ Age = 31

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

✗ Not a lexicographic range. Either visits useless rows or has to “bounce through” the index.

Search Key and Ordering, Pt 11

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - ✓ Age = 55 & Salary > 200
    - ✗ Age > 31 & Salary = 400
    - ✓ Age = 31
    - Age > 31

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

✗ Not a lexicographic range. Either visits useless rows or has to "bounce through" the index.

Search Key and Ordering, Pt 12

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - ✓ Age = 55 & Salary > 200
    - ✗ Age > 31 & Salary = 400
    - ✓ Age = 31
    - ✓ Age > 31

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

✗ Not a lexicographic range. Either visits useless rows or has to "bounce through" the index.

Search Key and Ordering, Pt 13

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - ✓ Age = 55 & Salary > 200
    - ✗ Age > 31 & Salary = 400
    - ✓ Age = 31
    - ✓ Age > 31
    - Salary = 300

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

✗ Not a lexicographic range. Either visits useless rows or has to “bounce through” the index.

Search Key and Ordering, Pt 14

- **Composite Keys:** more than one column
  - **Lexicographic order**
  - Search a *range?*
  - <Age, Salary>:
    - ✓ Age = 31 & Salary = 400
    - ✓ Age = 55 & Salary > 200
    - ✗ Age > 31 & Salary = 400
    - ✓ Age = 31
    - ✓ Age > 31
    - ✗ Salary = 300

SSN	Last Name	First Name	Age	Salary
123	Adams	Elmo	31	$300
443	Grouch	Oscar	32	$400
244	Oz	Bert	55	$140
134	Sanders	Ernie	55	$400
176	Grump	Donald	79	$300

✗ Not a lexicographic range. Either visits useless rows or has to “bounce through” the index.

Data Entry Storage Intro

- What is the representation of data in the index?
  - Actual data or pointer to the data
- How is the data stored in the data file?
  - Clustered or unclustered with respect to the index
- **Big Impact on Performance**
  - We'll learn each of these next

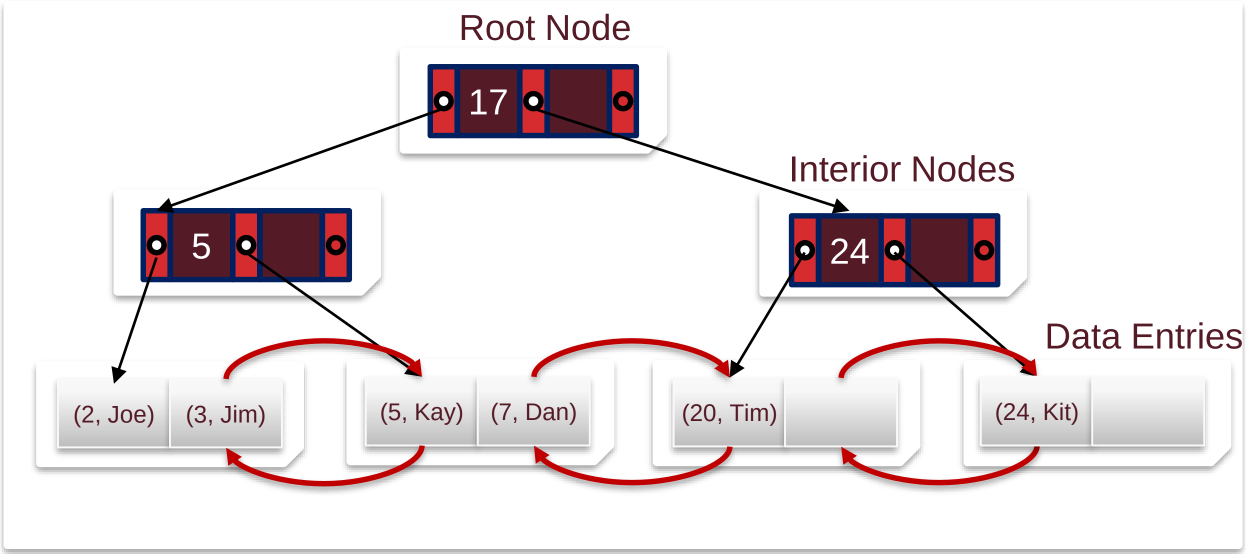
Three basic alternatives for data entries in any index

- Three basic alternatives for data entries in any index
  - Alternative 1: By Value
  - Alternative 2: By Reference
  - Alternative 3: By List of references
    - We’ll look in the context of B+-trees, but applies to any index

Alternative 1 Index (B+ Tree)

- Record contents are stored in the index file
  - No need to follow pointers

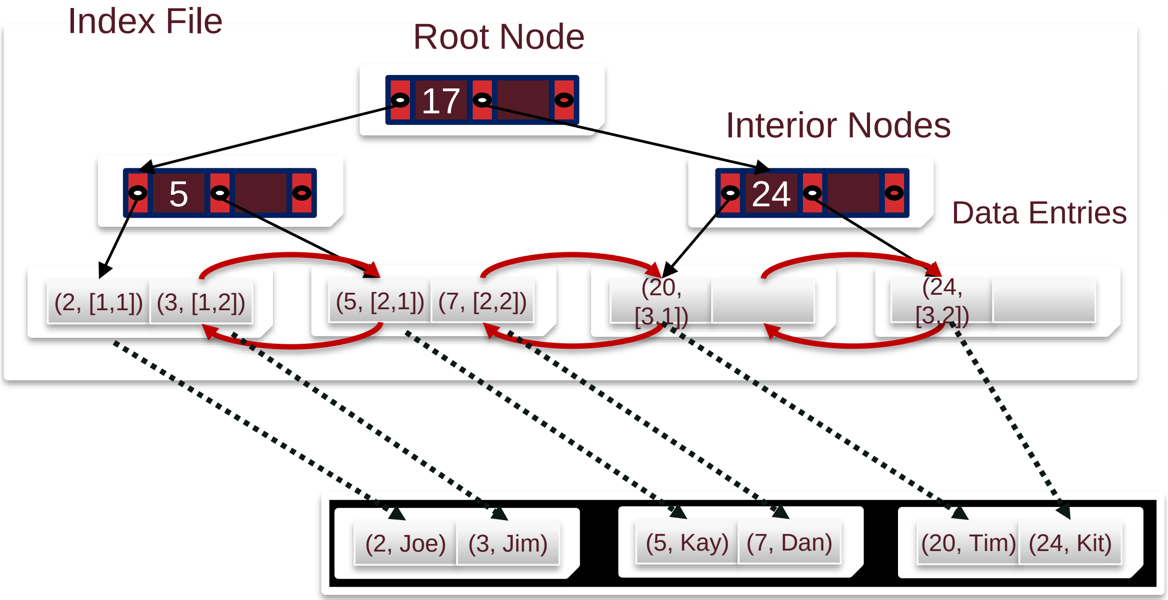
<u>uid</u>	name
2	Joe
3	Jim
5	Kay
7	Dan
20	Tim
24	Kit



Alternative 2 Index

- Alternative 2: **By Reference, <k**, rid of matching data record>
  - We used in slides above

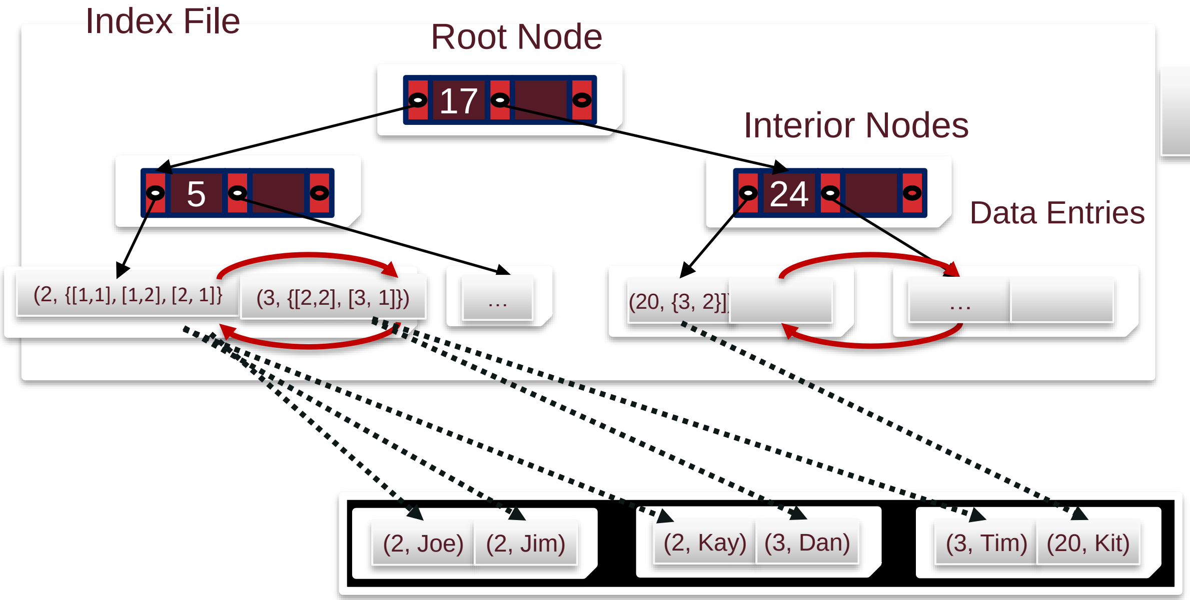
uid	name
2	Joe
3	Jim
5	Kay
7	Dan
20	Tim
24	Kit



Index Contains (Key, Record Id) Pairs

Alternative 3 Index

- Alternative 3: **By List of references, <k**, list of rids of matching data records>
  - Alternative 3 more compact than alternative 2
    - For very large rid lists, single data entry spans multiple blocks



Index Contains
(Key, {list of record Id}) Pairs

Key	Record Id
2	{[1,1], [1,2], [1,3]}
3	4

Indexing By Reference

- Both Alternative 2 and Alternative 3 index data *by reference*
- By-reference is *required* to support multiple indexes per table
    - Otherwise we would be replicating entire tuples
    - Replicating data leads to complexity when we're doing updates, so it's something we want to avoid

Alternative 2 vs Alternative 3 Table Illustration

Alternative 2
Index data entries

Key	Record Id
Gonzalez	[3, 1]
Gonzalez	[3, 2]
Gonzalez	[3, 3]
Hong	[3, 4]

SSN	Last Name	First Name	Salary
123	Gonzalez	Amanda	$400
443	Gonzalez	Joey	$300
244	Gonzalez	Jose	$140
134	Hong	Sue	$400

Alternative 3
Index data entries

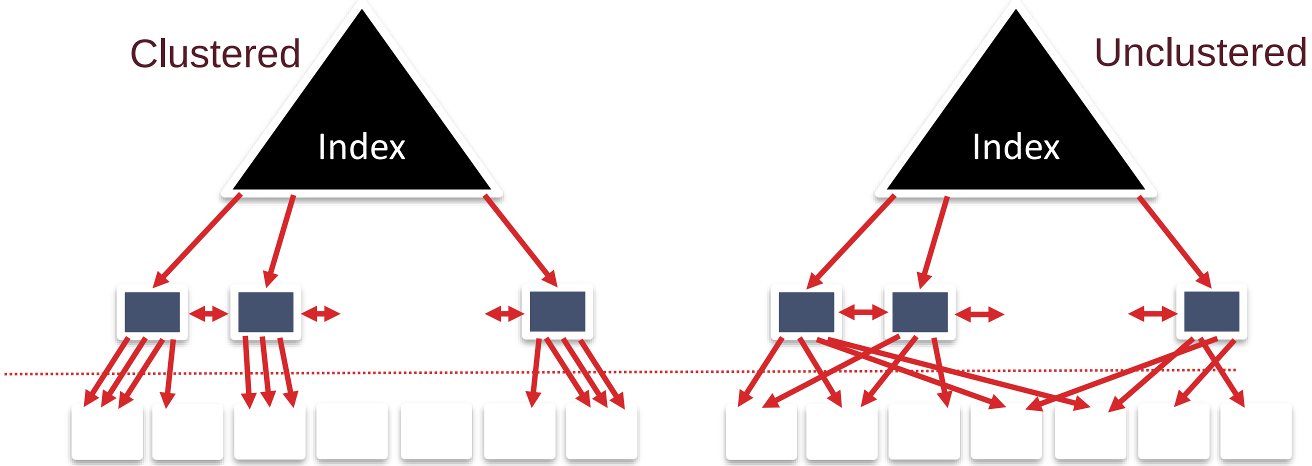
Key	Record Id
Gonzalez	[3, {1, 2, 3}]
Hong	[3,4]

Clustered vs. Unclustered Index

- By-reference indexes (Alt 2 and 3) can be *clustered* or *unclustered*
  - Really this is a property of the heap file associated with the index!
- Clustered index:
  - Heap file records are kept mostly ordered according to **search keys** in index
    - Heap file order need not be perfect: this is just a performance hint
    - Cost of retrieving data records through index varies greatly based on whether index is clustered or not!
- Note: different definition of “clustering” in AI:
  - grouping nearby items in *n*-space

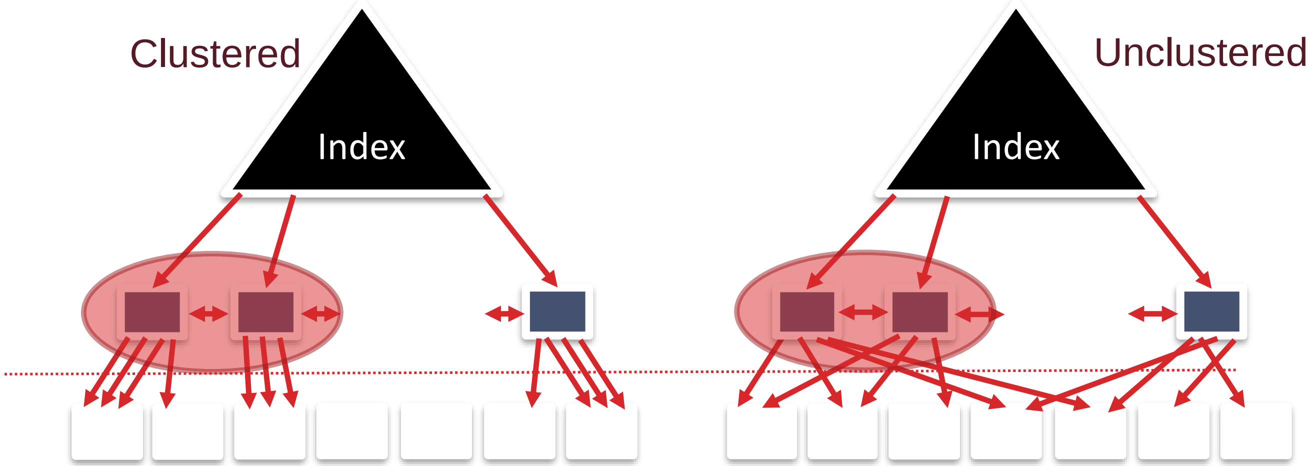
Clustered vs. Unclustered Index Visualization 1

- To build a clustered index, first sort the heap file
  - Leave some free space on each block for future inserts
  - Index entries direct search for data entries



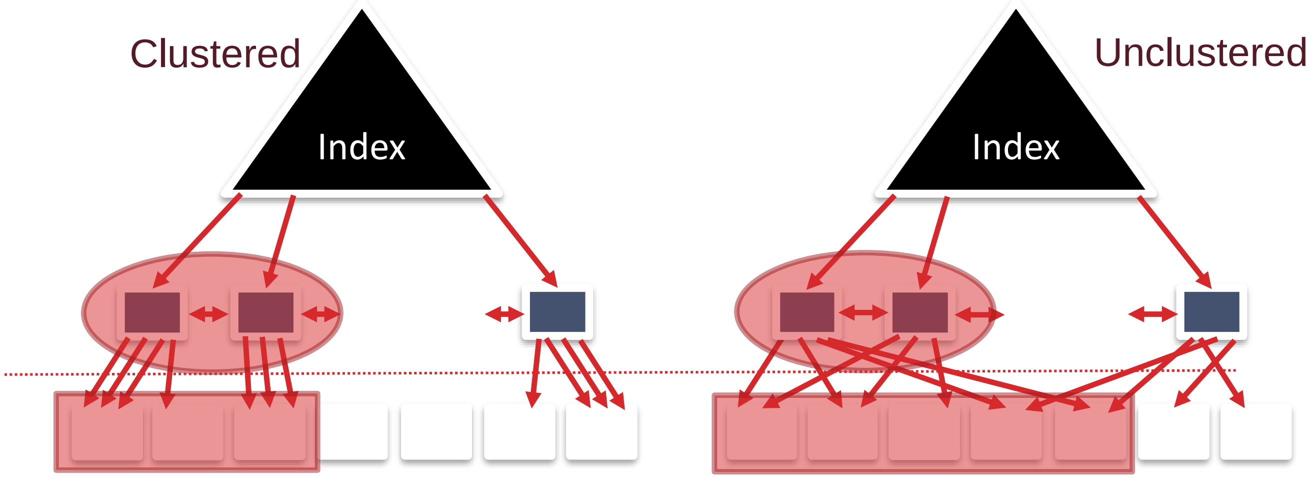
Clustered vs. Unclustered Index Visualization 2

- To build a clustered index, first sort the heap file
  - Leave some free space on each block for future inserts
  - Index entries direct search for data entries



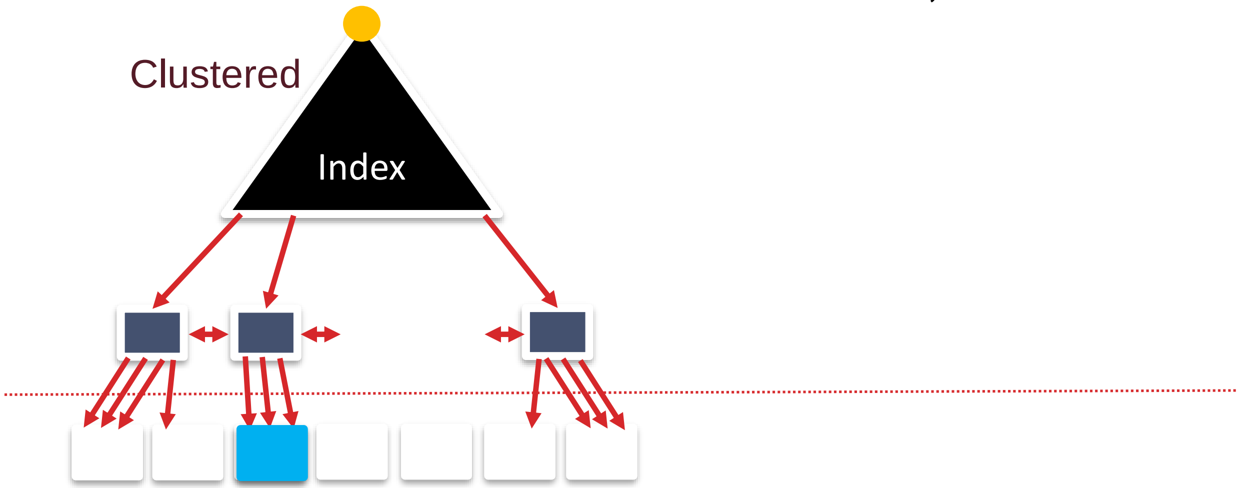
Clustered vs. Unclustered Index Visualization 3

- To build a clustered index, first sort the heap file
  - Leave some free space on each block for future inserts
  - Index entries direct search for data entries



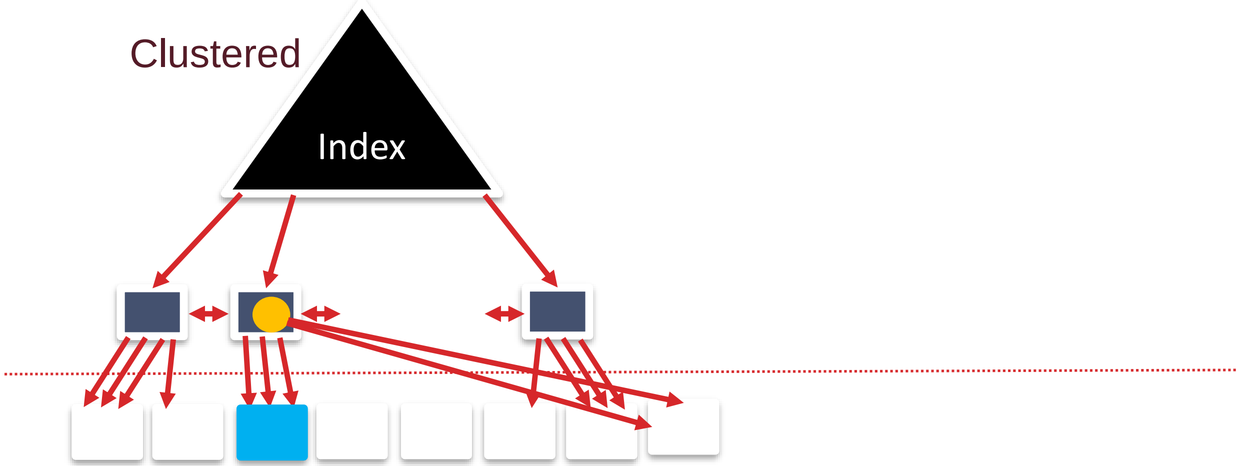
Clustered vs. Unclustered Index Visualization 5

- To build a clustered index, first sort the heap file
  - Leave some free space on each block for future inserts
- Blocks at end of file may be needed for inserts
  - Order of data records is “close to”, but not identical to, the sort order



Clustered vs. Unclustered Index Visualization 6

- To build a clustered index, first sort the heap file
  - Leave some free space on each block for future inserts
- Blocks at end of file may be needed for inserts
  - Order of data records is “close to”, but not identical to, the sort order



Clustered vs. Unclustered Indexes Pros

- Clustered Index Pros
  - Efficient for range searches
  - Potential locality benefits
    - Sequential disk access, prefetching, etc.
  - Support certain types of compression
    - More soon on this topic

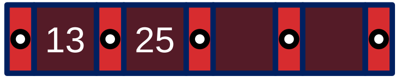
Clustered vs. Unclustered Indexes Cons

- Clustered Cons
  - More expensive to maintain
    - Need to periodically update heap file order
    - Solution: on the fly or “lazily” via reorganizations
  - Heap file usually only **packed to 2/3** to accommodate inserts

B+TREE REFINEMENT: VARIABLE-LENGTH KEYS

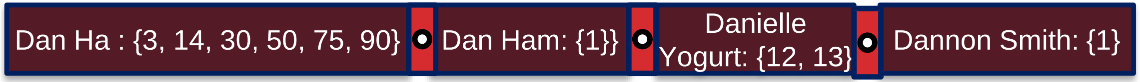
Variable Length Keys & Records

- So far we have been using integer keys



- **What would happen to our occupancy invariant with variable length keys?**

- What about data in leaf pages:



Redefine Occupancy Invariant

- Order (**d**) makes little sense with variable-length entries
  - Different nodes have different numbers of entries.
  - **Index pages** often hold many **more entries** than leaf pages
  - Even with fixed length fields, Alternative 3 gives variable length data entries

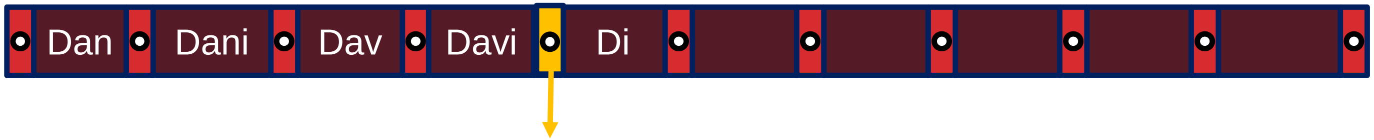
- Use a physical criterion in practice: ***at-least half-full***
  - Measured in **bytes**

- Many real systems are even sloppier than this
  - Only reclaim space when a page is completely empty.
  - Basically the deletion policy we described above…

Prefix Compress Keys?

- How can we get more keys on a page?

- What if we compress the keys?

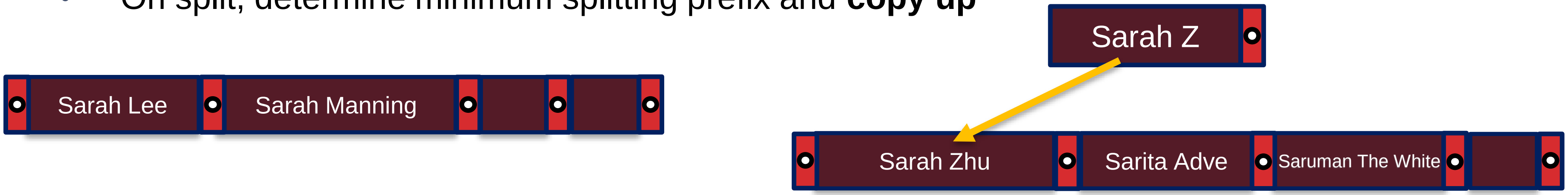


- Are these the same
  - David Jones?
  - Not the same partitioning of possible keys
  - But why would we care??

Prefix Key Compression

- What if we compress starting at leaf:

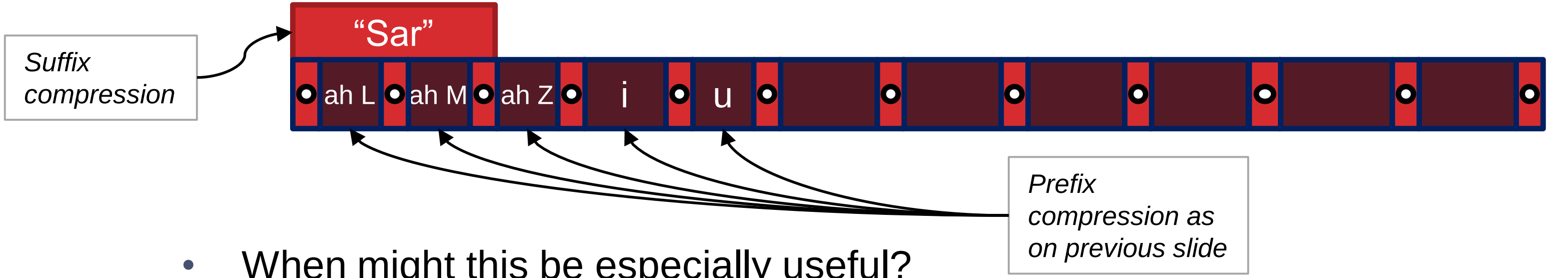
- On split, determine minimum splitting prefix and **copy up**



Suffix Key Compression

- All keys have large common prefix

- Move common prefix to header, leave only (compressed) suffix next to pointer



- When might this be especially useful?
  - Composite Keys. Example?
    - <Zip code, Last Name, First Name>

B+-TREE COSTS

Recall: Cost of Operations

	Heap File	Sorted File
Scan all records	B*D	B*D
Equality Search	0.5*B*D	$(\log_2 B)$*D
Range Search	B*D	$((\log_2 B)$+pages))*D
Insert	2*D	$((\log_2 B) + B)$*D
Delete	(0.5*B+1)*D	$((\log_2 B) + B)$*D

- **Can we do better with indexes?**

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block

Cost of Operations

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	
Equality Search	0.5*B*D	$(\log_2 B)$*D	
Range Search	B*D	$((\log_2 B)$+pages))*D	
Insert	2*D	$((\log_2 B)$ + B)*D	
Delete	(0.5*B+1)*D	$((\log_2 B)$ + B)*D	

- **Can we do better with indexes?**

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block

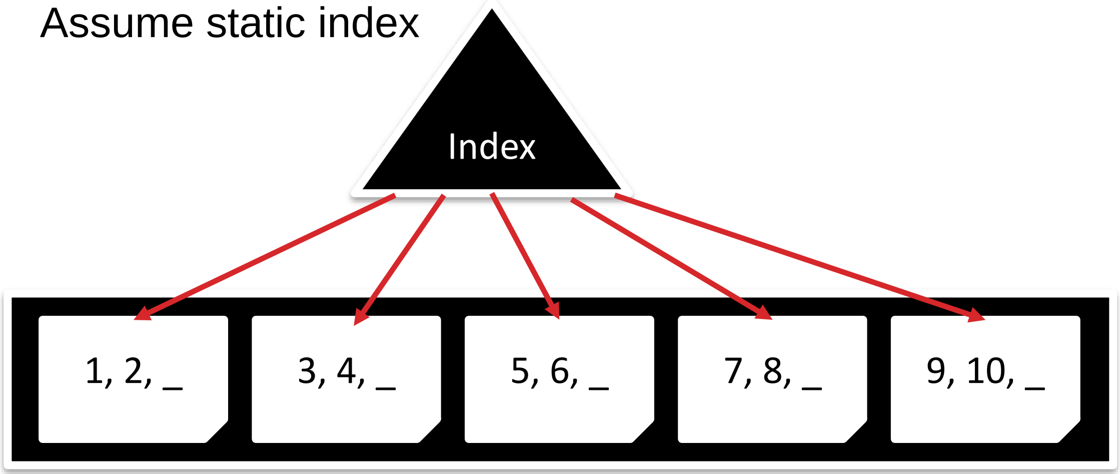
Cost of Operations, cont

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	
Equality Search	0.5*B*D	$(\log_2 B)$*D	
Range Search	B*D	$((\log_2 B)$+pages))*D	
Insert	2*D	$((\log_2 B)$ + B)*D	
Delete	(0.5*B+1)*D	$((\log_2 B)$ + B)*D	

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block

Clustered vs. Unclustered Index Assumptions

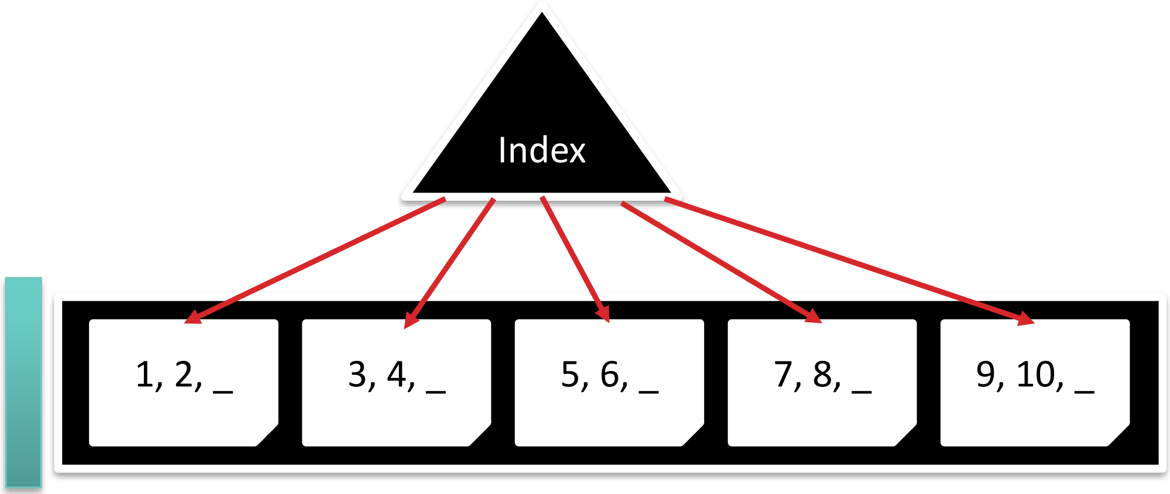
- Store data by reference (Alternative 2)
- Clustered index with 2/3 full heap file pages
  - Clustered → Heap file is initially sorted
  - **Fan-out** (F): relatively large. Why?
    - Page of <key, pointer> pairs ~ O(R)
  - Assume static index



Scan all the Records

- Do we need an Index?
  - No
- Cost? = 1.5 * B * D
  - Why?

Recall assumption from before regarding clustered indexes: heap file pages only **2/3** full.



Cost of Operations: Scan

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)$*D	
Range Search	B*D	$((\log_2 B)$+pages))*D	
Insert	2*D	$((\log_2 B)$ + B)*D	
Delete	(0.5*B+1)*D	$((\log_2 B)$ + B)*D	

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block

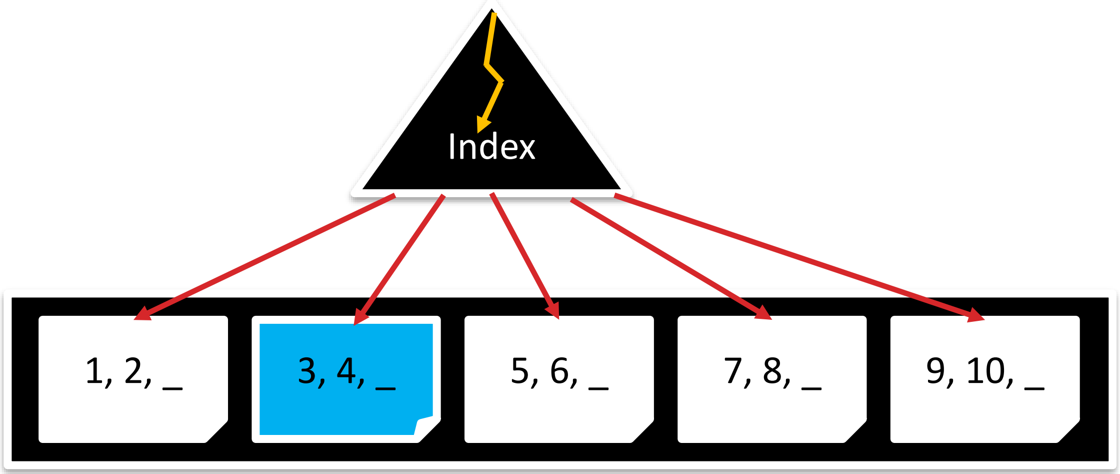
Cost of Operations: Equality Search?

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)$*D	
Range Search	B*D	$((\log_2 B)$+pages))*D	
Insert	2*D	$((\log_2 B) + B)$*D	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)$*D	

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

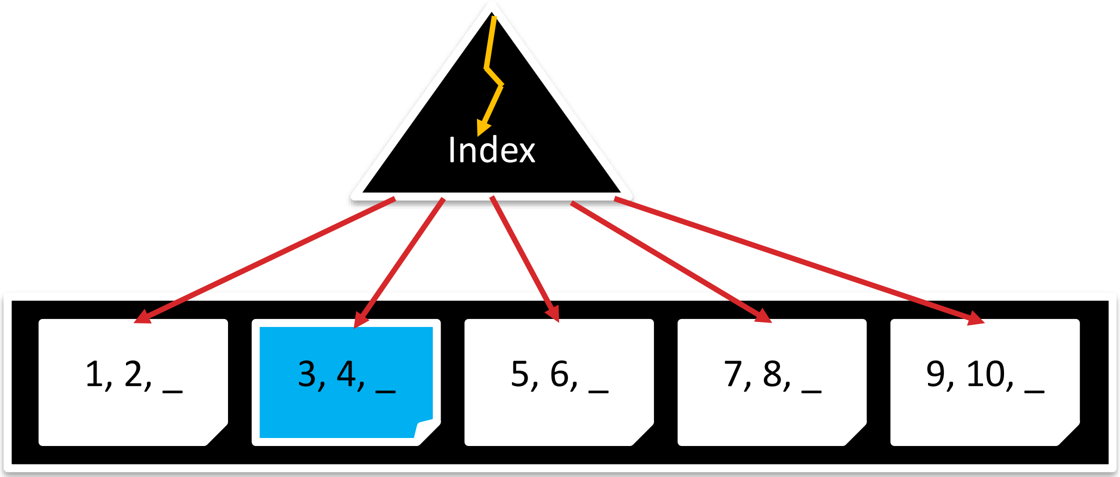
Find the record with key 3, pt 1

- Search the index:= ($\log_F$ (BR/E) + 1) * D
  - BR is the total number of records; E is the #records per leaf
  - the +1 is an “off by 1” thing: catches the cost of the root
  - E.g. F = 4, BR/E = 16: root, intermediate, leaf levels.
  - $\text{Log}_4(16)$ = 2, and I/O cost is 3!



Find the record with key 3, pt 2

- Search the index:= ($\log_F$ (BR/E) + 1) * D
- Lookup record in heap file by record-id = 1 * D
  - Recall record-id = <page, slot #>



Cost of Operations: Equality Search

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	$(\log_F(BR/E)+2)*D$
Range Search	B*D	$((\log_2 B)+pages))*D$	
Insert	2*D	$((\log_2 B) + B)*D$	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

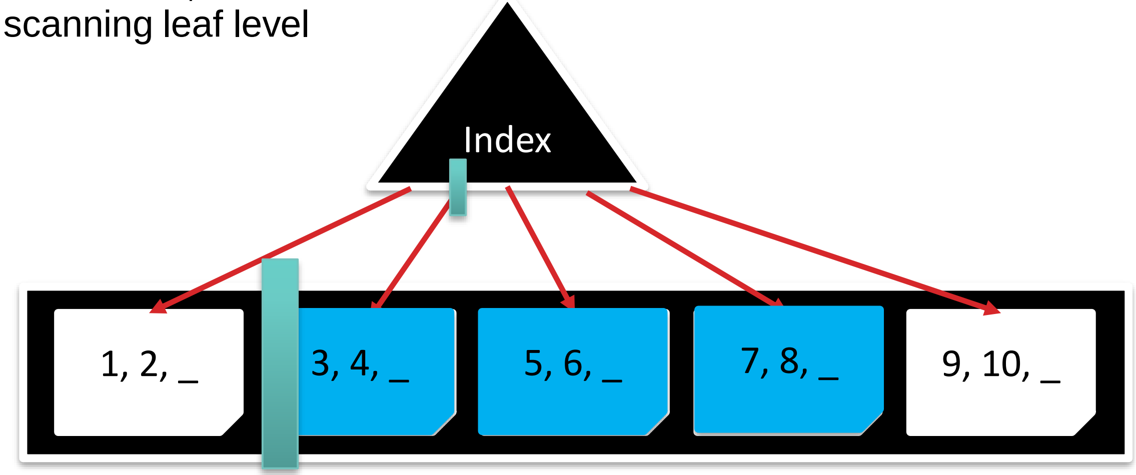
Cost of Operations: Range Search?

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)$*D	$(\log_F(BR/E)+2)$*D
Range Search	B*D	$((\log_2 B)$+pages))*D	
Insert	2*D	$((\log_2 B) + B)$*D	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)$*D	

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Find keys between 3 and 7

- Search the index: = $(\log_F(BR/E) + 1) * D$
- Scan the leaf level and lookup each matching record in the heap file by record-id
  - Recall record-id = <page, slot #>
- Heap file access: (3/2 * #pages) * D
- Scanning the leaf level is similar to heap file access: assume same (3/2 * #pages) * D
- In total $(\log_F(BR/E)$ + 3 * # pages) * D since one leaf page is overcounted in searching index and scanning leaf level



Cost of Operations: Range Search

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)$*D	$(\log_F(BR/E)+2)$*D
Range Search	B*D	$((\log_2 B)+pages))$*D	$(\log_F(BR/E)+3*pages)$*D
Insert	2*D	$((\log_2 B) + B)$*D	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)$*D	

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Cost of Operations: Insert?

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)$*D	$(\log_F(BR/E)+2)$*D
Range Search	B*D	$((\log_2 B)$+pages))*D	$(\log_F(BR/E)+3$*pages)*D
Insert	2*D	$((\log_2 B) + B)$*D	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)$*D	

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Cost of Operations: Insert

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	$(\log_F(BR/E)+2)*D$
Range Search	B*D	$((\log_2 B)+pages))*D$	$(\log_F(BR/E)+3*pages)*D$
Insert	2*D	$((\log_2 B) + B)*D$	$(\log_F(BR/E)+4)*D$
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Cost of Operations: Delete

Why “+4” in Insert/Delete?

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)$*D	$(\log_F(BR/E)+2)$*D
Range Search	B*D	$((\log_2 B)$+pages))*D	$(\log_F(BR/E)$+3*pages)*D
Insert	2*D	$((\log_2 B) + B)$*D	$(\log_F(BR/E)+4)$*D
Delete	(0.5*B+1)*D	$((\log_2 B) + B)$*D	$(\log_F(BR/E)+4)$*D

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Cost of Operations: Big O Notation

	Heap File	Sorted File	Clustered Index
Scan all records	O(B)	O(B)	O(B)
Equality Search	O(B)	$O(\log_2 B)$	$O(\log_F B)$
Range Search	O(B)	$O(\log_2 B)$	$O(\log_F B)$
Insert	O(1)	O(B)	$O(\log_F B)$
Delete	O(B)	O(B)	$O(\log_F B)$

- **B:** The number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Constant factors

- Assume you can do 100 sequential I/Os in the time of 1 random I/O
- For a particular lookup, is a B+-tree better than a full-table scan?
  - Had better be very “selective”
    - Visit < ~1% of pages!
  - Or do mostly sequential I/O at leaf level
    - Clustered index
  - Or use SSD
    - SSDs make indexes attractive
    - Especially for read-mostly workloads

Summary

- Query Structure
  - Understand composite search keys
  - Lexicographic order and search key prefixes
- Data Storage
  - Data Entries: Alt 1 (tuples), Alt 2 (recordIds), Alt 3 (lists of recordIds)
  - Clustered vs. Unclustered
    - Only Alt 2 & 3!

Summary Cont

- Variable length key refinements
  - Fill factors for variable-length keys
  - Prefix and suffix key compression
- B+-tree Cost Model
  - Attractive big-O
  - Don’t forget constant factors of random I/O
    - Hard to beat sequential I/O of scans unless very selective
  - Indexes beyond B+-trees for more complex searches