宽带信号：10HZ　100HZ　50HZ

①分帧　②加窗　③fft.

X[n] ⟶　160000 = N = length X

1　2　3　4 · · · · · · · · · · · N-2　N-1　N

二维Y　$\underbrace{\left(2\left\lfloor\frac{N}{512}\right\rfloor+1\right)}_{\text{行}}$，512 列的二维数组

具体：

第一帧：将 X[1] ~ X[512] 存在2维数组Y的第一行，

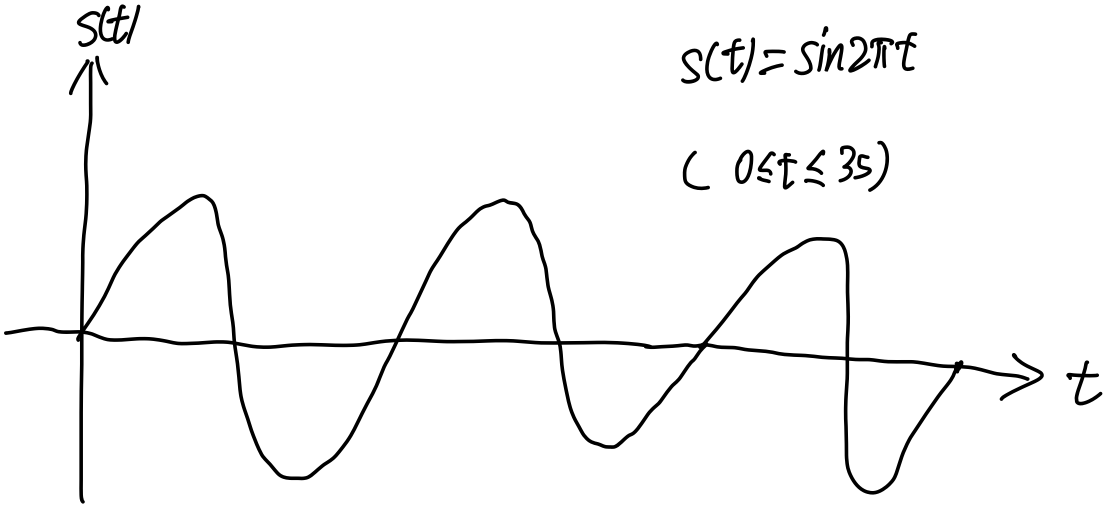
第二帧：将 X[257] ~ X[769] 存在2维数组Y的第二行

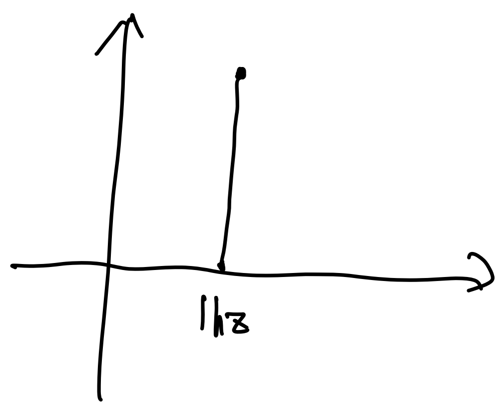
三　X[513] ~ X[1024]　三

$$y = \begin{bmatrix} x[1] & x[2] & x[3] & \cdots & x[512] \\ x[257] & x[258] & \cdots & & x[768] \\ x[513] & x[514] & \cdots & & x[1024] \\ x[769] & x[770] & \cdots & & \end{bmatrix}$$

$s(t) = \sin 2\pi t$

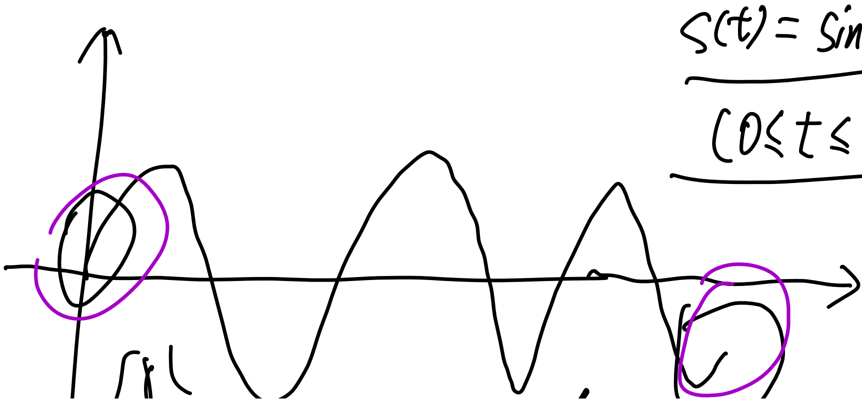
$(0 \le t \le 3s)$



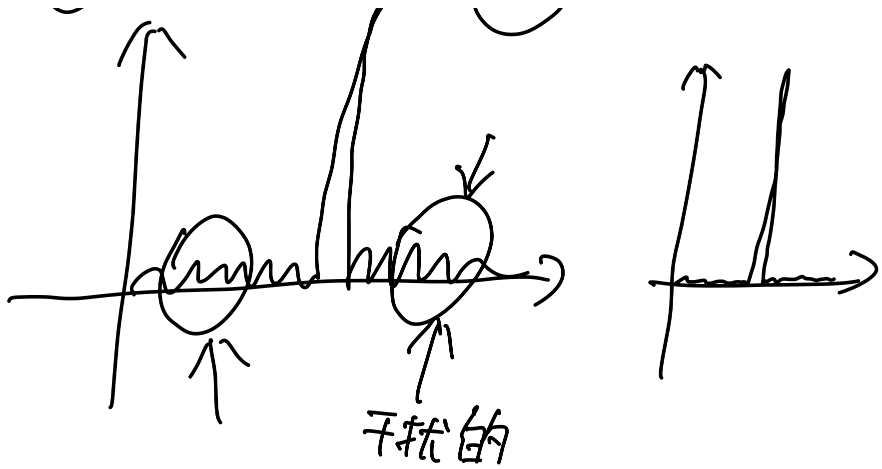


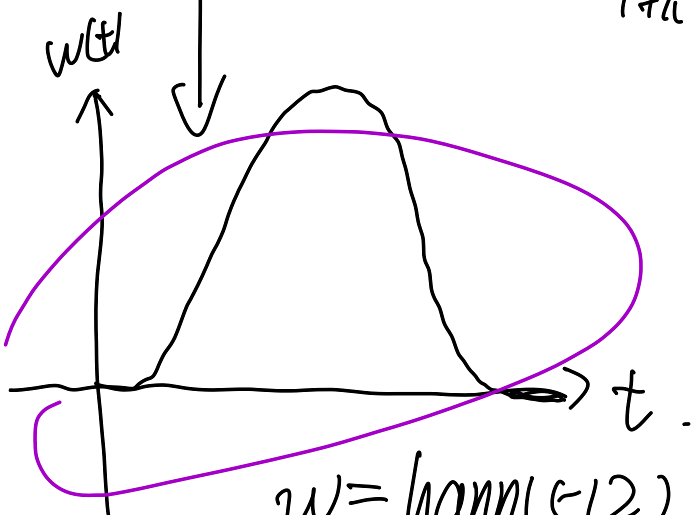
$s(t) = \sin(2\pi t)$

$(0 \le t \le 2.9s)$



512





S*W

$$w = \text{hann}(512)$$

$$y = \begin{bmatrix} w[1]x[1] & w[2]x[2] & w[3]x[3] & \cdots & w[512]x[512] \\ w[1]x[257] & w[2]x[258] & \cdots & & w[512]x[768] \\ w[1]x[513] & w[2]x[514] & \cdots & & w[512]x[1024] \\ w[1]x[769] & w[2]x[770] & \cdots & & \end{bmatrix}$$

fft

回到第二题:

4个麦克风信号，4个信号分别做短时傅里叶变换。

对于每个信号的每一帧你都会得到一个长度为512的数组。

$f_s = 44100Hz$

$$\mathrm{fft}\left(\underset{1}{\square}\ \ \underset{2}{\square}\ \ \underset{3}{\square}\ \cdots\cdots\ \underset{512}{\square}\right)$$

f(k)

$$\underline{\square}\ \ \underline{\square}\ \ \underline{\square}\ - - - \ \underline{\square} \quad 512$$

$$\frac{(k-1)}{512} f_s = 1 \qquad \frac{(k-1)}{N}\cdot f_s.$$

回到music算法

$\theta: -90 : 90$

Multiple Signal Classification algorithm (MUSIC algorithm)

Let $\boldsymbol{U}_n$ denote the $J\times(J-P)$ matrix containing the $J-P$ eigenvectors corresponding to all zero eigenvalues.

$$P_{music}(\theta)=\frac{1}{\sum_{i=1}^{J-P}\left|\underline{\mathbf{a}}^h(\theta)\underline{\boldsymbol{u}}_i\right|^2}=\frac{1}{\underline{\mathbf{a}}^h(\theta)\boldsymbol{U}_n\boldsymbol{U}_n^h\underline{\mathbf{a}}(\theta)}$$

$$a(\theta)=\begin{bmatrix} 1 \\ e^{-j\frac{2\pi d\sin\theta}{\lambda}} \\ e^{j\frac{4\pi d\sin\theta}{\lambda}} \\ e^{-j\frac{6\pi d\sin\theta}{\lambda}} \end{bmatrix}$$

The P largest peaks of $P_{music}(\theta)$ provide the source DOA, but the EVD of $\mathbf{A}\boldsymbol{R}_s\mathbf{A}^h$ is not available in practice .

$$\mathbf{A}\boldsymbol{R}_s\mathbf{A}^h\underline{\boldsymbol{u}}_i=\lambda\cdot\underline{\boldsymbol{u}}_i\Rightarrow\boldsymbol{R}_x\underline{\boldsymbol{u}}_i=(\mathbf{A}\boldsymbol{R}_s\mathbf{A}^h+\boldsymbol{R}_n)\underline{\boldsymbol{u}}_i=(\lambda+\sigma_n^2)\underline{\boldsymbol{u}}_i$$

So now $\underline{\boldsymbol{u}}_i$ is eigenvector of the smallest eigenvalues of $\boldsymbol{R}_x$ which means $\boldsymbol{U}_n$ contains the eigenvectors corresponding to the $J-P$ smallest eigenvalues of $\boldsymbol{R}_x$.

In practice, $\hat{\boldsymbol{R}}_x=\frac{1}{N}\sum_{k=1}^{N}\underline{\mathbf{x}}[k]\underline{\mathbf{x}}^h[k]$.

① $R_x=\frac{1}{N}x\,x^h$.

② 求R_x的最小两个特征值以及它对应的特征向量，

$$\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\\\alpha_4\end{pmatrix}\quad\begin{pmatrix}\beta_1\\\beta_2\\\beta_3\\\beta_4\end{pmatrix}$$

$$\boxed{U=\begin{pmatrix}\alpha_1 & \beta_1\\\alpha_2 & \beta_2\\\alpha_3 & \beta_3\\\alpha_4 & \beta_4\end{pmatrix}}$$

$$f_0=\frac{f_s}{512}.$$

对于每个 kf_0 $(k=1,2,\cdots,256)$

我们都可以算出对应的 $\hat{R}_x^{(kf_0)}$

$n[k+1] \to \frac{kf_s}{512}.$

512

$x^{(kf_0)}=$ 4x帧数

$$\begin{bmatrix} \text{第一帧fft的}kf_0\rightarrow(k+1)\text{项} & \text{第二帧fft的}kf_0\rightarrow\text{第}(k+1)\text{项} & \text{三} & \text{四} \\ \text{第一帧fft完的第}(k+1)\text{项},kf_0 & \sim & \sim & \cdots \\ \cdot & \sim & \sim & \sim \\ & & & \end{bmatrix} \begin{array}{l} \leftarrow \text{对于第一个麦克风} \\ \leftarrow \text{第二个} \\ \leftarrow \text{第三} \\ \leftarrow \text{四} \end{array}$$

对每一个 kf_0 $(k=0,1,2,\cdots)$

都可以算出 $\underline{R_x}$　两个特征值对应的特征向量

最小

$U^{(kf_0)}$

$$P(\theta)=\frac{1}{\sum_{k=1}^{256}\underline{a^h(\theta)}\,U_n^{(kf_0)}\,U_n^{h\,(kf_0)}\,\underline{a^h(\theta)}}$$

$$a^h(\theta)=\begin{pmatrix} e^{-2\pi j\frac{d\sin\theta}{\lambda}} \\ e^{-4\pi j\frac{d\sin\theta}{\lambda}} \\ e^{-6\pi j\frac{d\sin\theta}{\lambda}} \end{pmatrix}$$

music 算法的证明：

考虑4个麦克风，2个声源的情况。

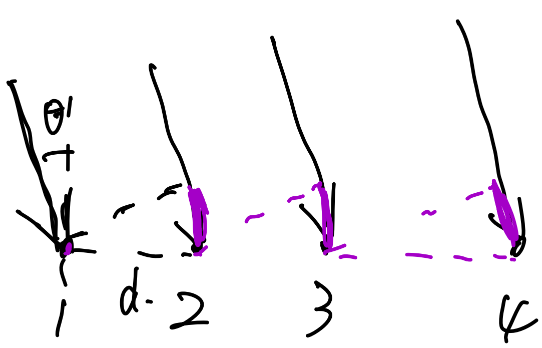
$$a(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi\frac{d\sin\theta}{\lambda}} \\ e^{-j4\pi\frac{d\sin\theta}{\lambda}} \\ e^{-j6\pi\frac{d\sin\theta}{\lambda}} \end{bmatrix}$$

source.

$v = \lambda f$.

$f = \frac{\omega}{2\pi}$ $\quad \omega = 2\pi f$

$\tau = \frac{d\sin\theta}{v}$. $\quad \lambda = \frac{v}{f}$



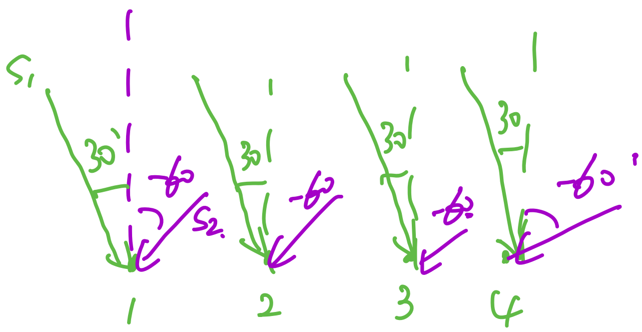
$\Delta\phi = \omega\tau$.

$= 2\pi f \tau$.

$$e^{-j\Delta\phi} = e^{-j\cdot 2\pi f \tau} = e^{-j 2\pi f \frac{d\sin\theta}{v}}$$

$$= e^{-j 2\pi \frac{d\sin\theta}{\lambda}}$$

有两个声源 $30°$, $-60°$.



对于S_1: $a_1(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi \frac{d\sin 30°}{\lambda}} \\ e^{-j4\pi \frac{d\sin 30°}{\lambda}} \\ e^{-j6\pi \frac{d\sin 30°}{\lambda}} \end{bmatrix}$

$\downarrow$

$x^1[k] = a_1(\theta)\cdot s_1[k]$

对于 Source 2:

$a_2(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi \frac{d\sin(-60°)}{\lambda}} \\ e^{-j4\pi \frac{d\sin(-60°)}{\lambda}} \\ e^{-j6\pi \frac{d\sin(-60°)}{\lambda}} \end{bmatrix}$

$\downarrow$

$x^2[k] = a_2(\theta)\cdot s_2[k]$

时 2个，环境噪声

$$x = x_1 + x_2 + \underset{\wedge}{n[k]} = A \cdot \underline{s}[k] + \underline{n}[k]$$

$$A = [a_1(\theta) \mid a_2(\theta)] = \begin{bmatrix} e^{-j2\pi \frac{d\sin(-60)}{\lambda}} & e^{-j2\pi \frac{d\sin 30}{\lambda}} \\ e^{-j4\pi \frac{d\sin(-60)}{\lambda}} & e^{-j4\pi \frac{d\sin 30}{\lambda}} \\ e^{-j6\pi \frac{d\sin(-60)}{\lambda}} & e^{-j6\pi \frac{d\sin 30}{\lambda}} \end{bmatrix}$$

$$s[k] = \begin{bmatrix} s_1[k] \\ \hline s_2[k] \end{bmatrix}$$

复指数信号

下面证明 $\hat{R}_x = \frac{1}{N} \cdot x \cdot x^h$ 存在两个很小的特征值..

在这里 期望 ~ 求平均.

$$\underline{R_x = \frac{1}{N} (A \cdot s + n)(A \cdot s + n)^h}$$

$R_x = \frac{1}{N}(As+n)(s^h A^h + n^h)$

$= \frac{1}{N}(A s \cdot s^h \cdot A^h + A \cdot s \cdot n^h + n \cdot s^h \cdot A^h + n \cdot n^h)$

$= \frac{1}{N}(\underline{A \cdot s \cdot s^h \cdot A^h} + n \cdot n^h)$

对于 $\frac{1}{N} A \cdot s \cdot n^h$ 和 $\frac{1}{N} n \cdot s^h \cdot A^h \approx 0$

<u>n 高斯. 白噪声</u>

<u>$n \ll s$</u> $E[n] \approx 0$.

<u>$\sum s[k] \approx 0$.</u>

$\lim\limits_{T \to +\infty} \frac{1}{T} \int_0^T e^{j\omega t} dt \approx 0$.

$\lim\limits_{T \to +\infty} \frac{1}{N} \sum_{n=0}^{+\infty} e^{j\omega n} \approx 0$.

$\underline{s_1(t) = e^{j(w_1 t + \phi_1)}}$ $\underline{s_2(t) = e^{j(w_1 t + \phi_2)}}$

f_s.

$$s_1 = \left[e^{j(w_1 \frac{1}{f_s} + \phi_1)} \quad e^{j(w_1 \frac{2}{f_s} + \phi_1)} \quad \cdots \quad e^{j(w_1 \frac{k}{f_s} + \phi_1)} \cdots \right]$$

$$s_2 = \left[e^{j(w_1 \frac{1}{f_s} + \phi_2)} \quad e^{j(w_1 \frac{2}{f_s} + \phi_2)} \cdots\cdots e^{j(w_1 \frac{k}{f_s} + \phi_1)} \cdots \right]$$

$f_s = 44100HZ$

$$\begin{bmatrix} s_1 \\ \hline s_2 \end{bmatrix} \left[s_1^h \,\vdots\, s_2^h \right] = \begin{bmatrix} L & L e^{j(\phi_2 - \phi_1)} \\ L e^{j(\phi_1 - \phi_2)} & L \end{bmatrix}$$

$$A \cdot s \cdot s^h \cdot A^h = u_i^h A \begin{bmatrix} L & L e^{j(\phi_1 - \phi_2)} \\ L e^{j(\phi_2 - \phi_1)} & L \end{bmatrix} (u_i A^h)^h = 0.$$

$\Rightarrow u_i^h A = 0$ 不存在 u_i s.t. $A^h u_i \neq 0$

$u_i^h A = 0$

$A^h u_i = 0$ $\qquad a_1$

$$\begin{bmatrix} 1 & e^{+j2\pi \frac{d\sin(-60)}{\lambda}} & e^{j4\pi \frac{d\sin(-60)}{\lambda}} & e^{j6\pi \frac{d\sin(-60)}{\lambda}} \\ 1 & e^{+j2\pi \frac{d\sin 30}{\lambda}} & e^{j4\pi \frac{d\sin 30}{\lambda}} & e^{j6\pi \frac{d\sin 30}{\lambda}} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = 0$$

Multiple Signal Classification algorithm (MUSIC algorithm)

Let $\boldsymbol{U}_n$ denote the $J \times (J-P)$ matrix containing the $J-P$ eigenvectors corresponding to all zero eigenvalues.

$$P_{music}(\theta) = \frac{1}{\sum_{i=1}^{J-P} \left|\mathbf{a}^h(\theta)\underline{\boldsymbol{u}}_i\right|^2} = \frac{1}{\mathbf{a}^h(\theta)\boldsymbol{U}_n\boldsymbol{U}_n^h\mathbf{a}(\theta)}$$

$\theta = -60$

$\theta = 30$.

$\frac{1}{\sigma_n^2 u_i}$

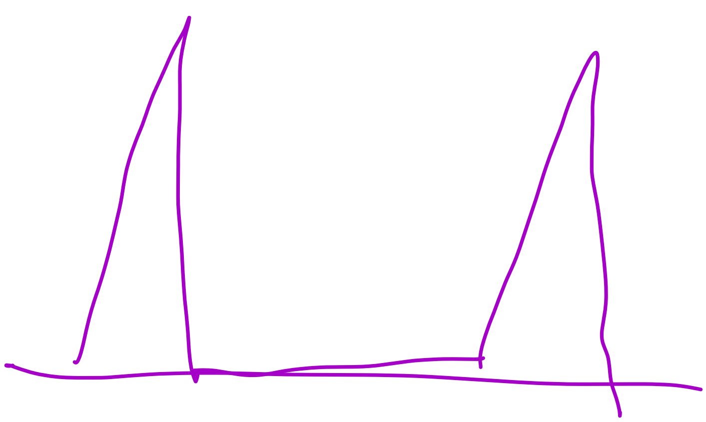
The P largest peaks of $P_{music}(\theta)$ provide the source DOA, but the EVD of $\mathbf{A}\boldsymbol{R}_s\mathbf{A}^h$ is not available in practice .

$$\mathbf{A}\boldsymbol{R}_s\mathbf{A}^h\underline{\boldsymbol{u}}_i = \lambda \cdot \underline{\boldsymbol{u}}_i \Rightarrow \boldsymbol{R}_x\underline{\boldsymbol{u}}_i = (\mathbf{A}\boldsymbol{R}_s\mathbf{A}^h + \boldsymbol{R}_n)\underline{\boldsymbol{u}}_i = (\lambda + \sigma_n^2)\underline{\boldsymbol{u}}_i$$

So now $\underline{\boldsymbol{u}}_i$ is eigenvector of the smallest eigenvalues of $\boldsymbol{R}_x$ which means $\boldsymbol{U}_n$ contains the eigenvectors corresponding to the $J-P$ smallest eigenvalues of $\boldsymbol{R}_x$.

In practice, $\hat{\boldsymbol{R}}_x = \frac{1}{N}\sum_{k=1}^{N}\underline{\mathbf{x}}[k]\underline{\mathbf{x}}^h[k]$.

$$R_x u_i = (\lambda + \sigma_n^2) u_i = \sigma_n^2 u_i$$
$\lambda = 0$



$$S_m(x) = \frac{a_0}{2} + \sum_{n=1}^{m} (a_n \cos nx + b_n \sin nx)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt \, dt$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt \, dt$$

$$S_m(x) = \frac{1}{\pi} \int_{0}^{\pi} f(t) \left[\frac{1}{2} + \sum_{n=1}^{m} \cos n(t-x) \right] dt.$$

$$\lim_{m \to +\infty} S_m(x) = f(x)$$

Riemann 引理:

Dirchlet 引理:

$\Rightarrow$ Dirchlet — Jordan 判别法

离散时间傅里叶级数

$\underline{x(t)} \xrightarrow{\text{sampling}} \underline{x[n]}$ f_s.

n为整数.

$x(t) = \sin 10\pi t$ $f_s = 10Hz$

$0 \le t \le 5s$

$x[n] = \sin \pi n$

$x[k] = x\left(\frac{k}{f_s}\right)$

$x[n]$.

$$x[n] = \underbrace{\sum_{k=1}^{N} a_k}\, e^{jk\left(\frac{2\pi}{N}\right)n} = \sum_{k=r+1}^{N+r} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

r

$$e^{jk\left(\frac{2\pi}{N}\right)(n+lN)} = e^{jk\left(\frac{2\pi}{N}\right)\cdot n} \cdot 1.$$

$$\sum_{n=1}^{N} e^{jk(2\pi/N)n} = \begin{cases} N, & k=0, \pm N, \pm 2N, \cdots \\ 0 & 其它. \end{cases}$$

$$\sum_{n=1}^{N} x[n] e^{-jr(2\pi/N)n} = \sum_{n=1}^{N} \sum_{k=1}^{N} a_k e^{j(k-r)2\pi/N \cdot n}$$

$$= N a_r.$$

$$a_r = \frac{1}{N} \sum_{n=1}^{N} x[n] e^{-jr(2\pi/N)n}.$$

$$\boxed{\begin{cases} x[n] = \sum_{k=1}^{N} a_k e^{jk(\frac{2\pi}{N})n} \\ a_k = \frac{1}{N} \sum_{n=1}^{N} x[n] e^{-jk(\frac{2\pi}{N})n} \end{cases}}$$

$a_k = a_{k+rN}$ $(r = 0, \pm 1, \pm 2, \cdots)$

$x[n]$ 实信号 $a_{+k} = a_{-k}^*$

$$a_{-k} = \frac{1}{N} \sum_{n=1}^{N} x[n] e^{jk\left(\frac{2\pi}{N}\right)n}$$

$$a_{+k} = \frac{1}{N} \sum_{n=1}^{N} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n}$$

$a_{+k} = a_{-k}^*$

$a_{+k} = a_{N-k}^*$

512.

$\cancel{2}56$ 257