# Heterogeneity, Inattention, and Bayesian Updates<sup>†</sup>

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We formulate a theory of expectations updating that fits the dynamics of accuracy and disagreement in a new survey of professional forecasters. We document new stylized facts, including the puzzling persistence of disagreement as uncertainty resolves. Our theory explains these facts by allowing for different channels of heterogeneity. Agents produce an initial forecast based on heterogeneous priors and are heterogeneously "inattentive." Updaters use Bayes' rule and interpret public information using possibly heterogeneous models. Structural estimation of our theory supports the conclusion that in normal times heterogeneous priors and inattention are enough to generate persistent disagreement, but not during the crisis. (JEL C53, D81, D83, D84, E31, E37)

I see a critical need for basic research on expectations formation. Understanding how persons update their expectations with receipt of new information often is a prerequisite for credible use of econometric decision models to predict behavior.

—Charles Manski (2004)

Expectations matter. [...] Yet how those expectations are formed, and how best to model this process, remains an open question.
—Olivier Coibion and Yuriy Gorodnichenko (2015)

Tow do agents update their expectations about an economic variable? Are their updates rational, in the sense of being consistent with Bayes' rule? How heterogeneous are agents? Do they copy each other (herd)? During a crisis, do they stick to their forecasting models/methods or are they willing to discard them and allow for a paradigm shift? We answer these questions by analyzing a previously unstudied dataset—*Bloomberg's ECFC survey of professional forecasters*—and by building and structurally estimating a theory of expectations updating.

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Our goal is to document new stylized facts about the dynamic evolution of forecast accuracy and disagreement and to build a theory that explains them simultaneously. While the theory we develop is simple and tightly parameterized, it fits the data remarkably well. In particular, it can explain the puzzling persistence of disagreement as the uncertainty about the forecasted variable resolves. The key ingredients of the theory are that agents formulate an initial forecast based on heterogeneous priors and update it infrequently (so they are "inattentive"). Agents who update use Bayes' rule to incorporate new information, and heterogenous attention generates persistent disagreement. Agents could additionally use heterogeneous models that bias the interpretation of public information. This means that there are three possible channels of heterogeneity that generate (persistent) disagreement: heterogeneity in priors, in attention and in the models used to interpret information. Our theory differs from the leading theories of expectations formation with information frictions, for example, sticky information (Mankiw and Reis 2002); noisy information (Sims 2003, Aghion et al. 2003); and heterogeneous priors (Patton and Timmermann 2010). However, it is close enough to them to allow for a comparative evaluation of their relative strengths and weaknesses in fitting the data.

The data we study consist of a panel of, on average, 75 professionals who forecast US annual (year-on-year) CPI inflation for the years 2007 to 2014. Each year corresponds to a different forecasting event, and for each of the 8 events, we observe the forecasters' updating behavior during the 18 months before the release of the figure (the forecasting period). Our main goal is to study the dynamics of updating *within* the forecasting period for each year. We do not study the dynamics from year to year, as our dataset only has eight years that moreover include the crisis. It would be straightforward to also model the year-to-year variation in a panel with a longer time series dimension.<sup>1</sup>

A preview of our estimation results is presented in Figure 1, which shows that our theory simultaneously fits the dynamics of disagreement (measured by the standard deviation of forecasts across participants) and accuracy (measured by minus the Root Mean Squared Error, henceforth RMSE) during the forecasting period for each year.<sup>2</sup> Even for the crisis years 2008–2009, the theory fits the dynamics of accuracy and can partly replicate the atypical pattern of increasing disagreement during the forecasting period. The theory is motivated by the key empirical regularities in the data. Forecasters in our data are "inattentive," in the sense that they update infrequently: on average, only about 40 percent–50 percent of forecasters update at least once a month, and the frequency varies across forecasters and over the forecasting period, but is similar across years.

We find that this type of inattention friction can be reconciled with the patterns of accuracy in the data. However, the heterogeneity induced by different updating histories is not enough to explain the patterns of disagreement. The fact that in the data disagreement persists even toward the end of the forecasting period, when

<sup>&</sup>lt;sup>1</sup>We provide a more detailed description of the data in Section I.

<sup>&</sup>lt;sup>2</sup>The forecasting period is reported backwards from 18 to 1 month before the release of the figure. The RMSE for each month is the square root of the average across forecasters of the squared difference between the forecast made on that month and the actual realization of annual inflation for that year.

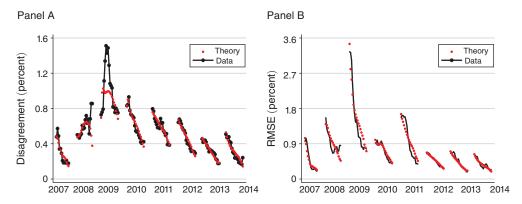


FIGURE 1. FIT OF THEORY FOR DISAGREEMENT (PANEL A) AND RMSE (PANEL B)

a lot of information about the forecasted variable is observed, suggests that there are deep idiosyncratic reasons why forecasters disagree: they could rely on different statistical models/methods for forecasting and/or have different industry- or individual-specific incentives, career concerns, pessimism/optimism biases, differences in private information, or different experience and skills. To capture these dimensions of heterogeneity parsimoniously, our theory allows for two additional channels of heterogeneity: heterogeneous priors to construct the initial forecast and heterogeneous statistical models that updaters use to interpret new public information. The three channels of heterogeneity in our theory are able to generate (persistent) disagreement.

The main novelty of our theory is to combine different channels of heterogeneity with Bayesian updating. As new information arrives, agents who update use Bayes' rule to incorporate public information. In reality, agents can observe a variety of public and private information, but two sources in particular stand out: the release of monthly inflation and the forecasts of other agents. Monthly inflation is clearly something that agents should pay attention to because it is a component of the variable they are forecasting. Similarly, other agents' forecasts are relevant, since they may not only include public, but also private information. In the baseline theory, we assume that the public information is monthly inflation. We also consider a "herding" variation of the theory that exploits a unique feature of Bloomberg's ECFC survey: upon logging onto the terminal, participants can see in real time the consensus (average) forecast. We leverage this feature to investigate whether forecasters use this information to update.

This paper contributes to the large and influential literature that empirically tests theories of informational rigidities (Mankiw, Reis, and Wolfers 2004; Coibion and Gorodnichenko 2012, 2015; Andrade and Le Bihan 2013). Our main contribution to this literature is twofold. First, we provide a simple theory that fits the dynamics of both accuracy and disagreement in the data while remaining within the confines of Bayesian updating. Second, our structural estimation results highlight the key

<sup>&</sup>lt;sup>3</sup> Indeed, consensus forecasts from various surveys have been documented to be remarkably accurate and are often used in policy and economic decision making (Ang, Bekaert, and Wei 2007).

theoretical ingredients that are necessary for fitting the data and provide an assessment of the strengths and weaknesses of a number of alternative theories. Our main findings can be summarized as follows.

First, multiple channels of heterogeneity are necessary to explain the fact that disagreement remains high even as the uncertainty about the forecasted variable diminishes. In normal times, it is enough to postulate that agents put high faith in their initial heterogeneous forecasts and are heterogeneously inattentive, but otherwise behave in a homogenous fashion. This conclusion is in line with previous findings in the literature and strengthens the case for including inattention and heterogeneous priors in any realistic theory of expectation formation. During the crisis, additional heterogeneity in the way agents interpret public information is needed to partly explain the high and persistent disagreement.

Second, a key parameter of the theory is agents' "faith" in their initial forecasts. Our estimates suggest that agents have high faith in their initial forecasts in non-crisis years, but this faith sharply decreases during the 2008–2009 crisis, as forecasters realize that they are facing a paradigm shift. Although we do not explicitly model agents' behavior during the 2008–2009 crisis, the results are compatible with the hypothesis that agents discard their initial models/methods. This can be viewed as lending support to Ortoleva's (2012) theory where decision-makers employ Bayes' rule in normal times but behave differently when they face "rare" events.

Third, in normal times, the type of public information that agents use to update could either be monthly inflation or the consensus forecast; however, this is clearly not true during the crisis. This is interesting because, even though forecasters' faith in their methods/models drops in crises, they do not seem to copy each other. This result contributes to the literature investigating whether agents herd in professional settings: the empirical evidence on professional forecasters is mixed. Ehrbeck and Waldmann (1996) rules it out, while Laster, Bennett, and Geoum (1999) reports evidence of herding among participants from banks and industrial corporations and Lamont (2002) among younger and less experienced forecasters.

Finally, our findings suggest that agents plausibly use Bayes' rule when updating. In contrast, Manzan (2011) and Lahiri and Sheng (2008) show that professional forecasters violate Bayesian updating. We can reconcile these findings by noting that in our theory only attentive agents are Bayesian. Our theory can further be viewed as providing a micro-foundation for the empirical finding of Lahiri and Sheng (2008) that heterogeneous weights on information are needed to match the disagreement in the data: in our theory, heterogeneous weights are due to heterogeneity in inattention. Some evidence of Bayesian updating is also reported in Coibion, Gorodnichenko, and Kumar (2015) who study a survey of firm's expectations and in an early empirical contribution by Caskey (1985).

The rest of the paper is organized as follows. Section I describes the data. Section II presents stylized facts. Section III describes the baseline theory. The results of the structural estimation and a counterfactual exercise are in Section IV. In Section V, we consider a number of alternative theories of expectation formation (Herding,

<sup>&</sup>lt;sup>4</sup>Formally, this is the precision of the initial forecast (inverse of the variance).

95) Chart	90 Histogram		age 1/7 Econor	nic Forecasts:	Contributors
United States	Consumer Prices (YoY	) z		<ul> <li>Yearly</li> </ul>	<ul><li>Quarterly</li></ul>
20) 2014 Actual	1.63		21) 2014 Foreca	ast 1.60	
			2015	2016	2017
Median			0.20	2.20	2.20
			0.36	2.21	2.3
Bloomberg Weighted Average			0.32	2.20	2.3
High			2.20	3.70	3.70
Low			-0.40	1.20	1.60
30) Apr. Survey			0.20	2.20	2.3
31) Mar. Survey			0.30	2.20	2.2
Contributors (74)		of	2015	2016	201
1) Participant 1		0/01	0.20	2.50	
2) Participant 2		0/01	0.00	2.20	
3) Participant 3		01	0.20	2.00	
4) Participant 4		5/01	0.20	2.50	
5) Participant 5		5/01	0.10	1.80	
6) Participant 6		5/01		2.30	
7) Participant 7		5/01	1.40	1.70	
8) Participant 8		1/30	0.20	2.50	
9) Participant 9	0	1/30	0.20	2.00	
10) Participant 10		1/30	0.10		2.2
III Participant 11	0	1/30	0.20	2.40	

FIGURE 2. BLOOMBERG EFCF SURVEY: A SNAPSHOT

Note: Participant names were removed for publication purposes.

Sticky Information, Noisy Information, Sticky-Noisy Information, and Patton and Timmermann 2010) and present results of their estimation. Section VI concludes. Appendix A contains details of some derivations and Appendix B additional illustrations of the results.

#### I. The Dataset: Bloomberg's ECFC Survey

We analyze updates of US annual (year-on-year) CPI inflation forecasts from a new dataset: the "Economic Forecasts ECFC" survey of professional forecasters conducted by Bloomberg. Users of the Bloomberg terminal can access it at any point in time (see Figure 2). Each forecast on the screen is associated with the name of the forecaster's institution as well as with the date of the last forecast update. For example, the screenshot on May 6, 2015 shows that Barclays last updated their forecast on May 1. The screen also displays in the first row the consensus forecast updated in real time. We obtained the data by a direct download from the terminal. To the best of our knowledge, we are the first to analyze this survey.

Participants.—A comparison of the ECFC survey to other surveys of professional forecasters considered in the literature is provided in Table 1. The table shows that the composition of the participants is comparable to other surveys but the number of participants is larger, updates can happen anytime, and the most recent forecasts of other participants are visible at any point in time. Participants are based in different countries and can be divided into three main categories: financial institutions (private banks, investment institutions, and large global investment banks),

	US SPF	Blue Chip	Consensus	Livingston	ECB SPF	HM Treasury	ECFC
Date of first survey Frequency	1968 Q	1978 M	1989 M	1946 Semiannual	1999 Q	1986 M	2002 Anytime
Participants (mean)	45	50	45	45	60	40	75
Anonymity	$\checkmark$	×	×	$\checkmark$	$\checkmark$	×	×
Observability of consensus	×	×	×	×	×	×	$\checkmark$
Financial inst.	37%	64%	49%	50%	49%	52%	60%
Economic cons.	29%	26%	24%	21%	22%	32%	26%
Univs. and gov.	34%	10%	27%	29%	29%	16%	14%

TABLE 1—COMMON SURVEYS OF PROFESSIONAL FORECASTERS

economic consulting firms, and others that include universities, research centers, and governmental agencies. On average, the percentage of participants from financial institutions is higher for ECFC and Blue Chip surveys, reaching 60 percent. The percentage of economic consulting firms is relatively in line with other surveys at 26 percent, while the number of universities, research centers, and other types of organizations is lower (14 percent) compared to the US SPF, Consensus Forecasts, Livingston, and the ECB's SPF.

Forecasts.—We focus on fixed-event forecasts of annual US inflation. For each year in the period 2007–2014, we consider the subset of forecasters who provide a forecast 18 months before the end of the year and who update at least 8 times during the forecasting period. This yields an average of 75 participants across years (in the analysis, we consider the actual number of participants for each year). Our panel dataset contains the history of forecast updates for all forecasters during the 18 months before the end of each year. Updates in the ECFC are irregularly spaced and more frequent than in other surveys; however, they are not frequent enough to allow us to conduct the analysis at a daily or weekly frequency since there are few updates in any given week. We therefore analyze updates at the monthly frequency and consider the forecasts available on the terminal the last day of each month. We index the horizon backwards, so for a given year, the index  $h = 18, \ldots, 1$  indicates that the forecast was produced h months before the end of the corresponding year.

Reward Structure and Incentives.—The survey manager of Bloomberg explained in private communications that participants are not explicitly rewarded nor ranked based on their accuracy. Active participants are, however, more likely to be cited in survey-related Bloomberg news stories and newsletters. The fact that forecasts are

<sup>&</sup>lt;sup>5</sup>More precisely, for each year, we remove forecasters who provided fewer than eight forecasts for either inflation, GDP, unemployment, or the Federal Funds Rate during the forecasting period. We then consider the group of forecasters who had a forecast on the system at the 18-months horizon, with the exception of the years 2007 and 2008, for which the initial forecasts are at the 13-months horizon and 16-months horizon, respectively. A small fraction of these forecasters have a forecast on the system before what we consider the initial horizon, but we ignore these earlier forecasts in the analysis.

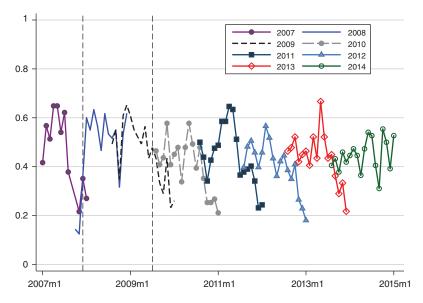


FIGURE 3. MONTHLY UPDATING FREQUENCIES

not anonymous could provide a stronger incentive for accuracy compared to other surveys (Table 1).

Survey Availability and Uses.—The survey is available on the Bloomberg terminals. In addition, its main results are divulged and published by Bloomberg in various newsletters, reports, and media outlets such as the monthly Bloomberg Briefs Economics Newsletter that are available to a broad array of users. Bloomberg collects data on the number of hits ECFC gets and on the number of users that subscribe to ECFC-related publications, but does not make these data publicly available.

# II. Stylized Facts

**Fact 1:** (Forecasters Are Inattentive)—Figure 3 plots the proportion of forecasters that update (change their forecast) at each horizon and year in our sample. On average, 40 percent to 50 percent of participants update at least once a month, which is in line with the figures in Dovern (2013) for the monthly Consensus Economics survey and slightly below the 60 percent to 70 percent documented by Andrade and Le Bihan (2013) for the survey of the ECB, which is quarterly. Notice that the proportion of updaters varies within a year but does not change dramatically between crisis (2008–2009) and other years.

The frequency with which forecasters update their forecast provides an empirical proxy for attention. Our dataset does not contain the time stamps of when each forecast was submitted, so we can tell that a forecaster logged in the system only when she decided to *change* her forecast. Because the absence of an update could be also due to the decision not to revise the forecast after acquiring new information, our

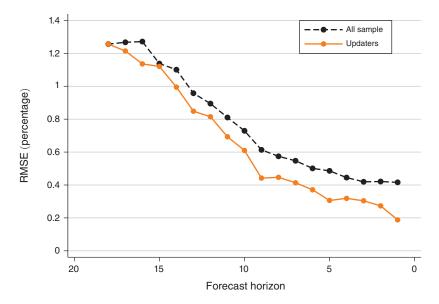


FIGURE 4. AVERAGE RMSE ACROSS YEARS: ALL SAMPLE AND UPDATERS

empirical measure can be viewed as a lower bound for attention. In order to measure attention, we construct an indicator variable capturing whether agent i updates her previous forecast h months before the end of the year:

(1) 
$$r_{i,h} = \begin{cases} 1 & \text{if } \hat{y}_{i,h} \neq \hat{y}_{i,h+1}, \\ 0 & \text{otherwise} \end{cases}$$

where  $\hat{y}_{i,h}$  is agent *i*'s forecast of annual inflation for a given year measured on the last day of month h and  $\hat{y}_{i,h+1}$  is her forecast measured on the last day of the previous month (h+1). We compute this indicator for each year 2007, ..., 2014 and for horizons  $h=17,\ldots,1$ . Then, each month the set of *updaters* consists of the forecasters for which  $r_{i,h}=1$ .

**Fact 2:** (Attention Improves Accuracy)—Figure 4 presents the evolution of the RMSE as the horizon shrinks from h=18 to h=1, averaged across the years. The RMSE for each horizon is the average across years of the square root of the average (across either the full sample of agents or just the updaters) of the squared difference between the individual forecast and the actual realization of annual inflation. As expected, as the forecast horizon is reduced, the accuracy of both groups improves since relevant new information accumulates over time. Figure 4 shows that updating improves accuracy at all horizons.

**Fact 3:** (Inattention Can Explain Violations of Forecast Rationality)—We test whether the rationality of the consensus forecast (the average forecast across agents)

Horizon	All sample	Updaters	
1	0.0000	0.1737	
2	0.0004	0.3365	
3	0.0946	0.0070	
4	0.5316	0.1614	
5	0.4158	0.0668	
6	0.4842	0.3406	
7	0.2420	0.1341	
8	0.1015	0.2957	
9	0.1204	0.6848	
10	0.0170	0.5145	
11	0.0041	0.4991	
12	0.1490	0.4697	
13	0.1214	0.4374	
14	0.2761	0.0696	
15	0.8849	0.7611	
16	0.0614	0.5078	
17	0.0000	0.0001	
18	0.0000	0.0000	

TABLE 2—p-VALUES OF RATIONALITY TESTS FOR THE CONSENSUS FORECAST

differs when we consider all forecasters or only updaters. As standard, we test rationality by a "Mincer-Zarnowitz" time series regression:

$$(2) y_s = \alpha + \beta \overline{y}_{s,h} + \varepsilon_s, \quad s = 2007, \dots, 2014,$$

where  $y_s$  is the realization of annual inflation for year s and  $\bar{y}_{s,h}$  is the consensus forecast h months before the end of the year, for either all forecasters or for only updaters. The consensus forecast is rational if  $\alpha=0$  and  $\beta=1$ . Table 2 reports the p-values for the test of the hypothesis of rationality obtained by estimating regression (2) for each horizon h separately across different years and considering HAC standard errors constructed using the Bartlett Kernel (with 2 lags). As the time series dimension only includes eight observations for each horizon, the results of the tests should be interpreted with caution.

The results confirm previously established findings in the literature (Pesaran and Weale 2006) that the consensus forecast (based on all forecasters) can fail rationality tests. Inattention—exhibited through infrequent updating—can partly explain this finding, as computing the consensus forecast considering only updaters can reverse this conclusion and support rationality for the majority of forecast horizons.

**Fact 4:** (Heterogeneous Inattention Cannot Explain Disagreement)—In the data, there is large disagreement in the forecasts at all horizons (measured by the standard deviation of individual forecasts), not only when considering all forecasters, but also when only focusing on updaters. Figure 5 shows the evolution of disagreement at each forecast horizon, averaged across the years in the sample.

Disagreement decreases with the forecast horizon, which is consistent with the fact that more information about annual inflation accumulates as the release date draws near. Updaters disagree less, but their disagreement is never zero. This implies that heterogeneity in attention is not enough to explain the persistent disagreement

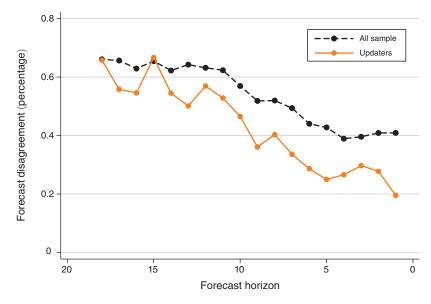


FIGURE 5. AVERAGE DISAGREEMENT ACROSS YEARS: ALL SAMPLE AND UPDATERS

observed in the data. There could be many other reasons why attentive forecasters disagree: they use different forecasting models, have different incentives (e.g., industry or individual-specific incentives), display pessimism or optimism biases, or have access to different sources of information or experience/skills. In the following section, we build a theory that allows for different channels of heterogeneity that generate disagreement.

#### III. A Theory of Expectations Updating

Heterogeneous Priors.—At month h=18, all agents provide an initial forecast. Agents provide heterogeneous forecasts, but assign the same precision to it:  $\hat{y}_{i,18}$  is

 $<sup>^{6}</sup>$ We drop the year subscript from y since our main focus is on the forecast updating within the forecasting period.

drawn from a normal  $N(\mu, \sigma_{\mu}^2)$ , while  $a_{i,18} = a_{18}$ . The parameters  $\mu$  and  $\sigma_{\mu}$  thus characterize the cross-sectional distribution of the initial forecasts, while the parameter  $a_{18}$  captures agents' faith in this initial forecast. This "heterogeneous prior" assumption is similar to that in Patton and Timmermann (2010). It allows the initial forecasts to be based on different statistical models/forecasting methods, without making explicit assumptions about what they are.

*Inattention.*—At each subsequent month h = 17, ..., 1 a fraction  $1 - \lambda_h$  of agents update their forecasts. As in Mankiw and Reis (2002),  $\lambda_h$  is exogenous.

Bayesian Updating.—Let  $\mathcal{I}_h$  denote the set of updaters at month h = 17, ..., 1. Updaters employ a signal  $z_{i,h}$  about annual inflation y with precision  $b_{i,h}$ :

$$z_{i,h} = y + \varepsilon_{i,h},$$

$$(4) \varepsilon_{i,h} \sim N(0,b_{i,h}^{-1}),$$

and use Bayes' rule to update. Normality and Bayes' rule imply that the updated forecast is a linear combination of the previous forecast and the signal  $z_{i,h}$ , where the weight on the signal is determined by the relative precision of the signal and the previous forecast. This implies that agents assign different precision to their previous forecast, depending on when it was last updated. The density forecast for an updater  $i \in \mathcal{I}_h$  is thus a normal  $N(\hat{y}_{i,h}, a_{i,h}^{-1})$  with

(5) 
$$\hat{y}_{i,h} = (1 - w_{i,h})\hat{y}_{i,h+1} + w_{i,h}z_{i,h},$$

(6) 
$$a_{i,h} = a_{18} + \sum_{j=h}^{17} \mathbf{1}_{ij} b_{i,j},$$

where  $\mathbf{1}_{ij} = 1$  if agent *i* updated at horizon *j* and 0 otherwise. The weight on the signal is

(7) 
$$w_{i,h} \equiv \frac{b_{i,h}}{a_{18} + \sum_{j=h}^{17} \mathbf{1}_{ij} b_{i,j}}.$$

The forecast for a non-updater  $i \notin \mathcal{I}_h$  is the previous month's forecast,  $\hat{y}_{i,h} = \hat{y}_{i,h+1}$ . Because during the forecasting period agents can observe a variety of information, there are several candidates for what would be reasonable signals for annual inflation. In this paper, we focus on two sources: monthly inflation releases in the "baseline" theory described in this section, and the consensus forecast in the "herding" variation described in Section VA. Both sources are public, but they differ in a fundamental way: the consensus forecast is common across agents, so  $z_{i,h} = z_h, \forall i$  in the herding theory. In the baseline theory, public information is used as an *input* into possibly heterogeneous statistical models, so  $z_{i,h} \neq z_{j,h}$ . This channel introduces heterogeneity in the *interpretation* of public information. We now explain the details.

Interpreting Information with Heterogeneous Models.—In the baseline theory, we assume monthly inflation is the information used to produce a signal for annual inflation. Because annual inflation y can be approximated by the sum of year-on-year monthly inflation  $x_h$  (henceforth simply "monthly inflation"),  $y \cong \sum_{h=0}^{11} x_h$ , monthly inflation is a key piece of information directly related to the forecasted variable. We assume that updaters observe monthly inflation and translate it into a signal about annual inflation by using an AR(1) model for  $x_h$ .

Models can be heterogeneous because of agent-specific intercepts  $c_i \sim N(c, \sigma_c^2)$ , which imply heterogeneous (subjective) conditional means of  $x_h$ :

(8) 
$$E^{(i)}[x_h|x_{h+1},\ldots] = c_i + \phi x_{h+1}.$$

The intercept  $c_i$  can be viewed as a reduced-form way to capture any agent-specific source of forecast bias relative to the AR(1) forecast that remains constant during the forecasting period, such as judgmental corrections or the fact that agents may use other predictors besides the lags of inflation in their model. Agents believe that their model is correctly specified, so they treat the monthly error  $v_{i,h} = x_h - c_i - \phi x_{h+1}$  as a white noise process,  $v_{i,h} \sim i.i.d. N(0, \sigma_v^2)$ .

Signals.—The signal  $z_{i,h}$  in the baseline theory is the (subjective) model-implied conditional mean of y based on the information available at month h, which is the entire history of monthly inflation up to the *previous* month  $h+1,^9 z_{i,h} = E^{(i)}[y|x_{h+1},x_{h+2},\dots]$ :

(9) 
$$z_{i,h} = \frac{12 c_i}{1-\phi} + \frac{\phi^{h-10} (1-\phi^{12})}{1-\phi} (x_{h+1} - \frac{c_i}{1-\phi})$$
 for  $h = 17, ..., 11$ ;

$$(10) \ z_{i,h} = \frac{(h+1)c_i}{1-\phi} + \frac{\phi(1-\phi^{h+1})}{1-\phi} \left(x_{h+1} - \frac{c_i}{1-\phi}\right) + \sum_{i=h+1}^{11} x_i \quad \text{for } h = 10, \dots, 1.$$

This construction underscores the fact that heterogenous models systematically bias the interpretation of new information, thus introducing *persistence* in disagreement.

The signal's precision  $b_{i,h}$  in equation (4) is the inverse of the variance of  $\varepsilon_{i,h} = y - z_{i,h}$ . Because we assume that heterogeneity is only in the conditional

$$x_h = \frac{1}{12} \left( \frac{cpi_h - cpi_{h+12}}{cpi_{h+12}} \right), \quad \forall h = 11, \dots, 0.$$

<sup>&</sup>lt;sup>7</sup> Annual inflation, e.g., for year 2007, is  $y = (\overline{cpi}_{2007} - \overline{cpi}_{2006})/\overline{cpi}_{2006}$ , where  $\overline{cpi}_{2007} = (1/12)\sum_{j=0}^{11} cpi_j$  and  $cpi_j$  is the consumer price index measured j months before the end of year 2007:  $y \cong \sum_{h=0}^{11} x_h$ , where

<sup>&</sup>lt;sup>8</sup>This assumption can be easily extended to incorporate other public information besides the lags of inflation, for example, by considering a vector autoregressive model. The effects of this would be to add several parameters to the model, thereby losing its parsimony and ease of interpretation. Also note that this would not change the implications for disagreement.

 $<sup>^{9}</sup>$ We don't include inflation for the current month h in the common information because forecasters may update before this information is released. In addition, official releases of monthly inflation are typically published with a lag.

means (and so in  $z_{i,h}$ ), the precision  $b_h$  is the same across agents and it is a known function of  $\phi$  and  $\sigma_v^2$ :

$$b_{h}^{-1} = \begin{cases} \frac{\sigma_{v}^{2}}{(1-\phi)^{2}} \left[ 12 - \frac{2\phi(1-\phi^{12})}{1-\phi} + \frac{\phi^{2}(1-\phi^{24})}{1-\phi^{2}} \right] + \frac{\phi^{2}(1-\phi^{12})^{2}(1-\phi^{2h-22})}{(1-\phi)^{3}(1+\phi)} \sigma_{v}^{2} & \text{if } h \geq 11 \\ \frac{\sigma_{v}^{2}}{(1-\phi)^{2}} \left( h + 1 - \frac{2\phi(1-\phi^{h+1})}{1-\phi} + \frac{\phi^{2}(1-\phi^{2(h+1)})}{1-\phi^{2}} \right) & \text{if } h \leq 10. \end{cases}$$

Full details about the derivation of  $z_{i,h}$  and  $b_h$  can be found in Appendix A.

#### **IV. Structural Estimation**

In this section, we discuss the estimation of the baseline theory. The parameters that we aim to estimate are  $\theta = (\mu, \sigma_{\mu}, a_{18}, c, \sigma_{c}, \phi, \sigma_{v})$ , while we calibrate  $\lambda_{h}$  using the observed fraction of updaters in the data. The structural estimation is performed by Simulated Method of Moments (SMM) as in Gouriéroux and Monfort (1996), Duffie and Singleton (1993), Ruge-Murcia (2012), which amounts to matching the theoretical disagreement and RMSE with their empirical counterparts.

The computation of the theoretical disagreement and RMSE proceeds as follows. For a given year, we randomly draw the initial point forecast  $\hat{y}_{i,18}$  for each of the N agents (N corresponds to the number of agents in the data for that year) from  $N(\mu, \sigma_{\mu}^2)$ . For horizons  $h = 17, \ldots, 1$ , we randomly draw a fraction  $1 - \lambda_h$  of agents who update, where  $1 - \lambda_h$  is the proportion of updaters observed in the data at horizon h for that year. The forecast for the non-updaters equals their forecast at horizon h + 1. The forecast for the updaters is obtained using equations (5)–(7), where  $z_{i,h}$  is derived using equations (9) and (10) and  $x_h$  is monthly inflation for that month and year computed using data from FRED. The signal's precision is given by (11). At each horizon h, we use the simulated forecasts to compute the disagreement across all agents. We then compute the RMSE as described in footnote 2, using the actual realization of annual inflation for the year under consideration obtained from the FRED dataset.

We estimate the theory either for each year separately or by pooling separately crisis and non-crisis years. For the year-by-year estimation, we repeat the simulation for  $\tau=300$  replicas of the year and estimate the parameters for each year separately to match the dynamics of both accuracy and disagreement within that year. For the pooled estimation across T years (where T=2 when pooling the crisis years 2008 and 2009, and T=6 when pooling the non-crisis years 2007 and 2010–2014), we repeat the simulation for each year but keep parameters constant across years (except  $\lambda_h$ , which is calibrated to the fraction of updaters in the data for that month and year). In this case, we obtain  $\tau T$  different samples with  $\tau=100$ , and we match the accuracy and disagreement on average over the T years. <sup>10</sup>

<sup>&</sup>lt;sup>10</sup> Although the choice of the number of replications  $(\tau)$  is arbitrary, following Duffie and Singleton (1993), the idea is to have a simulation sample,  $\tau T$ , where  $\tau T \to \infty$  as  $T \to \infty$ . As stressed by Gourieroux and Monfort

TABLE 3—ESTIMATED PARAMETERS OF THE BASELINE THEORY

Parameters	$\mu$	$\sigma_{\mu}$	$a_{18}$	c	$\sigma_c$	$\phi$	$\sigma_v$
Samples							
Non-crisis	1.629	0.677	67.401	0.007	0.0005	0.9713	0.0104
	(0.0868)	(0.0669)	(0.00001)	(0.0038)	(0.0283)	(0.0755)	(0.0053)
Crisis	3.751	0.611	36.41	0.00001	0.0105	0.932	0.007
	(0.763)	(0.111)	(0.00001)	(0.0826)	(0.0048)	(0.1455)	(0.0151)
Years							
2014	1.952	0.5272	69.074	0.057	0.0124	0.6001	0.0246
	(0.004)	(0.0108)	(0.00001)	(0.1595)	(0.0215)	(1.188)	(0.0796)
2013	1.986	0.462	42.592	0.0173	0.0031	0.7921	0.0332
	(0.0076)	(0.0289)	(0.00001)	(0.0156)	(0.0558)	(0.3413)	(0.0645)
2012	1.979	0.672	69.947	0.0306	0.0003	0.7769	0.0293
	(0.0456)	(0.0036)	(0.00001)	(0.0562)	(1.0201)	(0.309)	(0.0280)
2011	1.634	0.761	60.01	0.0675	0.0189	0.7938	0.0245
	(0.0315)	(0.0648)	(0.00001)	(0.0907)	(0.0009)	(0.0492)	(0.0463)
2010	2.189	0.832	45.257	0.0226	0.0177	0.729	0.0069
	(0.157)	(0.0012)	(0.00001)	(0.0110)	(0.0002)	(0.0621)	(0.0084)
2009	3.036	0.7025	15.732	0.00001	0.0382	0.6608	0.0298
	(0.0926)	(0.0588)	(0.00001)	(0.0734)	(0.0090)	(0.1469)	(0.0289)
2008	2.439	0.477	60.017	0.0888	0.0365	0.7286	0.0261
	(0.0984)	(0.0059)	(0.0001)	(0.2805)	(0.0805)	(0.6255)	(0.0729)
2007	1.879	0.482	45.252	0.0595	0.0011	0.8167	0.0209
	(1.114)	(0.517)	(0.00001)	(0.00001)	(0.1011)	(0.00001)	(0.0104)

*Note:* Results are based on SMM estimation, matching disagreement and accuracy either on average over non-crisis years (2007, 2010–2014) and crisis years (2008 and 2009) or separately for each year.

In the SMM estimation, we set the weighting matrix equal to the identity matrix. Based on the evidence about the small sample bias of method of moments estimators with many moment conditions discussed in, e.g., Tauchen (1986) and Altonji and Segal (1996), we restrict attention to a subset of horizons:  $h=1,3,7,11,15,18.^{11}$  This means that we have a total of 12 moments for accuracy and disagreement.

#### A. Estimation Results

Table 3 presents the estimated parameters and reveals several findings. First, heterogeneity in priors is a crucial feature of the theory, as evidenced by the significant estimates of  $\sigma_{\mu}$  (with the sole exception of 2007). Second, heterogeneity in models only seems to matter during and right after the crisis, as estimates of  $\sigma_c$  are only significant in the years 2008–2011. Third, the estimates of  $a_{18}$  are large, except for the crisis year 2009, suggesting that agents generally put high faith in their initial forecast but they lose this faith during turbulent years. Finally, the prior dispersion increases significantly during and right after the crisis and then goes back

<sup>(1996),</sup> when the number of replications tends to infinity, the SMM estimator coincides with GMM.

<sup>&</sup>lt;sup>11</sup> Patton and Timmermann (2010) also used a reduced number of horizons as moment conditions (6 horizons rather than their full set of 24).

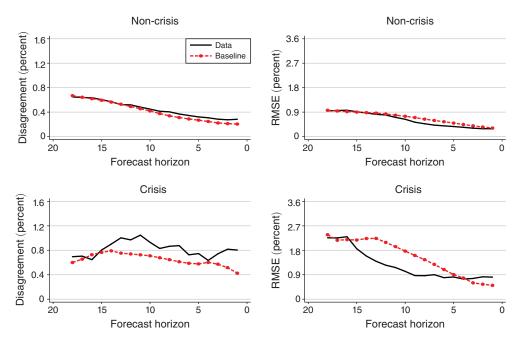


FIGURE 6. BASELINE THEORY FIT: DISAGREEMENT (LEFT) AND RMSE (RIGHT)

to pre-crisis levels, possibly reflecting the increase in uncertainty induced by the crisis.

We visualize the fit of the baseline theory in Figure 6 for the pooled estimation of crisis and non-crisis years (Appendix B shows for the fit of the year-by-year estimation, which magnifies the information in Figure 2). The baseline theory fits remarkably well the patterns of disagreement and accuracy in the data for the non-crisis years, but it underpredicts disagreement and accuracy during the crisis.

The estimation results provide insights into whether forecasters have a "double identity" during the six-month overlapping period when they forecast both current-and next-year inflation. In principle, the year-by-year estimation could lead to this conclusion because it allows agents to feed the same information (monthly inflation) into two different AR models when they update both forecasts during the overlap. The estimation results do not support this conclusion: the year-by-year estimates of the AR parameters are relatively constant and imposing constant parameters (and thus no "double identity") fits well into the non-crisis sample.

Figure 7 provides a possible explanation for the poor fit of the baseline theory during the crisis: it compares the estimated confidence intervals for the initial forecasts to the actual realization of inflation for that year. The figure shows that for most years the initial forecasts are on average close to 2 percent, the unofficial inflation target rate. In reality, however, we see that inflation experienced dramatic changes during and right after the crisis (e.g., it dropped from 3.85 percent in 2008 to -0.3 percent in 2009). It is thus plausible that forecasters

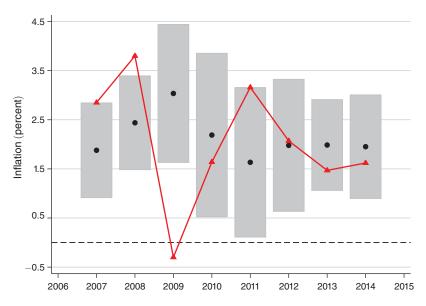


Figure 7. Yearly Estimates of  $\mu$  (Dots) with Confidence Intervals (Shaded Area) and Actual Inflation (Line)

not only lost faith in their initial forecast during this period, but also realized that a paradigm shift was under way and thus changed the way they forecast. The sharp decrease in RMSE in the early part of the forecasting period for crisis years shown in Figure 6, indeed, suggests that forecasters may have discarded their methods/models and obtained a large improvement in accuracy. Our theory cannot replicate these patterns, as it does not allow for changes within the forecasting period.

## B. Counterfactuals: Comparing Heterogeneity Channels

In this section, we perform counterfactual exercises to assess the relative importance of the different channels of heterogeneity that are embedded in the baseline theory. There are three such channels: heterogeneous priors, heterogeneous models used to interpret public information, and heterogeneous inattention, which implies that updaters assign different weights to the signal, depending on when it was last updated. We shut down each of the three channels in turn and simulate the baseline theory by either setting  $\sigma_{\mu}=0$  (homogeneous priors), or  $\sigma_c=0$  (homogeneous models), or  $\lambda_h=0$  (full attention). The remaining parameters are kept fixed to the values in Table 3. Figure 8 reports the fit of the counterfactuals. We see that by far the largest deterioration in the fit occurs when shutting down heterogeneous priors, followed by assuming that all forecasters update every month. Consistent with the estimation results, failing to account for heterogeneity in models does not affect the fit during normal times, while it does play a significant role in explaining the patterns of disagreement during the crisis.

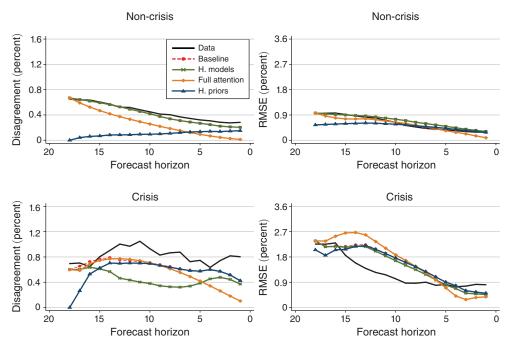


FIGURE 8. IMPORTANCE OF DIFFERENT HETEROGENEITY CHANNELS: DISAGREEMENT (LEFT) AND RMSE (RIGHT)

#### V. Other Theories

In this section, we compare the fit of the baseline theory to that of alternative theories. The estimation follows the procedure described in Section IV, subject to the assumptions made by each alternative theory.<sup>12</sup>

#### A. Herding

This variation assumes that the signal that agents use when updating is the consensus forecast, so they "herd." Relative to the baseline theory, this theory removes the heterogeneous-models component, as all updaters use the same signal. In our data the forecasts of other participants and the consensus forecast can be seen by updaters on the Bloomberg terminal. In addition, most updates occur in a given week each month, likely due to the fact that Bloomberg sends a monthly e-mail reminder to the participants. We thus let the signal in equations (5)–(7),  $z_{i,h} = z_h$ , be the consensus forecast measured on the day of the Bloomberg e-mail reminder in month h and the precision  $b_{i,h} = b_h$  be the inverse of the variance of the forecasts across agents measured on the same day, which is also observable and common across agents.

<sup>&</sup>lt;sup>12</sup>Note that we adapt alternative theories to our particular context, so "rejection" of a particular theory should be interpreted with caution as it may be due to the rejection of any of the additional assumptions we make.

### B. Sticky Information

This variation has a unique source of heterogeneity: agents have different forecasts because only a fraction  $1 - \lambda_h$  of them update at different horizons. Originally proposed by Mankiw and Reis (2002), in our setting, this theory implies that agents are inattentive and use homogeneous models to produce an initial forecast and to update. The forecast for the updaters is thus  $\hat{y}_{i,h} = z_h$  as in equations (9) and (10), for  $h = 18, 17, \ldots, 1$  with  $c_i = c$ . Note that this theory implies that there is no disagreement among updaters, something that is at odds with the data.

# C. Noisy Information

This variation also has a unique source of heterogeneity, due to signals that are observed with noise. The theory is inspired by Aghion et al. (2003) and Coibion and Gorodnichenko (2012), and in our context, it implies that all agents update every period but observe noisy signals. The variable observed with noise is monthly inflation  $x_h$ , which agents assume is an AR(1) as in equation (8), but with  $c_i = c$ . The noisy signals are

(12) 
$$s_{i,h} = x_h + u_{i,h},$$
$$u_{i,h} \sim N(0, \sigma_u^2) \quad \forall i, h,$$
$$E[u_{i,h}, v_\tau] = 0 \quad \forall i, h, \tau,$$

where  $v_t = x_\tau - c - \phi x_{\tau+1}$ ,  $v_\tau \sim i.i.d. N(0, \sigma_v^2)$ . The state-space form is  $\tilde{x}_h = \phi \tilde{x}_{h+1} + v_h$  and  $s_{i,h} = \tilde{x}_h + u_{i,h}$ , with  $\tilde{x}_h = x_h - c/(1 - \phi)$ . Agents compute the optimal estimate of  $x_h$  using the Kalman filter: letting  $\hat{x}_{i,h|h} \equiv E[\tilde{x}_h | s_{i,h}, s_{i,h+1}, \ldots]$  and  $P_{i,h|h} \equiv \text{var}[\tilde{x}_h | s_{i,h}, s_{i,h+1}, \ldots]$ , we have

$$\hat{x}_{i,h|h} = \hat{x}_{i,h|h+1} + P_{i,h|h+1} (P_{i,h|h+1} + \sigma_u^2)^{-1} (s_{i,h} - \hat{x}_{i,h|h+1}), 
P_{i,h|h} = P_{i,h|h+1} - P_{i,h|h+1} (P_{i,h|h+1} + \sigma_u^2)^{-1} P_{i,h|h+1},$$

where  $\hat{x}_{i,h|h+1} = \phi \hat{x}_{i,h+1|h+1}$  and  $P_{i,h|h+1} = \phi^2 P_{i,h+1|h+1} + \sigma_v^2$ . The optimal forecast  $\hat{y}_{i,h}$  for annual inflation is the conditional expectation (previously described by equations (9) and (10)) given the noisy signals for monthly inflation up to month h+1:

(13) 
$$\hat{y}_{i,h} = \frac{12c}{1-\phi} + \frac{\phi^{h-10}(1-\phi^{12})}{1-\phi} \hat{x}_{i,h+1|h+1} \quad \text{for } h = 18,\dots,11;$$

$$\hat{y}_{i,h} = \frac{(h+1)c}{1-\phi} + \frac{\phi(1-\phi^{h+1})}{1-\phi}\hat{x}_{i,h+1|h+1} + \hat{x}_{i,h+1|h+1} + \sum_{j=h+2}^{11} x_j$$

for 
$$h = 10, ..., 1$$
.

The last expression in (14) is equal to zero if  $h=10.^{13}$  For numerical stability, we initialize the Kalman filter 150 months before the 18 months forecast horizon and set the initial forecast and variance to  $\hat{x}_{i,167|168} = \phi \left( x_{168} - c / \left( 1 - \phi \right) \right)$  and  $P_{i,167|168} = \sigma_v^2 / \left( 1 - \phi^2 \right)$ . This implies that the filter's accuracy is constant across agents,  $P_{i,h|h} = P_{h|h}$ .

# D. Sticky-Noisy Information

Based on Andrade and Le Bihan (2013), in our context, this means that at month h a fraction  $1 - \lambda_h$  of agents observe a noisy signal. Those who update provide the forecast described in the "Noisy Information" variation. This theory has two sources of heterogeneity: inattention and noisy signals.

## E. Patton and Timmermann (2010)

This theory also has two sources of heterogeneity: different priors for the initial forecast and noisy signals. It assumes that all agents are attentive and the updated forecast at month h is a weighted average of the optimal forecast extracted from the Kalman Filter and the heterogenous initial priors,  $\hat{y}_{i,18}$ :

(15) 
$$\hat{y}_{i,h} = w_h \hat{y}_{i,18} + (1 - w_h) E[y | s_{i,h}],$$

(16) 
$$w_h = \frac{E[e_{ih}^2]}{\kappa^2 + E[e_{ih}^2]},$$

$$(17) e_{ih} \equiv y - E[y | s_{i.h}],$$

where  $E[y|s_{i,h}]$  is given by (13) and (14). In contrast to our theory, the weights  $w_h$  are not Bayesian but are given by an ad-hoc specification chosen to match the data (by choosing  $\kappa$ ). The assumed functional form has the property that as the filtered signal  $E[y|s_{i,h}]$  becomes more accurate, the weight attached to the signal increases.

Table 4 summarizes the key features of all the theories that we consider in the paper.

# F. Fit Comparison

Figures 9–13 report the fit of the different variations of the theory and of the baseline theory, for both crisis and non-crisis years.

The figures show that the baseline theory has the best overall fit. Figure 9 shows that the herding variation provides a comparable—if slightly worse—fit, but only in normal times. The herding theory is clearly not able to reproduce the high disagreement

<sup>&</sup>lt;sup>13</sup>These last two expressions in (14) imply that while the latest piece of available monthly inflation is observed with noise, the remaining historical releases of inflation are observed without it.

Table 4—Features of the Theories

Theory

Baseline Herding Sticky info Noisy info Sticky-noisy PT 2010

Non-crisis

Het. priors	Het. models	Inattention	Bayes' rule	Signal: Inflation	Signal: Consensus	Noisy signal
✓	✓	✓	✓	✓		
✓		✓	✓		✓	

Disagreement (percent) 3.6 1.6 Data RMSE (percent) Baseline 1.2 2.7 -- Herding 1.8 8.0 0.9 0.4 0 0 15 10 5 20 15 10 20 5 Forecast horizon Forecast horizon Crisis Crisis Disagreement (percent) 1.6 3.6 RMSE (percent) 1.2 2.7 0.8 1.8 0.9 0.4 0 15 10 Ó 15 10 20 20 Forecast horizon Forecast horizon

Non-crisis

FIGURE 9. HERDING FIT: DISAGREEMENT AND RMSE

during the crisis, something that the baseline theory can partly accomplish by allowing for heterogeneous signals. The sticky information variation (Figure 10) clearly underpredicts disagreement, particularly at long horizons, confirming that inattention alone does not introduce enough heterogeneity to explain the large amount of disagreement in the data. Allowing for noisy information (Figure 11) generates enough heterogeneity to almost replicate the initial disagreement, but cannot match the way that disagreement decreases with the forecast horizon. Combining the two frictions in the sticky-noisy variation (Figure 12) does not improve the fit, except for the last part of the forecasting period for crisis years. Patton and Timmermann (2010)'s variation in Figure 13 shows similar patterns to those for the noisy theory for non-crisis years, poorly matching the speed with which disagreement decreases during the forecasting period. For crisis years, instead, this variation provides better

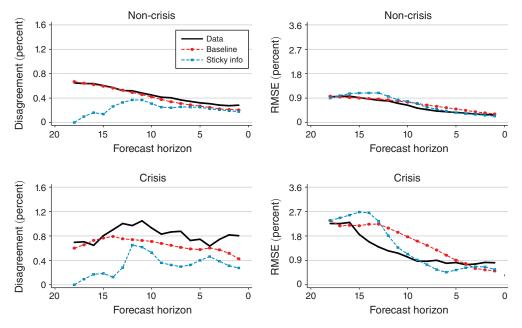


FIGURE 10. STICKY INFORMATION FIT: DISAGREEMENT AND RMSE

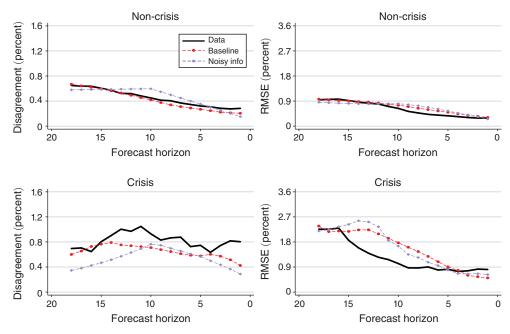


FIGURE 11. NOISY INFORMATION FIT: DISAGREEMENT AND RMSE

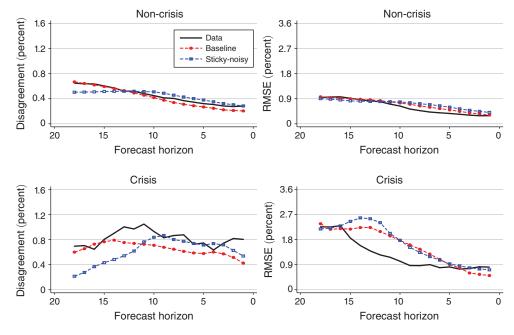


FIGURE 12. STICKY-NOISY INFORMATION FIT: DISAGREEMENT AND RMSE

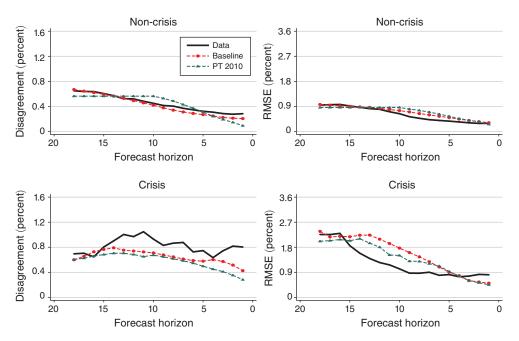


FIGURE 13. PT 2010 FIT: DISAGREEMENT AND RMSE

fit than the baseline theory for accuracy, probably due to the use of data-driven weights instead of Bayesian weights. Nonetheless, the disagreement generated by the baseline theory more closely resembles that in the data.

Summing up, the results support the following message: in normal times, agents base their initial forecasts on heterogeneous priors, to which they attach high faith. Updaters use Bayes' rule to incorporate homogeneous signals (either based on monthly inflation or the consensus forecast). Heterogeneity in inattention results in heterogeneous weights assigned to the signals. During a crisis, agents lose faith in their initial forecasts, no longer rely on homogeneous signals nor copy each other, and likely discard their forecasting methods/models.

#### VI. Conclusion

Expectations are a key determinant of economic decisions and a building block of macro and finance models. The increasing role of expectation manipulation in monetary policy makes it important not only to understand how expectations are formed and how they evolve over time, but also how they respond to communication of public information. This paper highlights the different sources of heterogeneity that, in addition to information rigidities, should be included in any theory of expectation formation that can explain the dynamic patterns of accuracy and disagreement observed in surveys of professional forecasters. Our theory is simple, assumes rational updating and it empirically outperforms competing theories of information rigidities.

#### APPENDIX A. BASELINE-MODEL DERIVATIONS

**Signal:** We present details on the derivation of the signal (the conditional mean of annual inflation) at different horizons h. Recall that  $y = \sum_{h=0}^{11} x_h$ . The AR(1) model assumption implies that the (subjective) unconditional mean of  $x_h$  is  $\tilde{\mu}_i = c_i/(1-\phi)$  and

$$x_{0} = \tilde{\mu}_{i} + \phi^{12}(x_{12} - \tilde{\mu}_{i}) + \sum_{j=0}^{11} \phi^{j} v_{i,j},$$

$$x_{1} = \tilde{\mu}_{i} + \phi^{11}(x_{12} - \tilde{\mu}_{i}) + \sum_{j=0}^{10} \phi^{j} v_{i,j+1},$$

$$\dots$$

$$x_{11} = \tilde{\mu}_{i} + \phi(x_{12} - \tilde{\mu}_{i}) + v_{i+11},$$

so that

$$y = 12 \tilde{\mu}_i + \frac{\phi(1-\phi^{12})}{1-\phi}(x_{12}-\tilde{\mu}_i) + \sum_{i=0}^{11} \frac{1-\phi^{i+1}}{1-\phi} v_{i,j}.$$

At horizon  $h \ge 12$ , we have:

(18) 
$$y = 12\tilde{\mu}_i + \frac{\phi^{h-11}(1-\phi^{12})}{1-\phi}(x_h - \tilde{\mu}_i) + \sum_{i=0}^{11} \frac{1-\phi^{j+1}}{1-\phi} v_{i,j} + \sum_{i=12}^{h-1} \frac{\phi^{j-11}(1-\phi^{12})}{1-\phi} v_{i,j},$$

whereas for  $h \leq 11$  some components of y are observed, and thus

(19) 
$$y = h\tilde{\mu}_i + \frac{\phi(1-\phi^h)}{1-\phi}(x_h - \tilde{\mu}_i) + \sum_{j=h}^{11} x_j + \sum_{j=0}^{h+1} \frac{1-\phi^{j+1}}{1-\phi} v_{i,j}.$$

The signal  $z_{i,h}$  is the conditional expectation of equations (18) and (19), respectively, given information up to horizon h + 1:

(20) 
$$z_{i,h} = 12\tilde{\mu}_i + \frac{\phi^{h-10}(1-\phi^{12})}{1-\phi}(x_{h+1}-\tilde{\mu}_i)$$
 for  $h = 18, \dots, 11$ ;

(21) 
$$z_{i,h} = (h+1)\tilde{\mu}_i + \frac{\phi(1-\phi^{h+1})}{1-\phi}(x_{h+1}-\tilde{\mu}_i) + \sum_{j=h+1}^{11} x_j \text{ for } h = 10,\ldots,1,$$

which correspond to equations (9) and (10) in the main text.

**Signal's Precision:** Given the signal  $z_{i,h} = y + \varepsilon_{i,h}$  and the expressions for y and  $z_{i,h}$ , the variance of the forecast error at horizon  $h \ge 12$  is derived as follows:

$$\varepsilon_{i,h} = y - z_{i,h} = \sum_{j=0}^{11} \frac{1 - \phi^{j+1}}{1 - \phi} v_{i,j} + \sum_{j=12}^{h} \frac{\phi^{j-11} (1 - \phi^{12})}{1 - \phi} v_{i,j},$$

$$E[\varepsilon_{i,h}^{2}] = \sigma_{\nu}^{2} \sum_{j=0}^{11} \frac{(1 - \phi^{j+1})^{2}}{(1 - \phi)^{2}} + \sigma_{\nu}^{2} \sum_{j=12}^{h} \frac{\phi^{2j-22} (1 - \phi^{12})^{2}}{(1 - \phi)^{2}},$$

where we rely on the assumption that  $v_{i,h} \sim i.i.d. N(0, \sigma_v^2)$  for all i and h. The first expression on the right-hand side of the previous equation is

$$\sigma_{\nu}^{2} \sum_{j=0}^{11} \frac{\left(1 - \phi^{j+1}\right)^{2}}{\left(1 - \phi\right)^{2}} = \frac{\sigma_{\nu}^{2}}{\left(1 - \phi\right)^{2}} \sum_{j=0}^{11} \left(1 - \phi^{j+1}\right)^{2}$$

$$= \frac{\sigma_{\nu}^{2}}{\left(1 - \phi\right)^{2}} \left[ (1 - \phi)^{2} + (1 - \phi^{2})^{2} + \dots + (1 - \phi^{12})^{2} \right]$$

$$= \frac{\sigma_{\nu}^{2}}{\left(1 - \phi\right)^{2}} \left[ 12 - 2(\phi + \phi^{2} + \dots + \phi^{12}) + (\phi^{2} + \phi^{4} + \dots + \phi^{24}) \right]$$

$$= \frac{\sigma_{\nu}^{2}}{\left(1 - \phi\right)^{2}} \left[ 12 - \frac{2\phi(1 - \phi^{12})}{1 - \phi} + \frac{\phi^{2}(1 - \phi^{24})}{1 - \phi^{2}} \right].$$

The second expression is

$$\sigma_{v}^{2} \sum_{j=12}^{h} \frac{\phi^{2j-22} (1-\phi^{12})^{2}}{(1-\phi)^{2}} = \sigma_{v}^{2} \frac{(1-\phi^{12})^{2}}{(1-\phi)^{2}} \sum_{j=12}^{h} \phi^{2j-22}$$

$$= \sigma_{v}^{2} \frac{(1-\phi^{12})^{2}}{(1-\phi)^{2}} (\phi^{2} + \phi^{4} + \dots + \phi^{2h-22})$$

$$= \sigma_{v}^{2} \frac{(1-\phi^{12})^{2}}{(1-\phi)^{2}} \frac{\phi^{2} (1-\phi^{2h-22})}{1-\phi^{2}}$$

$$= \sigma_{v}^{2} \frac{\phi^{2} (1-\phi^{12})^{2} (1-\phi^{2h-22})}{(1-\phi)^{3} (1+\phi)}.$$

Summing the two expressions, we obtain the expression for  $b_h^{-1}$  for  $h \ge 11$ ; while relying on the same derivation as in the first equation, we obtain the expression for  $h \le 10$ .

## APPENDIX B. BASELINE THEORY FIT: YEAR-BY-YEAR RESULTS

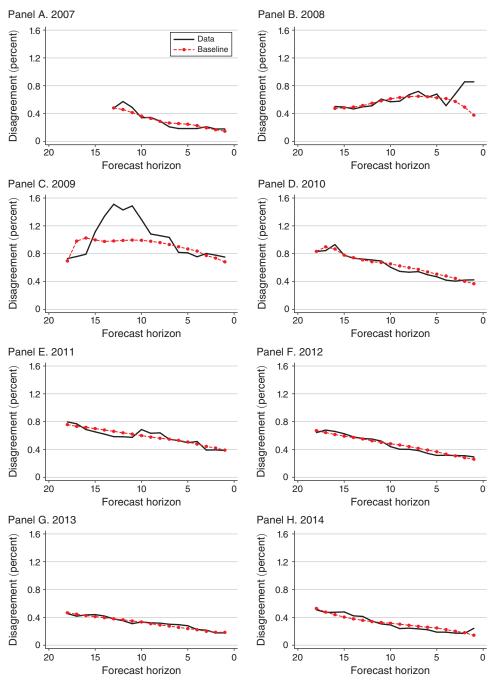


Figure 14. Disagreement, 2007-2014

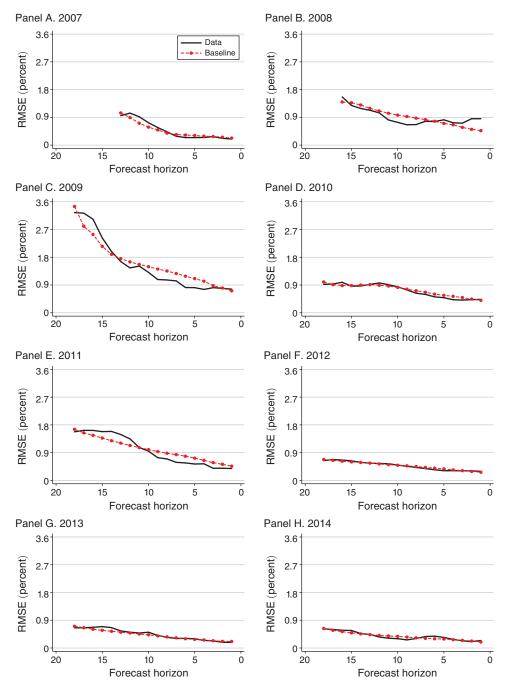


FIGURE 15. RMSE, 2007-2014

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