Understanding Expectation Formation from Probabilistic Surveys

Tao Wang
Johns Hopkins University

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Outline

1. Motivation
2. Theories
   - Sticky Expectation
   - Other theories
3. Data and Methodology
4. Stylized Facts
5. Appendix

What I am doing

- Use density information to
  - test expectation rigidity models
  - ... and identify differences in various theories
- Both individual and population moments
- Households and professional forecasters
  - drivers of difference in rigidity across two types of agents

Why density is important

- **Identification:** different theories have testable predictions on the second moments
  - Scenario 1. Two people think the chance of raining is 50%.
  - Scenario 2. One person thinks 100% and the other 0%.
- **Modeling Implications:** both mean and variance affect economic decisions
  - precautionary saving with income risks
  - portfolio choice with risky asset

Literature

- **Theory**
  - Sticky expectation [Carroll, 2003], [Reis, 2006]
  - Rational inattention [Sims, 2003], [Gabaix, 2014]
  - Noisy information [Lucas Jr, 1972], [Woodford, 2001]
  - Learning [Evans and Honkapohja, 2012]
  - Strategic interaction [Morris and Shin, 2002], [Hellwig and Veldkamp, 2009]
  - Diagostic expectation [Bordalo et al., 2018]
  - Model uncertainty [Hansen and Sargent, 2001], [Hansen and Sargent, 2008]
- **Empirics**
  - **Heterogeneity in Expectation:** [Mankiw et al., 2003]
  - **Testing Theories:** [Coibion and Gorodnichenko, 2012], [Fuhrer, 2018]

Unified Framework

h-period ahead density forecast by agent i at time t based on information set $I_{i,t}$

$$\widehat{f}_{i,t}(y_{t+h}|I_{i,t})$$

- Theories differ in $I_{i,t}$
- May also differ on information processing, i.e. $I_{i,t} \rightarrow \widehat{f}_{i,t}$

Definition and notation

- Individual
  - mean forecast $E_{i,t}(y_{t+h})$
  - forecast error $FE_{i,t+h|t} = y_{t+h} - E_{i,t}(y_{t+h})$
  - uncertainty $Var_{i,t}(y_{t+h})$
- Population
  - average forecast $\bar{E}_t(y_{t+h})$
  - average forecast error $\overline{FE}_t = y_{t+h} - \bar{E}_t(y_{t+h})$
  - cross-section disagreements $Var_t(E_{i,t}(y_{t+h}))$
  - average uncertainty $\overline{Var}_t(y_{t+h})$

Assumption about true process

$$y_{t+1} = \rho y_t + \omega_t$$

$$\omega_t \sim N(0, \sigma_\omega^2)$$

- $0 < \rho \leq 1$
- if $\rho = 0$, no way to forecast at all
- ω_t is i.i.d

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Sticky Expectation: assumptions

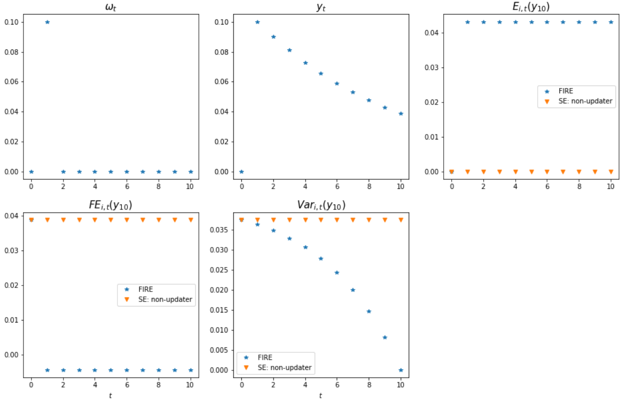
- At time t, agent i learns about y_t at a fixed Poisson rate λ
- A non-updater since $t - \tau$

$$E_{i,t}(y_{t+h}|y_{t-\tau}) = \rho^{h+\tau} y_{t-\tau}$$

- An updater is a special cae $\tau = 0$

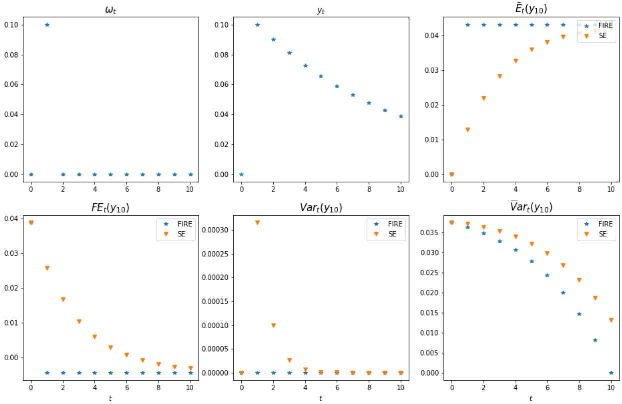
Impulse responses to shocks: individual moments

$$\text{True Process} \quad \rho = 0.9, \quad \sigma_\omega = 0.1, \quad \omega_1 = 0.1$$
$$\text{SE} \qquad \lambda = 0.4$$



Impulse responses to shocks: population moments

$$\text{True Process} \quad \rho = 0.9, \quad \sigma_\omega = 0.1, \quad \omega_1 = 0.1$$
$$\text{SE} \qquad \lambda = 0.4$$



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Noisy Information: assumptions

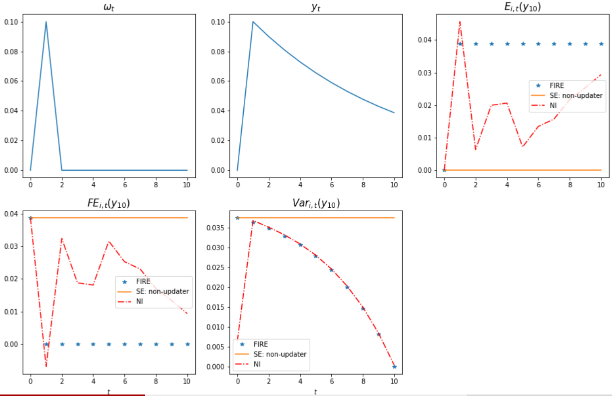
- Individual only observes noisy signals

$$
\begin{aligned}
&s_{i,t} = [s_t^{pb}, s_{i,t}^{pr}]' \in I_{i,t} \\
&\text{public signal:} \quad s_t^{pb} = y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \\
&\text{private signal:} \quad s_{i,t}^{pr} = y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0, \sigma_\xi^2)
\end{aligned}
$$

- Kalman filtering (simply normal updating if ρ=0)

Impulse responses to shocks: individual moments

$$\text{True Process} \quad \rho = 0.9, \quad \sigma_\omega = 0.1, \quad \omega_1 = 0.1$$
$$\text{SE:} \quad \lambda = 0.5; \quad \text{NI:} \quad \sigma_\xi = 0.1, \quad \sigma_\epsilon = 0.1$$



Impulse responses to shocks: population moments

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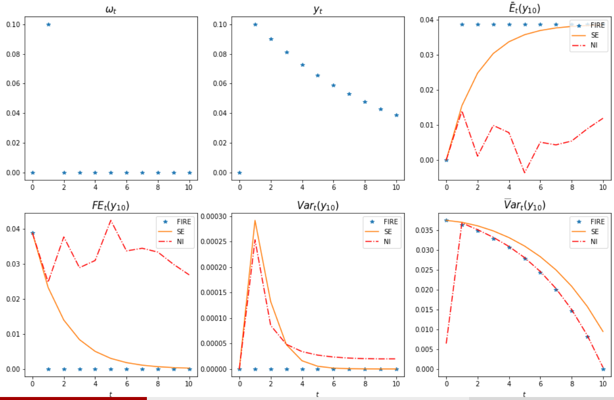
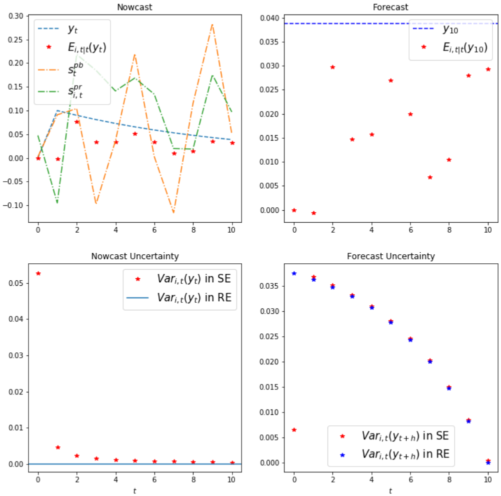
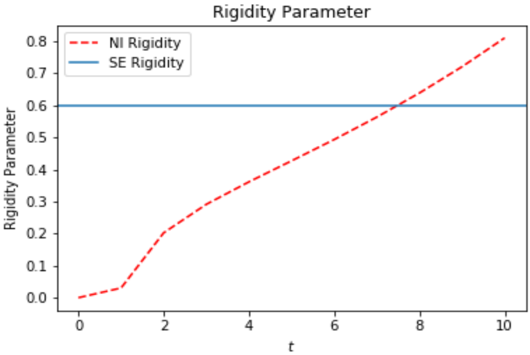


A detailed look into noisy information



Implied rigidity of different models



Other theories on to-do-list

- **Rational Inattention:** attentiveness endogenously respond to variances
- **Learning**: the structural parameter ρ is not known, thus the agent learns about it as if an econometrician does

Identification strategies 1: testing rigidity models

- [Coibion and Gorodnichenko, 2012]
  - FEs respond to shocks and serially correlated.
- **Additional in this paper**
  - Uncertainty does not depend on shocks; and serially correlated.

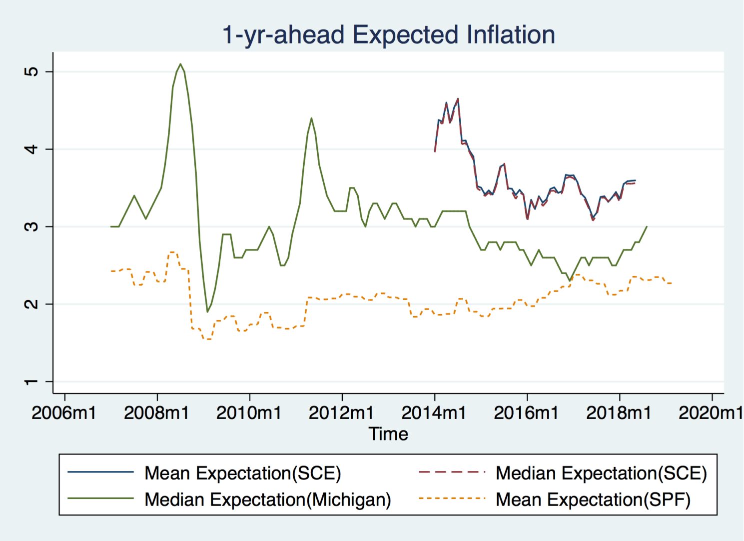
Identification strategies 2: differentiating theories

- [Coibion and Gorodnichenko, 2012]
  - FEs do not depend on past realizations according to baseline SE and NI; but do so according to heterogeneous priors or precision models.
  - Implied rigidity does not differ across shocks according to SE but differs according to NI.
  - Disagreements rise after shocks according to baseline SE, strategic interactions and heterogeneous priors but invariant according to baseline NI.
- **Additional in this paper**
  - Uncertainty do not depend on shocks per se according to baseline SE and NI, instead on degree of information rigidity.

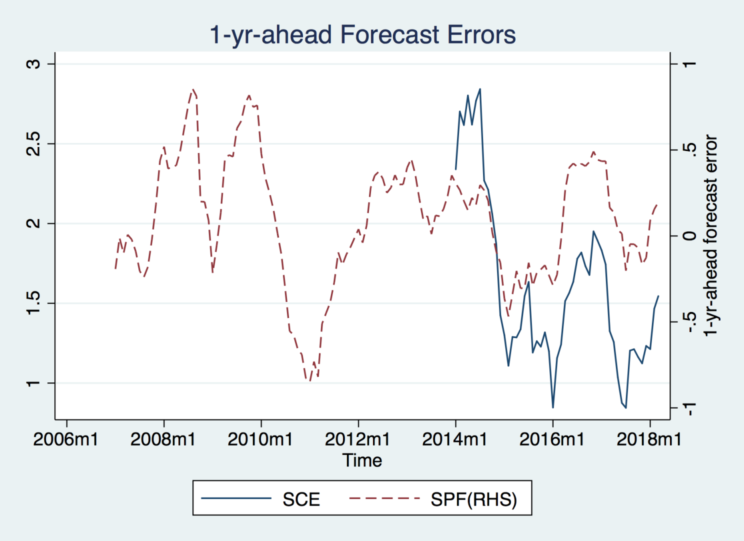
Data

	SCE	SPF
Time period	2013-present	2007-present
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Aggregate Var in Density	1-yr and 3-yr inflation	1-yr and 3-yr CPI and PCE
Pannel Structure	stay up to 12 months	average stay for 5 years
Demographic Info	Education, Income, Age	Industry

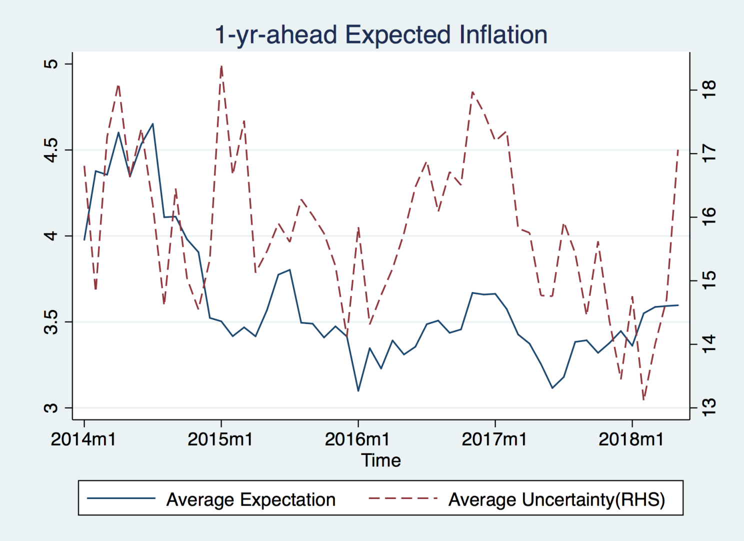
Population moments: average



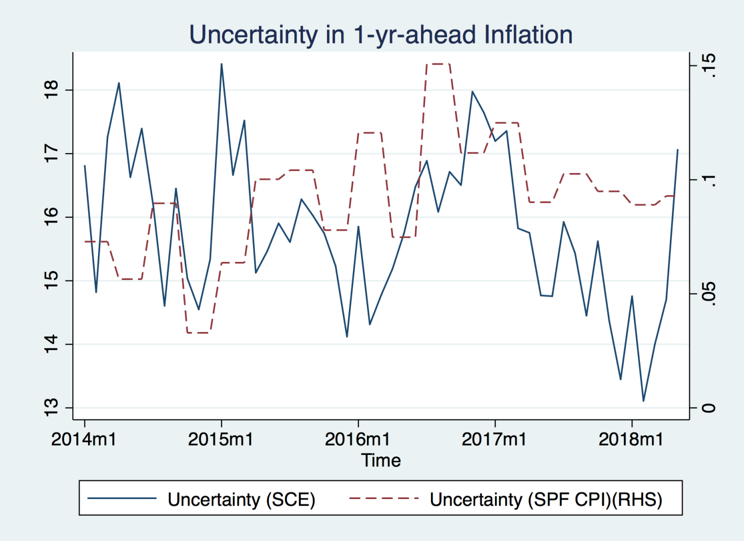
Population moments: average forecast errors



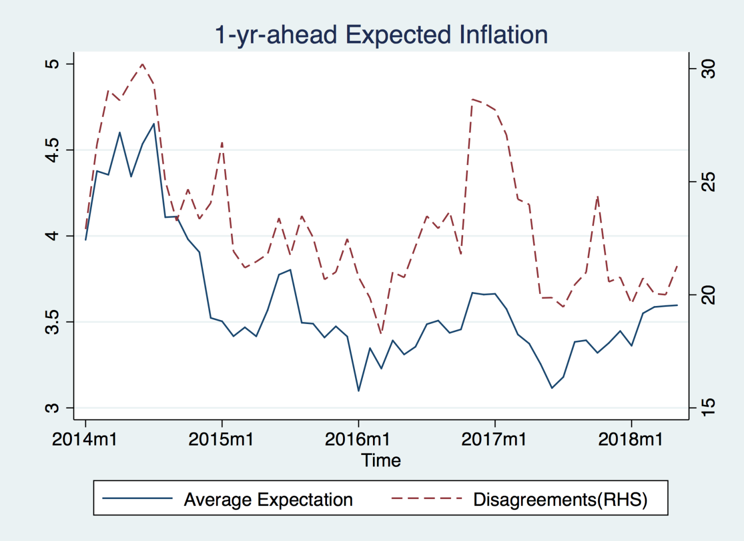
Population moments: average uncertainty



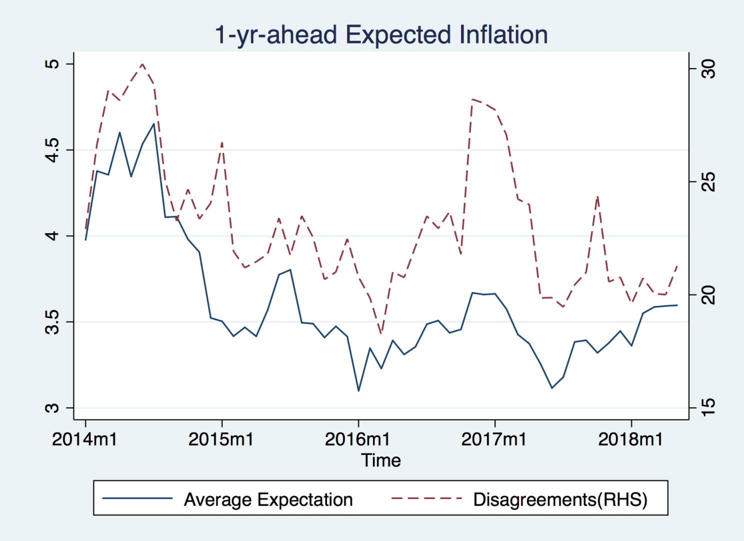
Population moments: average uncertainty



Population moments: disagreements



Population moments: disagreements



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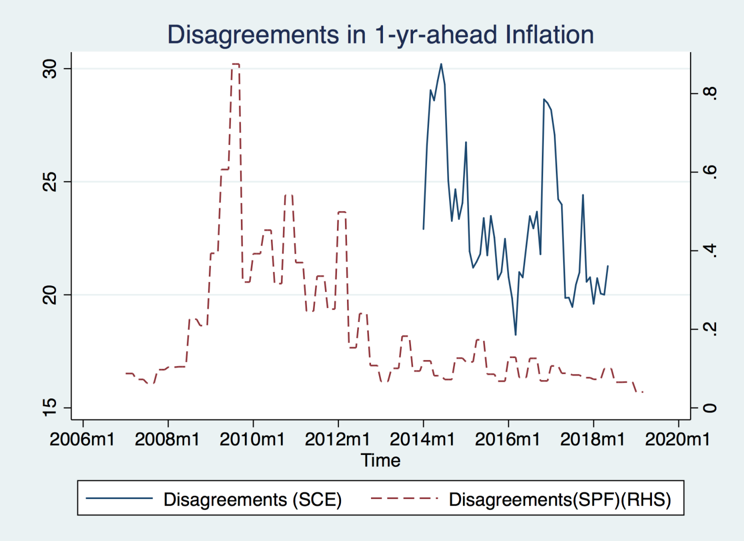
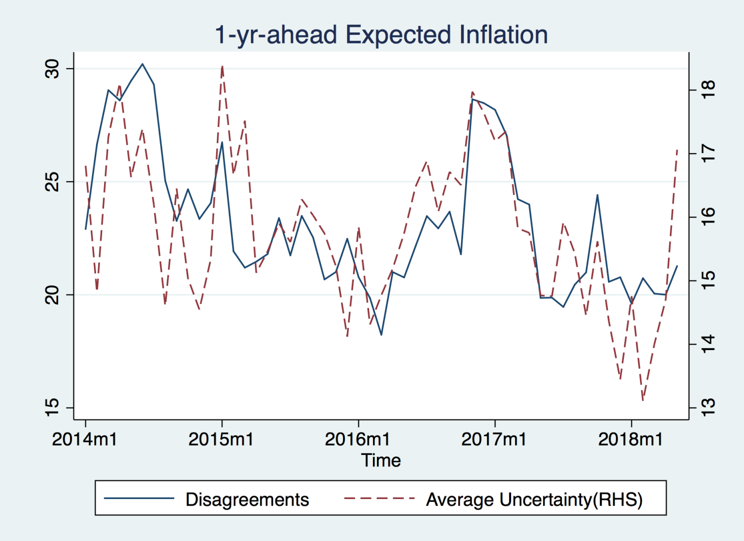
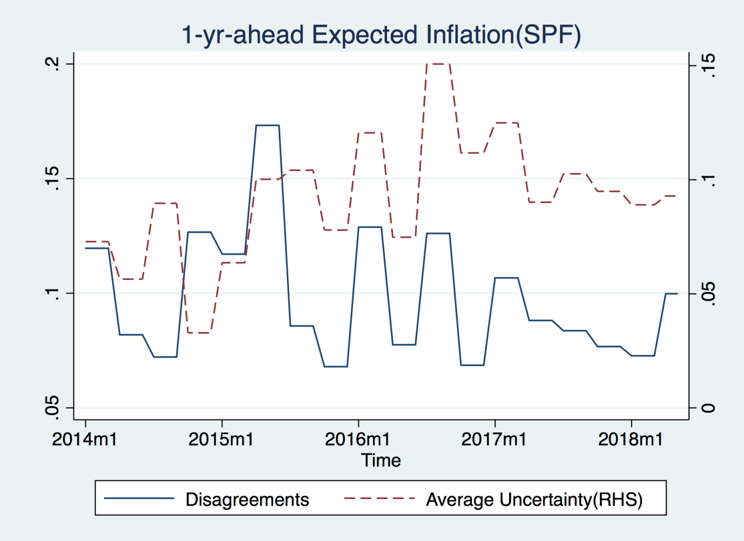


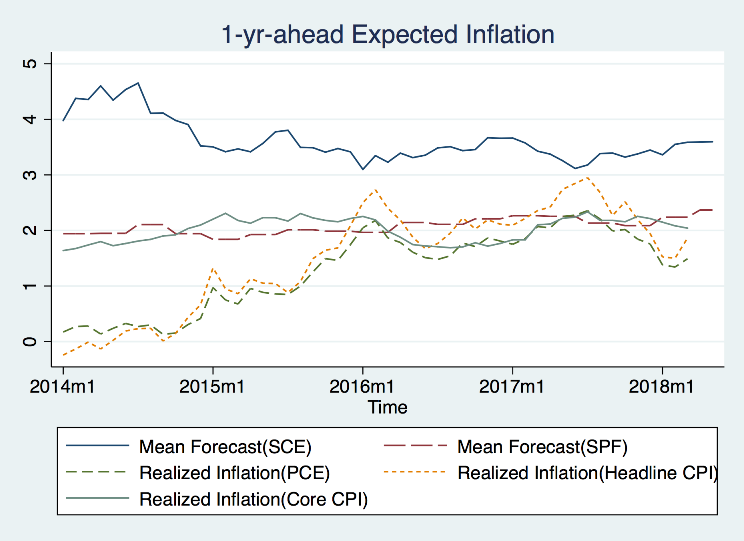
Population moments: uncertainty and disagreements



Population moments: uncertainty and disagreements



Population moments: forecast and realization



Empirical execution

- **Measurement Errors**: winsorization is still necessary
- **Density Estimation**: generalized beta estimation, [Engelberg et al., 2009]
- **Identification of Shocks**: following [Coibion and Gorodnichenko, 2012]

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Sticky Expectation: individual

For a non-updater since $t-\tau$ ($\tau=0$ for updater),

- **Mean**

$$E_{i,t}(y_{t+h}|y_{t-\tau}) = \rho^{h+\tau} y_{t-\tau}$$

- **Forecast Error**

$$FE_{i,t+h|t} = \underbrace{\sum_{s=0}^{h+\tau} \rho^s \omega_{t+h-s}}_{\substack{\text{weighted sum of} \\ \text{future realized shocks}}}$$

- **Variance**

$$Var_{i,t}(y_{t+h}|y_{t-\tau}) = \sum_{s=0}^{h+\tau} \rho^{2s} \sigma^2_\omega$$

Sticky Expectation: individual

$$\text{updater:} \quad \Delta Var_{i,t}(y_{t+h}|y_t) = \sum_{s=0}^{\tau} \rho^{2s}\sigma_\omega^2$$

$$\text{non-updater:} \ \Delta Var_{i,t|t-\tau-1}(y_{t+h}|y_{t-\tau-1}) = \sigma_\omega^2$$

1. Change in expectation(and variance) depends on if update or not
2. Cannot observe systematically sluggish response to shocks at individual level

Sticky Expectation: population

- **Average forecast**

$$\bar{E}_t(y_{t+h}) = \lambda \underbrace{E_t(y_{t+h})}_{\text{rational expectation at t}} + (1-\lambda) \underbrace{\bar{E}_{t-1}(y_{t+h})}_{\text{average expectation at } t-1}$$

$$= \lambda E_t(y_{t+h}) + (1-\lambda)(\lambda E_{t-1}(y_{t+h}) + ...)$$

$$= \underbrace{\lambda \sum_{s=0}^{\infty} (1-\lambda)^s E_{t-s}(y_{t+h})}_{\text{weighted sum of past rational expectations}}$$

- **Change in average forecast**

$$\Delta \bar{E}_t(y_{t+h}) = \underbrace{(1-\lambda)}_{\text{stickiness}} \Delta \bar{E}_{t-1}(y_{t+h}) + \lambda \rho^h \omega_t$$

Sticky Expectation: population

- **Disagreements**

$$Var_t(y_{t+h}) = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} (E_{t|t-\tau}(y_{t+h}) - \bar{E}_t(y_{t+h}))^2$$

- **Change in disagreements**

$$\Delta Var_t(y_{t+h}) = \rho^{2h}(1-\lambda)\lambda \underbrace{\omega_t^2}_{\text{shock at time t}}$$

1. Disagreements rise after the shock and then gradually decline
2. Response of disagreements depends on the size of the shock

Sticky Expectation: population

- **Average variance**

$$\overline{Var}_t(y_{t+h}) = (1-\lambda)\underbrace{\overline{Var}_{t-1}(y_{t+h})}_{\text{average variance at t-1}} + \underbrace{\lambda Var_t(y_{t+h})}_{\text{variance of updater at t}}$$

- **Change in average variance**

$$\Delta\overline{Var}_t(y_{t+h}) = \underbrace{(1-\lambda)\Delta\overline{Var}_{t-1}(y_{t+h}) - \lambda\rho^{2h}\sigma_\omega^2}_{\text{does not depend on shock at t}}$$

1. Average variance does not respond to shocks
2. Average variance has serial correlation with the same rigidity parameter $1-\lambda$

Noisy Information: individuals

- **Mean**

$$E_{i,t}(y_{t+h}) = \rho^h E_{i,t|t}(y_t)$$

$$E_{i,t|t}(y_t) = \underbrace{E_{i,t|t-1}(y_t)}_{\text{prior}} + P \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}}$$

$$= (1 - PH)E_{i,t|t-1}(y_t) + Ps_{i,t}$$

where $P = [P_\epsilon, P_\xi] = \Sigma^y_{i,t|t-1} H(H' \Sigma^y_{i,t|t-1} H + \Sigma^v)^{-1}$

where $\Sigma^y_{i,t|t-1}$ is the variance of y_t based on prior belief

and $\Sigma^v = \begin{bmatrix} \sigma^2_\epsilon & 0 \\ 0 & \sigma^2_\xi \end{bmatrix}$

Noisy Information: individuals

- **Change in mean**

$$\Delta E_{i,t|t}(y_{t+h}) = \underbrace{\rho^h(1-PH)\Delta E_{i,t-1|t-1}(y_t)}_{\text{Lagged response}} + \underbrace{\rho^h PH\Delta y_{i,t} + \rho^h P\Delta v_{i,t}}_{\text{Shocks to signals}}$$

1. Rigidity parameter $1-PH$
2. Serial correlation at individual level
3. Always respond to shocks

Noisy Information: individuals

- **Variance**

$$\Sigma^y_{i,t|t} = \Sigma^y_{i,t|t-1} - \Sigma^y_{i,t|t-1} H'(H\Sigma^y_{i,t-1} H' + \Sigma^v)^{-1} H\Sigma^y_{i,t|t-1}$$

- **Change in variance**

$$\Delta\Sigma^y_{i,t|t} < 0$$

1. It does not depend on the realizations of the signal.
2. It decreases unambiguously from $t-1$ to t.
3. The two properties carry through to h-period ahead forecast

Noisy Information: population

- **Mean**

$$\bar{E}_{t|t}(y_{t+h}) = \rho^h[(1-PH)\underbrace{\bar{E}_{t-1}(y_{t+h})}_{\text{Average prior}} + P\underbrace{\bar{s}_t}_{\text{Average Signals}}]$$
$$= (1-PH)\bar{E}_{t-1}(y_{t+h}) + P[\epsilon_t, 0]'$$
$$= (1-PH)\bar{E}_{t-1}(y_{t+h}) + P\epsilon_t$$

1. Same properties to the individual forecast

Noisy Information: population

- **Disagreements**

$$
\begin{aligned}
Var_t(y_{t+h}) &= E((E_{i,t|t}(y_{t+h}) - \bar{E}_t(y_{t+h}))^2) \\
&= \rho^{2h} P_\xi^2 \sigma_\xi^2
\end{aligned}
$$

1. increase with the forecast horizon
2. depends on noisiness private signals, but not on that of public signals and the variance of the true variable y
3. increase with the rigidity parameter P in this model

Noisy Information: population

- **Change in disagreements**

$$\Delta Var_t(y_{t+h}) = \rho^{2h}(1-\rho^2)P_\xi^2\sigma_\xi^2 > 0$$

1. disagreements increase as time goes from $t-1$ to t.
2. disagreements increase as approaching the variable of forecast

Noisy Information: population

- **Average variance**

$$\bar{V}ar_t(y_{t+h}) = \bar{\Sigma}^y_t$$

- **Change in average variance**

$$\Delta Var_t(y_{t+h}) < 0$$

1. average variance is the same as individual variance, not depend on signals
2. the variance unambiguously drop over time