

# Understanding Expectation Formation from Probabilistic Surveys

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# Outline

- 1 Motivation
- 2 Theories
  - Sticky Expectation
  - Other theories
- 3 Data and Methodology
- 4 Stylized Facts
- 5 Empirical Results
  - Test of Null Hypothesis of Rational Expectation
  - Professional forecasters as a baseline
- 6 Appendix

# What I am doing

- Use **density information** to
  - test expectation rigidity models
  - ... and identify differences in various theories
- Both **individual** and population moments
- **Households** and professional forecasters
  - drivers of difference in rigidity across two types of agents

# Why density is important

- **Identification:** different theories have testable predictions on the second moments
  - Scenario 1. Two people think the chance of raining is 50%.
  - Scenario 2. One person thinks 100% and the other 0%.
- **Modeling Implications:** both mean and variance affect economic decisions
  - precautionary saving with income risks
  - portfolio choice with risky asset

# Literature

## • Theory

- Sticky expectation [Carroll, 2003], [Reis, 2006]
- Rational inattention [Sims, 2003], [Gabaix, 2014]
- Noisy information [Lucas Jr, 1972], [Woodford, 2001]
- Learning [Evans and Honkapohja, 2012]
- Strategic interaction [Morris and Shin, 2002], [Hellwig and Veldkamp, 2009]
- Diagnostic expectation [Bordalo et al., 2018]
- Model uncertainty [Hansen and Sargent, 2001], [Hansen and Sargent, 2008]

## • Empirics

- **Heterogeneity in Expectation:** [Mankiw et al., 2003]
- **Testing Theories:** [Coibion and Gorodnichenko, 2012], [Fuhrer, 2018]

# Unified Framework

h-period ahead density forecast by agent  $i$  at time  $t$  based on information set  $I_{i,t}$

$$\hat{f}_{i,t}(y_{t+h}|I_{i,t})$$

- Theories differ in  $I_{i,t}$
- May also differ on information processing, i.e.  $I_{i,t} \rightarrow \hat{f}_{i,t}$

# Definition and notation

## • Individual

- mean forecast  $E_{i,t}(y_{t+h})$
- forecast error  $FE_{i,t+h|t} = y_{t+h} - E_{i,t}(y_{t+h})$
- uncertainty  $Var_{i,t}(y_{t+h})$

## • Population

- average forecast  $\bar{E}_t(y_{t+h})$
- average forecast error  $\overline{FE}_t = y_{t+h} - \bar{E}_t(y_{t+h})$
- cross-section disagreements  $Var_t(E_{i,t}(y_{t+h}))$
- average uncertainty  $\overline{Var}_t(y_{t+h})$

# Assumption about true process

$$y_{t+1} = \rho y_t + \omega_t$$

$$\omega_t \sim N(0, \sigma_\omega^2)$$

- $0 < \rho \leq 1$
- if  $\rho = 0$ , no way to forecast at all
- $\omega_t$  is i.i.d



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# Sticky Expectation: assumptions

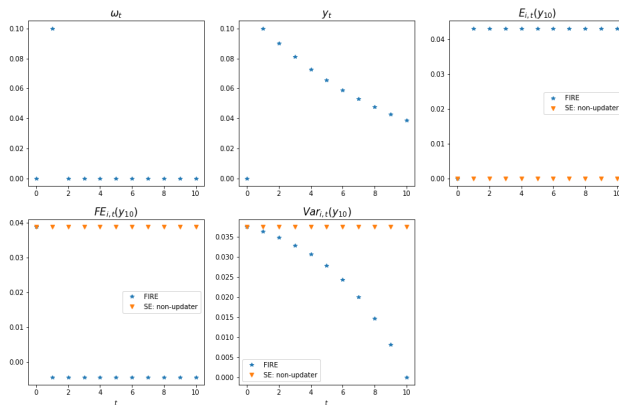
- At time  $t$ , agent  $i$  learns about  $y_t$  at a fixed Poisson rate  $\lambda$
- A non-updater since  $t - \tau$

$$E_{i,t}(y_{t+h}|y_{t-\tau}) = \rho^{h+\tau} y_{t-\tau}$$

- An updater is a special case  $\tau = 0$

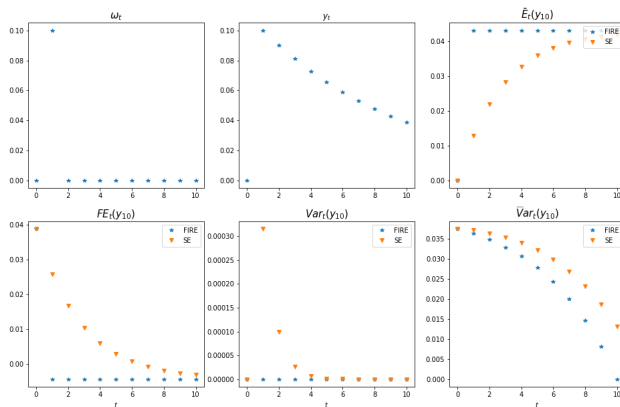
# Impulse responses to shocks: individual moments

True Process  $\rho = 0.9, \quad \sigma_{\omega} = 0.1, \quad \omega_1 = 0.1$   
 SE  $\lambda = 0.4$



# Impulse responses to shocks: population moments

True Process  $\rho = 0.9, \quad \sigma_{\omega} = 0.1, \quad \omega_1 = 0.1$   
 SE  $\lambda = 0.4$



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# Noisy Information: assumptions

- Individual only observes noisy signals

$$s_{i,t} = [s_t^{pb}, s_{i,t}^{pr}]' \in I_{i,t}$$

$$\text{public signal: } s_t^{pb} = y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

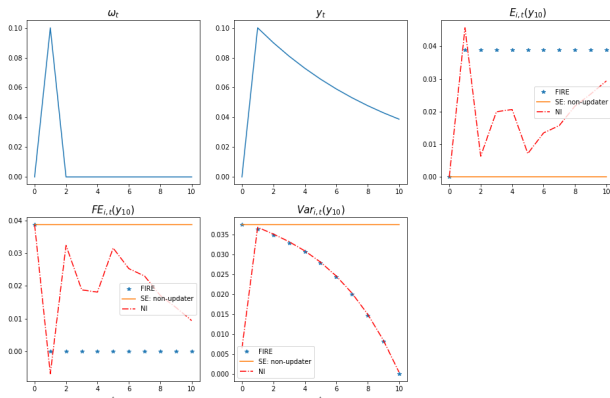
$$\text{private signal: } s_{i,t}^{pr} = y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0, \sigma_\xi^2)$$

- Kalman filtering (simply normal updating if  $\rho=0$ )

# Impulse responses to shocks: individual moments

True Process  $\rho = 0.9$ ,  $\sigma_{\omega} = 0.1$ ,  $\omega_1 = 0.1$

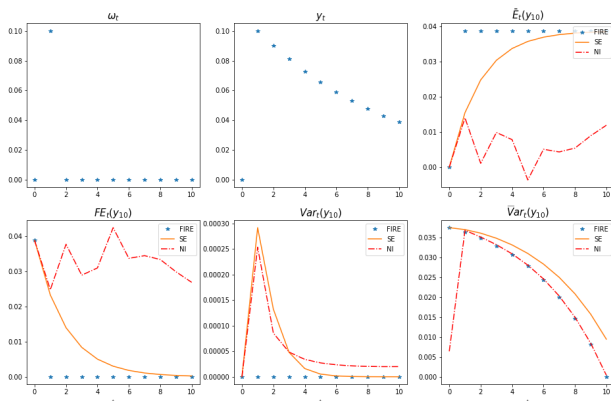
SE:  $\lambda = 0.5$ ; NI:  $\sigma_{\xi} = 0.1$ ,  $\sigma_{\epsilon} = 0.1$



# Impulse responses to shocks: population moments

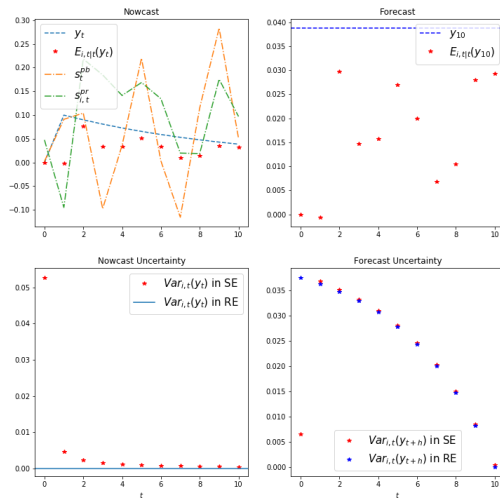
True Process  $\rho = 0.9$ ,  $\sigma_\omega = 0.1$ ,  $\omega_1 = 0.1$

SE:  $\lambda = 0.5$ ; NI:  $\sigma_\xi = 0.1$ ,  $\sigma_\epsilon = 0.1$

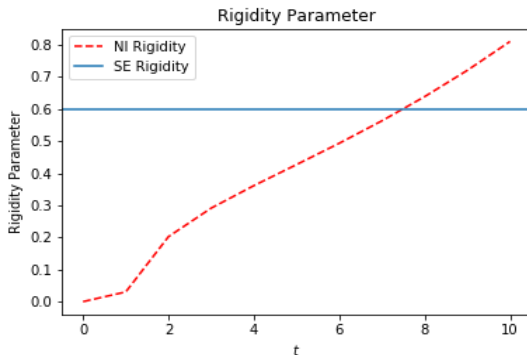




# A detailed look into noisy information



# Implied rigidity of different models



# Other theories on to-do-list

- **Rational Inattention:** attentiveness endogenously respond to variances
- **Learning:** the structural parameter  $\rho$  is not known, thus the agent learns about it as if an econometrician does

# Identification strategies 1: testing rigidity models

- [Coibion and Gorodnichenko, 2012]
  - **FEs** respond to shocks and serially correlated.
- **Additional in this paper**
  - **Uncertainty** does not depend on shocks; and serially correlated.

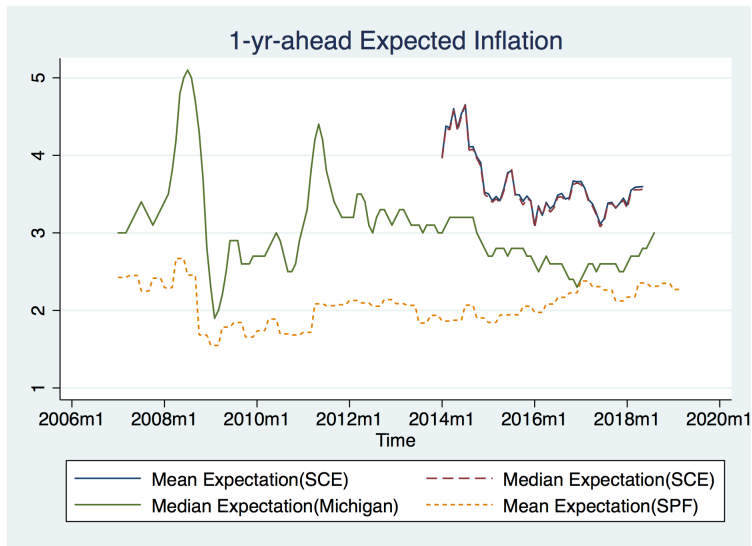
# Identification strategies 2: differentiating theories

- [Coibion and Gorodnichenko, 2012]
  - **FEs** do not depend on past realizations according to baseline SE and NI; but do so according to heterogeneous priors or precision models.
  - Implied rigidity does not differ across shocks according to SE but differs according to NI.
  - **Disagreements** rise after shocks according to baseline SE, strategic interactions and heterogeneous priors but invariant according to baseline NI.
- **Additional in this paper**
  - **Uncertainty** do not depend on shocks per se according to baseline SE and NI, instead on degree of information rigidity.

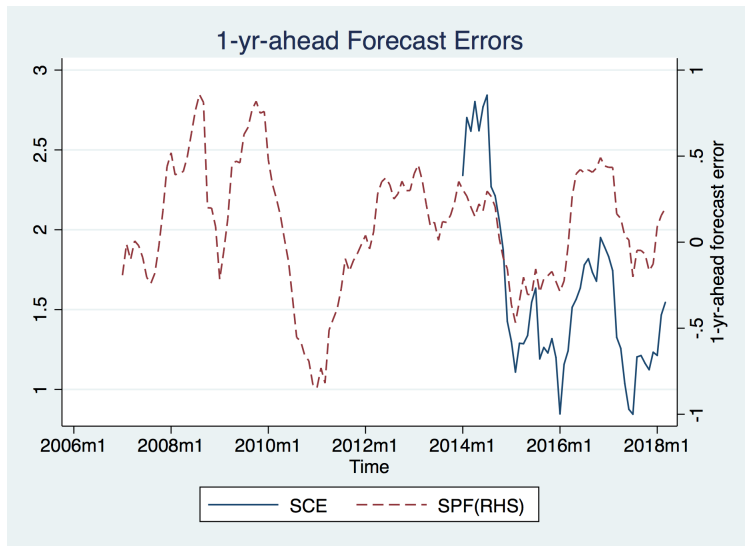
# Data

	SCE	SPF
Time period	2013-present	2007-present
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Aggregate Var in Density	1-yr and 3-yr inflation	1-yr and 3-yr CPI and PCE
Pannel Structure	stay up to 12 months	average stay for 5 years
Demographic Info	Education, Income, Age	Industry

# Population moments: average

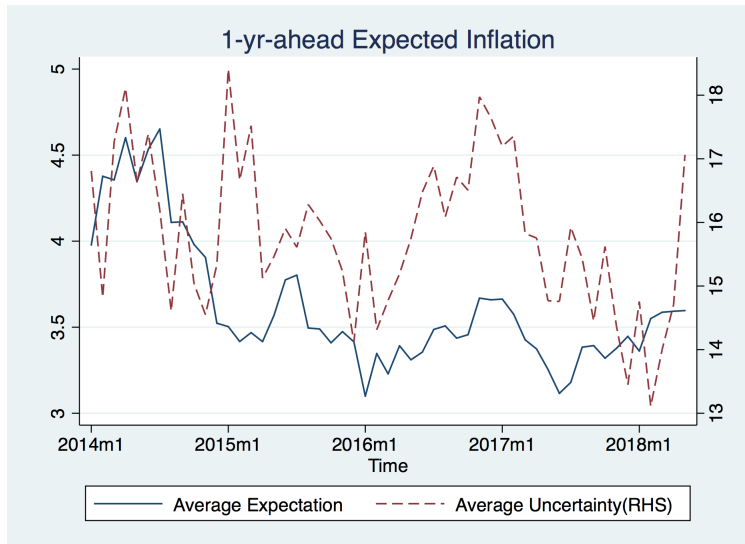


# Population moments: average forecast errors

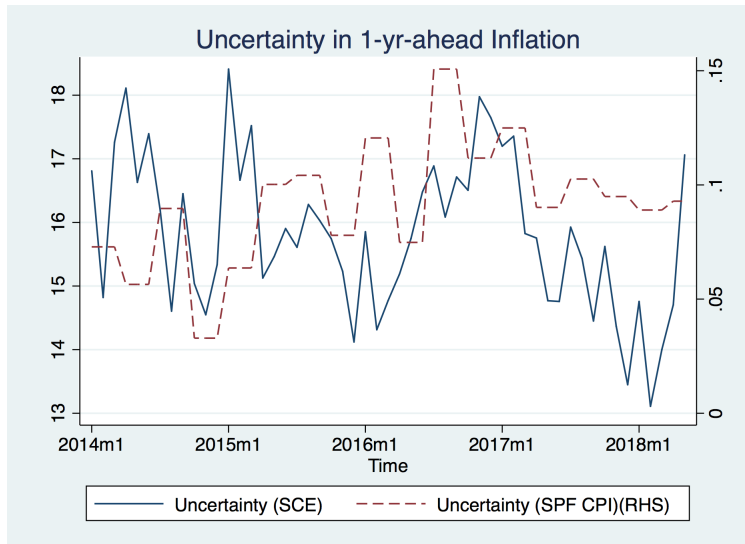




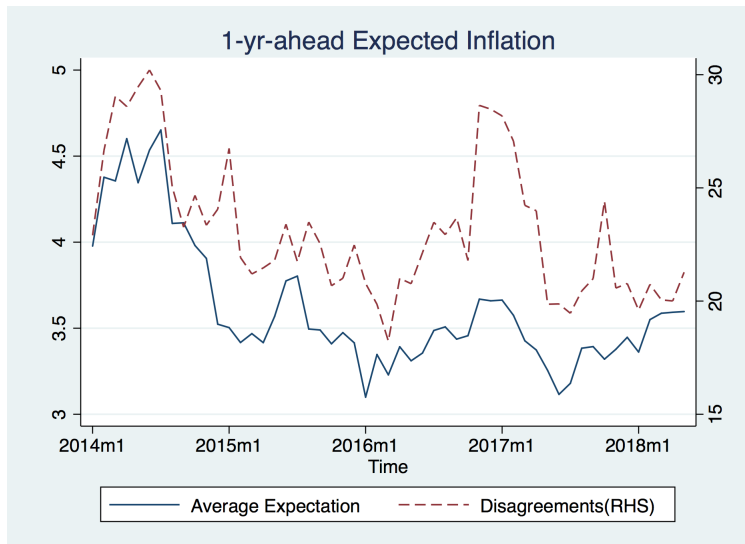
# Population moments: average uncertainty



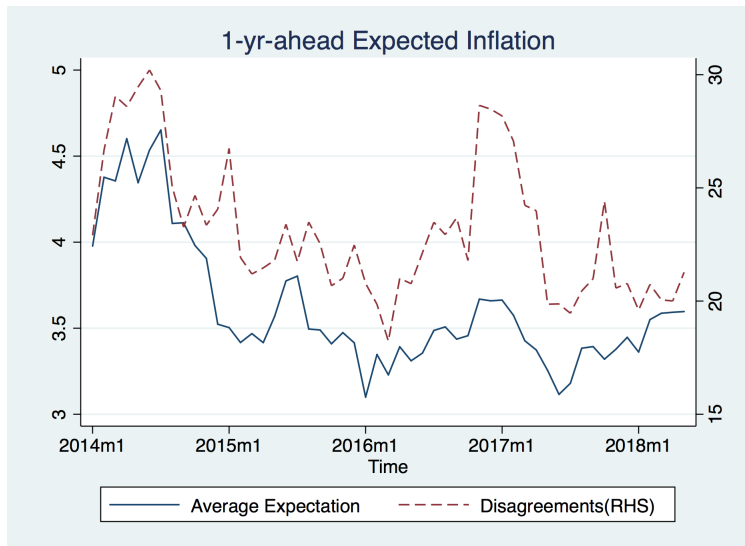
# Population moments: average uncertainty



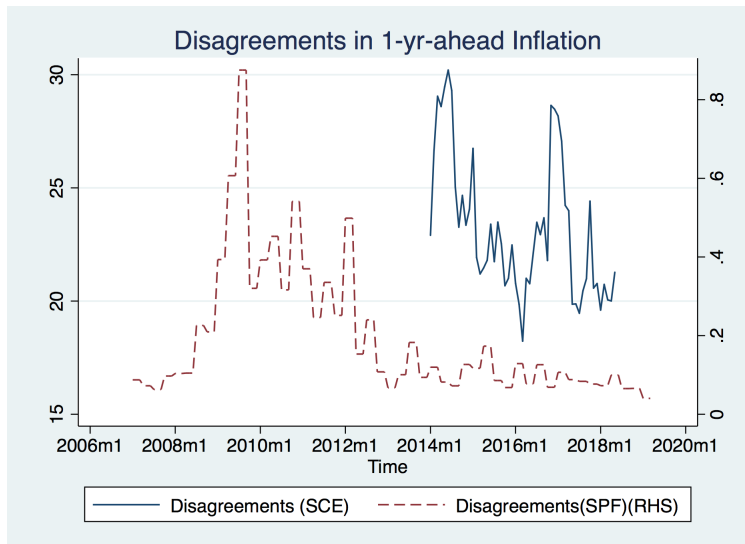
# Population moments: disagreements



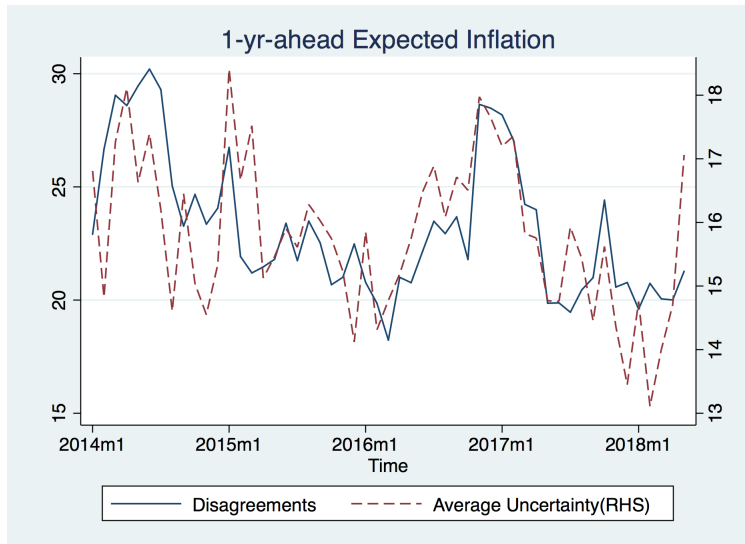
# Population moments: disagreements



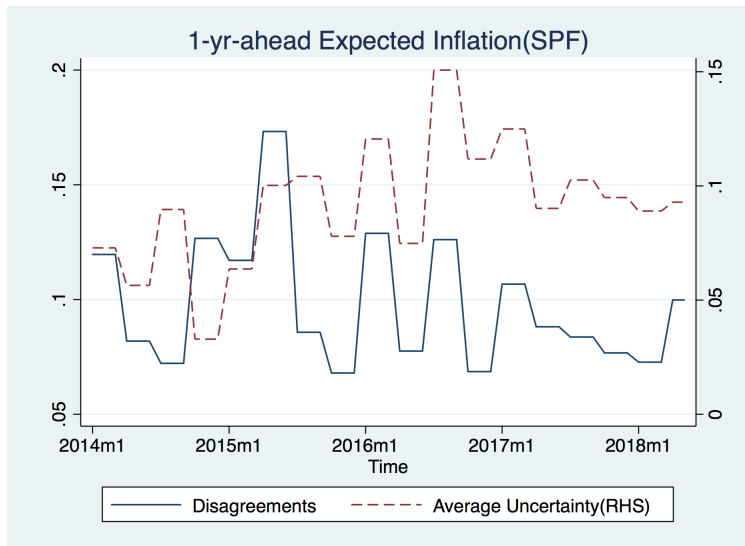
# Population moments: disagreements



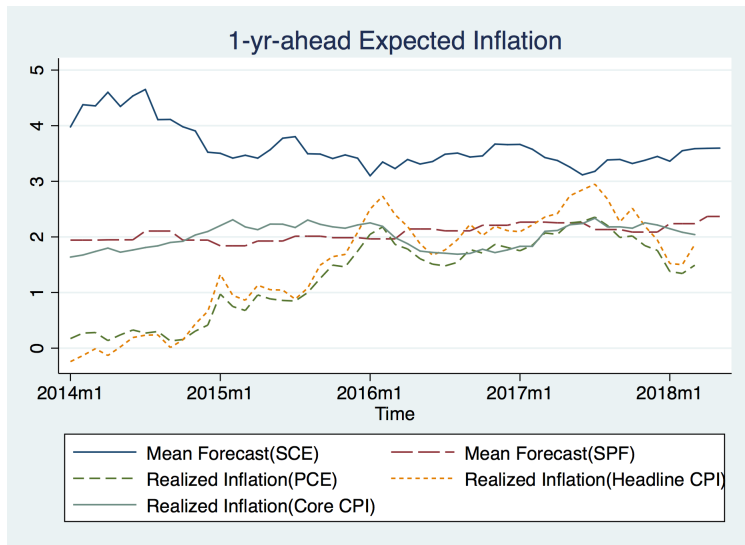
# Population moments: uncertainty and disagreements



# Population moments: uncertainty and disagreements



# Population moments: forecast and realization





# Empirical execution

- **Density Estimation:** generalized beta estimation, [Engelberg et al., 2009]
- **Identification of Shocks:** following [Coibion and Gorodnichenko, 2012] and monetary policy shocks.

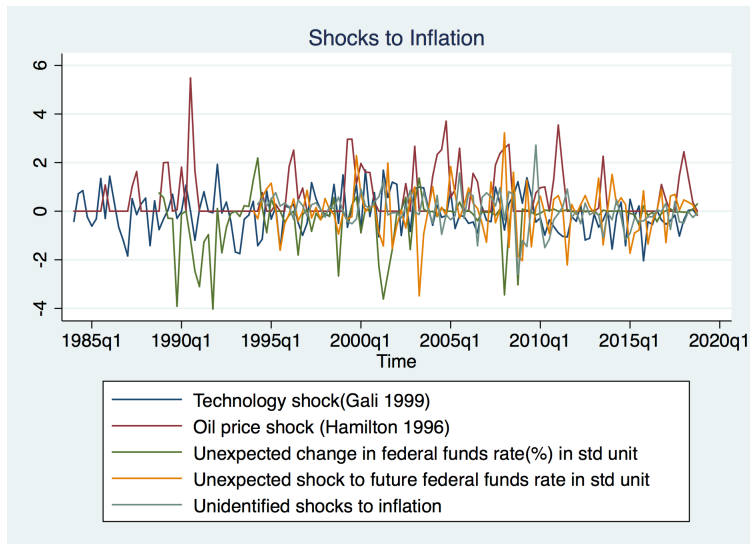
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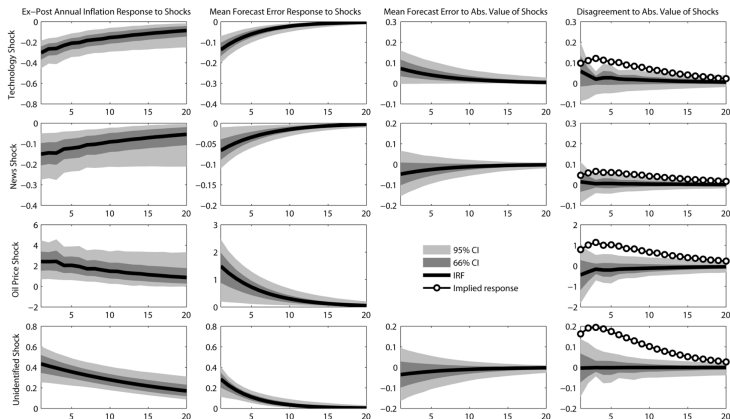
# Test of Rational Expectation

	SPFCPI_FE1vl	SPFPCE_FE1vl	SPFCPI_Vardiff	SPFPCE_Vardiff	SPFCPI_Disgdiff	SPFPCE_Disgdiff
L1.InfExp_FE	0.941*** (0.0840)	1.195*** (0.142)				
L2.InfExp_FE	-0.313** (0.116)	-0.625** (0.198)				
L3.InfExp_FE	0.114 (0.114)	0.0782 (0.196)				
L4.InfExp_FE	-0.137 (0.0820)	-0.0162 (0.131)				
L1.InfExp_Var_ch			-0.855*** (0.189)	-0.565** (0.162)		
L2.InfExp_Var_ch			-0.780** (0.220)	-0.452* (0.186)		
L3.InfExp_Var_ch			-0.556* (0.219)	-0.429* (0.189)		
L4.InfExp_Var_ch			-0.167 (0.188)	0.00385 (0.172)		
L1.InfExp_Disg_ch					-0.571*** (0.0699)	-0.640*** (0.127)
L2.InfExp_Disg_ch					-0.376*** (0.0764)	-0.0944 (0.141)
L3.InfExp_Disg_ch					-0.0455 (0.0661)	0.180 (0.138)
L4.InfExp_Disg_ch					-0.110* (0.0479)	-0.0364 (0.123)
N	143	41	44	44	146	44
R-sq	0.593	0.750	0.384	0.322	0.356	0.496

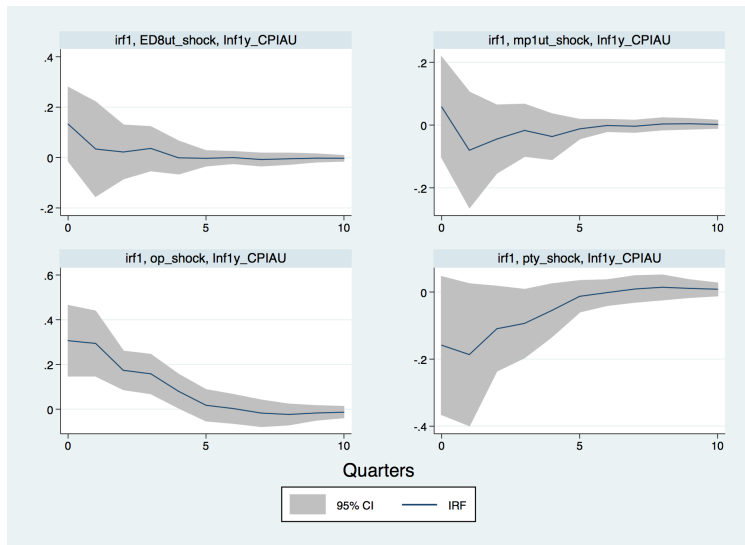
# Identified Inflation shocks



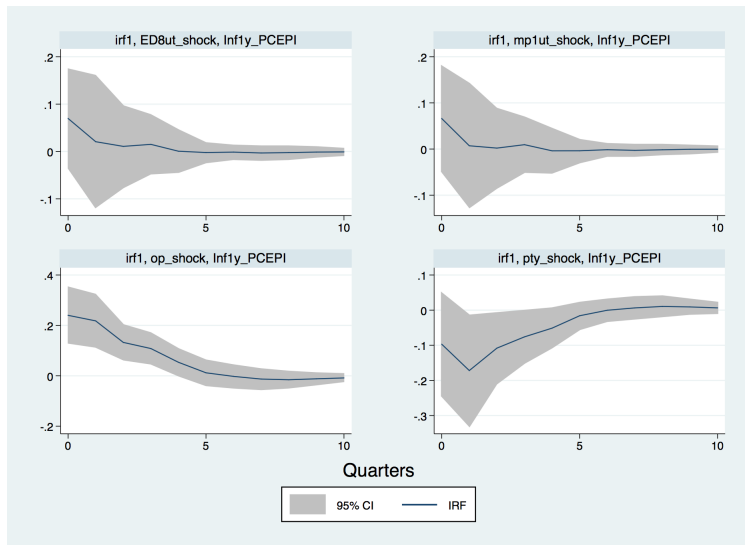
# Results from [Coibion and Gorodnichenko, 2012]



# Inflation IR to shocks



# Inflation IR to shocks

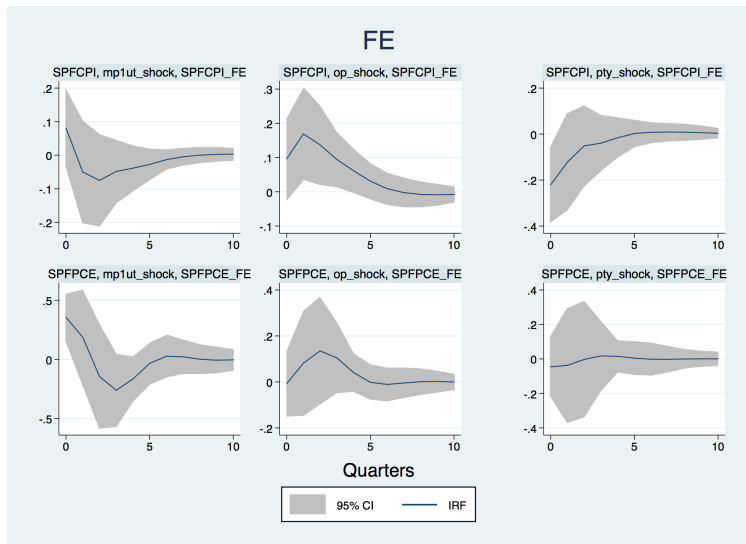


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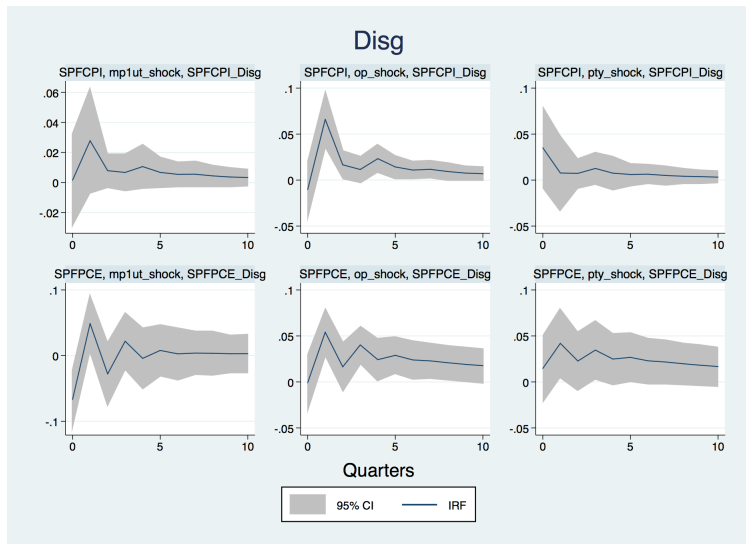
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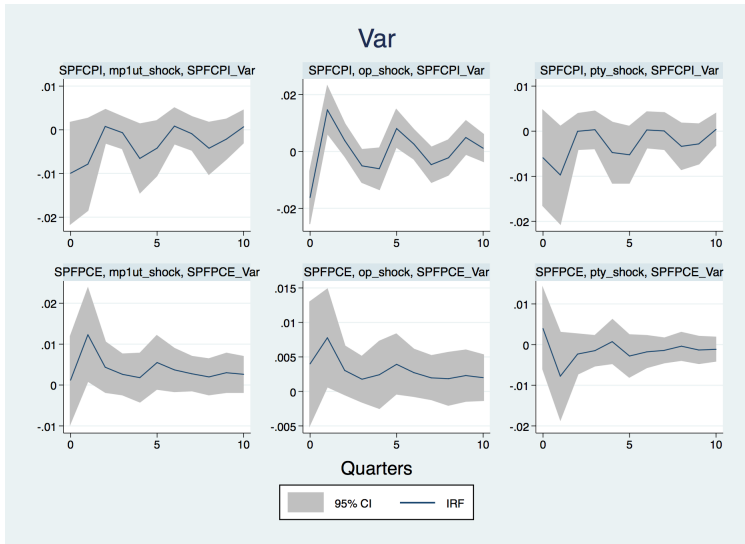
# Forecasting errors IR to shocks



# Disagreements IR to shocks



## Uncertainty IR to shocks





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*The Journal of Finance*, 73(1):199–227.



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Sims, C. A. (2003).

Implications of rational inattention.

*Journal of monetary Economics*, 50(3):665–690.



Woodford, M. (2001).

Imperfect common knowledge and the effects of monetary policy.

Technical report, National Bureau of Economic Research.

# Sticky Expectation: individual

For a non-updater since  $t - \tau$  ( $\tau = 0$  for updater),

- **Mean**

$$E_{i,t}(y_{t+h}|y_{t-\tau}) = \rho^{h+\tau} y_{t-\tau}$$

- **Forecast Error**

$$FE_{i,t+h|t} = \underbrace{\sum_{s=0}^{h+\tau} \rho^s \omega_{t+h-s}}_{\text{weighted sum of future realized shocks}}$$

- **Variance**

$$Var_{i,t}(y_{t+h}|y_{t-\tau}) = \sum_{s=0}^{h+\tau} \rho^{2s} \sigma_{\omega}^2$$



# Sticky Expectation: individual

$$\text{updater: } \Delta Var_{i,t}(y_{t+h}|y_t) = \sum_{s=0}^{\tau} \rho^{2s} \sigma_{\omega}^2$$

$$\text{non-updater: } \Delta Var_{i,t|t-\tau-1}(y_{t+h}|y_{t-\tau-1}) = \sigma_{\omega}^2$$

- ① Change in expectation(and variance) depends on if update or not
- ② Cannot observe systematically sluggish response to shocks at individual level

# Sticky Expectation: population

- Average forecast**

$$\begin{aligned}
 \bar{E}_t(y_{t+h}) &= \lambda \underbrace{E_t(y_{t+h})}_{\text{rational expectation at } t} + (1 - \lambda) \underbrace{\bar{E}_{t-1}(y_{t+h})}_{\text{average expectation at } t-1} \\
 &= \lambda E_t(y_{t+h}) + (1 - \lambda)(\lambda E_{t-1}(y_{t+h}) + \dots) \\
 &= \underbrace{\lambda \sum_{s=0}^{\infty} (1 - \lambda)^s E_{t-s}(y_{t+h})}_{\text{weighted sum of past rational expectations}}
 \end{aligned}$$

- Change in average forecast**

$$\Delta \bar{E}_t(y_{t+h}) = \underbrace{(1 - \lambda)}_{\text{stickiness}} \Delta \bar{E}_{t-1}(y_{t+h}) + \lambda \rho^h \omega_t$$

# Sticky Expectation: population

- Disagreements

$$\text{Var}_t(y_{t+h}) = \lambda \sum_{\tau=0}^{\infty} (1 - \lambda)^{\tau} (E_{t|t-\tau}(y_{t+h}) - \bar{E}_t(y_{t+h}))^2$$

- Change in disagreements

$$\Delta \text{Var}_t(y_{t+h}) = \rho^{2h}(1 - \lambda)\lambda \underbrace{\omega_t^2}_{\text{shock at time } t}$$

- Disagreements rise after the shock and then gradually decline
- Response of disagreements depends on the size of the shock

# Sticky Expectation: population

- Average variance**

$$\overline{Var}_t(y_{t+h}) = (1 - \lambda) \underbrace{\overline{Var}_{t-1}(y_{t+h})}_{\text{average variance at t-1}} + \underbrace{\lambda Var_t(y_{t+h})}_{\text{variance of updater at t}}$$

- Change in average variance**

$$\Delta \overline{Var}_t(y_{t+h}) = \underbrace{(1 - \lambda) \Delta \overline{Var}_{t-1}(y_{t+h}) - \lambda \rho^{2h} \sigma_\omega^2}_{\text{does not depend on shock at t}}$$

- 1 Average variance does not respond to shocks
- 2 Average variance has serial correlation with the same rigidity parameter  $1 - \lambda$

# Noisy Information: individuals

## • Mean

$$E_{i,t}(y_{t+h}) = \rho^h E_{i,t|t}(y_t)$$

$$E_{i,t|t}(y_t) = \underbrace{E_{i,t|t-1}(y_t)}_{\text{prior}} + P \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}}$$

$$= (1 - PH)E_{i,t|t-1}(y_t) + Ps_{i,t}$$

$$\text{where } P = [P_\epsilon, P_\xi] = \Sigma_{i,t|t-1}^y H (H' \Sigma_{i,t|t-1}^y H + \Sigma^\nu)^{-1}$$

where  $\Sigma_{i,t|t-1}^y$  is the variance of  $y_t$  based on prior belief

$$\text{and } \Sigma^\nu = \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\xi^2 \end{bmatrix}$$

# Noisy Information: individuals

- **Change in mean**

$$\Delta E_{i,t|t}(y_{t+h}) = \underbrace{\rho^h(1 - PH)\Delta E_{i,t-1|t-1}(y_t)}_{\text{Lagged response}} + \underbrace{\rho^h PH \Delta y_{i,t} + \rho^h P \Delta v_{i,t}}_{\text{Shocks to signals}}$$

- ① Rigidity parameter  $1 - PH$
- ② Serial correlation at individual level
- ③ Always respond to shocks

# Noisy Information: individuals

## • Variance

$$\Sigma_{i,t|t}^y = \Sigma_{i,t|t-1}^y - \Sigma_{i,t|t-1}^y H' (H \Sigma_{i,t-1}^y H' + \Sigma^v)^{-1} H \Sigma_{i,t|t-1}^y$$

## • Change in variance

$$\Delta \Sigma_{i,t|t}^y < 0$$

- ① It does not depend on the realizations of the signal.
- ② It decreases unambiguously from  $t - 1$  to  $t$ .
- ③ The two properties carry through to h-period ahead forecast

# Noisy Information: population

## • Mean

$$\begin{aligned}
 \bar{E}_{t|t}(y_{t+h}) &= \rho^h \left[ (1 - PH) \underbrace{\bar{E}_{t-1}(y_{t+h})}_{\text{Average prior}} + P \underbrace{\bar{s}_t}_{\text{Average Signals}} \right] \\
 &= (1 - PH) \bar{E}_{t-1}(y_{t+h}) + P[\epsilon_t, 0]' \\
 &= (1 - PH) \bar{E}_{t-1}(y_{t+h}) + P\epsilon_t
 \end{aligned}$$

- 1 Same properties to the individual forecast



# Noisy Information: population

## • Disagreements

$$\begin{aligned} \text{Var}_t(y_{t+h}) &= E((E_{i,t|t}(y_{t+h}) - \bar{E}_t(y_{t+h}))^2) \\ &= \rho^{2h} P_\xi^2 \sigma_\xi^2 \end{aligned}$$

- ① increase with the forecast horizon
- ② depends on noisiness private signals, but not on that of public signals and the variance of the true variable  $y$
- ③ increase with the rigidity parameter  $P$  in this model

# Noisy Information: population

- **Change in disagreements**

$$\Delta \text{Var}_t(y_{t+h}) = \rho^{2h}(1 - \rho^2)P_\xi^2\sigma_\xi^2 > 0$$

- 1 disagreements increase as time goes from  $t - 1$  to  $t$ .
- 2 disagreements increase as approaching the variable of forecast

# Noisy Information: population

- **Average variance**

$$\bar{Var}_t(y_{t+h}) = \bar{\Sigma}_t^y$$

- **Change in average variance**

$$\Delta Var_t(y_{t+h}) < 0$$

- 1 average variance is the same as individual variance, not depend on signals
- 2 the variance unambiguously drop over time