Understanding Expectation Formation from Probabilistic Surveys

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Outline

What I am doing

- Use density information to
  - test expectation rigidity models
  - ... and identify differences in various theories
- Both individual and population moments
- Households and professional forecasters
  - drivers of difference in rigidity across two types of agents

Why density is important

- **Identification:** different theories have testable predictions on the second moments
  - Scenario 1. Two people think the chance of raining is 50%.
  - Scenario 2. One person thinks 100% and the other 0%.
- **Modeling Implications:** both mean and variance affect economic decisions
  - precautionary saving with income risks
  - portfolio choice with risky asset

Literature

- **Theory**
  - Sticky expectation [Carroll, 2003], [Reis, 2006]
  - Rational inattention [Sims, 2003], [Gabaix, 2014]
  - Noisy information [Lucas Jr, 1972], [Woodford, 2001]
  - Learning [Evans and Honkapohja, 2012]
  - Strategic interaction [Morris and Shin, 2002], [Hellwig and Veldkamp, 2009]
  - Diagostic expectation [Bordalo et al., 2018]
  - Model uncertainty [Hansen and Sargent, 2001], [Hansen and Sargent, 2008]
- **Empirics**
  - **Heterogeneity in Expectation:** [Mankiw et al., 2003]
  - **Testing Theories:** [Coibion and Gorodnichenko, 2012], [Fuhrer, 2018]

Unified Framework

h-period ahead density forecast by agent i at time t based on information set $I_{i,t}$

$$\widehat{f}_{i,t}(y_{t+h}|I_{i,t})$$

- Theories differ in $I_{i,t}$
- May also differ on information processing, i.e. $I_{i,t} \rightarrow \widehat{f}_{i,t}$

Definition and notation

- Individual
  - mean forecast $E_{i,t}(y_{t+h})$
  - forecast error $FE_{i,t+h|t} = y_{t+h} - E_{i,t}(y_{t+h})$
  - uncertainty $Var_{i,t}(y_{t+h})$
- Population
  - average forecast $\bar{E}_t(y_{t+h})$
  - average forecast error $\overline{FE}_t = y_{t+h} - \bar{E}_t(y_{t+h})$
  - cross-section disagreements $Var_t(E_{i,t}(y_{t+h}))$
  - average uncertainty $\overline{Var}_t(y_{t+h})$

Assumption about true process

$$y_{t+1} = \rho y_t + \omega_t$$
$$\omega_t \sim N(0, \sigma_\omega^2)$$

- $0 < \rho \leq 1$
- if $\rho = 0$, no way to forecast at all
- ω_t is i.i.d

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Sticky Expectation: assumptions

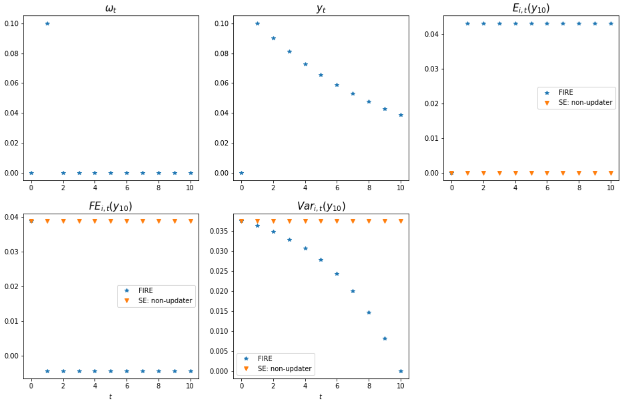
- At time t, agent i learns about y_t at a fixed Poisson rate λ
- A non-updater since $t-\tau$

$$E_{i,t}(y_{t+h}|y_{t-\tau}) = \rho^{h+\tau} y_{t-\tau}$$

- An updater is a special cae $\tau = 0$

Impulse responses to shocks: individual moments

$$\text{True Process} \quad \rho = 0.9, \quad \sigma_\omega = 0.1, \quad \omega_1 = 0.1$$
$$\text{SE} \qquad \lambda = 0.4$$



Impulse responses to shocks: population moments

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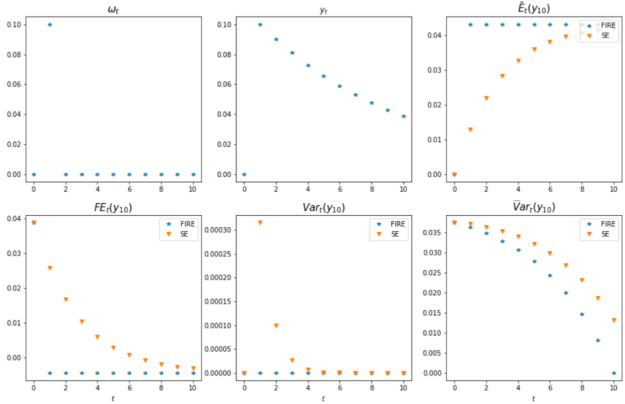

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Noisy Information: assumptions

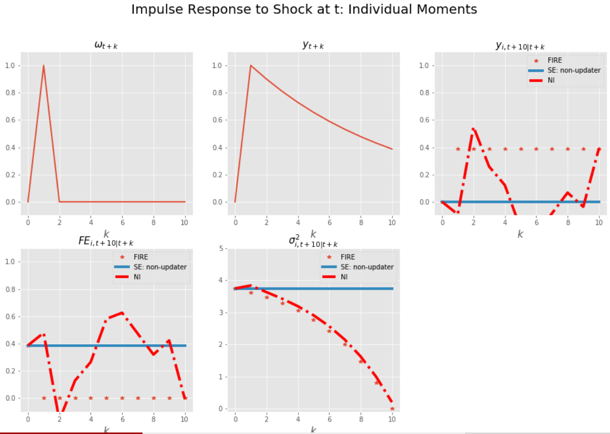
- Individual only observes noisy signals

$$
\begin{aligned}
&s_{i,t} = [s_t^{pb}, s_{i,t}^{pr}]' \in I_{i,t} \\
&\text{public signal:} \quad s_t^{pb} = y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \\
&\text{private signal:} \quad s_{i,t}^{pr} = y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0, \sigma_\xi^2)
\end{aligned}
$$

- Kalman filtering (simply normal updating if ρ=0)

Impulse responses to shocks: individual moments

$$\text{True Process} \quad \rho = 0.9, \quad \sigma_\omega = 0.1, \quad \omega_1 = 0.1$$
$$\text{SE:} \quad \lambda = 0.5; \quad \text{NI:} \quad \sigma_\xi = 0.1, \quad \sigma_\epsilon = 0.1$$



Impulse responses to shocks: population moments

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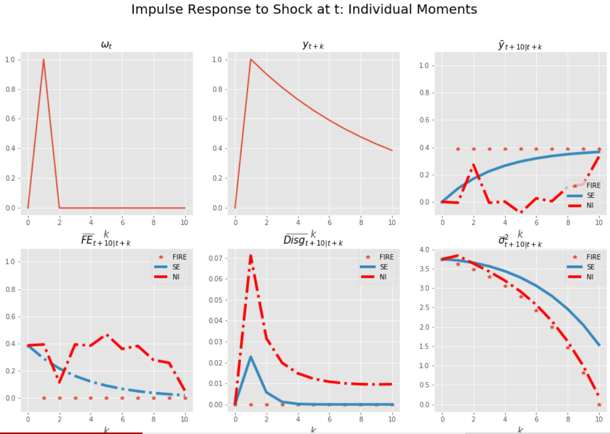
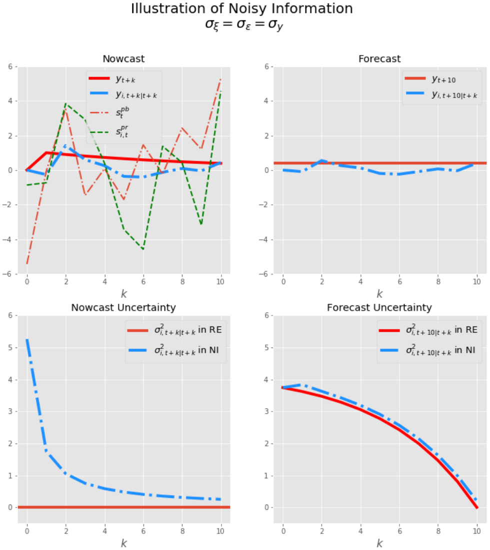
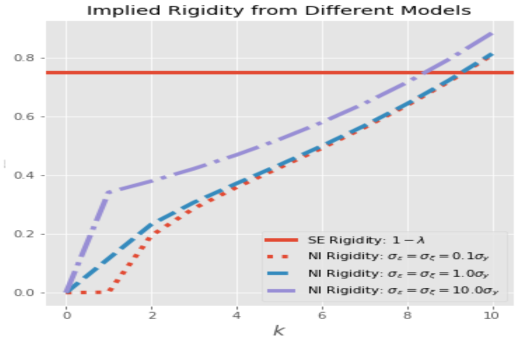


A detailed look into noisy information



Implied rigidity of different models



Other theories on to-do-list

- **Rational Inattention:** attentiveness endogenously respond to variances
- **Learning**: the structural parameter ρ is not known, thus the agent learns about it as if an econometrician does

Identification strategies 1: testing rigidity models

- [Coibion and Gorodnichenko, 2012]
    - FEs respond to shocks and serially correlated.
- **Additional in this paper**
    - Uncertainty does not depend on shocks; and serially correlated.

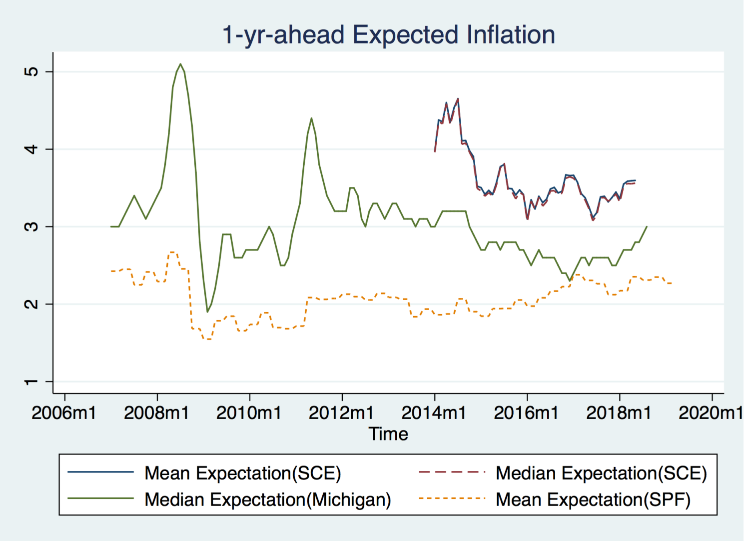
Identification strategies 2: differentiating theories

- [Coibion and Gorodnichenko, 2012]
  - FEs do not depend on past realizations according to baseline SE and NI; but do so according to heterogeneous priors or precision models.
  - Implied rigidity does not differ across shocks according to SE but differs according to NI.
  - Disagreements rise after shocks according to baseline SE, strategic interactions and heterogeneous priors but invariant according to baseline NI.
- **Additional in this paper**
  - Uncertainty do not depend on shocks per se according to baseline SE and NI, instead on degree of information rigidity.

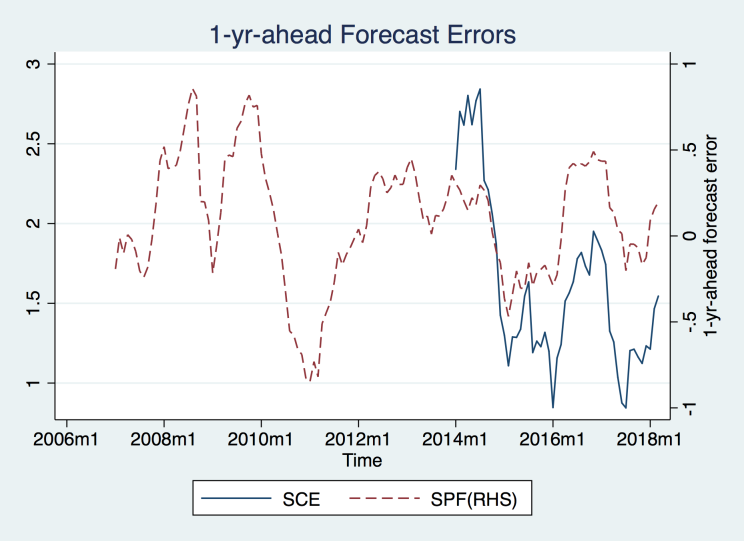
Data

	SCE	SPF
Time period	2013-present	2007-present
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Aggregate Var in Density	1-yr and 3-yr inflation	1-yr and 3-yr CPI and PCE
Pannel Structure	stay up to 12 months	average stay for 5 years
Demographic Info	Education, Income, Age	Industry

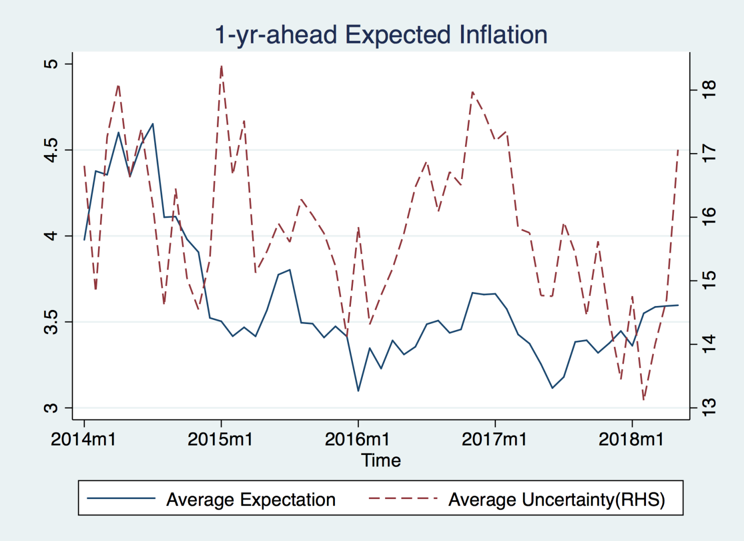
Population moments: average



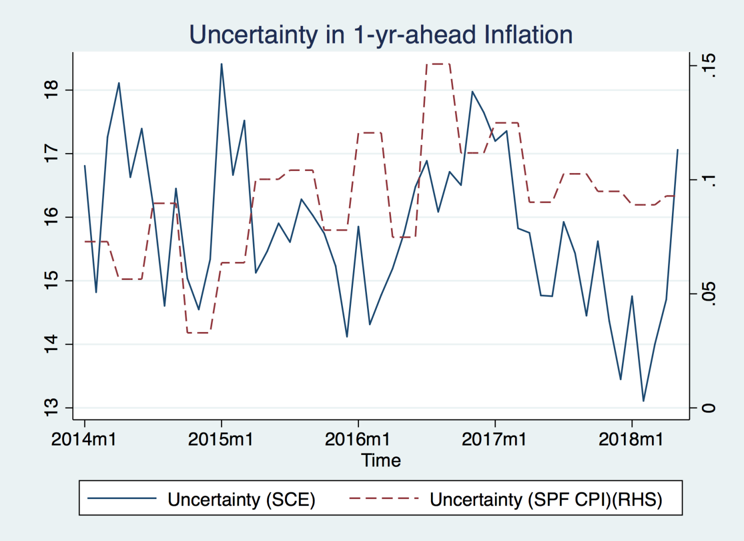
Population moments: average forecast errors



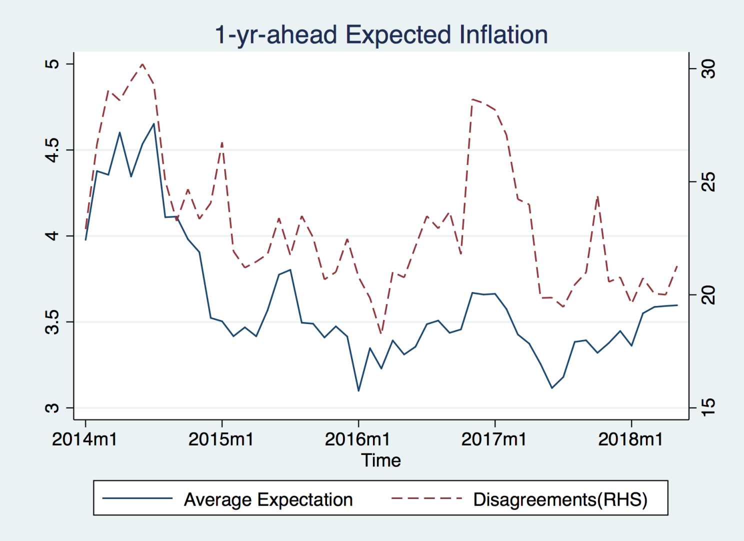
Population moments: average uncertainty



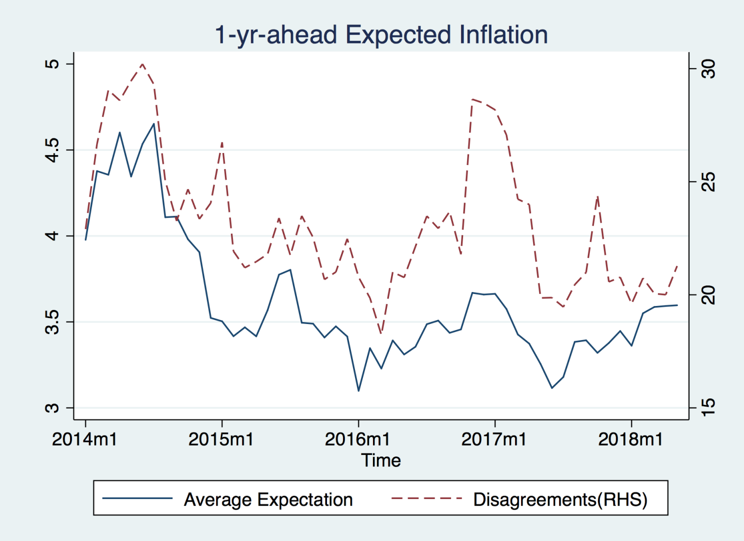
Population moments: average uncertainty



Population moments: disagreements



Population moments: disagreements



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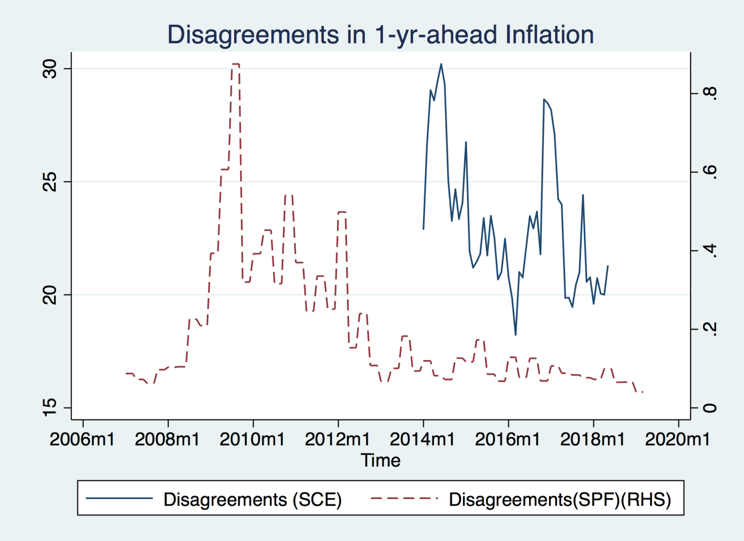
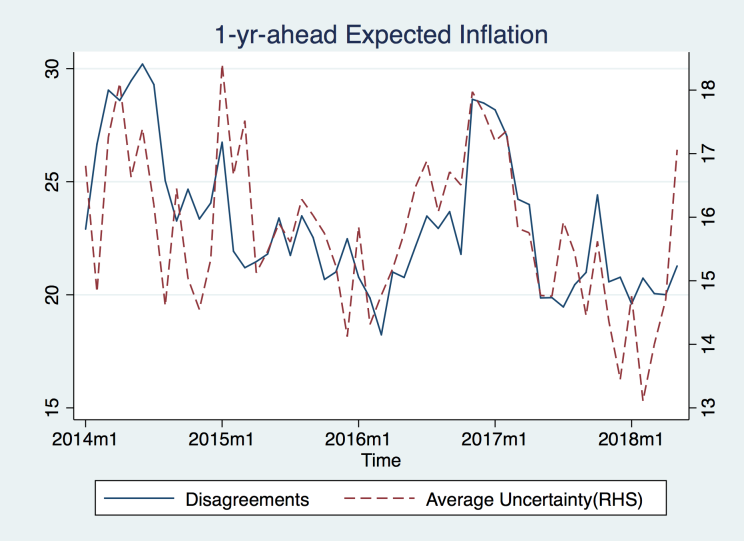


Population moments: uncertainty and disagreements



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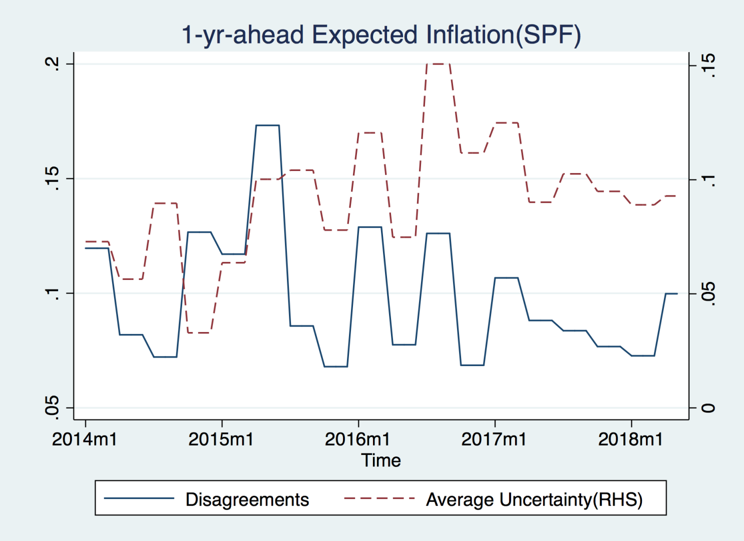
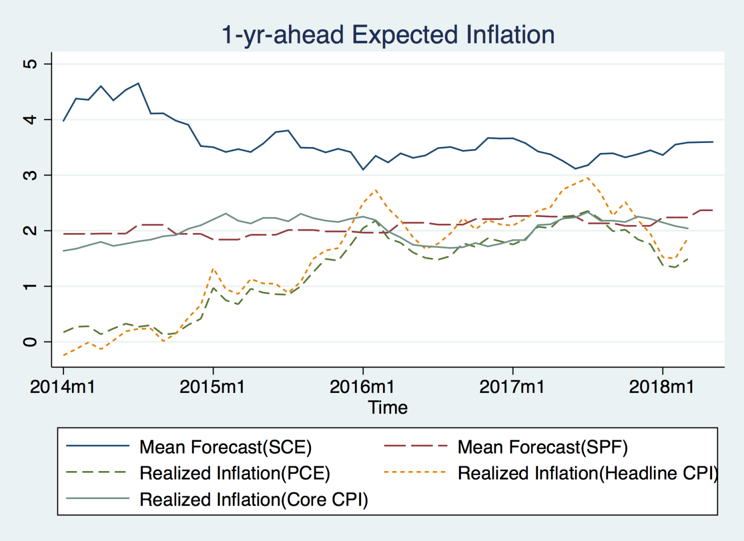


Population moments: forecast and realization



Empirical execution

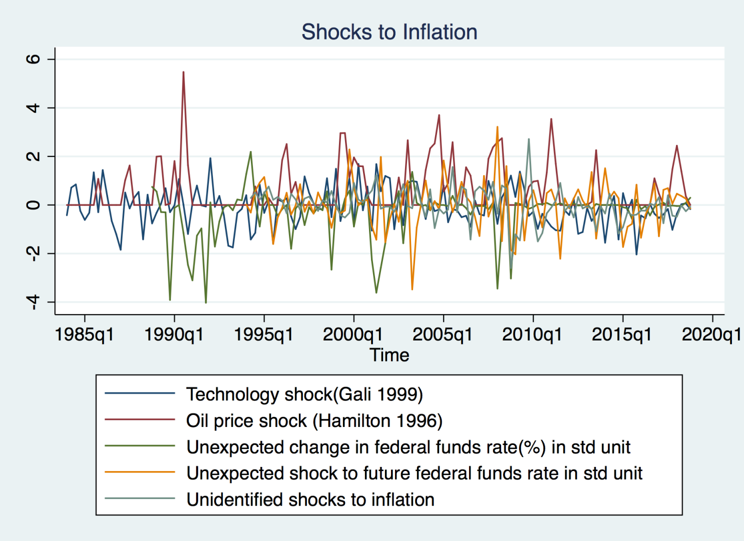
- **Density Estimation**: generalized beta estimation, [Engelberg et al., 2009]
- **Identification of Shocks**: following [Coibion and Gorodnichenko, 2012] and monetary policy shocks.

Outline

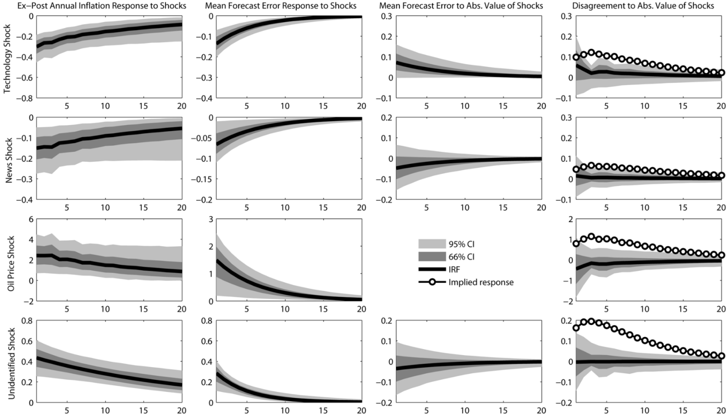
Test of Rational Expectation

	SPFCPI_FElvl	SPFPCE_FElvl	SPFCPI_Vardiff	SPFPCE_Vardiff	SPFCPI_Disgdiff	SPFPCE_Disgdiff
L.InfExp_FE	0.941***	1.195***				
	(0.0840)	(0.142)				
L2.InfExp_FE	-0.313**	-0.625**				
	(0.116)	(0.198)				
L3.InfExp_FE	0.114	0.0782				
	(0.114)	(0.196)				
L4.InfExp_FE	-0.137	-0.0162				
	(0.0820)	(0.131)				
L.InfExp_Var_ch			-0.855***	-0.565**		
			(0.189)	(0.162)		
L2.InfExp_Var_ch			-0.780**	-0.452*		
			(0.220)	(0.186)		
L3.InfExp_Var_ch			-0.556*	-0.429*		
			(0.219)	(0.189)		
L4.InfExp_Var_ch			-0.167	0.00385		
			(0.188)	(0.172)		
L.InfExp_Disg_ch					-0.571***	-0.640***
					(0.0699)	(0.127)
L2.InfExp_Disg_ch					-0.376***	-0.0944
					(0.0764)	(0.141)
L3.InfExp_Disg_ch					-0.0455	0.180
					(0.0661)	(0.138)
L4.InfExp_Disg_ch					-0.110*	-0.0364
					(0.0479)	(0.123)
N	143	41	44	44	146	44
R-sq	0.593	0.750	0.384	0.322	0.356	0.496

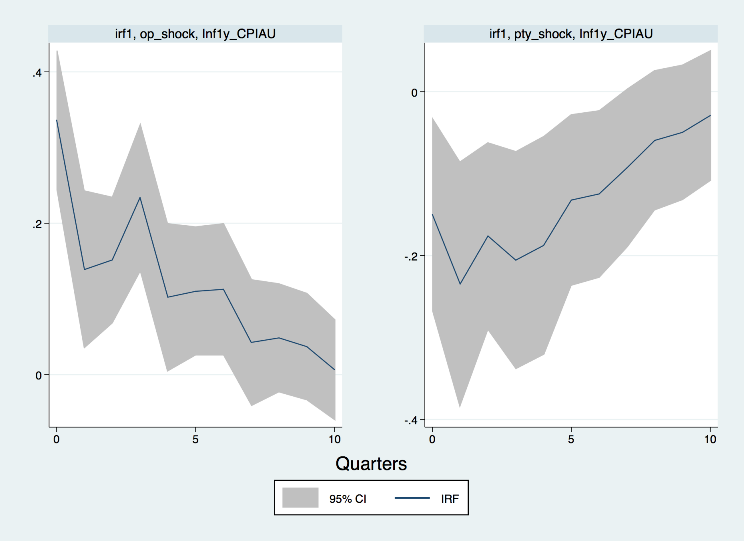
Inflation shocks



Results from [Coibion and Gorodnichenko, 2012]



Inflation IR to shocks exluding monetary policy



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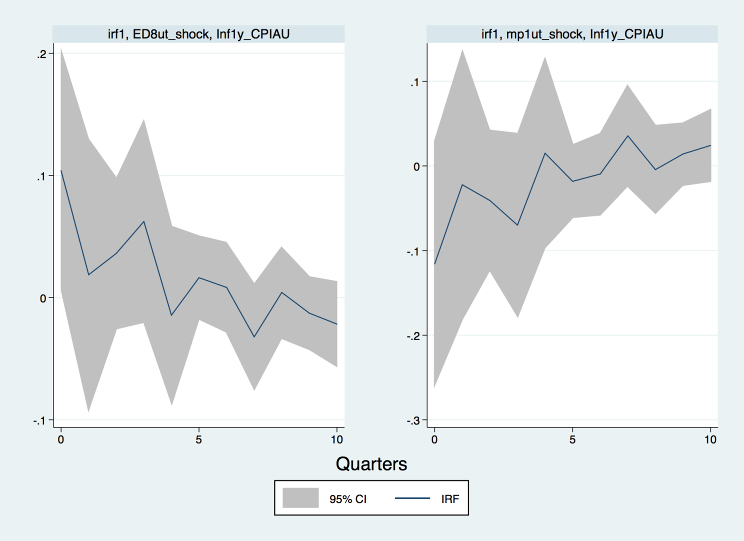

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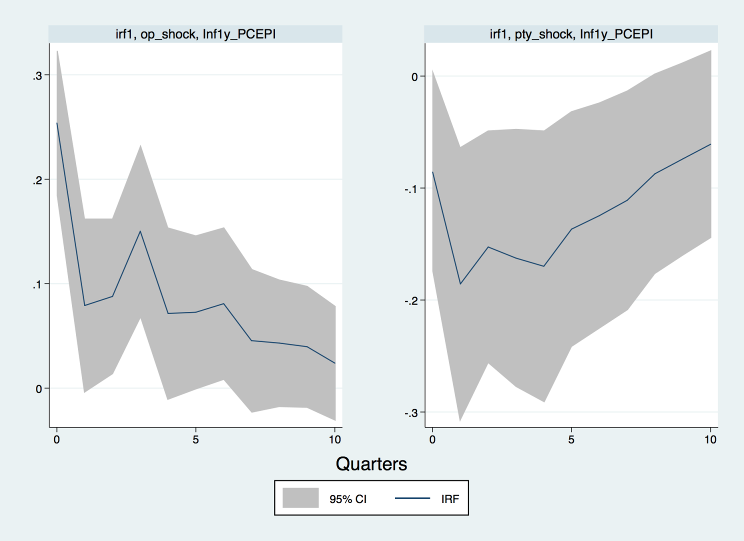

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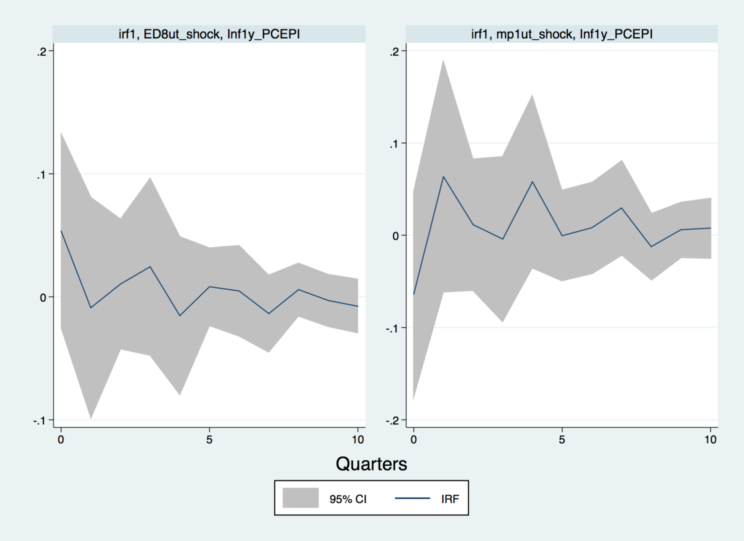
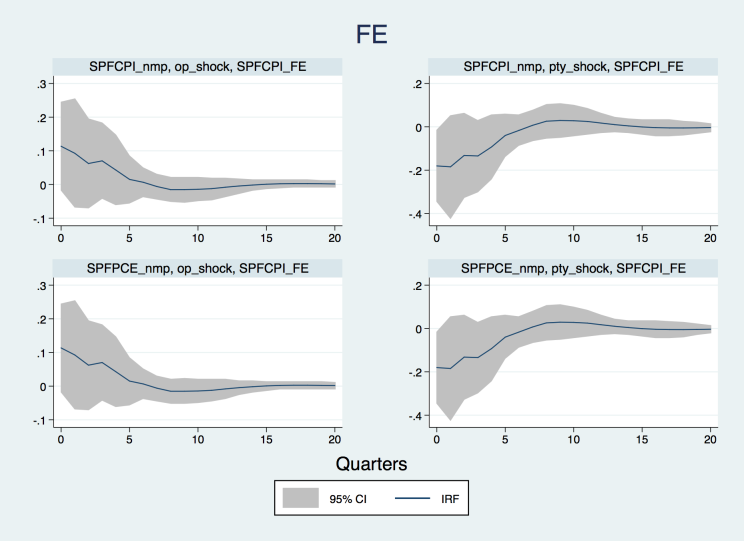
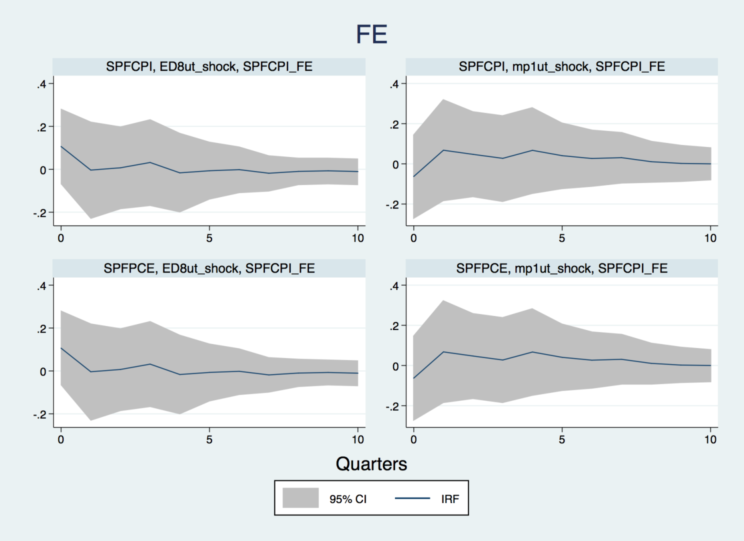


Outline

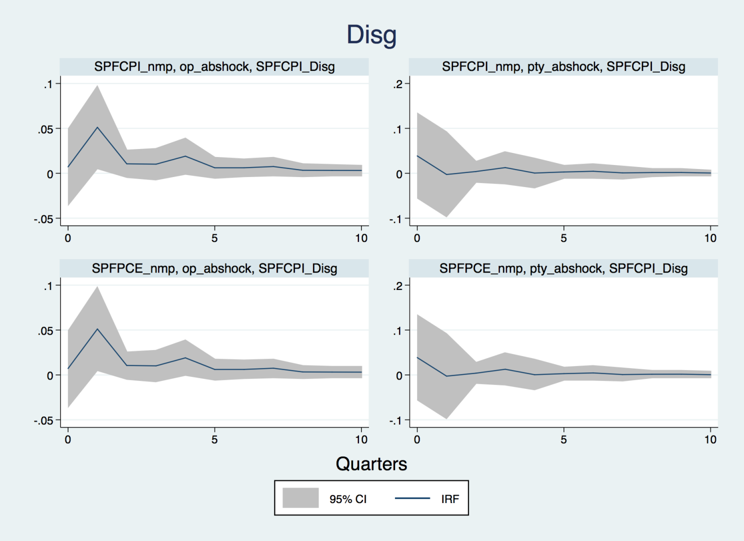
Forecasting errors IR to shocks excluding monetary policy



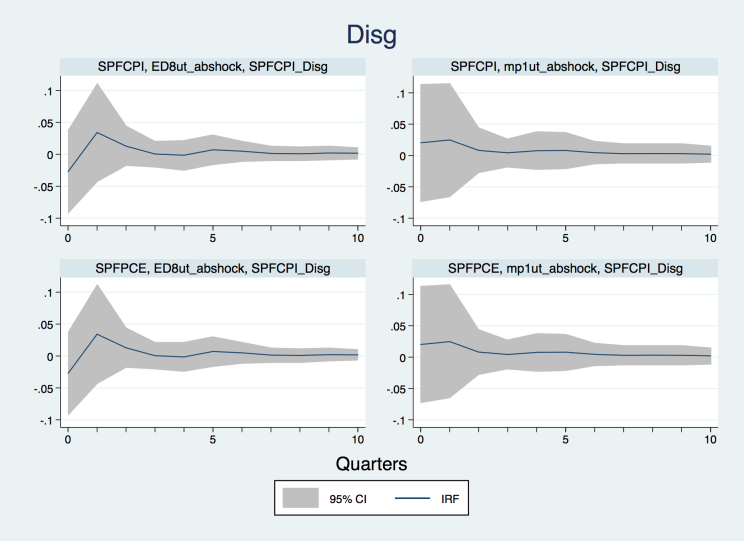
Forecasting errors IR to monetary policy shocks



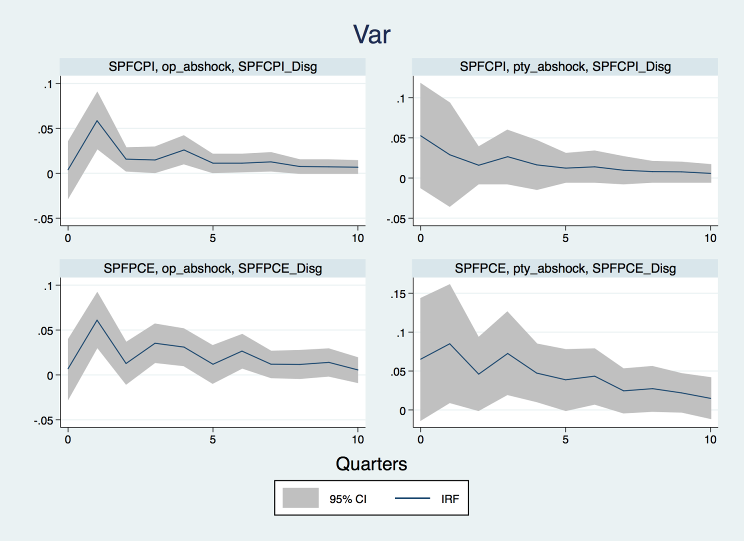
Disagreements IR to shocks excluding monetary policy



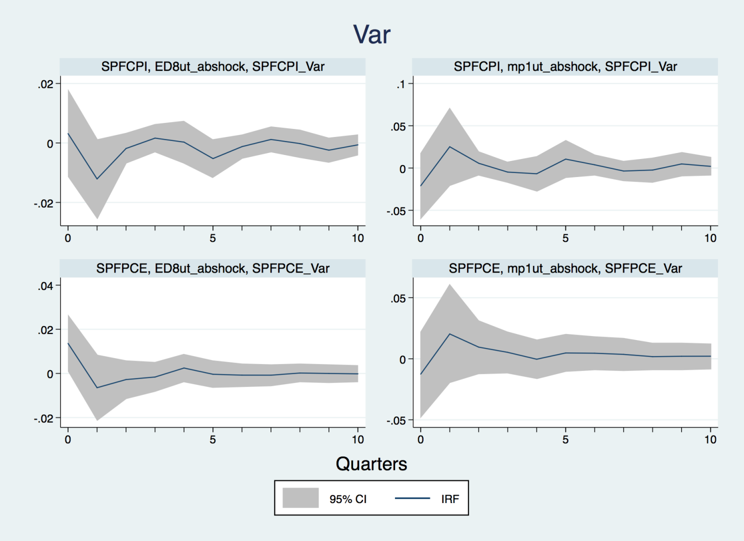
Disagreements IR to monetary policy shocks



Uncertainty IR to shocks excluding monetary policy



Uncertainty IR to monetary policy shocks



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Sticky Expectation: individual

For a non-updater since $t-\tau$ ($\tau = 0$ for updater),

- **Mean**

$$E_{i,t}(y_{t+h}|y_{t-\tau}) = \rho^{h+\tau} y_{t-\tau}$$

- **Forecast Error**

$$FE_{i,t+h|t} = \underbrace{\sum_{s=0}^{h+\tau} \rho^s \omega_{t+h-s}}_{\text{weighted sum of future realized shocks}}$$

- **Variance**

$$Var_{i,t}(y_{t+h}|y_{t-\tau}) = \sum_{s=0}^{h+\tau} \rho^{2s} \sigma_\omega^2$$

Sticky Expectation: individual

$$\text{updater:} \quad \Delta Var_{i,t}(y_{t+h}|y_t) = \sum_{s=0}^{\tau} \rho^{2s}\sigma_{\omega}^2$$

$$\text{non-updater:} \quad \Delta Var_{i,t|t-\tau-1}(y_{t+h}|y_{t-\tau-1}) = \sigma_{\omega}^2$$

1. Change in expectation(and variance) depends on if update or not
2. Cannot observe systematically sluggish response to shocks at individual level

Sticky Expectation: population

- **Average forecast**

$$\bar{E}_t(y_{t+h}) = \lambda \underbrace{E_t(y_{t+h})}_{\text{rational expectation at t}} + (1-\lambda) \underbrace{\bar{E}_{t-1}(y_{t+h})}_{\text{average expectation at } t-1}$$

$$= \lambda E_t(y_{t+h}) + (1-\lambda)(\lambda E_{t-1}(y_{t+h}) + ...)$$

$$= \underbrace{\lambda \sum_{s=0}^{\infty} (1-\lambda)^s E_{t-s}(y_{t+h})}_{\text{weighted sum of past rational expectations}}$$

- **Change in average forecast**

$$\Delta \bar{E}_t(y_{t+h}) = \underbrace{(1-\lambda)}_{\text{stickiness}} \Delta \bar{E}_{t-1}(y_{t+h}) + \lambda \rho^h \omega_t$$

Sticky Expectation: population

- **Disagreements**

$$Var_t(y_{t+h}) = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} (E_{t|t-\tau}(y_{t+h}) - \bar{E}_t(y_{t+h}))^2$$

- **Change in disagreements**

$$\Delta Var_t(y_{t+h}) = \rho^{2h}(1-\lambda)\lambda \underbrace{\omega_t^2}_{\text{shock at time t}}$$

1. Disagreements rise after the shock and then gradually decline
2. Response of disagreements depends on the size of the shock

Sticky Expectation: population

- **Average variance**

$$\overline{Var}_t(y_{t+h}) = (1-\lambda)\ \underbrace{\overline{Var}_{t-1}(y_{t+h})}_{\text{average variance at t-1}} + \underbrace{\lambda Var_t(y_{t+h})}_{\text{variance of updater at t}}$$

- **Change in average variance**

$$\Delta\overline{Var}_t(y_{t+h}) = \underbrace{(1-\lambda)\Delta\overline{Var}_{t-1}(y_{t+h}) - \lambda\rho^{2h}\sigma_\omega^2}_{\text{does not depend on shock at t}}$$

1. Average variance does not respond to shocks
2. Average variance has serial correlation with the same rigidity parameter $1-\lambda$

Noisy Information: individuals

- **Mean**

$$E_{i,t}(y_{t+h}) = \rho^h E_{i,t|t}(y_t)$$

$$E_{i,t|t}(y_t) = \underbrace{E_{i,t|t-1}(y_t)}_{\text{prior}} + P\ \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}}$$

$$= (1 - PH)E_{i,t|t-1}(y_t) + Ps_{i,t}$$

where $P = [P_\epsilon, P_\xi] = \Sigma^y_{i,t|t-1} H(H'\Sigma^y_{i,t|t-1} H + \Sigma^v)^{-1}$

where $\Sigma^y_{i,t|t-1}$ is the variance of y_t based on prior belief

and $\Sigma^v = \begin{bmatrix} \sigma^2_\epsilon & 0 \\ 0 & \sigma^2_\xi \end{bmatrix}$

Noisy Information: individuals

- **Change in mean**

$$\Delta E_{i,t|t}(y_{t+h}) = \underbrace{\rho^h(1-PH)\Delta E_{i,t-1|t-1}(y_t)}_{\text{Lagged response}} + \underbrace{\rho^h PH\Delta y_{i,t} + \rho^h P\Delta v_{i,t}}_{\text{Shocks to signals}}$$

1. Rigidity parameter $1-PH$
2. Serial correlation at individual level
3. Always respond to shocks

Noisy Information: individuals

- **Variance**

$$\Sigma^y_{i,t|t} = \Sigma^y_{i,t|t-1} - \Sigma^y_{i,t|t-1} H'(H\Sigma^y_{i,t-1}H' + \Sigma^v)^{-1} H\Sigma^y_{i,t|t-1}$$

- **Change in variance**

$$\Delta\Sigma^y_{i,t|t} < 0$$

1. It does not depend on the realizations of the signal.
2. It decreases unambiguously from $t-1$ to t.
3. The two properties carry through to h-period ahead forecast

Noisy Information: population

- **Mean**

$$
\begin{aligned}
\bar{E}_{t|t}(y_{t+h}) &= \rho^h[(1-PH)\underbrace{\bar{E}_{t-1}(y_{t+h})}_{\text{Average prior}} + P \underbrace{\bar{s}_t}_{\text{Average Signals}}] \\
&= (1-PH)\bar{E}_{t-1}(y_{t+h}) + P[\epsilon_t, 0]' \\
&= (1-PH)\bar{E}_{t-1}(y_{t+h}) + P\epsilon_t
\end{aligned}
$$

1. Same properties to the individual forecast

Noisy Information: population

- **Disagreements**

$$
\begin{aligned}
Var_t(y_{t+h}) &= E((E_{i,t|t}(y_{t+h}) - \bar{E}_t(y_{t+h}))^2) \\
&= \rho^{2h} P_\xi^2 \sigma_\xi^2
\end{aligned}
$$

1. increase with the forecast horizon
2. depends on noisiness private signals, but not on that of public signals and the variance of the true variable y
3. increase with the rigidity parameter P in this model

Noisy Information: population

- **Change in disagreements**

$$\Delta Var_t(y_{t+h}) = \rho^{2h}(1-\rho^2)P_\xi^2\sigma_\xi^2 > 0$$

1. disagreements increase as time goes from $t-1$ to t.
2. disagreements increase as approaching the variable of forecast

Noisy Information: population

- **Average variance**

$$\bar{V}ar_t(y_{t+h}) = \bar{\Sigma}_t^y$$

- **Change in average variance**

$$\Delta Var_t(y_{t+h}) < 0$$

1. average variance is the same as individual variance, not depend on signals
2. the variance unambiguously drop over time