Rigidity of Expectations: Additional Evidence from Density Forecasts of Professionals and Households

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Outline

Motivation

- there are various theories on "irrational expectation"
- different theories can be tested using survey data in a comparable manner (Coibion and Gorodnichenko (2012))
- a good theory needs to be (relatively) consistent in predictions across different moments
- higher moments, i.e. uncertainty, brings about one more restriction
- survey also contains information about data generating process itself

What this paper does

1. time series and cross-sectional pattern of uncertainty from **density** forecasts of the inflation
2. additional reduced-form tests of the full-information rationality null using the uncertainty
3. extend Coibion and Gorodnichenko (2012) in two ways
   - cross-moment estimation for each one of the particular theories on expectation
   - allowing for stochastic volatility of inflation process

Literature

- empirical tests on expectation formation
  - Mankiw et al. (2003), Carroll (2003), Branch (2004), Malmendier and Nagel (2015), Das et al. (2017), Coibion and Gorodnichenko (2012), Fuhrer (2018)
- density and probabilistic questions in expectation surveys
  - Manski (2004), Delavande et al. (2011), Manski (2018)
  - Bertrand and Mullainathan (2001), Van der Klaauw et al. (2008), Delavande (2014)
- different measures of uncertainty
  - Bachmann et al. (2013), Jurado et al. (2015), Binder (2017), Bloom (2009)

A generic framework

h-period ahead density forecast by agent i at time t based on information set $I_{i,t}$

$$f_{i,t+h|t} \equiv f_{i,t}(y_{t+h}|I_{i,t})$$

- theories of expectation differ in $I_{i,t}$
  - rational expectation (FIRE): $I_{i,t} = y_{i,t}$
  - sticky expectation (SE): $I_{i,t} = y_{t-\tau}$, τ being the most recent update date
  - noisy information (NI): $I_{i,t} = s_{i,t}(y_t)$, where $s_{i,t}$ is a vector of noisy signal(s)
- the process of variable determines the mapping from $I_{i,t}$ to $f_{i,t+h|t}$

Definition and notation

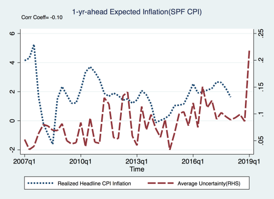
Individual moments	Population moments
Mean forecast: $y_{i,t+h\|t}$	Average forecast: $\bar{y}_{t+h\|t}$
Forecast error: $FE_{i,t+h\|t}$	Average forecast error: $\overline{FE}_{t+h\|t}$
Uncertainty: $Var_{i,t+h\|t}$	**Average uncertainty**: $\overline{Var}_{t+h\|t}$
	Disagreement: $\overline{Disg}_{t+h\|t}$

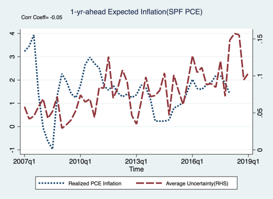
Data

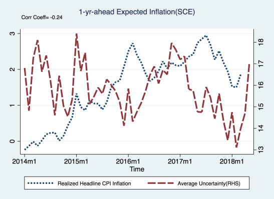
	SCE	SPF
Time period	2013M6-2018M6	2007Q1-2018Q4
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Aggregate Var in Density	1-yr-ahead inflation	1-yr and 3-yr core CPI and core PCE
Pannel Structure	stay up to 12 months	average stay for 5 years
Demographic Info	Education, Income, Age	Industry

- density estimation following (Engelberg et al. (2009))
- exclude top and bottom 5% values for forecast errors and uncertainty

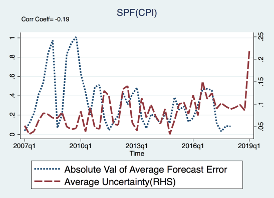
Basic patterns: uncertainty and realized inflation

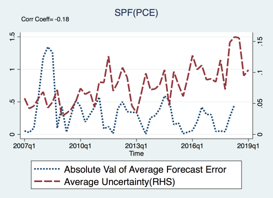


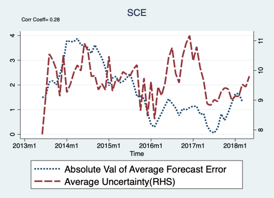




Basic patterns: uncertainty and the size of forecast errors

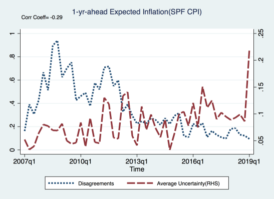


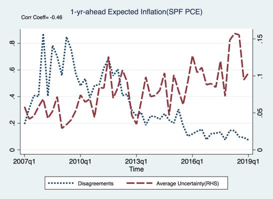


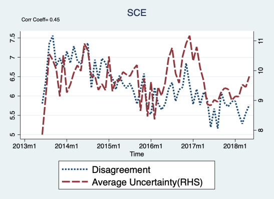


- no evidence for positive correlation betwen high ex ante uncertainty and ex post forecast errors.

Basic patterns: uncertainty and disagreement







- uncertainty are not the same as disagreement for professionals

Basic patterns: summary

- uncertainty varies across time
- uncertainty contains different information from widely proxies such as disagreement and forecast error

AR(1) model of inflation

- **Inflation process**

$$y_t = \rho y_{t-1} + \omega_t$$
$$\omega_t \sim N(0, \sigma^2_\omega)$$

- **Uncertainty**
  - FIRE: time-invariant

$$\overline{Var}^{*}_{t+h|t} = \sum_{s=1}^{h} \rho^{2s} \sigma^2_\omega$$

  - SE: time-invariant

$$\overline{Var}^{se}_{t+h|t} = \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \overline{Var}^{*}_{t+h|t-\tau}$$

  - NI: time-variant but quantitatively tiny due to highly efficient Kalman gain

$$\overline{Var}^{ni}_{t+h|t} = \rho^{2h} \overline{Var}^{ni}_{t|t} + \overline{Var}^{*}_{t+h|t}$$

Stocastic volatility (UCSV) inflation process (Stock and Watson (2007))

- **Inflation process**

$$
\begin{aligned}
y_t &= \theta_t + \eta_t, \quad \text{where } \eta_t = \sigma_{\eta,t}\xi_{\eta,t} \\
\theta_t &= \theta_{t-1} + \epsilon_t, \quad \text{where } \epsilon_t = \sigma_{\epsilon,t}\xi_{\epsilon,t} \\
\log \sigma^2_{\eta,t} &= \log \sigma^2_{\eta,t-1} + \mu_{\eta,t} \\
\log \sigma^2_{\epsilon,t} &= \log \sigma^2_{\epsilon,t-1} + \mu_{\epsilon,t}
\end{aligned}
$$

$$
\begin{aligned}
\xi_t &= [\xi_{\eta,t}, \xi_{\epsilon,t}] \sim N(0, I_2) \\
\mu_t &= [\mu_{\eta,t}, \mu_{\epsilon,t}]' \sim N(0, \gamma I_2)
\end{aligned}
$$

UCSV inflation process

- **Uncertainty**
  - FIRE: time-varying

$$\overline{Var}^{*}_{t+h|t} = \sum_{k=1}^{h} exp^{-0.5k\gamma_{\eta}} \sigma^2_{\eta,t} + exp^{-0.5h\gamma_{\epsilon}} \sigma^2_{\epsilon,t}$$

  - SE: time-varying

$$\overline{Var}^{se}_{t+h|t} = \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} \lambda \overline{Var}^{*}_{t+h|t-\tau}$$

  - NI (1-step-ahead): time-varying

$$\overline{Var}^{\theta}_{t|t-1} = \overline{Var}^{\theta}_{t-1|t-1} + Var^{*}_{t|t-1}(y_t)$$

Simulated method of moment estimation

$$\widehat{\Omega} = \underset{\{\Omega \in \Gamma\}}{argmin} (M_{\text{data}} - F^o(\Omega, Y)) W (M_{\text{data}} - F^o(\Omega, Y))'$$

- Ω: parameters of the particular $o \in \{fire, se, ni, de, seni\} \times \{ar, sv\}$
- Γ: constraints for the parameter.
- M_{data}: data moments
- F: simulated model moments according to a particular theory o, a function of parameters Ω as well as the Y, the real-time data (including history) up till each point of the time t.
  - unconditional moments, not specific to time
  - moments selected from average forecast, variance and autocovariance of forecasts, average diagreement, variance and autovariance of disagreement, average uncertainty, etc.
- W: weight matrix, identity matrix for now

Estimation procedure and algorithm

1. for each theory of expectation formation and the inflation process, start with an initial value for the parameter(s) of interest
2. simulate individual forecasts for a large enough ($N = 200$) number of forecasters
3. compute the average forecast errors, disagreement and average uncertainty across all agents
4. compute the time-series moments of the average forecast, disagreement, and uncertainty
5. compute the difference between the simulated moments and the data moments
6. keep searching the parameter value until reaching below a threshold of the loss

Two-step and joint estimation

1. two-step estimation: separately estimate inflation process parameters and then parameters of the inflation process
   - pros: computationally lighter
   - cons: potential misspecification. does not utilize the expectation data to understand inflation process per se.
2. joint estimation: targeting both moments of realized inflation series and moments of forecasts to simultaneously estimate both the inflation process and the parameter of expectation formation
   - pros: additional information gain from expectations data about inflation process itself
   - cons: more computation burden

Scoring card for a theory of expectation formation

To look if the parameter and goodness of fit is robust to

1. use of different moments in estimation
2. alternative assumption about the underlying process
3. two-step estimation or joint-estimation
4. relatively fit with professionals and households

Outline

SE parameter estimate

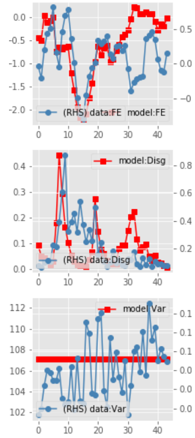
Table: SMM Estimates of SE

0	1	2	3		SE: $\hat{\lambda}_{SPF}$(Q)	SE: $\hat{\lambda}_{SPF}$(Q)	SE: ρ	SE: σ	SE: $\hat{\lambda}_{SCE}$(M)	SE: $\hat{\lambda}_{SCE}$(M)	SE: ρ	SE: σ
FEVar	FEATV				0.47	0.36	1	0.08	0.2	0.5	0.84	0.25
FEVar	DisgATV	DisgVar			0.47	0.38	1	0.1	0.21	0.54	0.92	0.18
FEVar	FEATV	DisgVar	DisgATV	Var	0.47	0.36	1	0.08	0.21	0.5	0.84	0.25

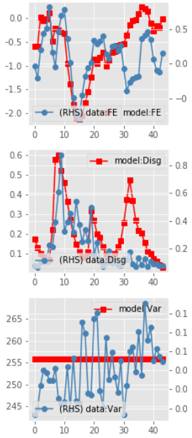
- λ: update rate in SE

Professionals and SEAR

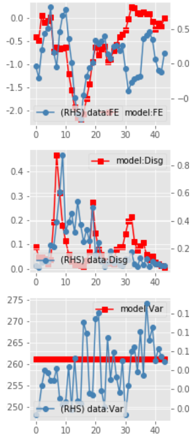
(a) FE



(b) Disg

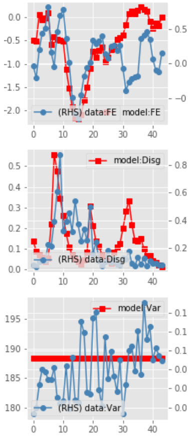


(c) FE/Disg



Professionals and SEAR: joint estimation

(a) FE



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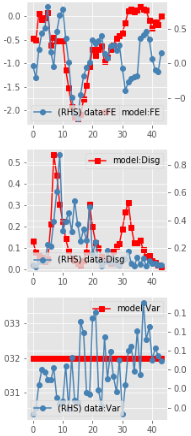

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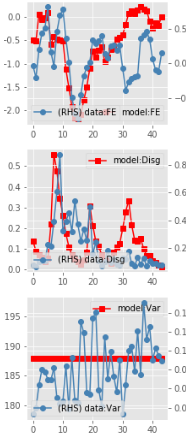
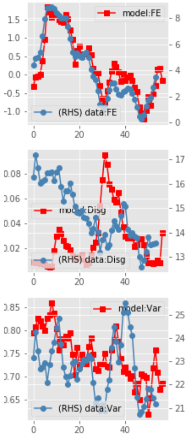
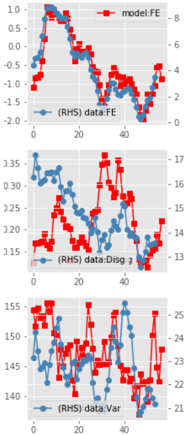


Households and SEAR

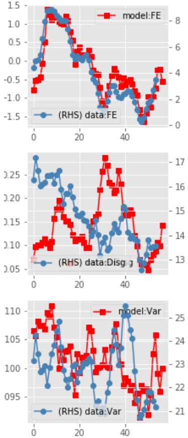
(a) FE



(b) Disg

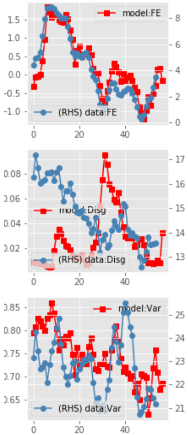


(c) FE/Disg



Households and SEAR: joint estimates

(a) FE



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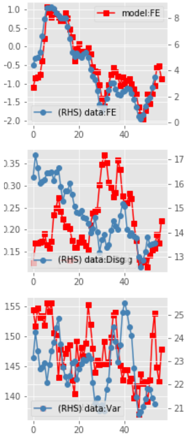
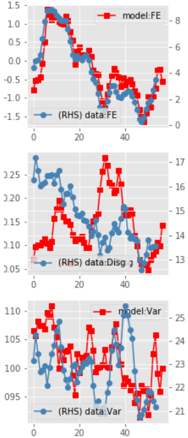


(c) FE/Disg



DE parameter estimate

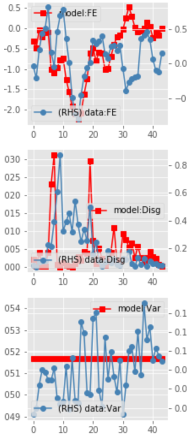
Table: SMM Estimates of DE

0	1	2	3	4	5	6	DE: θ_{SPF}	DE: $\sigma_{\theta,SPF}$	DE: θ_{SPF}	DE: $\sigma_{\theta,SPF}$	DE: ρ	DE: σ	DE: θ_{SCE}	DE: $\sigma_{\theta,SCE}$	DE: θ_{SCE}	DE: $\sigma_{\theta_S CE}$	DE: ρ	DE: σ
FE	FEVar	FEATV					-0.23	0.22	NA	NA	NA	NA	9.35	10.65	0.82	0.85	1	0
FE	FEVar	FEATV	Disg	DisgVar	DisgATV		-0.26	1.41	-0.14	1.44	0.99	0.16	8.2	9.52	4.79	4.59	0.58	0.55
FE	FEVar	FEATV	Disg	DisgVar	DisgATV	Var	-0.24	1.43	-0.17	1.44	0.99	0.16	4.78	3.01	NA	NA	NA	NA

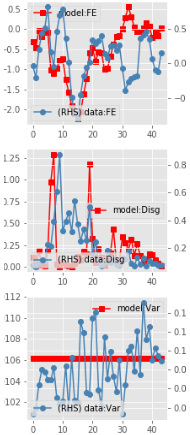
- θ: representativeness parameter, $\theta > 0$ according to DE.
- σ_θ: dispersion of representativeness across population

Professionals and DEAR

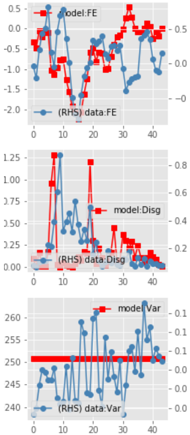
(a) FE



(b) Disg



(c) FE/Disg



Professionals and DEAR: joint estimate

(a) FE

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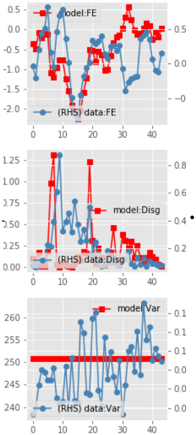
(b) Disg



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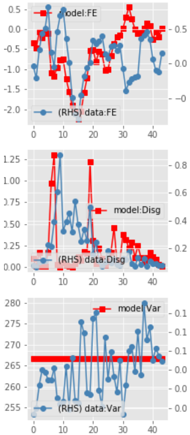
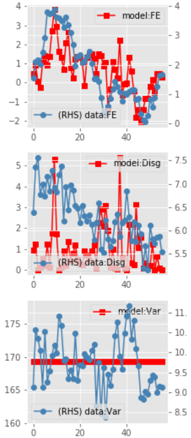
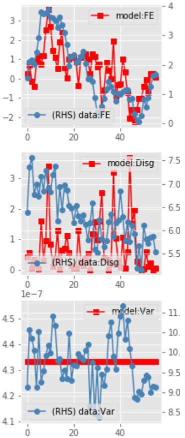


Households and DEAR

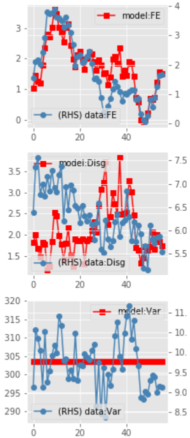
(a) FE



(b) FE/Disg

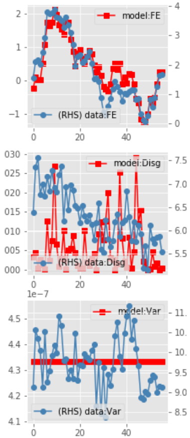


(c) FE/Disg/Var

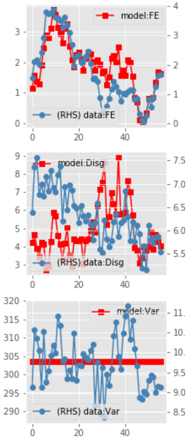


Households and DEAR: joint estimates

(a) FE



(b) FE/Disg



(c) FE/Disg/Var

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NIAR parameters

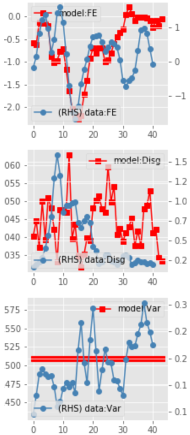
Table: SMM Estimates of NI

0	1	2	3	4	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	NI: ρ	NI: σ	NI: $\hat{\sigma}_{pb,SCE}$	$\hat{\sigma}_{pr,SCE}$	NI: $\hat{\sigma}_{pb,SCE}$	$\hat{\sigma}_{pr,SCE}$	NI: ρ	NI: σ
FEVar	FEATV				0.09	2.77	0.093	1.408	0.911	0.422	3.4	15.4	3.397	15.395	0.997	0.027
FEVar	FEATV	DisgVar	DisgATV		0.09	2.77	0.093	1.408	0.911	0.422	3.4	15.4	3.397	15.395	0.997	0.027
FEVar	FEATV	DisgVar	DisgATV	Var	0.14	3.85	0.133	1.359	0.911	0.422	4.9	22.4	4.860	22.367	0.997	0.027

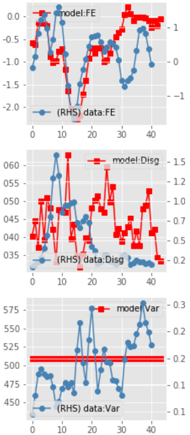
- σ_{pb}: noisiness of public signals in NI
- σ_{pr}: noisiness of private signals in NI

Professionals and NIAR

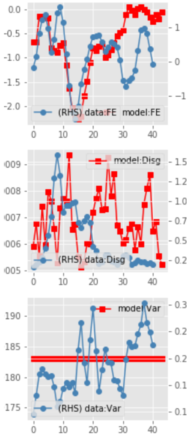
(a) FE



(b) Disg

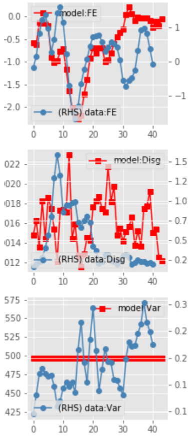


(c) FE/Disg/Var



Professionals and NIAR: joint estimates

(a) FE



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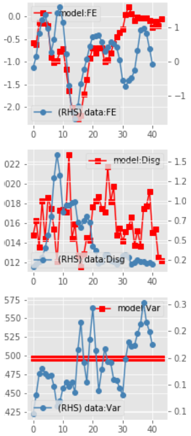
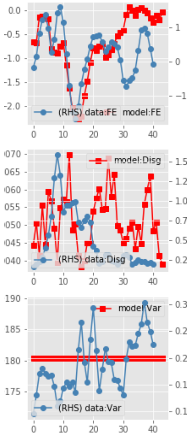
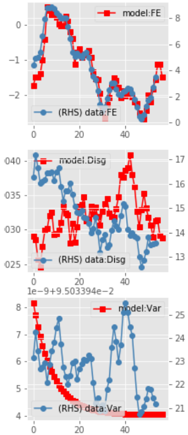


(c) FE/Disg

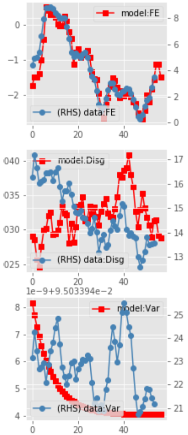


Households and NIAR

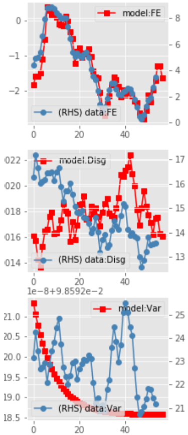
(a) FE



(b) FE/Disg

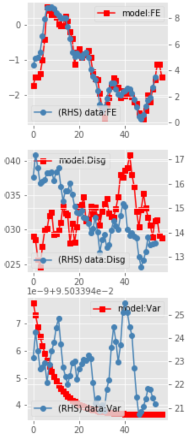


(c) FE/Disg/Var

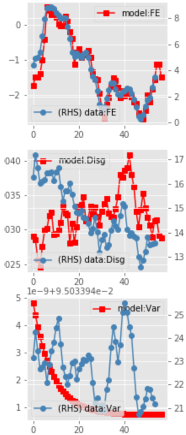


Households and NIAR: joint estimates

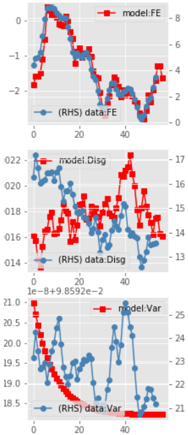
(a) FE



(b) FE/Disg



(c) FE/Disg/Var



Outline

SESV parameters

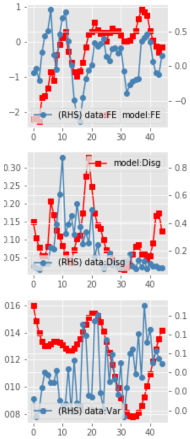
Table: SMM Estimates of Parameters of SESV

0	1	2	3	SE: $\hat{\lambda}_{SPF}$(Q)	SE: $\hat{\lambda}_{SPF}$(Q)	SE: γ	SE: $\hat{\lambda}_{SCE}$(M)	SE: $\hat{\lambda}_{SCE}$(M)	SE: γ
DisgATV	Var			0.3	0.46	2.52	0.09	0.09	0.7
FEATV	DisgVar	DisgATV		0.3	0.46	2.53	0.07	0.07	0.26
FEATV	DisgVar	DisgATV	Var	0.3	0.46	1.26	0.07	0.07	0.26

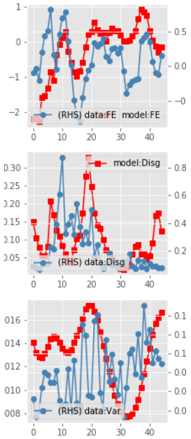
- λ: update rate in SE
- γ: size of the innovation to volatility

Professionals and SESV

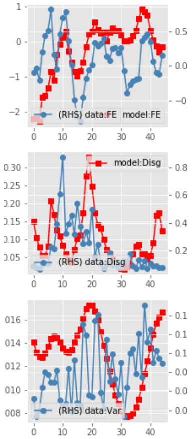
(a) FE



(b) Disg

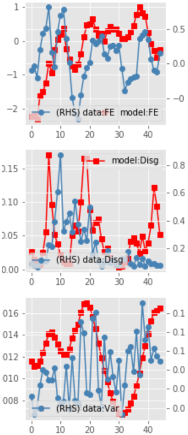


(c) FE/Disg

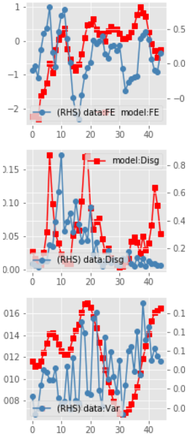


Professionals and SESV: joint estimates

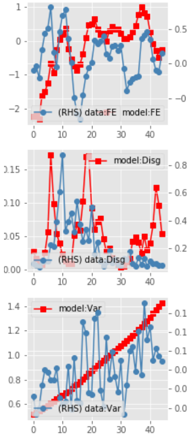
(a) FE



(b) Disg

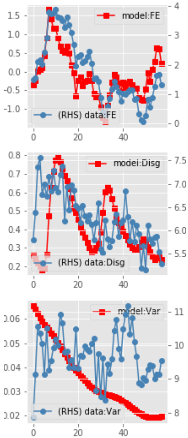


(c) Disg

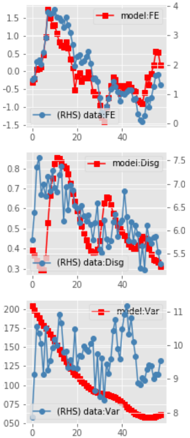


Households and SESV

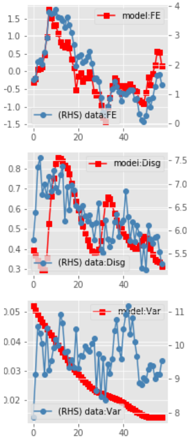
(a) Disg/Var



(b) FE/Disg

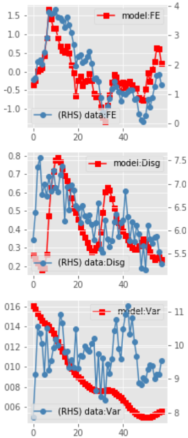


(c) FE/Disg/Var

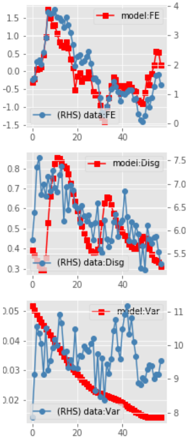


Households and SESV: joint estimates

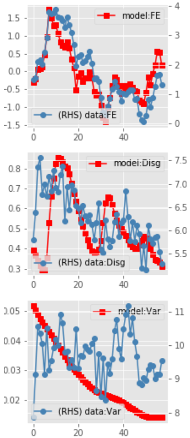
(a) Disg/Var



(b) FE/Disg



(c) FE/Disg/Var



DESV parameters

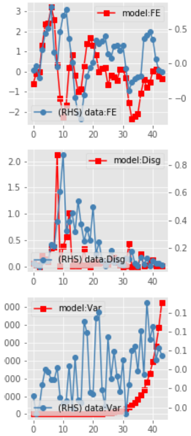
Table: SMM Estimates of Parameters of DESV

0	1	2	3	4	DE: θ	σ_θ	DE: θ	σ_θ	γ	DE: θ	σ_θ	DE: θ	σ_θ	γ
FE	FEVar	FEATV			-0.44	0.36	-0.43	1.03	0.13	7.81	4.39	7.81	2.99	0.7
FEVar	FEATV	DisgVar	DisgATV		-0.44	0.27	-0.44	0.27	0.3	7.64	6.46	7.64	6.46	0.7
FEVar	FEATV	DisgVar	DisgATV	Var	-0.43	0.26	-0.43	0.26	0.14	1.03	0	1.03	0	0.2

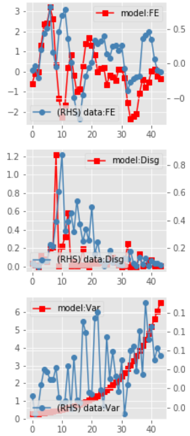
- θ: representativeness parameter
- σ_θ: dispersion of representativeness across population
- γ: size of the innovation to volatility

Professionals and DESV

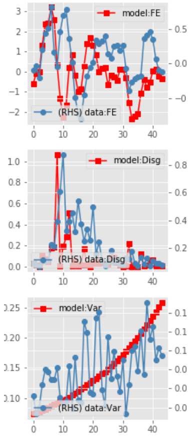
(a) FE



(b) FE/Disg

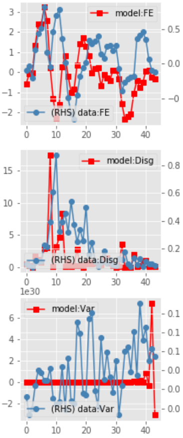


(c) FE/Disg/Var

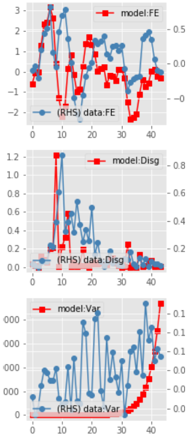


Professionals and DESV: joint estimates

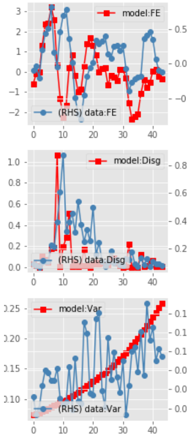
(a) FE



(b) FE/Disg

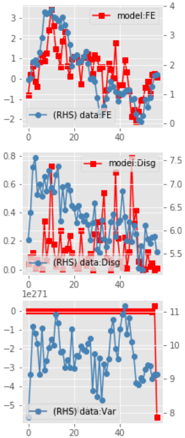


(c) FE/Disg/Var

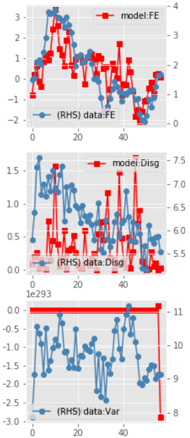


Households and DESV

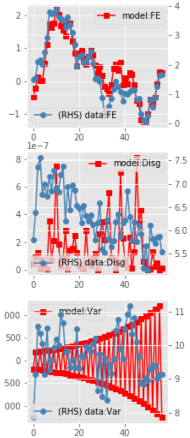
(a) FE



(b) FE/Disg

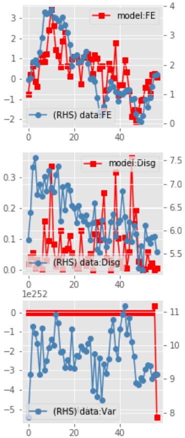


(c) FE/Disg

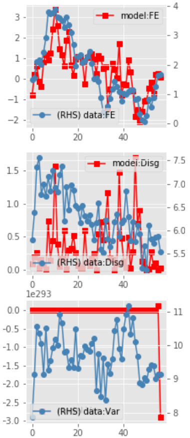


Households and DESV: joint estimates

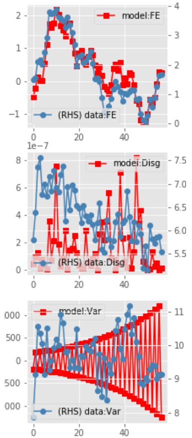
(a) FE



(b) FE/Disg



(c) FE/Disg/Var



NISV parameters

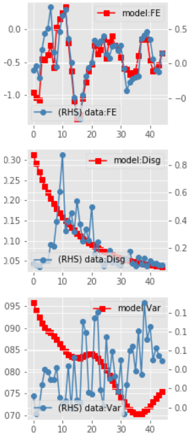
Table: SMM Estimates of Parameters of NISV

0	1	2	3	4	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	γ	NI: $\hat{\sigma}_{pb,SCE}$	$\hat{\sigma}_{pr,SCE}$
FEVar	FEATV	Var			2.35	2	2.04	23.01	2.53	2.00014E+14	3.63
FEVar	FEATV	DisgVar	DisgATV	Var	3.33	1.71	2.04	22.96	2.53	1.09884E+13	3.63

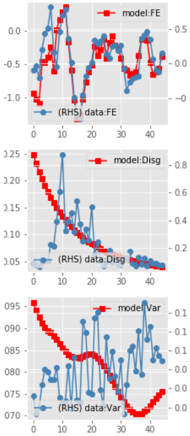
- σ_{pb}: noisiness of public signals in NI
- σ_{pr}: noisiness of private signals in NI
- γ: size of the innovation to volatility

Professionals and NISV

(a) FE/Var

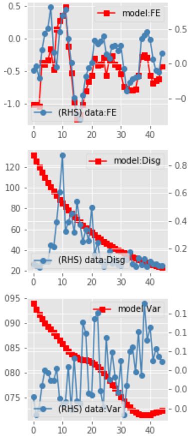


(b) FE/Disg/Var

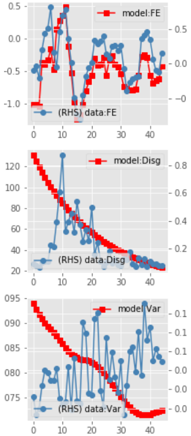


Professionals and NISV: joint estimates

(a) FE/Var

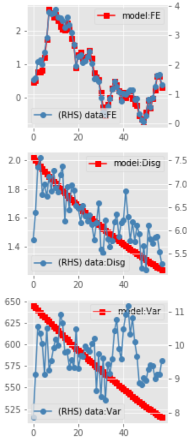


(b) FE/Disg/Var

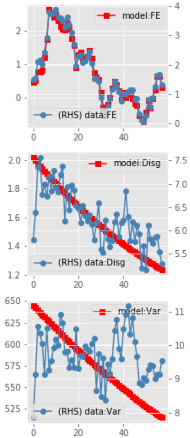


Households and NISV

(a) FE/Var

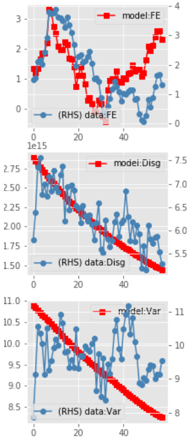


(b) FE/Disg/Var

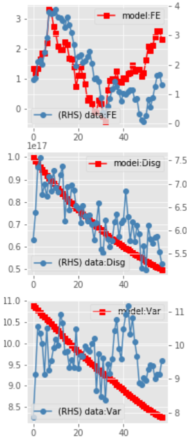


Households and NISV: joint estimates

(a) FE/Var



(b) FE/Disg/Var



Taking stock

Table: A scoring card of different theory

Criteria	SE	NI	DE
Sensitive to moments used for estimation?	No	No	No
Sensitive to the assumed inflation process?	No	Yes	No
Sensitive to a two-step or joint estimate?	No	Yes	No
Sensitive to the type of agents?	Yes	Yes	Yes
Matching with FE	Yes	Yes	Yes
Matching with disagreement	Yes	No	Unsure
Matching with uncertainty	Unsure	No	No

Conclusion

- Sticky expectation (SE) augmented with stochastic volatility of inflation process matches data of inflation and expectations better than other theories for both professionals and households.
- Within each model, households are more inconsistent compared to professionals
- Incorporating higher moments, i.e. uncertainty, helps "discipline" theories on expectation formation
- Higher moments from surveys also contain useful information about the inflation dynamics itself

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