Understanding Expectation Formation from Probabilistic Questions

Tao Wang
Johns Hopkins University

April, 2019

Outline

What I want to do

- Use density information to identify differences in various theories
- Both individual and population moments
- Households and professional forecasters

Why density is important

- **Identification:** different theories have testable predictions on the second moments
  - Scenario 1. Two people think the probability of raining is 50%.
  - Scenario 2. One person thinks 100% and the other 0%.
- **Modeling Implications:** both mean and variance affect economic decisions
  - precautionary saving with income risks
  - portfolio choice with risky asset

Literature

- **Theory**
  - Sticky information [Carroll, 2003], [Reis, 2006]
  - Rational inattention [Sims, 2003], [Gabaix, 2014]
  - Noisy information [Lucas Jr, 1972], [Woodford, 2001]
  - Learning [Evans and Honkapohja, 2012]
  - Strategic interaction [Morris and Shin, 2002], [Hellwig and Veldkamp, 2009]
  - Diagostic expectation [Bordalo et al., 2018]
  - Model uncertainty [Hansen and Sargent, 2001], [Hansen and Sargent, 2008]
- **Empirics**
  - **Heterogeneity in Expectation:** [Mankiw et al., 2003], [Coibion et al., 2018]
  - **Testing Theories:** [Coibion and Gorodnichenko, 2012], [Fuhrer, 2018]

Unified Framework

h-period ahead density forecast by agent i at time t based on information set $I_{i,t}$

$$\widehat{f}_{i,t}(y_{t+h}|I_{i,t})$$

- Theories differ in what is in $I_{i,t}$
- May also differ on information processing, i.e. $I_{i,t} \rightarrow \widehat{f}_{i,t}$

Definition and notation

- Individual
  - mean forecast $E_{i,t}(y_{t+h})$
  - forecast error $FE_{i,t+h|t} = y_{t+h} - E_{i,t}(y_{t+h})$
  - uncertainty $Var_{i,t}(y_{t+h})$
- Population
  - average forecast $\bar{E}_t(y_{t+h})$
  - average forecast error $\overline{FE}_t = y_{t+h} - \bar{E}_t(y_{t+h})$
  - cross-section disagreements $Var_t(y_{t+h})$
  - average uncertainty $\overline{Var}_t(y_{t+h})$

Assumption about true process

$$y_{t+1} = \rho y_t + \omega_t$$
$$\omega_t \sim N(0, \sigma_\omega^2)$$

- $0 < \rho < 1$
- if $\rho = 0$, random walk, no way to forecast at all
- ω_t is i.i.d

Outline

Sticky Expectation: assumptions

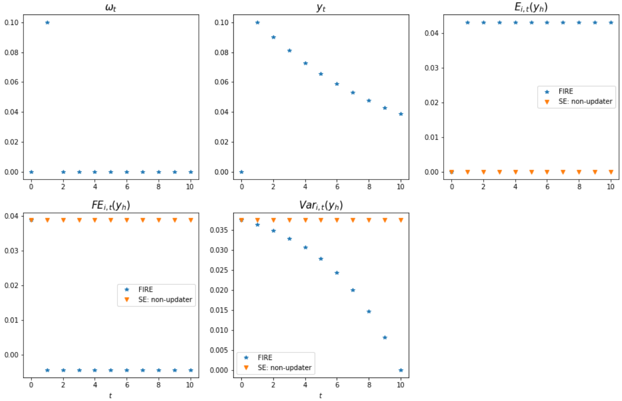
- At time t, agent i learns about y_t at a fixed Poisson rate λ
- For a non-updater since $t-\tau$

$$E_{i,t}(y_{t+h}|y_{t-\tau}) = \rho^{h+\tau} y_{t-\tau}$$

- Full information rational expectation is a special case: $\tau = 0$

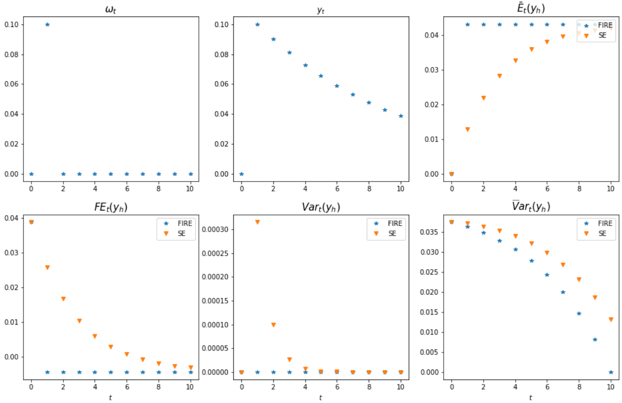
Impulse responses to shocks: individual moments

$$\omega_t = 0.1, \quad h = 10, \quad \rho = 0.9, \quad \lambda = 0.5$$



Impulse responses to shocks: population moments

$$\omega_t = 0.1, \quad h = 10, \quad \rho = 0.9, \quad \lambda = 0.5$$



Outline

Noisy Information: assumptions

- Individual only observes noisy signals

$$
\begin{aligned}
s_{i,t} &= [s_t^{pb}, s_{i,t}^{pr}] \in I_{i,t} \\
s_t^{pb} &= y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \\
s_{i,t}^{pr} &= y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0, \sigma_\epsilon^2)
\end{aligned}
$$

- Or in vector form

$$
s_{i,t} = Hy_t + v_{i,t} \quad \text{where } H = [1,1]' \text{ and } v_{i,t} = [\epsilon_t, \xi_{i,t}]'
$$

- Kalman filtering (simply normal updating if ρ=0)

Noisy Information: predictions

- **Similar to Sticky Expectation**
  1. **Macro rigidity**: population forecasts partially respond to shocks
  2. **Non-response of variance**: both individual and population variance does not respond to shocks.
- **Different from Sticky Expectation**
  1. **Micro rigidity**: both individual and population forecast partially respond to shocks
  2. **Horizon-sensitive rigidity**: rigidity parameter decreases with horizon
  3. **Increasing disagreements:** population disagreements increase over time as approaching $t + h$
  4. **Shock-specific responses:** different impacts of fundamental shocks, or simply news shocks

where is variance

Other theories on to-do-list

- **Rational Inattention:** attentiveness endogenously respond to variances
- **Parameter Learning**: the structural parameter ρ is not known, thus the agent learns about it as if an econometrician does

Data

how the data is used?

	SCE	SPF
Time period	2013-present	2007-present
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Aggregate Var in Density	1-yr and 3-yr ahead inflation	1-yr CPI and PCE
Pannel Structure	stay up to 12 months	average stay for 5 years
Demographic Info	Education, Income, Age	Industry

Empirical execution

- **Measurement errors**: winsorization is still necessary
- **Density Estimation**: generalized beta estimation, [Engelberg et al., 2009]
- **Identification of Shocks**: following [Coibion and Gorodnichenko, 2012]

Bordalo, P., Gennaioli, N., and Shleifer, A. (2018).
Diagnostic expectations and credit cycles.
The Journal of Finance, 73(1):199–227.

Carroll, C. D. (2003).
Macroeconomic expectations of households and professional forecasters.
the Quarterly Journal of economics, 118(1):269–298.

Coibion, O. and Gorodnichenko, Y. (2012).
What can survey forecasts tell us about information rigidities?
Journal of Political Economy, 120(1):116–159.

Coibion, O., Gorodnichenko, Y., and Kumar, S. (2018).
How do firms form their expectations? new survey evidence.
American Economic Review, 108(9):2671–2713.

Engelberg, J., Manski, C. F., and Williams, J. (2009).
Comparing the point predictions and subjective probability distributions of professional forecasters.

Journal of Business & Economic Statistics, 27(1):30–41.

Evans, G. W. and Honkapohja, S. (2012).
Learning and expectations in macroeconomics.
Princeton University Press.

Fuhrer, J. C. (2018).
Intrinsic expectations persistence: evidence from professional and household survey expectations.

Gabaix, X. (2014).
A sparsity-based model of bounded rationality.
The Quarterly Journal of Economics, 129(4):1661–1710.

Hansen, L. and Sargent, T. J. (2001).
Robust control and model uncertainty.
American Economic Review, 91(2):60–66.

Hansen, L. P. and Sargent, T. J. (2008).
Robustness.
Princeton university press.

Hellwig, C. and Veldkamp, L. (2009).
Knowing what others know: Coordination motives in information acquisition.
The Review of Economic Studies, 76(1):223–251.

Lucas Jr, R. E. (1972).
Expectations and the neutrality of money.
Journal of economic theory, 4(2):103–124.

Mankiw, N. G., Reis, R., and Wolfers, J. (2003).
Disagreement about inflation expectations.
NBER macroeconomics annual, 18:209–248.

Morris, S. and Shin, H. S. (2002).
Social value of public information.
american economic review, 92(5):1521–1534.

Reis, R. (2006).
Inattentive consumers.
Journal of monetary Economics, 53(8):1761–1800.

Sims, C. A. (2003).
Implications of rational inattention.
Journal of monetary Economics, 50(3):665–690.

Woodford, M. (2001).
Imperfect common knowledge and the effects of monetary policy.
Technical report, National Bureau of Economic Research.

Sticky Expectation: individual

For a non-updater since $t-\tau$ ($\tau = 0$ for updater),

- **Mean**

$$E_{i,t}(y_{t+h}|y_{t-\tau}) = \rho^{h+\tau} y_{t-\tau}$$

- **Forecast Error**

$$FE_{i,t+h|t} = \underbrace{\sum_{s=0}^{h+\tau} \rho^s \omega_{t+h-s}}_{\substack{\text{weighted sum of} \\ \text{future realized shocks}}}$$

- **Variance**

$$Var_{i,t}(y_{t+h}|y_{t-\tau}) = \sum_{s=0}^{h+\tau} \rho^{2s} \sigma_\omega^2$$

Sticky Expectation: individual

$$\text{updater:} \quad \Delta Var_{i,t}(y_{t+h}|y_t) = \sum_{s=0}^{\tau} \rho^{2s}\sigma_\omega^2$$

$$\text{non-updater:} \; \Delta Var_{i,t|t-\tau-1}(y_{t+h}|y_{t-\tau-1}) = \sigma_\omega^2$$

1. Change in expectation(and variance) depends on if update or not
2. Cannot observe systematically sluggish response to shocks at individual level

Sticky Expectation: population

- **Average forecast**

$$\bar{E}_t(y_{t+h}) = \lambda \underbrace{E_t(y_{t+h})}_{\text{rational expectation at t}} + (1-\lambda) \underbrace{\bar{E}_{t-1} y_{t+h}}_{\text{average expectation at } t-1}$$

$$= \lambda E_t(y_{t+h}) + (1-\lambda)(\lambda E_{t-1}(y_{t+h}) + \ldots)$$

$$= \underbrace{\lambda \sum_{s=0}^{\infty} (1-\lambda)^s E_{t-s}(y_{t+h})}_{\text{weighted sum of past rational expectations}}$$

- **Change in average forecast**

$$\Delta \bar{E}_t(y_{t+h}) = \underbrace{(1-\lambda)}_{\text{Stickiness Parameter}} \Delta \bar{E}_{t-1}(y_{t+h}) + \lambda \rho^h \omega_t$$

Sticky Expectation: population

- **Disagreements**

$$Var_t(y_{t+h}) = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} (E_{t|t-\tau}(y_{t+h}) - \bar{E}_t(y_{t+h}))^2$$

- **Change in disagreements**

$$\Delta Var_t(y_{t+h}) = \rho^{2h}(1-\lambda)\lambda \underbrace{\omega_t^2}_{\text{shock at time t}}$$

1. Disagreements rise after the shock and then gradually decline
2. Response of disagreements depends on the size of the shock

Sticky Expectation: population

- **Average variance**

$$\overline{Var}_t(y_{t+h}) = (1-\lambda)\underbrace{\overline{Var}_{t-1}(y_{t+h})}_{\text{average variance at t-1}} + \underbrace{\lambda Var_t(y_{t+h})}_{\text{variance of updater at t}}$$

- **Change in average variance**

$$\Delta\overline{Var}_t(y_{t+h}) = \underbrace{(1-\lambda)\Delta\overline{Var}_{t-1}(y_{t+h}) - \lambda\rho^{2h}\sigma^2_\omega}_{\text{does not depend on shock at t}}$$

1. Average variance does not respond to shocks
2. Average variance has serial correlation with the same rigidity parameter $1-\lambda$

Noisy Information: individuals

- **Mean**

$$
\begin{aligned}
E_{i,t}(y_{t+h}) &= \rho^h E_{i,t|t}(y_t) \\
E_{i,t|t}(y_t) &= \underbrace{E_{i,t|t-1}(y_t)}_{\text{prior}} + P \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}} \\
&= (1 - PH)E_{i,t|t-1}(y_t) + Ps_{i,t}
\end{aligned}
$$

where $P = [P_\epsilon, P_\xi] = \Sigma^y_{i,t|t-1} H (H' \Sigma^y_{i,t|t-1} H + \Sigma^v)^{-1}$

where $\Sigma^y_{i,t|t-1}$ is the variance of y_t based on prior belief

and $\Sigma^v = \begin{bmatrix} \sigma^2_\epsilon & 0 \\ 0 & \sigma^2_\xi \end{bmatrix}$

Noisy Information: individuals

- **Change in mean**

$$\Delta E_{i,t|t}(y_{t+h}) = \underbrace{\rho^h(1-PH)\Delta E_{i,t-1|t-1}(y_t)}_{\text{Lagged response}} + \underbrace{\rho^h PH\Delta y_{i,t} + \rho^h P\Delta v_{i,t}}_{\text{Shocks to signals}}$$

- Rigidity parameter $1-PH$
- Serial correlation at individual level
- Always respond to shocks

enumerate

Noisy Information: individuals

- **Variance**

$$\Sigma^y_{i,t|t} = \Sigma^y_{i,t|t-1} - \Sigma^y_{i,t|t-1} H'(H\Sigma^y_{i,t-1} H' + \Sigma^v) H\Sigma^y_{i,t|t-1}$$

- It does not depend on the realizations of the signal.
- **Change in variance**
- It decreases unambiguously from $t-1$ to t.

$$\Delta\Sigma^y_{i,t|t} < 0$$

- The two properties carry through to h-period ahead forecast

Noisy Information: population

- **Mean**

$$\bar{E}_{t|t}(y_{t+h}) = \rho^h[(1-PH)\underbrace{\bar{E}_{t-1}(y_{t+h})}_{\text{Average prior}} + P\underbrace{\bar{s}_t}_{\text{Average Signals}}]$$
$$= (1-PH)\bar{E}_{t-1}(y_{t+h}) + P[\epsilon_t, 0]'$$
$$= (1-PH)\bar{E}_{t-1}(y_{t+h}) + P\epsilon_t$$

- Same properties to the individual forecast

Noisy Information: population

- **Disagreements**

$$
\begin{aligned}
Var_t(y_{t+h}) &= E((E_{i,t|t}(y_{t+h}) - \bar{E}_t(y_{t+h}))^2) \\
&= \rho^{2h} P_\xi^2 \sigma_\xi^2
\end{aligned}
$$

- increase with the forecast horizon
- depends on noisiness private signals, but not on that of public signals and the variance of the true variable y
- increase with the rigidity parameter P in this model

Noisy Information: population

- **Change in disagreements**

$$\Delta Var_t(y_{t+h}) = \rho^{2h}(1-\rho^2)P_\xi^2\sigma_\xi^2 > 0$$

- disagreements increase as time goes from $t-1$ to t.
- disagreements increase as approaching the variable of forecast

Noisy Information: population

- **Average variance**

$$\bar{V}ar_t(y_{t+h}) = \bar{\Sigma}^y_t$$

- **Chnage in average variance**

$$\Delta Var_t(y_{t+h}) < 0$$

- average variance is the same as individual variance, not depend on signals
- the variance unambiguously drop over time