Testing Theories of Expectation Formation using Probabilistic Forecasts

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1 Introduction

The theories on how different agents form expectations have proliferated over the past decade. On one hand, these theories built upon different micro foundations produce some similar macro patterns. For instance, the notion of information rigidity, or expectation rigidity could be micro founded by many different theories, but mostly generate sluggish response to new information.

On the other hand, however, there are important and subtle differences in testable predictions from these theories for both individual forecasts and aggregate moments of the forecasts. My goal of this exercise is to derive the testable predictions based on various theories, and utilize the probablistic questions from both professional forecasters survey and household survey to test them.

The potential contribution lies in the extra insights I may get from using probablistic questions.[1] One obvious advantage of density expectations compared to point one is availability of higher moments, i.e. variance of the forecast indicates the dispersion of the forecasts. Along time dimension, the dynamics of conditional relative likelihood is a useful indicator of information gain or forecasting efficiency. Besides, as in a typical signal-extraction model, the prediction that new information reduce conditional variance can be tested only using probability distributions. Although the detailed implementation of these tests involves technical difficulties, the ideas are quite clear. Lastly, with second moments in individual level expectations, we are allowed to potentialy explore the heterogeneity across agents within one theory. For instance, sticky information typically assume a posson update rate for all agents in the economy. This is partly due to the fact that available data only allows us to recover single parameter of regidity instead of inter-group heterogeneity.

[1]For an insightful survey on the importance of measuring subjective expectations using probablistic surveys, see [Manski, 2004].

2 Theories of Expectation Formation

2.1 Definition of moments

An agent i is forming expectations about a stochastic variable y^i_{t+h}. The supscript i can be dropped if it is an aggregate (i.e. inflation) variable instead of individual specific (i.e. household income or house value). This paper focuses on forecasting of aggregate variables, in particular, inflation. So we can simply denote the variable as y_{t+h}. To put it another way, all agents are foreasting the same object. [2]

Denote $f_{i,t+h|t}$ as agent i's h-period-ahead density forecast. $f_{i,t+h|t}$ is the conditional density of $y_{i,t+h}$ given the information set $I_{i,t}$ available at time t.

$$f_{i,t+h|t} \equiv f_{i,t}(y_{t+h}|I_{i,t})$$

$I_{i,t}$ is the information set available for individual i at time t. The information set can be agent specific, thus it has subscript i. The specific content contained in I_t varies from different models of expectation. For instance, sticky expectation and rational inattention literature all assume that agents are not able to update new information instataneously. So the information set may not contain the most recent realization of the variable of forecast y_t. [3]

Accordingly, h-period ahead mean forecast at t, denoted as $y_{i,t+h|t}$, is the conditional expectation of y_{t+h} by the agent i.

$$y_{i,t+h|t} \equiv E_{i,t}(y_{t+h}) = \int f_{i,t+h|t} dy_{t+h}$$

Similarly, individual forecasting variance $\sigma_{i,t+h|t}$, hereafter termed as indiviual uncertainty in this paper, is the conditional variance.

$$\sigma^2_{i,t+h|t} \equiv Var_{i,t}(y_{t+h})$$

Individual forecast error $FE_{i,t+h|t}$ is the difference of ex post realized value of y_{t+h} and individual foreacast at time t.

$$FE_{i,t+h|t} = y_{i,t+h|t} - y_{t+h}$$

The population analogues of individual mean forecast, uncertainty and forecast errors are simply the average of the individual moments taken across agents. Denote them as $\bar{y}_{t+h|t}$, $\bar{\sigma}^2_{t+h|t}$, and $\overline{FE}_{t+h|t}$, respectively. Hereafter, they are termed as population mean forecast, population uncertainty and population forecast error, respectively. In addition, disagreements is defined as the cross-sectional variance of mean forecasts of individual agents. Denote it as $\overline{Var}_{t+h|t}(y_{i,t+h|t})$. To simplify notation, let us directly call it $\overline{Disg}_{t+h|t}$. [4]

[2] Only in the context of forecasting aggregate variable, it is meaningful to study the average expectations and disagreements across agents.

[3] Given the same information set available to agents, different theories may also differ in the underlying models each agent use to form the conditional density of the variables by agent i. Examples of such include multi-prior or model uncertainty. [**?**].

[4] This is the same terminology used by [**?**].

Table 1: Definition and Notation of Moments

Individual Moments	Population Moments
Mean forecast: $y_{i,t+h\|t}$	Average forecast: $\bar{y}_{t+h\|t}$
Forecast error: $FE_{i,t+h\|t}$	Average forecast error: $\overline{FE}_{t+h\|t}$
Uncertainty: $\sigma^2_{i,t+h\|t}$	Average uncertainty: $\bar{\sigma}^2_{i,t+h\|t}$
	Disagreements: $\overline{Disg}_{t+h\|t}$

In summary, we have 3 individual moments and 4 population moments listed in below in Table 2.1.

Finally, we assume the underlying true process of y_t is $AR(1)$ with persistency parameter $0 < \rho < 1$ and i.i.d. shock ω_t.

$$y_{t+1} = \rho y_t + \omega_t$$

$$\omega_t \sim N(0, \sigma^2_\omega)$$

2.2 Benchmark of full-information rational expectation(FIRE)

In the FIRE benchmark, it is assumed that all agents perfectly observe y_t at time t and understand the true process of y. Therefore, individual forecast is $\rho^h y_t$, which is shared by all agents. Therefore, it is also equal to average forecast.

Both individual and population forecast errors are simply the realized shocks between $t+1$ to $t+h$.

$$\overline{FE}^*_{i,t+h|t} = \sum_{s=1}^{h-1} \rho^s \omega_{t+h-s}$$

I use supscrit of $*$ to denote all the moments according to FIRE. It is easy to see that the forecast error is orthogonal to information available till time t. This provides the a well known null hypothesis of FIRE. [**?**]

The second implication from FIRE here is that forecast errors of non-overlapping horizon forecasts are not correlated. For instance, forecast error at time t and that at time $t+h$ or further are not serially correlated. This is not the case within h periods as the realized shocks in overlapping periods enter both forecast errors.

$$Cov(\overline{FE}^*_{t+h|t}, \overline{FE}^*_{t+s+h|t+s}) = 0 \quad \forall s \geq h$$

FIRE also has predictions about population disagreements. [5] As agents perfect update the same information, there is no disagreements at any point of the time.

[5] For instance, [Mankiw et al., 2003] documents substantial time-varying disagreements about future inflation using consumers and professional forecasters' survey.

$$\overline{Disg}^*_{t+h|t} = 0 \quad \forall t$$

A corollary is that disagreements do not respond to realized shocks any any point of the time. [6]

In addition to forecast error and disagreements, according to FIRE all individual shares the same degree of uncertainty. The uncertainty simply comes from uncertainty about future unrealized shocks between t and $t+h$. To put it another way, according to FIRE, there is neither disagreements about mean, nor disagreements about the uncertainty [7]

$$\bar{\sigma}^{*2}_{t+h|t} = \sum_{s=1}^{h} \rho^{2s}\sigma^2_{\omega}$$

The time series behavior of uncertainty according to FIRE is less obvious. Weather it is time-variant depends on if the variance of the shocks ω_t is time varying. Although in our baseline case we make such an assumption, in general it may not be true. [8]

In the same time, the serial correlation of uncertainty across different horizons of forecasting does follow testable pattern according to FIRE. In particular, consistent with our current notation, let us denote the forecast of y_{t+h} over horizon of $h-k$ as $y_{t+h|t+k} \quad \forall k = 0, 1...h$.

$$\bar{\sigma}^{*2}_{t+h|t+k} = \sum_{s=1}^{h-k} \rho^{2s}\sigma^2_{\omega}$$

As the forecasting horizon decreases relative to a given future point of time, there is a pure efficiency gain or reduction in uncertainty as more and more shocks have realized. So the uncertainty should drop unambigously.

An equivalent interpretation to this is through the perspective of revision. Defining the forecast revision as the difference of different vintages of foreacsts fixing the terminate date, the revision in uncertainty should be always negative and the drop should be exactly equal to discounted variance of the realized shocks. [9]

$$\bar{\sigma}^{*2}_{t+h|t+1} - \bar{\sigma}^{*2}_{t+h|t} = -\rho^{2h}\sigma^2_{\omega}$$

2.3 Sticky Expectation (SE)

The theory of sticky expectation ([?], [?]), regardless of its different microfoundations by various authors, builds upon the assumption that agents do not update information instantaneously as they do in FIRE. One of the most tractable

[6] Empricial tests of this kind include [?].[?] etc.

[7] This is the same to []'s terminology.

[8] [?] on time-varying volatility.

[9] Compared to other null hypotheses discussed above, this is a weaker test. It is true that rational foreacsting implies reduction in uncertainty with shorter horizon. But there is no direct test if the drop in uncertainty is efficient enough so as to be consistent with FIRE.

models assume there is a homogenous Possion rate λ of updating among the population. Specificaly, at any point of time t, each agent learns about the up-to-date realization of y_t with probability of λ; otherwise, it forms the expectation based on the most recent up-to-date realization of $y_{t-\tau}$, where τ is the time experienced since previous update.

2.3.1 Individual moments

Denote the mean forecast of a non-updater since $t-\tau$ as $y_{i,t+h|t-\tau}$ since her forecast conditions upon the information up till $t-\tau$.

$$y_{i,t+h|t-\tau} = \rho^{h+\tau} y_{t-\tau}$$

Now her information set is not up to date, the uncertainty to a non-updater is higher than an updater and it increases with the duration of non-updating τ.

$$\sigma^2_{i,t|t-\tau} = \sum_{s=1}^{h+\tau} \rho^{2s}\sigma^2_\omega$$

In FIRE, updating at each period t resolves only the uncertainty about the shocks realizing in t. In constrast, in SE each updating resolves the uncertainty about all the realized shocks since last update. Fixing the terminate date, i.e. forecasting y_{t+1} at time t relative to forecasting y_{t+1} at $t-\tau$, the revision in uncertainty is

$$\sigma^2_{i,t+1|t} - \sigma^2_{i,t+1|t-\tau} = \rho^2\sigma^2_\omega - \sum_{s=1}^{\tau+1} \rho^{2s}\sigma^2_\omega = -\sum_{s=2}^{\tau+1} \rho^{2s}\sigma^2_\omega$$

FIRE assumes $\tau = 1$ for all the agents and all the time, namely all agents' last update takes place in previous period. So setting $\tau = 1$ in above equation gives the reduction in uncertainty in FIRE. One can easily see that the reduction in ucnertainty is greater in SE for any $\tau > 1$ than FIRE.

In summary, the key difference between FIRE and SE with respect to behavior of uncertainty is that the later does not reduce uncertainty as efficiently as in the former primarily because of the rigidity incorporating new information. One qualitative pattern from this difference is that any evidence of ambiguous pattern or non-negative revision in uncertainty is inconsistent with FIRE. [10]

2.3.2 Population moments

It is important to note that the rigidity in updating according to SE cannot be systamatically observable in the individual level, both in terms of forecasts errors and uncertainty. This is because the behaviors of each individual foreacst specifically depends on if the she updates or not in that period.

[10] Because of this, the difference in average responses in variance to new information may speak to potential heterogeneity in information rigidity. According to the theory above, higher information rigidity implies high volatility of variance responses.

Relying upon the Law of Large Numbers, we can derive testable predictions about population moments that allow us perform tests of sticky expectation and recover rigidity parameter λ. [11]

One well known prediction from SE literature is that the average foreacast is a weighted average of update-to-date rational expectation and lagged average expectation as reproduced below. [12] It can be also expressed as a weighted average to all the past realizations of y. Setting $\lambda = 1$, then the SE collapses to FIRE and the average forecast is equal to y's long-run mean of zero.

$$
\begin{aligned}
\bar{y}_{t+h|t} &= \lambda \underbrace{y^*_{t+h|t}}_{\text{rational expectation at t}} + (1-\lambda) \underbrace{\bar{y}_{t+h|t-1}}_{\text{average forecast at } t-1} \\
&= \lambda \sum_{s=0}^{\infty} (1-\lambda)^s y^*_{t+h|t-s} \\
&= \lambda \sum_{s=0}^{\infty} (1-\lambda)^s \rho^{s+h} y_{t-s}
\end{aligned}
\tag{1}
$$

It is also easy to show that the average forecast errors are serially correlated. Setting $\lambda = 1$, then the SE collapses to FIRE, in which there is no serial correlation between forecast errors and it fully respond to newly realized shocks at time t. [13]

$$
\overline{FE}_{t+h|t} = (1-\lambda)\overline{FE}_{t+h-1|t-1} + \lambda \rho^h \omega_t \tag{2}
$$

[Coibion and Gorodnichenko, 2012] also derives predictions about the disagreements in rigidity models. Unlike zero disagreements in FIRE, SE is characterized by non-zero dispersion in forecasting because of different lags in updating across populations. [14]

$$
\overline{Disg}_{t+h|t} = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} (y_{t+h|t-\tau} - \bar{y}_{t+h|t}))^2 \tag{3}
$$

From time t to $t+1$, the change in dispersion comes from two sources. One is newly realized shock at time $t+1$. The other component is from people who did not update at time t and update at time $t+1$.

In SE, disagreements rise after realization of the shock and gradually returns to its steady state level. In the same time, it does not depend upon the particular realization of the shock. [Coibion and Gorodnichenko, 2012] derive the impulse response of dispersion at time $t+j$ to a shock that realized at t.

[11] [Carroll, 2003] is one notable example of this.
[12] See [Coibion and Gorodnichenko, 2012] or appendix of this paper for detailed steps.
[13] Need to check if is revision or change from
[14] need to check if correct also.

$$\rho^{2(h+j)}(1-\lambda^{j+1})\lambda^{j+1}\omega_t^2 \tag{4}$$

Finally, SE predicts a sluggish behavior of revision in uncertainty with a similar spirit of forecast errors.

$$\begin{aligned}\bar{\sigma}^2_{t+h|t} &= \sum_{\tau=0}^{+\infty} \underbrace{\lambda(1-\lambda)^{\tau}}_{\text{fraction who non-updater until } t-\tau} \underbrace{\sigma^2_{t+h|t-\tau}}_{\text{uncertainty of most recent update at } t-\tau} \\ &= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \sum_{s=0}^{h+\tau} \rho^{2s}\sigma^2_{\omega}\end{aligned} \tag{5}$$

Correspondingly, the revision in average uncertainty will be below. [15]

$$\bar{\sigma}^2_{t+h|t+1} - \bar{\sigma}^2_{t+h|t} = (1-\lambda)(\bar{\sigma}^2_{t+h|t} - \bar{\sigma}^2_{t+h|t-1}) - \lambda\rho^{2h}\sigma^2 \tag{6}$$

Revision in uncertainty is now a weighted average of most recent resolutions of shocks and its lagged counterpart. In particular, the second component is the information gain from most recent realization of the shock underweighted ($\lambda < 1$). The first component is the inefficient sourced from stickiness of updating The higher rigidity (lower λ), the efficieny gain or uncertainty reduction from one vintage of forecast to the next is less compared to its FIRE correspondents.

2.3.3 Summary of predictions of information rigidity

- Individual expectation may or may not change depend upon if updating.
- Individual variances changes non-monotonically depending on if updating. Always increase with arrival of new information.
- Population mean forecast responds shocks with lags.
- Forecast errors are serially correlated.
- Population disagreements rise in response to new shocks and return to steady state level gradually.
- Population average uncertaity revision is serially correlated and underreacts to volatility of the shocks.

[15] Make sure you have clarify the difference between revision and change.

2.4 Noisy information

A class of so-called noisy information model describes the expectation formation as a process extracting or filtering true fundamental state variable y_t from a sequence of realized signals. The starting assumption is that agent cannot observe the true variable perfectly. Unlike information rigidity model, it is assumed that agents keep track of the realizations of the signals instantaneously all the time.

We assume agent i observe two signals s^{pb} and s^{pr}_i, with s^{pb} being public signal common to all agents, and s^{pr}_i private signals being individual specific. The generating process of two signals are

$$\begin{aligned} s^{pb}_t &= y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2_\epsilon) \\ s^{pr}_{i,t} &= y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0, \sigma^2_\epsilon) \end{aligned} \tag{7}$$

We can stack the two signals into one vector $s_{i,t} = [s^{pb}_t, s^{pr}_{i,t}]'$ and $v_{i,t} = [\epsilon_t, \xi_{i,t}]'$. So in a compact form, it can be written as

$$\begin{aligned} s_{i,t} &= Hy_t + v_{i,t} \\ &\text{where } H = [1, 1]' \end{aligned} \tag{8}$$

In our general framework, the noisy information implies that the information set $I_{i,t}$ available to individual i at time t only includes past and recent realizations of the signals. The individual density forecast of y_{t+h} is

$$\widehat{f}_{i,t}(y_{t+h}|I_{i,t}) = \widehat{f}_{i,t}(y_{t+h}|s_{i,t}, s_{i,t-1}...) \equiv \widehat{f}_{i,t|t}(y_{t+h})$$

We use $t|k$ to denote the moments at time t based on information(signals) till time k.

Then we are ready to apply Kalman Filter in this context. The posterior distribution of y_t after seeing all signals till t is

$$\widehat{f}_{i,t|t}(y_{t+h}) \sim N(E_{i,t|t}(y_{t+h}), Var_{i,t|t}(y_{t+h})) \tag{9}$$

where the expectation and variances are functions of noisiness of signals and fundamentals. The expectation also depends on the realized values. But this is not the case for variance.

2.4.1 Individual moments

1. Expectation

   Now any agent trying to forecast future variables will have to form her expectation of the contemporaneous state variable, $E_{i,t|t}(y_t)$. Then the best h-period ahead forecast is simply iterated h periods forward based on the AR(1) process.

Thus, we first work out $E_{i,t|t}(y_t)$.

$$
\begin{aligned}
E_{i,t|t}(y_t) &= \underbrace{E_{i,t|t-1}(y_t)}_{\text{prior}} + P \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}} \\
&= (1-PH)E_{i,t|t-1}(y_t) + Ps_{i,t} \\
&= (1-PH)E_{i,t|t-1}(y_t) + PHy_{i,t} + Pv_{i,t} \\
\text{where the Kalman gain } P = [P_\epsilon, P_\xi] &= \Sigma^y_{i,t|t-1}H(H'\Sigma^y_{i,t|t-1}H + \Sigma^v)^{-1} \\
&\text{where } \Sigma^y_{i,t|t-1} \text{ is the variance of } y_t \text{ based on prior belief} \\
&\text{and } \Sigma^v = \begin{bmatrix} \sigma^2_\epsilon & 0 \\ 0 & \sigma^2_\xi \end{bmatrix}
\end{aligned}
\tag{10}
$$

The h-period ahead forecast is

$$
E_{i,t|t}(y_{t+h}) = \rho^h E_{i,t|t}(y_{t+h}) \tag{11}
$$

Individual forecast partially responds to new signals, i.e. $P < 1$. $P = 1$ is a special case when both signals are perfect thus $\Sigma^v = 0$, then the formula collapses to full rational expectation.

Now, the rigidity parameter is governed by $1 - PH$ with multiple signals. It is a function of variance of y from the prior of previous period and noisiness of the signals. Therefore, it is time variant as the variance is updated by the agent each period.

There are a few important distinctions between noisy information and sticky expectation.

• First, the persistence of expectation exists at individual level. There is serially correlation between $E_{i|t|t}(y_t)$ and $E_{i,t|t-1}(y_t)$, or more generally, between $E_{i,t|t}(y_{t+h})$ and $E_{i,t|t-1}(y_{t+h})$. This pattern can be only observed from population moments according to sticky expectation models.

To see this, the change in individual forecast from $t-1$ to t is

$$
\Delta E_{i,t|t}(y_{t+h}) = \underbrace{\rho^h(1-PH)\Delta E_{i,t-1|t-1}(y_t)}_{\text{Lagged response}} + \underbrace{\rho^h PH\Delta y_{i,t} + \rho^h P\Delta v_{i,t}}_{\text{Shocks to signals}} \tag{12}
$$

The serial correlation is $\rho^h(1-PH)$, it does not only depend on PH, but also the forecast horizon h. Therefore, one testable assumption is to see auto regression of change in forecast to see if the coefficient depends on horizon.

• Second, the expectation adjusts in each period as long as there is new information. In sticky expectation, however, the expectation adjusts only when the agent updates.

2. Variance

   The posterior variance at time t is a linear function of prior variance and variance of signals.

$$\Sigma^y_{i,t|t} = \Sigma^y_{i,t|t-1} - \Sigma^y_{i,t|t-1} H'(H\Sigma^y_{i,t-1}H' + \Sigma^v)H\Sigma^y_{i,t|t-1} \tag{13}$$

   There are a few important properties in the variance.

   - First, it does not depend on the realizations of the signal.

   - Second, it decreases unambigously from $t-1$ to t. To see this

$$\Sigma^y_{i,t|t} - \Sigma^y_{i,t|t-1} = -\Sigma^y_{i,t|t-1} H'(H\Sigma^y_{i,t-1}H' + \Sigma^v)H\Sigma^y_{i,t|t-1} < 0 \tag{14}$$

   These two properties carry through to the h-period ahead forecast as well. As the forecast variance is the following

$$Var_{i,t|t}(y_{t+h}) = \rho^{2h} \underbrace{Var_{i,t}(y_t)}_{\Sigma_{i,t|t}} + \sum_{s=0}^{h} \rho^{2s}\sigma^2_\omega \tag{15}$$

$$\Delta Var_{i,t|t}(y_{t+h}) = \rho^{2h}\Delta\Sigma_{i,t|t} - \rho^{2h}\sigma^2_\omega \tag{16}$$

   From t to $t+1$, when $h \geq 1$, the decline in variance come from two sources. The first source is the pure gain from the new signals, i.e. $\Delta\Sigma^y_{i,t|t}$. It is scalled by the factor ρ^{2h}. The second source is present in full information rational expectation model: as time goes from $t-1$ to t, there is a reduction of uncertainty about ω_t.

2.4.2 Population moments

1. Average forecast

$$\begin{aligned} \bar{E}_{t|t}(y_{t+h}) &= \rho^h[(1-PH)\underbrace{\bar{E}_{t-1}(y_{t+h})}_{\text{Average prior}} + P \underbrace{\bar{s}_t}_{\text{Average Signals}}] \\ &= (1-PH)\bar{E}_{t-1}(y_{t+h}) + P[\epsilon_t, 0]' \\ &= (1-PH)\bar{E}_{t-1}(y_{t+h}) + P\epsilon_t \end{aligned} \tag{17}$$

2. Change in average forecast

$$\Delta \bar{E}_t|t(y_{t+h}) = \rho^h(1-PH)\Delta \bar{E}_{t-1}(y_{t+h}) + \rho^h P \Delta \epsilon_t \tag{18}$$

   Same to the individual forecast, the change in average forecasts has serial correlation with the same auto regression parameter $\rho^h(1-PH)$.

3. Cross-sectional disagreements

   In this model, the only disagreements across agents come from the difference in realized private signals. Therefore, in short-cut, the disagreements are

$$\begin{aligned} Var_t(y_{t+h}) &= E((E_{i,t|t}(y_{t+h}) - \bar{E}_t(y_{t+h}))^2) \\ &= \rho^{2h} P_\xi^2 \sigma_\xi^2 \end{aligned} \tag{19}$$

   Several properties.

   • First, the disagreements increase with the forecast horizon. • Second, the disagreements depends on noisiness private signals, but not on that of public signals and the variance of the true variable y. • Third, similar to sticky expectation model, the disagreements also increase with the rigidity parameter P in this model.

4. Change in disagreements

$$\Delta Var_t(y_{t+h}) = \rho^{2h}(1-\rho^2) P_\xi^2 \sigma_\xi^2 > 0 \tag{20}$$

   The disagreements increase as time goes from $t-1$ to t. Also, as the time approaches $t+h$, the disagreements increase. This seems counterintuitive. But the reason is that here the disagreenments always exist simply because agents receive private signals, this disagreements is actually amplified as time goes forward.

5. Average variance

   Since the variance does not depend on signals and the precision is the same aross the agents, average variance is equal to the variance of each individual.

$$\bar{V}ar_t(y_{t+h}) = \bar{\Sigma}_t^y \tag{21}$$

   Also, same as the individual variance, the variance unambiguiously drop as time goes by.

$$\Delta Var_t(y_{t+h}) < 0 \tag{22}$$

2.4.3 Summary of predictions from noisy information

• Invididual expectation adjusts in each period, but only partially adjusts to new information.
• Unlike sticky expectation, slugishness in adjustment or serial correlation of adjustment exists in individual level. The correlation parameter decreases with forecast horizon, which is not the case in sticky expectation.
• Individual variance unambiguously drops each period as one approaches the period of realization. In sticky expectation, it increases regardless of updating or not.
• Population average forecast partially adjusts to news and has serial correlation as the individual level.
• Population disagreements rise in each period as time approaches the period of realization. Disagreetments will never be zero.
• Average variance declines unambiguously each period.

2.5 Other Theories

1. Rational inattention.
   - Chris Sim's model.[Sims, 2003] Information serves the role of uncertainty reduction measured by relative entropy. Agents optimally trade off the fixed cost of being attentative versus the gain from uncertainty gain.
   - Ricardo Reis's model. [Reis, 2006]
   - Xavier Gabaix's sparse matrix model. [Gabaix, 2014]
2. Epidemiologic view. [Carroll, 2003] Regardless of the microfoundations of the information rigidity, households infrequently get access to more rational-and-up-to-date expectations from professional forecasters with a poisson process.
3. Strategic behaviors. Second order belief, i.e., what you believe of what others believe. [Angeletos and Jennifer, 2009]

3 Empirical Results

3.1 Data

- **New York Fed Survey of Consumer Expectation**
  - Include not only perceived probabilities of binary event as in Michigan Survey does, but also elicit densitity forecasts.
- **Professional Forecasters**

A summary of the data information is as below.

Table 2: Information of Data

	SCE	SPF
Time period	2013-present	2007-present
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Aggregate Var in Density	1-yr and 3-yr ahead inflation	1-yr CPI and PCE
Individual Var in Density	1-yr earning growth	No
Pannel Structure	stay up to 12 months	average stay for 5 years
Demographic Info	Education, Income, Age, Location	Industry

3.2 Density Estimates

3.3 Test of Null Hypothesis of Rational Expectation

3.3.1 Replicating [?]

3.4 Professional Forecasters as a Benchmark

3.4.1 Replicating [Coibion and Gorodnichenko, 2012]

3.4.2 Additional Evidence from Uncertainty

3.5 Evidence from Households

4 Conclusion

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