A Discussion of *Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades*

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Highlights of the Paper

- **information cascade**: simply follows previous agent's action regardless of private signals
- not just localized conformity, but also sudden changes and reversal of the fashion
- an informational mechanism beyond traditional mechanisms
    - punishment of deviations
    - positive externality
    - conformity preference
    - communication

A Toy Model

- n agents;
- each one makes discrete choice of adoption or rejection;
- fixed cost of adoption C, set to be $1/2$;
- value of adoption being 1 and 0 with equal probability 1/2;
- general decision rule: adopt if $\pi \times V + (1 - \pi) \times 0 > 1/2$, where π is the posterior for $V = 1$, thus the pivotal posterior is $1/2$;
- imperfect singal for agent i is p generated based on following table, for instance $Prob(H|V = 1) = p$, assume $p > 1/2$, the signal is useful.

	High Signal	**Low Signal**
V=1	p	$1 - p$
V=0	$1 - p$	p

- individual signal conditionally independent;
- individual i can only observe previous actions;

Deision sequence

Each agent's action is a function of previous action and her own signal $a_i(A_{i-1}, s_i)$

- 1st adopts if H, rejects if L;
- 2nd:
  - if 1st adopts, 2nd adopts if H, indifferent if L;
  - if 1st rejects, 2nd rejects if L, indifferent if H;
- 3nd
  - if first two adopt, adopts **regardless of signal**
  - if first two reject, rejects **regardless of signal**
  - if one reject and one adopt, indifferent if one rejects and one adopts before him
- ...

In [2]:
```python
### Simulation
n_sim = 10      # nb of simulations
p= 1/2+0.000000001    # prob(H|V=1)
nb = 20   # nb of decision maker
#pb_cascade=[]
#pb_c_cascade=[]
#pb_w_cascade=[]
for i in range(n_sim):
    nb_adopt = 0
    adopt=[]
    sig=np.random.choice(2,nb,p)
    for ii in range(nb):
        adopt.append(adopt_rule(nb_adopt,ii,sig[ii],p))
        nb_adopt=nb_adopt+adopt_rule(nb_adopt,ii,sig[ii],p)
    print('Total number of adoption is '+str(nb_adopt))
    print('The sequence of decision is '+str(adopt))
```

```
Total number of adoption is 20
The sequence of decision is [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]
Total number of adoption is 2.0
The sequence of decision is [0, 0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0]
Total number of adoption is 0
The sequence of decision is [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0]
Total number of adoption is 17.0
The sequence of decision is [1, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1]
Total number of adoption is 20
The sequence of decision is [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]
Total number of adoption is 19.5
The sequence of decision is [1, 0.5, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1]
Total number of adoption is 19.5
The sequence of decision is [1, 0.5, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
```

```
1, 1, 1, 1, 1]
Total number of adoption is 20
The sequence of decision is [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]
Total number of adoption is 19.5
The sequence of decision is [1, 0.5, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1]
Total number of adoption is 20
The sequence of decision is [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1]
```

till the 3rd, conditional probability of information cascade

- Given V=1

$$\text{prob(correct cascade)} = \underbrace{p * p}_{\text{prob}(HH)} + \underbrace{p(1-p) * 1/2}_{\text{prob(HL but the 2st adopts)}}$$

$$\text{prob(wrong cascade)} = \underbrace{(1-p) * (1-p)}_{\text{prob}(LL)} + \underbrace{(1-p) * p * 1/2}_{\text{prob(LH but the 2st rejects)}}$$

correct cascade	no cascade	wrong cascade
$\frac{p(1+p)}{2}$	$p - p^2$	$\frac{(p-2)(p-1)}{2}$

...

till the (n+1)th, ex ante probability of information cascade

- cascade only happens after even numbers of people:
  - think about after 3 individuals: either it has happend after 2, or the third one is still using her own signal.
- in correct cascade after n people:

$$\underbrace{\text{prob(in correct cascade after n people)}}_{\mathbf{P}^n}=\underbrace{\text{prob(in correct cascade after n-2 people)}}_{\mathbf{P}^{n-2}}$$

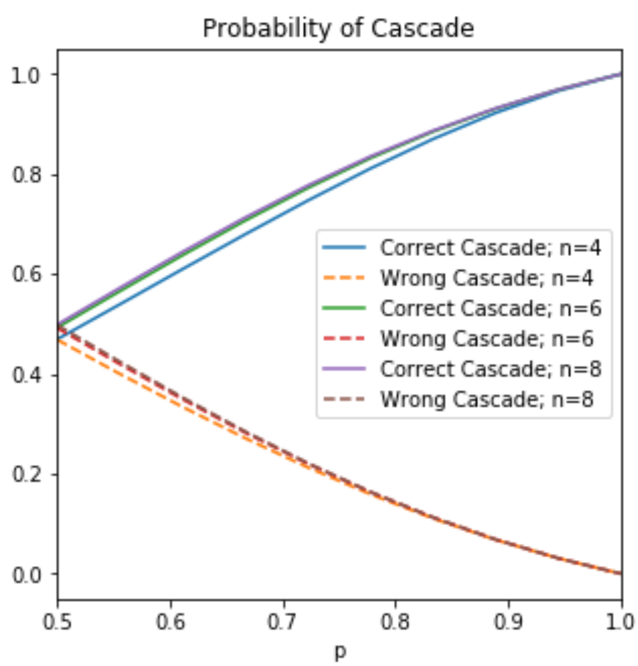
$$+\underbrace{\text{prob(not in correct cascade after n-2 pe}}_{1-\mathbf{P}^{n-2}}$$

correct cascade	no cascade	wrong cascade
$\frac{p(p+1)[1-(p-p^2)^{n/2}]}{2(1-p+p^2)}$	$(p-p^2)^{n/2}$	$\frac{(p-2)(p-1)[1-(p-p^2)^{n/2}]}{2(1-p+p^2)}$

In [3]:
```
# Plot
plt.figure(figsize=[5,5])

p = np.linspace(0.5001,0.9999,10)
n_list = np.array([4,6,8])

for n in n_list:
    prob_co = p*(p+1)*(1-(p-p**2)**(n/2))/2/(1-p+p**2)
    prob_wr = (p-2)*(p-1)*(1-(p-p**2)**(n/2))/2/(1-p+p**2)
    plt.plot(p,prob_co,label="Correct Cascade; n="+str(n))
    plt.plot(p,prob_wr,'--',label="Wrong Cascade; n="+str(n))
    plt.legend(loc=0)
    plt.xlabel('p')
    plt.xlim([0.5,1])
    plt.title("Probability of Cascade")
plt.show()
```



Questions

- how often information cascade occurs?
- what are those conditions?
- is it fragile?
- what factors reverse the cascade

Generalized Model

- n agents;
- a finite number of values: $v_1, v_2 .. v_l ... v_s$
- no trivial decision $v_1 < C < v_s$
- a finite number of signals: $x_1, x_2 ... x_q ... x_R$
- prior of value bing v_l is μ_l;
- conditional probability: $p_{ql} = prob(x_q | v_l)$
- Perfect Bayesian Equilibrium: sequentially rational and consistent.

Two conditions for which information cascade eventually arise as n goes to ∞

- **Condition 1** MLRO properties of signal and values.
$$\frac{p_{q+1,l+1}}{p_{q,l+1}} \geq \frac{p_{q+1,l}}{p_{q,l}} \quad \forall q < R$$

- **Condition 2** No long-run ties. $\forall q \; v_q \neq C$. As agent gets closer to true value, she is still not indifferent to adopting and rejecting.

Result 1.

If condition 1 and 2 hold, as $n \to \infty$, $\text{prob}(\text{information cascade occurs}) \to 1$

```
An information cascade eventually begins.
```

Result 2. Different precisions of signal(different p's in binary case)

- **Lower precision** of leader makes later followers better off. It makes later followers use their own information.

Fragility of Cascades

Result 3. public release of information

- Release of **noisy enough** public information before the first individual's decision can make some individuals worse off.

the depth of cascade

- Result 4. If all individuals' signals are drawn from the same distribution, then after the cascade has begun, all individuals welcome public information.
- Result 5. The realse of a **noisier public signal** than private information can shatter a long-lasting cascade.

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Millions of smoke need not discourage investigation of side effects of sm
oking.
```

- Result 6. If there is a non-zero probability of release of public information before everyone makes the decision, then eventually the population settles into the correct cascade.

reversal of cascades

- Probability of cascade reversal is higher than the probability of correct choice. Higher than the full information regime.

Examples

model assumptions

1. sequential actions
2. observe private signals plus previous actions
3. observable actions, not verbal communications
4. no other mechanisms enforcing conformity

model implications

1. local or idiosyncratic
2. fragility
3. some agents ignore private information

phenomenon

- Zoology
- Medical practice and scientific theory
- Finance
- Peer influence and stigma