How Do Agents Form Inflation Expectations? Evidences from the Forecast Uncertainty

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Abstract

Density forecasts of macroeconomic variables provide one additional moment restriction, uncertainty, for testing and exploring the implications of theories about how people form expectations differently from full-information rationality benchmark. This paper first documents the persistent dispersion in inflation uncertainty of professionals and households, and how it conveys different information from the widely used proxies to uncertainty such as cross-sectional disagreement and forecast errors. Second, utilizing the panel data structure of both surveys, I provide additional reduced-form test results as well as structural estimates for three workhorse theories of "irrational expectation": sticky expectation (SE), noisy information (NI), and diagnostic expectation (DE) by jointly accounting for its predictions for different moments. This is a natural extension of Coibion and Gorodnichenko (2012), which examines different moments separately. Also, motivated by the time-varying pattern of the uncertainty observed from surveys, I considers an alternative inflation process featuring stochastic volatility. These extensions allow me to match the joint dynamics of inflation and forecast moments in better goodness of fit. It also testifies how incorporating higher moments from survey data helps understand both the expectation formation mechanisms and inflation dynamics.

Keywords: Inflation, Expectation Formation, Rigidity, Overreaction

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1 Introduction

Theories on how agents form expectations in ways deviating from rational expectations (RE) have proliferated over the past decade. On one hand, various theories built upon different micro-foundations produce somewhat similar macro patterns. For instance, the expectation rigidity documented by Coibion and Gorodnichenko (2012, 2015), i.e. the sluggish response in aggregate expectations to new information, can be micro-founded by both Sticky Expectation(SE) and Noisy Information (NI). On the other hand, there are important and subtle differences in testable predictions from various theories regarding both individual forecasts and aggregate moments. For instance, in contrast with the models featuring expectation rigidity at the aggregate level, the theory of Diagnostic Expectation (DE) (Bordalo et al., 2018, 2020) predicts overreaction to the news at the individual level.

One of the crucial steps ahead in advancing this literature is to testing these theories better using expectation surveys. This paper undertakes a cross-moment estimation of each theory that jointly accounts for its predictions about different moments of the forecasts such as forecast errors, cross-sectional disagreements, and uncertainty. Although reduced-form tests focused on first moments have been powerful in rejecting the null of the full-information rational expectation (FIRE), identifying the differences among these non-FIRE theories require more information from second or higher moments and needs to rely on more restrictions across moments. Such an idea builds on the seminal work Coibion and Gorodnichenko (2012), which compares different theories by examining if the impulse responses of the inflation expectations to externally estimated shocks align with the qualitative predictions of these models.

With comparable estimates of different theories, I can evaluate to what extent each theory fits the observed expectation data and inflation dynamics. In addition, I can evaluate each model of expectation formation in terms of its sensitivity with respect to four criteria (1) what moments are used for estimation, i.e. only the forecast error or higher moments such as disagreements and uncertainty. (2) the specification of the underlying process of the inflation, i.e. an AR(1) with constant volatility or one with different components of the time-varying volatility. (3) if estimating the underlying process and expectations separately or jointly. The former basically recovers the inflation process only based on inflation data while the latter let the expectations provide information for estimating the inflation process. (4) if it accounts for different agents such as households and forecasters' expectations equally well.

This is not the first paper that estimates theories on expectation formation using cross-moment restrictions (For instance, Giacomini et al. (2020)). But the key novelty of this paper lies in utilizing an additional moment of forecasts in estimation, i.e. the uncertainty. Existing work that studies expectation formation utilizing survey data mostly focuses on the individual and cross-sectional patterns of the mean forecast and forecast errors. But there are additional insights from the forecast uncertainty, which is only available if each surveyee is asked to assign their own perceived probabilities to a range of values of the variable. For instance, the dispersion of uncertainty across individual forecasters, as that of the average forecasts, are inconsistent with the benchmark full-information rational expectation as the latter assumes that agents agree on the generating process of the data and have the common knowledge of available information. Dynamically, across different vintages of the forecast, the revision in uncertainty is a measure of information gain or the degree of forecasting efficiency. Besides, the relationship between uncertainty with other moments such as forecast errors and disagreements provides a way to check if the surveyed expectations are consistent with certain theories of expectation formation. In addition, although some theories have similar qualitative predictions about the forecasting moments, the configurations of the model parameters consistent with the theory may not be empirically realistic.

With the framework presented above at hand, this paper evaluates three workhorse theories on expectation formation and some hybrid versions seen in the literature. (1) Sticky expectation (SE). (2) Noisy information (NI). (3) Diagnostic expectations (DE). (4) Diagnostic Expectation/Noisy Information DENI).[1] One major distinction between the first two and DE is that the former predict underreaction due to rigidity and the latter predicts overreaction. The distinction between SE and NI are quantitative. The test of the two models relies upon which model can generate patterns consistent with the data with a realistic configurations of parameters. In particular, the estimation in this paper suggests that an unrealistic high degree of noisiness of the signals are needed for the rigidity in NI while a more realistic updating frequency is sufficient for the SE.

Other findings from the paper are the following.

- Overall, sticky expectation (SE) augmented with inflation of stochastic volatility (SV) fits the joint dynamics of inflation expectation and inflation better than other theories, for both households and professionals.
- Three theories perform indistinguishably well in matching dynamics of forecast errors but differ in matching dynamics of disagreement and uncertainty.
- All of the three theories' parameter estimates are not particularly sensitive to the use of moments in estimation.
- Theories differ in their sensitivity to if jointly estimating inflation process and to assumed inflation process. In both fronts, SE and DE have similar performances, but NI coefficient estimate changes substantially.
- Within each theory, households show a limited degree of consistency compared professionals. This evidence of inconsistency adds to the evidence rejecting full-information-rationality based on reduced-form tests.

Because this paper adds uncertainty to the moments used for estimation, another contribution of this paper is that I extend all of these theories on expectation formation to explore its respective implications for uncertainty. For theories whose benchmark model does not have distinctive predictions about uncertainty from the FIRE, such as DE, I explore the extensions of the model to characterize extrapolation beyond uncertainty. Then I explicitly derive predictions of various theories of expectation formation about the dynamics of uncertainty.

[1] For instance, Bordalo et al. (2020) embeds the DE in a NI model, which is called a "a diagnostic Kalman filtering problem".

In particular, in SE, the extra uncertainty in addition to the uncertainty associated with rational forecasting arises from non-updating of the most recent information. While in NI model, the uncertainty endogenously determines agents' degree of reaction to the news in the Kalman filtering problem. Different from the two, the state-of-art model of DE maintains the constant variance assumption. It poses a question if the extrapolation is also relevant in the second moment. Different models featuring information rigidity in incorporating new information predict the inefficiency in forecast revisions. This paper shows that such inefficiency can not only be seen in the average forecast, but also in the forecast uncertainty. This imposes additional restrictions on parametric configurations that produce the degree of rigidity seen in the data.

On the empirical side, besides structural estimation, I also provide additional tests of the null of full-information rational expectation using the revision of uncertainty. I first replicate the results of tests based on forecast errors using microdata of households and professionals following the existing literature. Then, I undertake autoregression-based tests for revisions in mean forecasts and uncertainty, respectively. The new result from this paper is that the revision in uncertainty has a serial correlation that is not consistent with the level of forecast efficiency predicted by rational expectation. This provides an additional result that rejects the null of the rational expectation hypothesis.

1.1 Related Literature

This paper is related to four strands of literature. First, it is realted to a series of empirical work directly testing and evaluating various theories on expectations formation using survey data. For instance, Mankiw et al. (2003), Carroll (2003), Branch (2004). More recent examples include Coibion et al. (2018) on firms' managers. In addition to testing particular sets of theories, there is also a number of papers that show people's expectations are driven by idiosyncratic demographics, cognitive abilities and macroeconomic histories experienced(Malmendier and Nagel (2015), Das et al. (2017) and D'Acunto et al. (2019), etc.). In terms of the methodology, this paper is closest to Giacomini et al. (2020), which estimates theories of expectation formation using cross-moments restrictions. However, all of these studies simply rely upon point forecasts instead of density forecast or surveyed uncertainty. This is one theme on which this paper differs from the existing literature.

Second, Manski (2004), Delavande et al. (2011), Manski (2018) and many other papers have advocated long for eliciting probabilistic questions measuring subjective uncertainty in economic surveys. Although the initial suspicion concerning to people's ability in understanding, using and answering probabilistic questions is understandable, Bertrand and Mullainathan (2001) and other work have shown respondents have the consistent ability and willingness to assign a probability (or "percent chance") to future events. Armantier et al. (2017) have a thorough discussion on designing, experimenting and implementing the consumer expectation surveys to ensure the quality of the responses[2]. Broadly speaking, the advocators have argued that going beyond the revealed preference approach, availability to survey data provides economists with direct information on agents' expectations and helps avoids imposing arbitrary as-

[2]Other literature includes Van der Klaauw et al. (2008) and Delavande (2014), etc.

sumptions. This insight holds for not only point forecast but also and even more importantly, for uncertainty, because for any economic decision made by a risk-averse agent, not only the expectation but also the perceived risks matter a great deal.

Third, by approximating subjective uncertainty directly from density responses, this paper contributes to the literature that develops and uses a variety of measures of uncertainty, especially in the macroeconomic context. There is a long tradition of approximating uncertainty by measures that are more directly available in survey data or that can be estimated by econometric methods. For instance, Bachmann et al. (2013) use ex-ante disagreement and ex-post forecast errors computed from forecasters' surveys as proxies of uncertainty. Jurado et al. (2015) define the time-varying uncertainty as conditional volatility of the unforecastable component of a variable and estimate it using multiple macroeconomic series. Binder (2017) approximate uncertainty from rounding in survey data based on the insights from cognitive literature. Besides, the text-based approach such as Bloom (2009) constructs indices of policy uncertainty based on texts of newspaper reporting. Although these proxies are all meant to capture the notion of uncertainty, as shown in Section 2.3, cross-validation seems to suggest they are statistically uncorrelated or even negatively correlated.

Fourth, the literature that has been originally developed under the theme of forecast efficiency provides a framework analyzing the dynamics of uncertainty useful for the purpose of this paper. The focus of the forecasting efficiency literature is evaluating forecasters' performance and improving forecasting methodology, but it can be adapted to test the theories of expectation formation of different types of agents. This is especially relevant to this paper as I focus on the uncertainty.

The paper is organized as followed. Section 2 shows the stylized patterns of different forecasting moments of professional forecasts and households. Section 3 first sets up a unified framework in which testable predictions of different theories can be compared. Also, I derive a various moment conditions from these theories. Section 4 undertakes reduced-form time-series regressions that test the null hypothesis of FIRE and implications of different theories. Section 5 includes results from estimating the theory-specific parameters using simulated method of moments. It also evaluates the sensitivity of the model specification. Section 6 concludes the paper and discusses the plan for the next step.

2 Data and Facts

2.1 Definition and notation

An agent i is forming expectations about a stochastic macroeconomic variable y_{t+h} [3], the inflation in this paper. Denote $f_{t+h|t}$ as agent i's h-period-ahead density forecast. $f_{i,t+h|t}$ is the conditional density of y_{t+h} given the information set $I_{i,t}$ available at time t.

$$f_{i,t+h|t} \equiv f_{i,t}(y_{t+h}|I_{i,t})$$

[3] Only in the context of aggregate variable, it makes sense to study the population moments such as average expectations and disagreements. Studying expectations of idiosyncratic variables requires individual panel data, as well as the idiosyncratic realizations of the variable.

The information set could be agent specific, thus it has subscript i. The specific content contained in I_t varies from different models of expectation. For instance, sticky expectation (SE) and rational inattention literature all assume that agents are not able to update new information instantaneously. So the information set may not contain the most recent realization of the variable of forecast y_t. In contrast, NI assumes that the information set only contains noisy signals of the underlying variables. Different theories may also differ in terms of the mapping from information to conditional density forecasts. [4] For instance, DE deviates from Bayesian learning by allowing agents to assign updating weights based on the salience or representativeness of the new information.

Accordingly, h-period-ahead mean forecast at t, denoted as $y_{i,t+h|t}$, is the conditional expectation of y_{t+h} by the agent i.

$$y_{i,t+h|t} \equiv E_{i,t}(y_{t+h}) = \int f_{i,t+h|t} dy_{t+h}$$

Similarly, individual forecasting variance $Var_{i,t+h|t}$, hereafter termed as individual uncertainty in this paper, is the conditional variance corresponding to the forecast density distribution.

Individual forecast error $FE_{i,t+h|t}$ is the difference of individual forecast at time t and ex post realized value of y_{t+h}. By definition. positive(negataive) forecast errors mean overpredict (underpredict) the variables.

$$FE_{i,t+h|t} = y_{i,t+h|t} - y_{t+h}$$

The population analogs of individual mean forecast, uncertainty and forecast errors are simply the average of the individual moments taken across agents. Denote them as $\bar{y}_{t+h|t}$, $\overline{Var}_{t+h|t}$, and $\overline{FE}_{t+h|t}$, respectively. Hereafter, they are termed as the population mean forecast, population uncertainty and population forecast error, respectively. In addition, disagreement is defined as the cross-sectional variance of mean forecasts of individual agents, denoted as $\overline{Disg}_{t+h|t}$.

To slightly abuse the terminology, I refer to the 3 individual indicators and 4 population indicators defined above as moments and they are listed in Table 1.

[INSERT TABLE 1 HERE]

2.2 Data

The focus of this paper naturally restricts my options of the survey data to use compared to other empirical literature of testing theories of expectation formation. The surveys that have elicited density forecasts of macroeconomic variables for a sufficiently long period are rare. Rarer, for the purpose of the paper, is the data structure that

[4] There are other theories that fall into this category. underlying models each agent uses to form the conditional density of the variables by agent i. For instance, Patton and Timmermann (2010) finds that the disagreements are driven by not the only difference in information but also heterogeneity in prior and models. More theoretical work includes multi-prior or model uncertainty such as Hansen and Sargent (2001), Hansen and Sargent (2008), etc.

allows for comparing the revisions across vintages that have a fixed terminate date of realization, either in individual level or aggregate level.

Survey of Professional Forecasters(SPF) meet both criteria thanks to the density forecasts of a number of macroeconomic variables, including core CPI and core PCE inflation they have started eliciting since 2007, as well as a series of GDP deflator dating back to 1968. This paper focuses on the first two, for which both forecasts for current-year inflation, basically nowcast, and one-year-ahead forecast are based on densities. This makes it possible to directly test the implications of the revisions in uncertainty across different vintages of forecasts.

The New York Fed Survey of Consumer Expectation(SCE) that started in 2013 meet the first criteria and half of the second. In particular, households are asked to provide their perceived probabilities about 1-year-ahead and 3-year-ahead inflation for various ranges of values[5]. This allows for comparing 3-year-ahead forecast at time $t-3$ with 1-year-ahead forecast at $t-1$. Since the maximum duration for households to stay in the panel is 12 months (for about one-third of the households), forecast revision can be only examined in the population level. The advantage of SCE compared to SPF is its monthly frequency. This provides an invaluable chance to explore the dynamics of uncertainty. A summary of the two surveys is in Table 2.

Converting expressed probability forecasts based on externally divided bins into an underlying subjective distribution requires a density distribution. I follow Engelberg et al. (2009) to estimate the density distribution of each individual surveyee for SPF. Answers with positive probabilities assigned to three bins is fit with a generalized beta distribution. Depending on if there is open-ended bin on either side with positive probability, 2-parameters or 4-parameters of the beta distribution are estimated. For those with only two bins with positive probabilities and adjacent, it is fit with a triangular distribution. For only one bin with probability, a uniform distribution over the positive probability bin is fit. See my online appendix for the detailed steps of estimation of python codes. This is the same approach adopted by the New York Fed researchers Armantier et al. (2017) for SCE and directly provided their estimate of uncertainty. I directly use them.

To avoid the biases introduced by extreme answers or data errors, I undertake some winsorization for both data set. For SPF, I drop the outliers of mean forecast and uncertainty estimates at both the top and bottom one percentile as these are typically abnormals that are due to measurement errors or other reasons. For SCE, I drop the top and bottom 5 percent of mean forecasts and uncertainty as households mean forecast are inclined to give extreme values. All the results in this paper are robust to a different threshold such as 10 and 1 percentile. [6]

[INSERT TABLE 2 HERE]

Throughout the paper, I use three measures of inflation: headline CPI, core CPI, and core PCE . Depending on the specific variable of forecast in the survey series, the realization of the corresponding inflation is used to compute moments such as

[5] Most importantly, they are kindly reminded that all the probabilities need to add up to one. The online questionnaire interface is designed such restriction is imposed.

[6] For mean forecasts and uncertainty, respectively, this means dropping 6528 and 5096 observations, out of 68887 observations in total.

forecast errors. Specifically, SPF has density forecasts for both core CPI and core PCE [7]. For SCE, as the households responders are asked about the overall inflation, It is the most appropriate to be interpreted as headline CPI inflation. To simplify the expression, from now on, core CPI and core PCE are simply referred to as CPI and PCE, respectively.

2.3 Stylized facts

Although the sheer magnitudes of the differences between professionals' moments and those of households are so big that a direct comparison of the two seems redundant, their respective within-agent-type correlation serves a ready checking device of some statistical consistency. Figure 1a, 1b, and 1c plot the population uncertainty against realized inflation, forecast errors, and disagreements in the first, second and third rows, respectively.

It is widely documented in literature [8] and resonated by anecdotal narratives that high inflation is typically associated with high inflation uncertainty. However, the observable correlation of realized inflation and the directly estimated average uncertainty from both professionals and households are at most weakly positive during the period between 2007-2019. In particular, the correlation coefficients 0.14, 0.12 and -0.24 for SPF's CPI forecast, SPF's PCE forecast, and SCE's CPI expectation. This may suggest that during a period of persistently low and stable inflation, the conventional positive redux of inflation and uncertainty is not a good description of the relation of the two.

Figure 1b looks into the relationship between size of the forecast error and uncertainty. Although according to our benchmark framework in Section 3, there is no mathematical correlation between the size of the forecast errors and uncertainty as the former depends on the realized shocks and the later depends on the volatility of the shock, it is worth checking if in the data a greater ex-ante uncertainty implies bigger ex post forecast errors. The correlation coefficients of the two are -0.19, -0.18 and 0.27 for SPF CPI forecasts, SPF PCE forecasts, and SCE's forecasts, respectively. The two indicators turn out to be negatively correlated for professional forecasts. Only households forecasts exhibit such a positive correlation.

Figure 1c examines the relationship between disagreement and uncertainty. Many empirical literature in macroeconomics use disagreements and uncertainty as if they are similar concepts[9]. Such confusion in practice is partly due to the difficulty of finding appropriate measures of uncertainty in the first place. But as we have presented in 3, the two are concepts with distinct statistical definitions.[10] The empirical pattern

[7]SPF also has density forecasts for GDP deflator(for GNP prior to 1992: Q1) going back to 1968. Since the ranges of the values and the definition are not consistent over time, I do not use it in this paper.

[8]The list literature showing positive inflation and uncertainty is long. For example Ball et al. (1990).

[9]For instance, Bachmann et al. (2013) used ex-ante disagreement and ex-post forecast errors as two measures of uncertainty and find that both uncertainty indicators lead to a reduction in real economic activity.

[10]This point was made very clearly by Zarnowitz and Lambros (1987). Manski (2018) also points out many empirical work has confused the dispersion with uncertainty.

of the two, as shown in the plots, also confirms that the two are different objects. The correlation coefficients of the two turn out to be negative, -0.31 and -0.37 for SPF forecasts. In stark contrast, households seem to bear a strong correlation of 0.43 between the two moments.

Overall, professional forecasters' moments exhibit patterns more consistency from a statistical point of view. In contrast, the positive correlation across households' ex-ante uncertainty, ex-post forecast errors, and cross-section disagreements cannot be easily reconciled by the framework we set up in Section 3.

Persistent disagreement in expectations has been used as important stylized evidence inconsistent with the assumption of identical expectation embedded in FIRE (Mankiw et al. (2003)). A similar fact-checking can be done with respect to individual uncertainty. FIRE predicts individual share an equal degree of uncertainty. In contrast, SE predicts that uncertainty of individuals differ in that agents are not equally updated at a point of the time (Equation 8). NI allows for the possibility of homogeneity in uncertainty only under the stringent conditions of an equal precision of signals and the same prior for uncertainty (Equation 22). Therefore, the presence of dispersion of uncertainty across agents is not consistent with predictions from FIRE.

Figure 2 plots the median inflation expectation along with its 25/75 percentiles in the left and its counterpart in uncertainty in the right column. Not only there is long-lasting dispersion in individual forecasts, i.e. disagreement, but also notable heterogeneity in uncertainty across agents. And not surprisingly, the dispersion of both forecasts and uncertainty of households are both of a much greater magnitude than that of the professionals. The 25/75 inter-quantile-range of households point forecasts is 4-5 percentage points compared to 1 percentage point of professionals. And the IQR of the uncertainty of households is around 150-200 times(12-14 times for standard deviation) as that of professional forecasters.

Besides, in terms of the distribution of uncertainty, there are households/professional difference and similarity. Households' uncertainty is more skewed toward the right (higher uncertainty), meaning there is a wide dispersion in the high values of uncertainty. This can be also seen in Figure 3, where I plot the kernel density estimated distribution of uncertainty by year. What is common for both types of agents is that the dispersion in uncertainty is persistent over time and do not show much time-variation [11].

Another pattern worth discussing in Figure 2 is that there is a notable rise in the dispersion of professional forecasts in the recent 2-3 years, primarily driven by an increase of upper side of the forecast (i.e. 75 percentile forecast increases from 1% to 2%). [12] This is consistent with the observation in the top left two graphs of Figure 3 that the distribution of inflation forecasts in recent years have become flattened.

FIRE also predicts an unambiguous reduction in uncertainty as one approaches the date of realization, where the drop is exactly equal to the volatility of the realized shocks. Although quantitatively it is hard to check this, one can look if the distribution of the uncertainty revision concentrates in the negative range. Figure 4 plots

[11] Kumar et al. (2015) also presents the dispersion in uncertainty using a shorter-period of sample for SCE.

[12] This should be interpreted with caution since the disagreements of SPF forecasts shown in Figure 1c actually exhibits a gradual decline.

the average revision in mean forecasts and uncertainty from 1-year-ahead forecast in year $t-1$ to the current-year nowcast in year t. The more negative range in which the revision lies, the more "rational" of the forecast. Looking from the histograms, uncertainty revision shows left-skewness relative to zero. This implies on average, forecasters feel more certain for her nowcasts relative to her forecast made one year before. Unfortunately, since SCE does not provide the data structure for this purpose, I cannot make a comparison between two types of agents. A formal test of revision equal to zero or being negative will be carried out in Section 4.

[INSERT FIGURES 1a, 1b, 1c HERE]

3 Theories of Expectation Formation

We start by assuming an underlying process of the inflation. In the benchmark scenario, we assume the underlying true process of y_t is $AR(1)$ with persistence parameter $0 < \rho < 1$ and i.i.d. shock ω_t.

$$y_t = \rho y_{t-1} + \omega_t \tag{1}$$

$$\omega_t \sim N(0, \sigma^2_\omega)$$

The assumption of i.i.d. shock with constant volatility turns out insufficient to match the joint dynamics of inflation and observed forecast moments. Therefore, as an extension, I also consider an alternative assumption that inflation consists of two unobservable components with stochastic volatility. I leave specifying this explicitly for later so as to focus on comparing different models of expectation formation under a simple AR(1) process.

3.1 Benchmark of full-information rational expectation(FIRE)

In the FIRE benchmark, it is assumed that all agents perfectly observe y_t at time t and understand the true process of y. Therefore, the individual forecast is $\rho^h y_t$, which is shared by all agents. Therefore, it is also equal to the average forecast.

Both individual and population forecast errors are simply the realized shocks between $t+1$ to $t+h$.

$$\overline{FE}^*_{t+h|t} = -\sum_{k=0}^{h-1} \rho^k \omega_{t+h-k} \tag{2}$$

I use superscript of $*$ to denote all the moments according to FIRE. It is easy to see that the forecast error is orthogonal to information available till time t. This provides a well known null hypothesis of FIRE.

The second implication from FIRE here is that forecast errors of non-overlapping horizon are not correlated. (Equation 3). For instance, forecast error at time t and that at time $t+h$ or further are not serially correlated. This is not the case within h periods as the realized shocks in overlapping periods enter both forecast errors. These FE-based restrictions of FIRE provide the foundations for the tests used in Section 4.

$$Cov(\overline{FE}^*_{t+h|t}, \overline{FE}^*_{t+s+h|t+s}) = 0 \quad \forall s \geq h \tag{3}$$

Concerning uncertainty, the first simple implication by FIRE is that all individual shares the same degree of uncertainty. The uncertainty about future y simply comes from uncertainty about unrealized shocks between t and $t+h$. With the same model in mind (Equation 1) and the same information y_t, everyone's uncertainty is equal to the weighted sum of the future volatility before its realization (Equation 4). In FIRE, there are neither disagreements about mean, nor disagreements about the uncertainty [13].

$$\text{Var}^*_{t+h|t} = \sum_{s=1}^{h} \rho^{2s}\sigma^2_\omega \tag{4}$$

The time series behavior of h-year-ahead uncertainty, i.e. $\text{Var}_{t+h|t}$, $\text{Var}_{t+h+1|t+1}$, etc, depends on the true process of y. Specifically, it depends if σ^2_ω is time-varying. If time-invariant, h-period-ahead uncertainty is simply as constant. In baseline case, I make such an assumption, in general it may not be true. In the later section, I make the alternative assumptions of the inflation process allowing for stochastic volatility.[14]

The testable implication of rationality lies in the revision of uncertainty. Hereafter, we refer revision (instead of change) as the difference of moments across vintages of the forecast with the fixed terminate date of realization. For instance, the difference between the uncertainty about y_{t+h} at time t and the uncertainty about y_{t+h} at time $t-1$.

Moving from t to $t+1$, for instance, revision in uncertainty is simply a negative constant independent from the time. There is an unambiguous reduction in uncertainty (or information gain in the forecasting literature) as more and more shocks have realized.

$$\begin{aligned} \overline{\text{Var}}^*_{t+h|t+1} - \overline{\text{Var}}^*_{t+h|t} &= -\rho^{2h}\sigma^2_\omega \\ &= \rho(\overline{\text{Var}}^*_{t+h|t} - \overline{\text{Var}}^*_{t+h|t-1}) \end{aligned} \tag{5}$$

Lastly, FIRE has predictions about disagreements. As agents perfect update the same information, there is no disagreement at any point of the time.

$$\overline{Disg}^*_{t+h|t} = 0 \quad \forall t \tag{6}$$

[13]This is the same to Jurado et al. (2015)'s terminology.

[14]For example, Justiniano and Primiceri (2008), Vavra (2013) on time-varying volatility of inflation.

3.2 Sticky Expectation (SE)

The theory of sticky expectation (Mankiw and Reis (2002), Carroll (2003) etc.), regardless of various micro-foundations, builds upon the assumption that agents do not update information instantaneously as they do in FIRE. One tractable assumption is that there is a homogenous Poisson rate λ of updating among the population. Specifically, at any point of time t, each agent learns about the up-to-date realization of y_t with the probability of λ; otherwise, it forms the expectation based on the most recent up-to-date realization of $y_{t-\tau}$, where τ is the time experienced since the last update.

Denote the mean forecast of a non-updater since $t-\tau$ as $y_{i,t+h|t-\tau}$ since her forecast conditions upon the information up till $t-\tau$.

$$y^{se}_{i,t+h|t-\tau} = \rho^{h+\tau} y_{t-\tau} \tag{7}$$

Now her information set is not up to date, the uncertainty to a non-updater is higher than an updater and it increases with the duration of non-updating τ.

$$\text{Var}^{se}_{i,t|t-\tau} = \sum_{s=1}^{h+\tau} \rho^{2s} \sigma^2_\omega \tag{8}$$

FIRE basically assumes $\tau = 1$ for all the agents and all the time, namely all agents' last update takes place in the previous period. So setting $\tau = 1$ in the above equation gives the uncertainty in FIRE.

In the individual level, the key difference between FIRE and SE is that the later does not reduce uncertainty as efficiently as in the former primarily because of the rigidity incorporating new information. But note that the rigidity in updating according to SE cannot be systematically observable in the individual level, both in terms of forecasts errors and uncertainty. This is because the behaviors of each individual forecast specifically depend on if she updates or not in that period.

Relying upon the law of large numbers, one can derive testable predictions about population moments that allow us to conduct tests of sticky expectation and recover rigidity parameter λ.

One well-known prediction from SE is that the average forecast is a weighted average of update-to-date rational expectation and lagged average expectation as reproduced below. [15] It can be also expressed as a weighted average to all the past realizations of y. Setting $\lambda = 1$, then the SE collapses to FIRE and the average forecast is equal to y's long-run mean of zero.

$$\begin{aligned} \bar{y}^{se}_{t+h|t} &= \lambda \underbrace{y^*_{t+h|t}}_{\text{rational expectation at t}} + (1-\lambda) \underbrace{\bar{y}^{se}_{t+h|t-1}}_{\text{average forecast at } t-1} \\ &= \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^\tau \rho^{h+\tau} y_{t-\tau} \end{aligned} \tag{9}$$

[15] See Coibion and Gorodnichenko (2012) or appendix of this paper for detailed steps.

It is easy to show that the average forecast errors are serially correlated (Equation 10). Setting $\lambda = 1$, SE collapses to FIRE, as seen in Equation 2, in which there is no serial correlation between forecast errors and it fully responds to newly realized shocks at time t.

$$\overline{FE}^{se}_{t+h|t} = (1-\lambda)\rho\overline{FE}^{se}_{t+h-1|t-1} - \lambda\sum_{k=1}^{h}\rho^k\omega_{t+k} \tag{10}$$

Regarding uncertainty, average uncertainty at any point of time is now a weighted average of uncertainty to agents whose last updates have taken place in different periods of past. Since at any point of the time, there are agents who have not updated the recent realization of the shocks, thus with higher uncertainty, the population uncertainty is unambiguously higher than the case of FIRE.

$$\begin{aligned}
\overline{\mathrm{Var}}^{se}_{t+h|t} &= \sum_{\tau=0}^{+\infty} \underbrace{\lambda(1-\lambda)^\tau}_{\text{fraction of non-updater until } t-\tau} \underbrace{\overline{\mathrm{Var}}^{*}_{t+h|t-\tau}}_{\text{uncertainty of most recent update at } t-\tau} \\
&= \sum_{\tau=0}^{+\infty}\lambda(1-\lambda)^\tau\sum_{s=0}^{h+\tau}\rho^{2s}\sigma^2_\omega \\
&\geq \overline{\mathrm{Var}}^{*}_{t+h|t}
\end{aligned} \tag{11}$$

With respect to revision, the inefficiency of reducing uncertainty in SE takes the following form in the aggregate level. Since not all agents incorporate the recently realized shocks, the revision in average uncertainty exhibits serial correlation described in Equation 12. It is a weighted average of the resolution of uncertainty from the most recent shocks and its lagged counterpart.

$$\overline{\mathrm{Var}}^{se}_{t+h|t+1} - \overline{\mathrm{Var}}^{se}_{t+h|t} = (1-\lambda)(\overline{\mathrm{Var}}^{se}_{t+h|t} - \overline{\mathrm{Var}}^{se}_{t+h|t-1}) - \lambda\rho^{2h}\sigma^2 \tag{12}$$

In particular, the second component is the information gain from the most recent realization of the shock underweighted by $\lambda < 1$. The first component is the inefficiency sourced from the stickiness of updating. The higher rigidity (lower λ), the smaller the efficiency gain or uncertainty reduction compared to in FIRE.

Lastly, SE also predicts non-zero disagreements and sluggish adjustment compared to FIRE. This is because of different lags in updating across populations.

$$\overline{Disg}^{se}_{t+h|t} = \lambda\sum_{\tau=0}^{\infty}(1-\lambda)^\tau(y_{t+h|t-\tau} - \bar{y}_{t+h|t}))^2 \tag{13}$$

From time t to $t+1$, the change in disagreements comes from two sources. One is newly realized shock at time $t+1$. The other component is from people who did not update at time t and update at time $t+1$. [16].

[16] Coibion and Gorodnichenko (2012) derive the impulse response of dispersion at time $t+k$ to a shock that realized at t. Disagreements increase after realization of the shock and gradually returns to its steady-state level. $\rho^{2(h+k)}(1-\lambda^{k+1})\lambda^{k+1}\omega^2_t$

3.2.1 Summary of moment conditions of SE

- Average forecast errors are zero mean across time but positively serially correlated and have a higher variance across time than in FIRE.
- Population disagreements are positive on average across time, positive serial correlation, and higher variance than the FIRE case.
- Population average uncertainty is higher than FIRE and its revision under-reacts to the volatility of the shocks.

3.3 Noisy information(NI)

A class of models (Lucas Jr (1972), Sims (2003), Woodford (2001), etc.), noisy information(NI) hereafter, describes the expectation formation as a process of extracting or filtering true variable y_t from a sequence of realized signals. The starting assumption is that the agent cannot observe the true variable perfectly. Unlike SE, it is assumed that agents keep track of the realizations of signals instantaneously all the time.

We adopt a general framework like in Coibion and Gorodnichenko (2015) and assume that agent i observe two signals s^{pb} and s^{pr}_i, with s^{pb} being public signal common to all agents, and s^{pr}_i private signals being individual specific with subscript i. The generating process of two signals is assumed to be the following.

$$\begin{aligned} s^{pb}_t &= y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2_\epsilon) \\ s^{pr}_{i,t} &= y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0, \sigma^2_\xi) \end{aligned} \tag{14}$$

Stacking the two signals into one vector $s_{i,t} = [s^{pb}_t, s^{pr}_{i,t}]'$ and $v_{i,t} = [\epsilon_t, \xi_{i,t}]'$, the equations above can be rewritten as

$$\begin{aligned} s_{i,t} &= H y_t + v_{i,t} \\ &\text{where } H = [1, 1]' \end{aligned} \tag{15}$$

Now any agent trying to forecast future y has to form her expectation of the contemporaneous y. Denote it as $y^{ni}_{i,t|t}$, which needs to be inferred from the signals particular to agent i. The agent's best h-period ahead forecast is simply iterated h periods forward based on the AR(1) process and it is equal to $\rho^h y_{i,t|t}$. This is the same as FIRE.

What is different from FIRE is that the agent makes her best guess of y_t using Kalman filtering at time t. Specifically, the mean forecast of individual i is the posterior mean based on her prior and realized signals $s_{i,t}$.

$$\begin{aligned} y^{ni}_{i,t|t} &= \underbrace{y^{ni}_{i,t|t-1}}_{\text{prior}} + P \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}} \\ &= (1 - PH) y^{ni}_{t|t-1} + PH y_t + P v_{i,t} \end{aligned} \tag{16}$$

where the Kalman gain P is a vector of size of two that determines the degrees of reaction to signals.

$$P = [P_\epsilon, P_\xi] = \text{Var}_{i,t|t-1} H (H' \text{Var}_{i,t|t-1} H + \Sigma^v)^{-1} \tag{17}$$

$\text{Var}_{i,t|t-1}$ is the forecast variance of y_t based on prior belief till the previous period $t-1$. And Σ^v is a 2-by-2 matrix indicating the noisiness of the two signals.

$$\Sigma^v = \begin{bmatrix} \sigma^2_\epsilon & 0 \\ 0 & \sigma^2_\xi \end{bmatrix} \tag{18}$$

Individual forecast partially responds to new signals as $PH < 1$. $PH = 1$ is a special case when both signals are perfect thus $\Sigma^v = 0$, then the formula collapses to FIRE.

A comparable parameter with $1 - \lambda$ in SE that governs rigidity in NI is $1 - PH$. It is a function of previous period uncertainty about y_t and noisiness of the signals determined by Σ^v. Note beyond steady state, P is time-variant as the variance is updated by the agent each period. Since the volatility of inflation process is deterministic under AR(1), we can focus on the steady state Kalman gain corresponding to a constant variance. Therefore, I drop time t from P.

What differentiates average forecast from individual's is the role played by private signals. On average, private signals cancel out across agents, therefore, only public signals enter the average forecast, thus, average forecast errors (Equation 19).

$$\begin{aligned} \bar{y}^{ni}_{t+h|t} &= \rho^h [(1 - PH) \underbrace{\bar{y}^{ni}_{t+h|t-1}}_{\text{Average prior}} + P \underbrace{\bar{s}_t}_{\text{Average Signals}}] \\ &= (1 - PH) \bar{y}^{ni}_{t+h|t-1} + P\epsilon_t \end{aligned} \tag{19}$$

Kalman filtering also updates the variance recursively according to the rule of normal updating. The posterior variance at time t is a linear function of uncertainty in the previous period and variance of signals.

$$\text{Var}^{ni}_{i,t|t} = \text{Var}^{ni}_{i,t|t-1} - \text{Var}^{ni}_{i,t|t-1} H' (H \text{Var}^{ni}_{i,t|t-1} H' + \Sigma^v)^{-1} H \text{Var}^{ni}_{i,t|t-1} \tag{20}$$

The unconditional nowcasting variance can be solved as the steady-state value of the Equation 21 above. In the steady-state, there is no heterogeneity across agents in forecasting uncertainty and the nowcasting uncertainty becomes a constant. Thus we can drop the subscript i. The unconditional average uncertainty increases with the noisiness of the two signals as well as the volatility of inflation itself. Its closed-form solution is given in the Appendix.

Beyond steady-state, the Equation 20 also directly gives the revision in uncertainty from time $t-1$ to t. The newly arrived information, although noisy, still brings about information gains, thus leading to an unambiguously drop in uncertainty. But due to the signal is not perfect, i.e. $\Sigma^v \neq 0$, there is inefficiency in reducing uncertainty compared to FIRE.

$$\mathrm{Var}^{ni}_{i,t|t} - \mathrm{Var}^{ni}_{i,t|t-1} = -\mathrm{Var}^{ni}_{i,t|t-1} H'\left(H \mathrm{Var}^{ni}_{i,t|t-1} H' + \Sigma^{v}\right)^{-1} H \mathrm{Var}_{i,t|t-1} < 0 \tag{21}$$

In order for it to be directly comparable with the revision in uncertainty in FIRE (Equation 5) and SE (Equation 12), the nowcasting uncertainty about y_t needs to be converted to h-period-ahead forecasting uncertainty. This is simply to add the nowcasting uncertainty discounted by ρ^{2h} to the uncertainty about future unrealized shocks.

$$\mathrm{Var}^{ni}_{i,t+h|t} = \rho^{2h} \mathrm{Var}^{ni}_{i,t|t} + \sum_{s=1}^{h} \rho^{2s} \sigma^2_{\omega} \geq \mathrm{Var}^{*}_{t+h|t} \tag{22}$$

As a result, the revision in h-period-ahead uncertainty from $t-1$ to t only partially reacts to the resolution of uncertainty from newly realized shock ω_t in the past period.

NI also predicts non-zero disagreement in the presence of private signals. The size of the disagreement increases with the noisiness of the private signals.

$$\overline{Disg}^{ni}_{t+h|t} > 0 \tag{23}$$

At the same time, different from SE, there is no serial correlation in disagreements in NI. This is because the dispersion of private signals is time-invariant and independent across periods. Since the disagreement in forecast only comes from the disagreement in nowcastings, NI also implies that the disagreements is smaller for longer forecast horizons.

3.3.1 Summary of moment conditions from NI

- Individual and population forecast errors are zero on average across time but have positive serial correlation and higher variance than that in FIRE.
- The steady-state/unconditional uncertainty of both individual and population are higher than FIRE and increase with the noisiness of private and public signals. Uncertainty drops when the forecast horizon decreases, i.e. negative uncertainty revision, but the reduction is smaller in size than in FIRE due to the underreaction to noisy signals.
- Population disagreement is positive on average across time and it increases with the size of the volatility of private signals. There is no serial correlation in disagreement across time but it has higher variance across periods.

3.4 Diagnostic Expectations (DE)

Different from the previous two theories featuring informational rigidity, diagnostic expectation (Bordalo et al. (2018)) introduces an extrapolative mechanism in expectation formation that results in overreactions to the news (Bordalo et al. (2020)). Both

SE and NI deviate from FIRE in terms of the information set available to the agents, while DE deviates from FIRE in terms of the processing of an otherwise fully updated information set.

Skipping over its micro foundation, the following equation captures the essence of DE's mechanism. Each individual i's h-period-ahead forecast consists of two components. The first component can be considered as a rational forecast based on the fully updated y_t. The second component corresponds to the potential overreaction to the unexpected shock to inflation at time t, i.e. the forecast error. The degree of overreaction is governed by the parameter θ. The premise of DE models is that $\theta \geq 0$. The forecast collapses to the FIRE when $\theta = 0$. Any $\theta > 0$ implies cases in which the agent revises her forecast overly to the realized forecast error in the same direction.

$$\bar{y}^{de}_{i,t+h|t} = \rho^h y_t + \theta_i(\rho^h y_t - \bar{y}^{de}_{i,t+h|t-1}) \tag{24}$$

There is no room for disagreement with homogenous degree of overraction. To account for the possibility of a positive disagreement, I assume θ to be different across different agents, therefore, I add the subscript i to the parameter. Since agents are equally informed about the realizations of the variable, the only room for disagreement to rise is either difference in the initial information set or the degree of overreaction differs across people. To capture this, I assume θ_i to follow a normal distribution across the population with a variance of σ^2_θ. So the DE model has two parameters. Disagreement increases with the dispersion of overreaction term governed by σ^2_θ.

Expressing the equation above in terms of forecast errors, one can immediately see the forecast errors have negative serial correlation.

$$FE^{de}_{i,t+h|t} = FE^{*}_{t+h|t} - \theta_i \rho FE^{de}_{i,t+h-1|t-1} \tag{25}$$

And at population level, the average forecast errors takes a similar form except that the overreaction parameter is now the population average θ.

$$\overline{FE}^{de}_{t+h|t} = \overline{FE}^{*}_{t+h|t} - \theta\rho\overline{FE}^{de}_{t+h-1|t-1} \tag{26}$$

Finally, as to the uncertainty, since the mechanism of extrapolation in DE does not change the agent's perceived distribution of the future shocks, benchmark DE theory predicts the forecast uncertainty to remain the same as the rational expectation.

$$\overline{Var}^{de}_{t+h|t} = \overline{Var}^{*}_{t+h|t} \tag{27}$$

3.4.1 Summary of moment conditions from DE

- Average forecast errors are zero on average across time but have **negative** serial correlation and higher variance than FIRE.
- The average uncertainty is equal as that in FIRE unless the extrapolation also takes place in forecast uncertainty.
- Population disagreements are positive on average across time only if there is heterogeneity in the degree of overreaction. But there is no serial correlation.

3.5 Diagnostic Expectation (DE) augmented with Heterogeneous Information (DENI)

Bordalo et al. (2020) embeds heterogeneous information in a standard DE model. Their motivation is primarily to generate cross-sectional disagreement in forecasts because the baseline version predicts zero-dispersion unless heterogeneity in the degree of overreaction is introduced in the first place as we did in the previous section. The framework is essentially a hybrid of the NI and DE. It maintains the assumption regarding how agents overreact to new information at individual levels but the information is no longer the real-time realization of the variable but a noisy signal of it. Since the mechanism is explicitly discussed in previous sections, I omit the detailed derivations here.

3.5.1 Summary of moment conditions from DENI

- Population forecast errors are zero on average across time. The sign of the auto-correlation depends on the relative size of the positive serial correlation induced by rigidity and negative one induced by overreaction.

- Average uncertainty is now the same as NI due to noisiness of the signal and it increases with the noisiness of signals and volatility of inflation per se.

- Disagreement is positive on average due to the presence of private information. It also has positive auto-correlation and higher variance across time than FIRE.

3.6 An inflation process with stochastic volatility (SV)

This section considers an alternative inflation process for which the volatility of shocks to the inflation is stochastic, following the unobservable components/stochastic volatility (UCSV) model in Stock and Watson (2007). The purpose is twofold. First, the time-varying pattern of observed forecast uncertainty seen in Section 2 seems to suggest that the assumption of constant volatility is deficient. Second, it allows me to evaluate the sensitivity test of each theory of expectation formation with respect to the alternative underlying process of the forecast variable.

In particular, UCSV assumes that the inflation has an unobserved permanent and a transitory component.

$$\begin{aligned} & y_t = \zeta_t + \eta_t, \quad \text{where } \eta_t = \sigma_{\eta,t}\xi_{\eta,t} \\ & \zeta_t = \zeta_{t-1} + \epsilon_t, \quad \text{where } \epsilon_t = \sigma_{\epsilon,t}\xi_{\epsilon,t} \\ & \log \sigma^2_{\eta,t} = \log \sigma^2_{\eta,t-1} + \mu_{\eta,t} \\ & \log \sigma^2_{\epsilon,t} = \log \sigma^2_{\epsilon,t-1} + \mu_{\epsilon,t} \end{aligned} \tag{28}$$

The shocks to levels of the two components and their volatility are drawn from following distributions, respectively. The only parameter of the model is γ, which determines the smoothiness of the time-varying volatility.

$$\begin{aligned} \xi_t &= [\xi_{\eta,t}, \xi_{\epsilon,t}] \sim N(0, I) \\ \mu_t &= [\mu_{\eta,t}, \mu_{\epsilon,t}]' \sim N(0, \gamma I) \end{aligned} \tag{29}$$

The information set necessary for forecasting is different in SV from in AR(1) process. Consider first the benchmark case of FIRE. At time t, the FIRE agent sees the most recent and past realization of all stochastic variables as of t, including y_t, ζ_t, η_t, $\sigma_{\eta,t}$, $\sigma_{\epsilon,t}$. Using $*sv$ stands for FIRE benchmark under the stochastic volatility assumption.

$$\overline{y}^{*sv}_{t+h|t} \equiv y^{*sv}_{t+h|i,t} = \zeta_t \tag{30}$$

Under FIRE, forecast error is simply the cumulative sum of unrealized permanent and transitory shocks from t to $t+h$, which is equal to

$$\overline{FE}^{*sv}_{t+h|t} = \sum_{s=1}^{h} (\eta_{t+s} + \epsilon_{t+s}) \tag{31}$$

Disagreement is zero across agents in FIRE. h-step-ahead conditional variance, or the forecast uncertainty is time-varying as the volatility is stochastic now.

$$\begin{aligned} \overline{Var}^{*sv}_{t+h|t} \equiv Var^{*sv}_{t+h|i,t} &= \sum_{k=1}^{h} E_{i,t}(\sigma^2_{\eta,t+k}) + E_{i,t}(\sigma^2_{\epsilon,t+k}) \\ &= \sigma^2_{\eta,t} \sum_{k=1}^{h} exp^{-0.5k\gamma} + \sigma^2_{\epsilon,t} exp^{-0.5h\gamma} \end{aligned} \tag{32}$$

Under the sticky expectation (SE), an agent whose most recent up-do-date update happened at $t-\tau$ only have seen the realizations of y, ζ, η, σ_η, σ_ϵ till $t-\tau$. Her forecast is hence the permanent component at $t-\tau$.

$$y^{sesv}_{t+h|i,t-\tau} = \zeta_{t-\tau} \tag{33}$$

The lag in updating is also reflected in a higher forecast uncertainty than in FIRE.

$$Var^{sesv}_{t+h|i,t-\tau} = \sigma^2_{\eta,t-\tau} \sum_{k=1}^{h+s} exp^{-0.5k\gamma} + \sigma^2_{\epsilon,t-\tau} exp^{-0.5(h+\tau)\gamma} \tag{34}$$

The population average of the two are, respectively, a weighted average of people whose the most update was in $t, t-1 ... t-s, t-\infty$, respectively. The key difference in SV from AR(1) is that the average uncertainty exhibits positive serial correlation under SV. Expectations being sticky further increases the positive serial correlation compared to that in FIRE due to the lag in updating of the shocks to the volatility. The predictions regarding both forecast errors and disagreements under SV are the same.

Under noisy information (NI), the agent at time t needs to forms her best real-time nowcast for the permanent component ζ_t using noisy signals, $\bar{\zeta}_{t|t}$, via Kalman filtering. We assume again that the noisy signals of ζ_t consists of a public signal s_t^{pb} and a private signal $s_{i,t}^{pr}$ around the true realization of ζ_t.

$$y_{t+h|t}^{nisv} \equiv y_{t+h|i,t}^{nisv} = \bar{\zeta}_{t|t} \tag{35}$$

The prediction regarding forecast errors and disagreements remain the same as in AR(1). What is different under time-varying volatility is that there is no steady-state Kalman gain and uncertainty independent from time because the underlying volatility of the variable is time-varying. This also implies that the rigidity induced by the noisiness of information is state-dependent. At each period, the agents in the economy will update their forecast based on the realized volatility. In periods with high (low) fundamental volatility, the Kalman gain from noisy signals is larger (smaller) thus the agents will be more (less) responsive to the new information. There is no such state-dependence of rigidity in SE.

The mechanisms of DE and DENI exactly mimic that under AR(1) except that the average volatility is time varying now. Therefore, I leave the derivations in the Appendix.

3.7 Comparing Theories

We summarize the distinctions across different theories in their moment conditions. The three theories all predict average population FEs across time to be zero and have higher variances than in FIRE. The distinction lies in that two rigidity models SE and NI imply a positive serial correlation while DE implies a negative serial correlation. DENI allows the two forces to counteract with each other. Therefore, the sign of the serial correlation is ambiguous.

In terms of disagreement across agents, both SE and NI predict non-zero disagreement, positive serial correlation, and higher volatility. Non-zero disagreement can only arise in DE only if heterogeneity in overreaction is introduced. But this still leads to a zero time serial correlation. The prediction of DENI is the same as that of NI.

Forecast uncertainty also contains useful distinctions in moment conditions across theories. In particular, although SE and NI both predict higher uncertainty than FIRE, the magnitudes of that depend on the size of structural parameters in the respective model. In contrast, baseline DE predicts equal uncertainty to FIRE. DENI has similar predictions as NI.

The general takeaway from these is that not only the first moment, i,e. forecast errors but also higher moments, disagreement and uncertainty contains restrictions to identify the model parameters within each theory. We will utilize these moment conditions to estimate each theory in Section 5.

4 Reduced-form Tests of Full-Informational Rationality

This section first reproduces a number of statistical tests of FIRE seen in existing literature in Table 3, primarily following Mankiw et al. (2003), and then extends the tests relying on uncertainty in Table 4 and 5, in the spirit of forecasting efficiency by Nordhaus (1987). It is an extension of revision tests on mean forecasts by Fuhrer (2018) to the average uncertainty.

The first set of tests, hereafter, referred to as FE-based tests, utilize the moment restrictions on forecast errors. In plain words, the null hypotheses of the three tests are the following. First, since the forecasts are on average unbiased according to FIRE, forecast errors across agents should converge to zero in a large sample. Second, forecast errors of non-overlapping forecasting horizon are not serially correlated (Equation 3). Third, forecast errors cannot be predicted by any information available at the time of the forecast, including the mean forecast itself and other variables that are in the agent's information set. This follows from Equation 2. In addition, I include what is called weak version of the FE-based test which explores the serial correlation of forecast errors in overlapping periods, i.e. 1-year-ahead forecasts within one year. The forecast errors are correlated to the extent of the realized shocks in the overlapping periods. So the positive serial correlation does not directly violate FIRE. But the correlation of overlapping forecast errors still contains useful information about the size of the realized shocks.

FE-based tests results are presented in Table 3. Individual-level data are used thanks to the panel structure of both surveys. Since test 2 and 3 requires individual forecasts in vintages that are more than one year apart while SCE only surveys each household for 12 months, the two tests are done for only SPF forecasts of CPI and PCE. Also, the regressions are adjusted accordingly depending on the quarterly and monthly frequency of SPF and SCE. Since these regressions are based on 1-year inflations in overlapping periods, white standard error is computed for hypothesis testing.

First, all three forecast series easily reject the unbiasedness test at the significance level of 0.1%. There is upward bias across professional forecasters and households [17], while unsurprisingly, the bias is almost 20 times of that for professional forecasters(2.2 versus 0.12) for headline CPI.

Second, the point forecast one year ago predicts the forecast errors for professionals in the significance level of 0.1%. For headline CPI inflation, for instance, one percentage point inflation forecast corresponds to 0.3 percentage points of the forecast errors one year later. Thus test 2 in Table 3 easily rejects the second hypothesis test of FIRE that past information does not predict future forecast errors.

Third, forecast errors can be predicted by forecast errors one year ago with a significant coefficient of around 0.08 for headline CPI and 0.05 for PCE, as seen in test 3 of Table 3. Errors of non-overlapping forecasting periods are correlated, against the null of FIRE.

Lastly, test 4 in Table 3 presents a higher serial correlation of forecast errors produced within a year. For SPF forecasts, the serial correlation does not exist beyond

[17] Coibion et al. (2018) finds the same upward bias for firms' managers.

1 quarter, implying relative efficiency of forecasts. For the households, the forecast errors are more persistent over the entire year in that current forecast errors are correlated with all past forecast errors over the past three quarters. Again, although the persistence of 1-year forecast errors within one year does not directly violate FIRE, the fact that households' forecast errors being more persistent than professionals provide useful clues about the relative rigidity of the two types of agents.

[INSERT TABLE 3 HERE]

The second batch of tests in Table 4 focus on estimating forecasting efficiency using revisions of mean forecasts and uncertainty, hereafter referred as revision-based tests. In plain words, the revision from 1-year-ahead forecast to nowcast of current-year inflation is efficient according to two criteria (1) Forecast revision does not depend on past information, including the past revisions. (2) Drop in uncertainty is sufficiently rapid to reflect the uncertainty of all realized shocks.

The mean revision test by Fuhrer (2018) takes the following form (using 1 period as an example):

$$y_{i,t+1|t+1} - y_{i,t+1|t} = \alpha + \beta(y_{i,t+1|t} - y_{i,t+1|t-1}) + \epsilon_{i,t+1} \tag{36}$$

In the above equation $\beta = 0$ according to FIRE, because rational forecast revision only responds to newly realized shocks thus it is not predictable by past revisions.[18]. Since we have four vintages of the forecasts from SPF, the above specification can include lagged revisions up to 4 quarters.

The test with uncertainty simply replaces the revision of forecast with revision in uncertainty $\sigma^2_{i,t+1|t+1} - \sigma^2_{i.t+1|t}$, for instance. This regression follows from Equation 12 for SE and Equation 21 for NI. Although it cannot be directly used as a test against FIRE null, the autocorrelation coefficient speaks to the speed of the drop in uncertainty. Depending on the model, one can interpret it as the particular structural parameter of rigidity.

The top panel in Table 4 presents the results for the mean forecast. Following Fuhrer (2018), I include the median forecast available at time t and $t-1$ as an indicator of past information for the revision regression. In the first column of each panel, I report the regression on a constant.

What's surprising is the mean revision in forecast being negative and significant. Forecasts on average make downward adjustment of 1.26 percentage points of CPI and 1.1 percentage points of PCE from her previous year forecast of the same-period inflation.

The second to fourth columns of each panel in Table 4 checks autocorrelation of revisions including different lags. Revisions of forecasts are serially correlated over 4 quarters and the coefficients are all positive and significant. Also, the median forecasts as the past information always predict a negative revision with significant coefficients. This is evidence against the null hypothesis of FIRE and my estimates are comparable with those by Fuhrer (2018).

[18] Adding $y_{t+1|t}$ to both sides of Equation 36 gives an equivalent null hypothesis used by Fuhrer (2018): coefficient of regression of $y_{t+1|t+1}$ on $y_{t+1|t}$ is $1 - \beta = 1$.

The bottom panel reports autoregression results for revision in uncertainty. Again, the first column first test the mean revision against the null being zero. The mean revisions in uncertainty are both negative (0.5-0.6 percentage points equivalence in standard deviation of uncertainty) and statistically significant, confirming our observation from Figure 4 that forecasters are more certain about current inflation compared her previous year forecast.

The second to fourth column shows a positive serial correlation of revision in uncertainty for both CPI and PCE forecasts. The revision to CPI seems more efficient as serial correlation is with only one-quarter lag. For PCE, the revisions in uncertainty are serially correlated with all past three quarters.

[INSERT TABLE 4]

Table 5 presents the results with the revision replaced with change in mean forecasts and uncertainty, i.e. from $y_{t|t-1}$ to $y_{t+1|t}$. As we have discussed in Section 3, the auto-correlation of change in mean forecast and uncertainty do not bear testable predictions from FIRE. But if the forecasts and uncertainty are persistent in its first difference, it may imply that the agent does not react to the news and newly realized shocks sufficiently enough. In addition, the auto-correlation regressions of this kind is a useful characterization of the time series dynamics of forecasts. With the variable being the first difference, the panel structure of SCE and SPF allows for calculating changes in individual levels for greater sample size, especially for households. Besides autoregression, I also report the constant estimate of the changes in the first column of each sub-panel.

The most noticeable pattern for both professionals and households, and for both mean forecast and uncertainty is that past change predict future changes with universally negative coefficients across different horizons. Most of the negative coefficients are statistically significant in the 1% level. For instance, one percentage point of increase in SPF's CPI forecast in the previous quarter predicts around 0.29 percentage points decrease in the next quarter. This negative correlation is smaller in size but remain significant into further past, i.e. -0.24 for two quarters lag and -0.1 for three quarters lag. Data on SCE is monthly, so lags are included up to six months. The negative correlation between past and current changes are all negative and significant for households. The sizes of the correlation coefficients are comparable with professional forecasts when the monthly coefficients are converted to their quarterly average, i.e. -0.3 to -0.4.

Such an auto-correlation of change in mean forecast is very much reflected in the same regression for uncertainty, as reported in the bottom panel of Table 5. For SPF of CPI and PCE, respectively, one unit increase in uncertainty about 1-year-ahead inflation in previous quarter predicts around a 0.39 and 0.44 unit of the drop in the next quarter. The effect holds up to two quarters for professionals and 5 months for households.

These evidence suggest that both the mean forecast and uncertainty of individuals are mean-reverting. An essentially equivalent explanation is that both series are realizations of noisy signals around their respective long-run mean. This will lead to the exact negative correlation of first differences we have seen. The mean-reverting patterns might also explain what is observed from Figure 3, according to which, there are no significant changes in the distribution across different years.

The second noticeable result lies in the constant regressions reported in the first column of each sub-panel in Table 5. It implies that households constantly lower their mean forecasts as well as uncertainty from month to month, while professional forecasts do not behave in such a pattern. In particular, the constant regression of the change in the mean forecast for SCE gives an estimated coefficient of -0.05 which is significant in the 5% level. Individual households' 1-year-ahead inflation expectations keep being downward adjusted each month compared to their previous answer. What is more interesting is that their uncertainty about 1-year-ahead inflation also decreases each month. The size of the downward adjustment is -1.39 unit and statistically significant in the level of 0.1%. This negative significant and constant coefficient remains throughout all auto-regressions, implying it is not driven by time-varying changes.

The most natural explanation for this that repeatedly surveyed households have become more informative about inflation over time. Given the unconditional forecast errors of inflation by households are positive, a downward adjustment of inflation stands for a less-biased forecast. [19]

[INSERT TABLE 5]

In summary, the major additional insights that arise from the empirical tests of this section is that rigidity of incorporating new information in forming expectations imply noticeable inefficiency of revisions in forecasts and drop in uncertainty.

5 Model Estimation and Sensitivity Analysis

5.1 SMM Estimation

The reduced-form tests in Section 4 are sufficient in rejecting the null hypothesis of FIRE. But there are two limitations with these tests in terms of identifying differences among non-FIRE theories. First, the coefficient estimates from the reduced-form regression cannot always be mapped into a structural parameter of the particular model. Second, even if it does so, the tests fall short of simultaneously utilizing all the restrictions across moments implied by a particular non-FIRE theory, as discussed in great detail in Section 3. In this section, I undertake a structural estimation that jointly accounts for cross-moment restrictions.

Since many of the moment conditions cannot be easily derived as a closed-form function of parameters, I adopt the simulated method of moment (SMM). In a nutshell, the estimation chooses the best set of model parameters by minimizing the weighted distances between the data moments and the model-simulated moments. For a given

[19] The possibility that the surveys' information set is influenced by the survey itself is a double-edged sword. On one hand, this poses a methodological challenge to the survey designers of expectations as to if surveys can objectively elicit the "true" expectations held by the respondents. On the other hand, researchers can use the survey as a meaningful intervention tool to identify the effect of factors such as information provision and attention. Recent examples of this type of research includes: Coibion et al. (2018) for firms and Coibion et al. (2019) for households.

process of inflation, and a particular theory of expectation formation, the vector of the parameters estimates is defined as the minimizer of the following objective function.

$$\widehat{\Omega}^o = \underset{\{\Omega^o \in \Gamma^o\}}{argmin}(M_{\text{data}} - F^o(\Omega^o, H))W(M_{\text{data}} - F^o(\Omega^o, H))'$$

where Ω^o stands for the parameters of the particular pair of theory of expectation and inflation process, i.e. $o \in \{se, ni, de, deni\} \times \{ar, sv\}$. Γ^o represents the corresponding parameter space respecting the model-specific restrictions. M_{data} is a vector of the unconditional moments that is computed from data on expectations and inflation. F^o is the simulated model moments under the theory pair o. W is the weighting matrix used for the SMM estimation. I report estimation results using the 2-step feasible SMM approach, in which the inverse of the variance-covariance matrix from the 1st-step estimation using identity matrix is used as the W in the second step, which has been shown to give asymptotically efficient estimates of the model parameters.

Crucially, notice that the model-implied moments F^o are not just a function of model parameters Ω^o, but also a function of the corresponding information set available to the forecasters. I use H to represent the historical realizations of the variables in the agents' information set that are used as the inputs for forecasts. Although the real-time history is the same across models, the mapping between the history to data moments depends on model specifics. For instance, although the real-time inflation is the only variable in the information set for AR(1) process, information set in SV contains both permanent component of the inflation and the realized levels of volatility. Since the different components are not directly observed from historical data, I estimate it using the Markov Chain Monte Carlo (MCMC) procedure developed by Stock and Watson (2007) in this context.

It is also important to mimic the information set that were truly available to the agents at each point of the time in history. Therefore, I use the real-time data on historical inflation that was publicly available at each point of the time instead of the latest release of the historical data, since it is well known among macroeconomists that the later typically incorporates many rounds of revisions over time. I obtain the data from the Real-Time Data Research Center hosted by Philadelphia Fed[20].

The estimation is also specific to the choices of moments included in computing the distances. I focus on the unconditional moments of (independent from time) at the population level defined in Table 1. In particular, they include the mean (FE), variance($FEVar$) and auto-covariance($FEATV$) of population forecast error, the mean($Disg$), variance($DisgVar$) and auto-covariance($DisgATV$) of disagreement, and the mean (Var), variance($VarVar$) and auto-covariance($VarATV$) of uncertainty. When the joint estimation is done, two moments of inflation series are used, the variance($InfVar$) and auto-covariance($InfVar$).

The model implied moment conditions also implicitly depend on the parameters of the inflation process for a given model. This point is illustrated well in Bordalo et al. (2020). For instance, the observed overreaction in DE is lower for an AR(1) process with higher persistence. In recovering the model parameters associated with the expectation formation, it is important to take into account the information contained in

[20]https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/.

expectation data regarding the process of inflation per se. To handle this, I undertake both 2-step and joint estimation. The former refers to the first externally estimating the inflation process and then estimate expectation formation separately treating the inflation parameter as given. The latter refers to jointly estimating parameters of inflation and expectation.

These alternative specifications of the estimation also serve as a model sensitivity analysis with respect to following criteria: (1) different choices of moments; (2) AR(1) and SV for the process of inflation. (3) two-step and jointly. (4) for both professionals and households. A reasonable theory of expectation formation ought to be relatively robust to these criteria.

5.2 Moments matching and parameter estimates

I first focus on the professionals' expectations as a benchmark since professionals are supposed to be more rational. A first look at the data moments of SPF already provides clues as to the degree of the deviations from FIRE benchmark predictions laid out in Section 3. When separately estimated, the core CPI inflation during 1995-2019 followed an AR(1) process with the persistence parameter $\hat{\rho} = 0.99$ and volatility $\hat{\sigma} = 0.22$. The FIRE benchmark predicts zero forecast errors, zero disagreement, an uncertainty level identical to the volatility of inflation shocks, and zero time-variation and auto-correlation of these population forecasting moments. But in the data, quarterly professional forecasts core CPI inflation has a mildly positive average forecast errors 0.09 percentage points, a mild degree of disagreement of 0.2, and an uncertainty of 0.08, larger than the inflation volatility $0.04 \approx 0.22^2$ estimated from the same period. These are indeed deviations from FIRE predictions. But at the same time, it is worth noting that the variance of all forecast moments and their auto-covariance of between two consecutive years (non-overlapping horizons) are all close to zero, implying that the deviation is not severe. These altogether suggest that the most important source of identification among FIRE alternatives of expectation formation may come from persistent disagreement and inflation uncertainty.

Table 6 presents the SMM estimates of different model parameters for professionals. For each theory, I estimate the theory both in two steps or jointly using expectations and inflation data. Different rows within each panel reports the estimates depending on various choices of moments used for estimation: forecast errors only, forecast error and disagreement, and the two plus uncertainty.

[INSERT TABLE 6 HERE]

5.2.1 Cross-moment consistency within a theory of expectation

As discussed in Section 3, each theory of expectation formation has distinctive predictions about the unconditional moments of forecast error, disagreement, and uncertainty. Cross-moment restrictions help identify the parameter values for the particular theory. Comparing the sensitivity of parameter estimation from utilizing information from different moments serves as an over-identification test.

All theories under consideration in this paper have parameter estimates that are broadly consistent within the theory no matter what moments are used, although additional moments do cause variation in the magnitudes of the model parameters. This pattern is the most salient in the estimation for professional forecasts in Table 6.

For SE, as the top panel of the table shows, the estimated annual updating rate is 0.35-0.36 across different combinations of the moments, suggesting information updating every four months. Such a medium degree of information rigidity seems to reflect the counteracting evidence both for and against the stickiness of the updating. On one hand, average forecast errors with higher variance (0.13) than the inflation volatility $0.23^2 \approx 0.04$, and the positive disagreement of 0.20 and higher uncertainty of 0.08 than inflation volatility are all in line with the patterns of the information rigidity. On the other hand, the little serial correlation in average forecast errors and disagreements is inconsistent with the predictions of SE.

For NI, various cross-moment estimations point to an estimated noisiness of the public signals σ_{pb} between $0.77-2.18$ and that of private signals σ_{pr} between $0.94-2.56$. These are highly noise signals compared to the conditional standard deviation of the inflation shock 0.23. since forecast errors at the population level could be due to both public and private noises and their consequent under-reaction to the news of equal degree, forecast-error moments only are not sufficient to identify the two model parameters separately. The estimation based on FE and Disg suggests both signals have a standard deviation above 2 percentage points. But incorporating information from uncertainty reduces the noisiness of both signals by half from above 2 to less than 1, leading to more sensible estimates. By assumption of NI, the higher noisiness of the signals and higher inflation volatility result in higher steady-state uncertainty in NI, as shown in the Equation 20. This suggests that the uncertainty reveals a lower degree of information incompleteness and information rigidity endogeoustly generated due to underreaction to the noisy information. Another observation worth making is that throughout specifications, public signals have higher precision than private ones. This confirms that professional forecasters are highly attentive to macroeconomic news.

DE estimates demonstrate good within-model consistency across specifications, although the results suggest under-reaction instead of overreaction as in the canonical DE. With only information from forecast error, the estimated overreaction parameter of DE θ is around -0.36 on average for all forecasters. At the same time, the estimated dispersion in the overreaction parameter is 0.68, which implies only a small fraction of the population does overreact to the news to inflation ($\theta > 0$). Information from higher moments such as disagreement helps better identify the population dispersion in the degree of overreaction since the heterogeneity in responsiveness to the news is the only source of disagreement in DE. The small size of disagreement only needs to be matched by a more modest degree of heterogeneity in the overreaction parameter, a standard deviation of the θ of 0.39 for a similar mean estimate of -0.48. With additional moments from uncertainty, the estimates stay similar. This is not surprising because DE does not have identification information for the two model parameters. These findings are consistent with the rigidity/underreaction seen in the population level(Coibion et al. (2018)) in contrast to the overreaction seen at the individual level (Bordalo et al. (2018)).

Compared to the previous theories, DENI sees less cross-moment consistency for professional forecasts. With the non-identification with only forecast error moments, additional information from disagreement helps narrow parameters to a sensible range of values of DENI. As dispersion of private signals serves as the source of the positive disagreement, the overreaction parameter could "focus on" matching the forecast errors and their variation. This reveals a positive overreaction parameter θ around 0.05 while a noisiness of both public and private nosiness of signals of 1.5 percentage points. This is in line with the original premise of the DENI model as in Bordalo et al. (2020), which reconciles both overreactions to news and a positive disagreement. However, incorporating information from uncertainty again leads to a negative overreaction parameter estimate $\theta = -1.87$. Uncertainty revealed from professional forecasts leads to a lower estimate of the noisiness of both signals to 0.19 and 0.73 respectively, which further implies higher responsiveness to the news. Therefore, the behaviors of forecast errors need to be again matched by some level of underreaction to the news instead of overreaction.

5.2.2 Interaction between expectation formation and inflation dynamics

The specific mechanisms of expectation formation could also interact with the underlying process of inflation. With our benchmark AR(1) process, both the persistence of the shock to inflation ρ and the overall volatility of the inflation shock σ determine what the FIRE forecasts moments should be and model-specific forecasting moments are not only a function of the model parameters but also the inflation process parameters itself. The differences between 2-step estimation and joint estimation reveal such inter-dependence. I impose a loose lower bound for the persistence parameter ρ to be 0.9 to limit the possibility the overfitting.

For SE, letting professional forecasts reveal information about the inflation process leads to a less persistent and less volatile AR(1) process. The joint estimate of the persistence parameter ρ hit the externally set lower bound 0.9 and lower inflation volatility of $0.13-0.16$, compared to 0.99 and 0.23 when estimated separately. The joint estimation with disagreement or uncertainty under such a less volatile process generates an update rate estimate of $0.62-0.64$, implying a degree of the rigidity half of that from separate estimation. This suggests that the underlying information rigidity could have been lower if the inflation process is less persistent and less volatile. This is consistent with the underlying mechanisms of SE. Think about an extreme case where shocks to inflation are purely transitory, i.e. $\rho = 0$, the infrequent updating of the realized shocks will not lead to any differences in forecasts and the uncertainty will be exactly equal to the inflation uncertainty. It means that less rigidity is required in the expectation formation to justify the observed level of disagreement and uncertainty. Another way of interpreting such changes is via the question of how expectation moments shed light upon the inflation process itself. Since both disagreement and uncertainty increase with the inflation volatility σ, such change in estimates seem to suggest that the low disagreement and uncertainty as shown by professionals need to be justified by simply a lower inflation volatility.

NI shows more sensitivity toward the estimation procedure. Although across different moments, joint-estimate reveals exactly the same estimate of the persistence parameter $\rho = 0.99$, the revealed inflation volatility and noisiness of signals all see sizable changes. For instance, the inflation volatility σ is estimated to be zero in joint estimation and the nosiness of signals becomes smaller with FE+Disg moments used and bigger when information from uncertainty is also used. This reflects some horse races between the noisiness of the signals and the volatility of inflation itself. This is not surprising because, in NI models, both shocks to inflation itself or to the signals contribute to forecast errors and forecast uncertainty. Although it is ex-ante clearly distinguishable from the point of view of the modeler, the distinction between a shock to inflation itself and simply to signals may be indistinguishable for the agents who try to form the forecasts. These results suggest that the data-implied information rigidity according to NI could be potentially sensitive to the assumed volatility to inflation itself.

In terms of the sensitivity toward the inflation process itself, DE estimates perform rather well, although the results reveal underreaction instead of overreaction as shown in the separate estimation. Letting DE-consistent expectation to speak to the inflation process itself leads to a lower persistence ($0.9 - 0.92$) and lower volatility ($0.12 - 0.15$) of inflation, but the average overreaction parameter θ remain in the range of -0.36 to -0.48 and its cross-sectional dispersion σ_θ stays in the range of $0.39 - 0.56$. Across moments conditions, most of the forecasters under-react to the news.

DENI performs the worst in terms of its sensitivity against the estimation procedure, implying it is very dependent on the persistence and volatility of the inflation process itself. Taking the most likely credible estimate that utilizes information from all moments, a mild degree of underreaction $\theta = -0.11$, a very precise public signals and a modestly noisy private signals and a less persistent inflation process $\rho = 0.9$ and equal degree of volatility $\sigma = 0.23$ fit the data the joint dynamics of inflation and professional forecasts the best.

5.2.3 Professionals versus households

In contrast with professionals, raw household forecasts see a more substantial deviation from FIRE in every dimension. During the sample period of 2013-2020, the monthly headline CPI inflation is estimated to have a persistence of 0.98 and a volatility of 0.41[21]. The household's forecasts had an average forecast error of 1.75, a disagreement of 6.15, and an average uncertainty of 9.70. It is obvious that these deviations are difficult to be directly accounted for by the theories under consideration because of their sheer sizes. In order for the theory to fit better with data, I first control for the individual fixed effects on individual forecasts, forecast errors, and uncertainty before computing the population average of these moments. This follows from the emerging evidence[22] that individual-specific effects such as demographics and experience play important roles in driving systematic differences in expectations. After controlling for this ex-ante heterogeneity in beliefs, households' expectations exhibit similar yet still more distorted expectations as that of professionals. It has an average forecast error close to zero, but a substantially larger disagreement of 2.48, and a slightly higher uncertainty of 1.72 than professionals.

[INSERT TABLE 7 HERE]

[21] The higher volatility compared to that Core CPI is due to higher frequency and inclusion of more volatile items in the CPI basket.

[22] Malmendier and Nagel (2015), Das et al. (2017), D'Acunto et al. (2019) and so forth.

Table 7 presents the estimates for households after excluding the fixed effects on forecasts, forecast errors, and uncertainty. Overall, the estimates are surprisingly very similar to that from professionals and within each model, and, in some sense, show even more model consistency across moment conditions and estimation procedures. Most noticeably, the updating rate in SE falls into a very small range of values of $0.31 - 0.35$, equivalent to an updating every four months. This is extremely close to our estimates for professionals. It is well documented in the literature that households are more irrational than professionals. But the results of our estimates show that the major differences are not least mostly due to the differences in updating rate in information.

The second intuitive pattern implied from the households estimates is that different from professionals, the NI-model estimates reveal a much larger noisiness of public signals (hitting the upper bound 3) than that of private signals ($0.45 - 0.65$) across all moments and both 2-step and join estimations. This is very consistent with the fact that households have been found to pay very little attention to commonly accessible macroeconomic news than to the individual-specific information that is immediate to them.

Similar to that for professionals, the canonical DE model-based estimates suggest an over-reaction parameter values between -2(lower bound) to -0.99 on average and a way larger degree of heterogeneity across populations with standard deviation between 3.88 to 5.

As to DENI, it shows more model consistency for households. Across specifications, the estimated overreaction parameter is negative between -0.93 to -0.3. At the same time, the nosiness of both noisy signals is very similar to that of NI model estimates. It all suggests that households not only underreact to news in general but also face highly noisy public signals and more precise private signals.

5.2.4 Alternative inflation process with stochastic volatility

Table 8 and 9 show the estimation of the model-specific parameters allowing the alternative inflation process featuring stochastic volatility in two separate unobserved components of different persistence. Compared to the benchmark AR(1) process, there are two crucial implications for expectation formation. The first is that now the SV model admits time-varying volatility, which has more potential to be consistent with the time-varying pattern of the forecast uncertainty, primarily in the expectations of the household. The second is that given the permanent component is a random walk, the shock to the permanent component is permanent instead of persistent. We examine if the alternative inflation process accommodates similar estimates of the model-specific parameters.

Among all theories, SE gives the closest parameter estimates to that of the benchmark. Surprisingly enough, admitting stochastic volatility reveals an almost identical information rigidity in SE, i.e. an annual updating rate of $0.34 - 0.35$ for both households and professionals. This is not a mechanical coincidence since the SVSE model assumes agents do not only infrequently update realized shocks but also the shocks to the volatility, as explicitly discussed in the section 3.6. Therefore, the dynamics of uncertainty seen from data do provide useful additional information now to identify information rigidity than the benchmark model. This suggests that SE has a very good consistency against to the assumed inflation process. Such information rigidity is also indirectly confirmed by the revealed underreaction according to DESV estimates. On average, both households and professionals underreact instead of overreacting to inflation news even under the stochastic volatility model of inflation. More specifically, professionals show slightly more underreaction with SV than the benchmark estimates, and households show less.

The estimates for NI and DENI agumented with SV are a lot different from that from benchmark estimates. Since SV effectively allows the inflation to be more volatile and the shock to be more persistent, the estimated nosiness of both public and private signals for professionals increase in the sizes compared to the $AR(1)$ so much that they both hit the externally imposed upper bound of 3. But for households, the sizes of the public signals and privates signals flipped the relationship, i.e. the nosiness of public signals becomes 0 while privates signals are much noisier. This suggests that both NI and DENI have poor sensitivity toward alternative inflation process.

[INSERT TABLE 8 and TABLE 9 HERE]

5.3 The scoring card of different theories

Table 10 reports my evaluation of the four theories under consideration based on four sensitivity criteria laid out in the previous section. According to this evaluation, the SE seems to capture the average behavior of expectations better than the other two theories. DE shows a similar degree of performance according to these criteria, but the average estimation does not show overreaction in line with the premise of the theory, instead, underreaction, consistent with the existence of the information rigidity.

In comparison, NI and DENI perform less well in terms of these four criteria except for the sensitivity against the moment conditions used for estimation. It is worth giving some diagnoses of the crucial challenges of the two theories. The central issue that makes NI be able to explain data well is that the level of information rigidity consistent with NI formulation requires too big sizes of the nosiness of the signals. And it is also very sensitive to the assumed level of inflation volatility and the degree of its persistence. Such a problem carries on to the DENI model, which in theory could have to reconcile the observed coexistence of information rigidity and overreaction better, as argued in Bordalo et al. (2020).

[INSERT TABLE 10 HERE]

6 Conclusion

Most of the research thus far on expectation formation and how it deviates from the rational benchmarks have focused on the first moment, namely the mean forecast. This paper testifies that surveyed forecasting uncertainty by professionals and households provides useful information to understand the exact mechanisms of expectation formation. It does not only provide additional reduced-form testing resulting in rejecting FIRE, such as persistent disagreements in forecasting uncertainties and its inefficient revisions, it also provides additional moment restrictions to any particular model of expectation formation.

At least two lines of questions remain unresolved in this paper. First, throughout the analysis, we maintain the normality assumptions of the shocks and ignore the beliefs in tail events or even higher moments. It is a natural extension of this paper to explore how different theories of expectation formation may contain different predictions on tail beliefs. Second, although this paper focuses only on macroeconomic expectations regarding inflation, it is worth asking if the belief formation regarding individual variables such as income bear similar mechanisms.

Table 1: Definition and Notation of Moments

Individual Moments	Population Moments
Mean forecast: $y_{i,t+h\|t}$	Average forecast: $\bar{y}_{t+h\|t}$
Forecast error: $FE_{i,t+h\|t}$	Average forecast error: $\overline{FE}_{t+h\|t}$
Uncertainty: $\text{Var}_{i,t+h\|t}$	Average uncertainty: $\overline{\text{Var}}_{t+h\|t}$
	Disagreements: $\overline{Disg}_{t+h\|t}$

Table 2: Information of Data

	SCE	SPF
Time period	2013-2019	2007-2019
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Var in Density	1-yr and 2-yr-ahead inflation	1-yr-ahead GDP deflator, Core CPI and Core PCE
Panel Structure	stay up to 12 months	average stay for 5 years
Individual Info	Education, Income, Age, Location	Industry

Table 3: Tests of Rationality and Efficiency Using Forecast Errors

	SPF CPI	SPF PCE	SCE
Test 1: Unbiasedness			
Constant	0.122***	0.586***	2.220***
	(0.017)	(0.061)	(0.019)
N	4697	1208	67380
Test 2: FE does not depend on past information			
Forecast 1-yr before	0.307***	0.586***	NA
	(0.020)	(0.061)	NA
Constant	-0.655***	-0.777***	NA
	(0.060)	(0.116)	NA
N	3429	1208	NA
R^2	0.0721	0.118	NA
Test 3: FEs of non-overlapping forecast horizons not serially correlated			
Forecast Error 1-year before	0.0756***	0.0503***	NA
	(0.020)	(0.035)	NA
Constant	0.145***	0.275***	NA
	(0.021)	(0.035)	NA
N	3356	1208	NA
R^2	0.00591	0.00264	NA
Test 4: Overlapping FEs are only weakly serially correlated			
Forecast Error 1-q before	0.657***	0.834***	0.297***
	(0.025)	(0.037)	(0.021)
Forecast Error 2-q before	0.0282	-0.0858	0.308***
	(0.027)	(0.048)	(0.046)
Forecast Error 3-q before	-0.0244	-0.0555	0.311***
	(0.025)	(0.038)	(0.045)
Constant	0.0626***	0.113***	0.742***
	(0.019)	(0.026)	(0.097)
N	2536	1004	2836
R^2	0.439	0.552	0.232

Note: white standard errors reported in the parentheses of estimations. *** $p<0.001$, ** $p<0.01$ and * $p<0.05$.

Table 4: Tests of Revision Efficiency Using Mean Revision and Uncertainty

	SPF CPI				SPF PCE				SCE				
Test 1. Revision efficiency of mean forecast													
	Mean revision	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Mean_rv		0.539***	0.418***	0.387***		0.606***	0.435***	0.369***	L.InfExp_Mean_rv		0.884***	0.750***	0.685**
		(0.031)	(0.043)	(0.052)		(0.034)	(0.042)	(0.049)			(0.084)	(0.171)	(0.211)
L2.InfExp_Mean_rv			0.218***	0.166**			0.261***	0.246***	L2.InfExp_Mean_rv			0.206	0.199
			(0.040)	(0.053)			(0.047)	(0.058)				(0.173)	(0.185)
L3.InfExp_Mean_rv				0.134**				0.116	L3.InfExp_Mean_rv			-0.073	-0.122
				(0.048)				(0.069)				(0.191)	(0.264)
SPFCPI_ct50		-0.444***	-0.391**	-0.454**					L4.InfExp_Mean_rv				0.055
		(0.105)	(0.124)	(0.138)									(0.217)
SPFPCE_ct50						-0.432***	-0.413***	-0.504***	L5.InfExp_Mean_rv				0.073
						(0.109)	(0.111)	(0.138)					(0.242)
									L6.InfExp_Mean_rv				-0.016
													(0.140)
Constant	-1.257***	0.329	0.351	0.546*	-1.095***	0.365	0.428*	0.641**	Constant	-0.263***	-0.016	-0.010	-0.005
	(0.045)	(0.191)	(0.237)	(0.269)	(0.039)	(0.188)	(0.191)	(0.228)		(0.038)	(0.029)	(0.033)	(0.042)
N	1337	1045	822	652	1111	867	683	549	N	41	40	38	35
R^2	0.000	0.335	0.355	0.372	0.000	0.409	0.444	0.452	R^2	0.000	0.717	0.732	0.690
Test 2. Revision efficiency of uncertainty													
	Mean revision	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Var_rv		0.290*	0.529***	0.581***		0.577***	0.477***	0.344*	L.InfExp_Var_rv		0.731***	0.723***	0.785***
		(0.122)	(0.117)	(0.145)		(0.080)	(0.130)	(0.148)			(0.108)	(0.159)	(0.161)
L2.InfExp_Var_rv			-0.059	-0.209			0.360*	0.205*	L2.InfExp_Var_rv			0.283	0.174
			(0.125)	(0.127)			(0.143)	(0.098)				(0.197)	(0.246)
L3.InfExp_Var_rv				0.353**				0.390*	L3.InfExp_Var_rv			-0.336	-0.410
				(0.121)				(0.149)				(0.186)	(0.266)
									L4.InfExp_Var_rv				0.270
													(0.219)
									L5.InfExp_Var_rv				-0.065
													(0.248)
									L6.InfExp_Var_rv				-0.082
													(0.172)
Constant	-0.034***	-0.011**	-0.008*	-0.005	-0.039***	-0.019**	-0.010**	-0.007*	Constant	-0.590**	-0.186	-0.199	-0.163
	(0.005)	(0.004)	(0.003)	(0.004)	(0.006)	(0.006)	(0.003)	(0.003)		(0.174)	(0.155)	(0.164)	(0.171)
N	1189	877	663	504	1082	801	604	458	N	41	40	38	35
R^2	0.000	0.124	0.284	0.408	0.000	0.353	0.583	0.723	R^2	0.000	0.549	0.597	0.619

Standard errors are clustered by date. *** p<0.001, ** p<0.01 and * p<0.05.

Table 5: Weak Tests of Revision Efficiency Using Change in Forecasts and Uncertainty

	SPF CPI				SPF PCE					SCE			
Test 3. Weak revision efficiency of change in forecast													
	Mean change	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Mean_ch		-0.295***	-0.344***	-0.367***		-0.303***	-0.348***	-0.364***	L.InfExp_Mean_ch		-0.433***	-0.586***	-0.642***
		(0.034)	(0.044)	(0.045)		(0.043)	(0.059)	(0.062)			(0.01)	(0.013)	(0.025)
L2.InfExp_Mean_ch			-0.179***	-0.242***			-0.162*	-0.200**	L2.InfExp_Mean_ch			-0.336***	-0.439***
			(0.047)	(0.049)			(0.061)	(0.067)				(0.018)	(0.031)
L3.InfExp_Mean_ch				-0.097**				-0.088*	L3.InfExp_Mean_ch			-0.143***	-0.270***
				(0.032)				(0.036)				(0.012)	(0.027)
									L4.InfExp_Mean_ch				-0.183***
													(0.027)
									L5.InfExp_Mean_ch				-0.096***
													(0.021)
									L6.InfExp_Mean_ch				-0.044**
													(0.013)
Constant	-0.005	-0.004	-0.011	-0.015	0.001	0.008	-0.002	-0.007	Constant	-0.055*	-0.034	-0.001	-0.002
	(0.023)	(0.024)	(0.026)	(0.026)	(0.020)	(0.020)	(0.022)	(0.022)		-0.023	-0.023	-0.028	-0.033
N	1636	1430	1266	1141	1402	1190	1022	898	N	53016	43166	28850	14445
R^2	0.000	0.086	0.112	0.128	0.000	0.090	0.112	0.120	R^2	0.000	0.202	0.273	0.306
Test 4. Weak revision efficiency of change in uncertainty													
	Mean change	t-1	t-1- t-2	t-1-t-3	Mean change	t-1	t-1- t-2	t-1-t-3		Mean change	t-1	t-1- t-2	t-1-t-3
L.InfExp_Var_ch		-0.393**	-0.568***	-0.543**		-0.444***	-0.602***	-0.658***	L.InfExp_Var_ch		-0.382***	-0.565***	-0.652***
		(0.136)	(0.146)	(0.177)		(0.094)	(0.127)	(0.145)			-0.015	-0.022	-0.037
L2.InfExp_Var_ch			-0.322**	-0.278*			-0.289*	-0.404**	L2.InfExp_Var_ch			-0.300***	-0.406***
			(0.104)	(0.132)			(0.110)	(0.137)				-0.021	-0.031
L3.InfExp_Var_ch				0.048				-0.292	L3.InfExp_Var_ch			-0.123***	-0.265***
				(0.096)				(0.154)				-0.012	-0.027
									L4.InfExp_Var_ch				-0.130***
													-0.025
									L5.InfExp_Var_ch				-0.058**
													-0.018
									L6.InfExp_Var_ch				-0.025
													-0.012
Constant	-0.002	-0.001	0.004	0.004	0.000	0.002	0.004	0.005	Constant	-1.339***	-1.324***	-1.139***	-0.839***
	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)		-0.123	-0.11	-0.104	-0.163
N	1202	950	765	625	1078	842	657	519		53016	43166	28850	14445
R^2	0.000	0.120	0.265	0.242	0.000	0.233	0.321	0.385		0	0.182	0.278	0.321

Standard errors are clustered by date. *** p<0.001, ** p<0.01 and * p<0.05.

Table 6: SMM Estimates of Different Models: Professionals

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.36	0.99	0.23	0.37	0.9	0.13				
FE+Disg	0.35	0.99	0.23	0.64	0.9	0.13				
FE+Disg+Var	0.34	0.99	0.23	0.62	0.9	0.16				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	NI: ρ	NI: σ		
FE	3	2.14	0.99	0.23	1.52	1.59	0.99	0		
FE+Disg	2.18	2.56	0.99	0.23	0.83	2.09	0.99	0		
FE+Disg+Var	0.77	0.94	0.99	0.23	0.85	1.79	0.99	0		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_θ	ρ	σ	$\hat{\theta}$	σ_θ	ρ	σ		
FE	-0.36	0.68	0.99	0.23	-0.48	0.56	0.9	0.13		
FE+Disg	-0.48	0.39	0.99	0.23	-0.5	0.38	0.92	0.12		
FE+Disg+Var	-0.44	0.38	0.99	0.23	-0.48	0.39	0.9	0.15		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	-0.81	2.91	3	0.99	0.23	-0.18	1.27	1.99	0.99	0
FE+Disg	0.05	1.5	1.54	0.99	0.23	0.98	1.93	1.12	0.99	0
FE+Disg+Var	-1.87	0.19	0.73	0.99	0.23	-0.11	0	0.32	0.9	0.23

Table 7: SMM Estimates of Different Models: Households

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.35	0.99	0.41	0.33	0.93	0.27				
FE+Disg	0.33	0.99	0.41	0.31	0.93	0.26				
FE+Disg+Var	0.34	0.99	0.41	0.33	0.93	0.24				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ		
FE	3	0.65	0.99	0.41	3	0.45	0.9	0.31		
FE+Disg	3	0.45	0.99	0.41	3	0.51	0.9	0.36		
FE+Disg+Var	3	0.5	0.99	0.41	3	0.48	0.9	0.34		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_θ	ρ	σ	$\hat{\theta}$	σ_θ	ρ	σ		
FE	-2	5	0.99	0.41	-1.02	0.51	0.9	0.32		
FE+Disg	-0.99	3.88	0.99	0.41	-0.99	3.88	0.9	0.32		
FE+Disg+Var	-0.99	3.88	0.99	0.41	-1.14	4.01	0.91	0.3		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	-0.93	3	0.08	0.99	0.41	-0.51	3	0.07	0.9	0.31
FE+Disg	-0.45	3	0.15	0.99	0.41	-0.79	3	0.11	0.96	0.21
FE+Disg+Var	-0.3	2.47	0.53	0.99	0.41	NA	NA	NA	NA	NA

Table 8: SMM Estimates of Different Models under Stochastic Volatility: Professionals

SE				
Moments Used	2-Step Estimate			
	$\hat{\lambda}$	γ		
FE	0.35	0.2		
FE+Disg	0.34	0.2		
FE+Disg+Var	0.34	0.2		
NI				
Moments Used	2-Step Estimate			
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	γ	
FE	3	3	0.2	
FE+Disg	3	3	0.2	
FE+Disg+Var	3	2.59	0.2	
DE				
Moments Used	2-Step Estimate			
	$\hat{\theta}$	σ_{θ}	γ	
FE	-0.63	0.79	0.2	
FE+Disg	-0.64	0.21	0.2	
FE+Disg+Var	-0.64	0.21	0.2	
DENI				
Moments Used	2-Step Estimate			
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	γ
FE	-0.62	2.17	3	0.2
FE+Disg	-0.55	3	3	0.2
FE+Disg+Var	-0.56	2.78	3	0.2

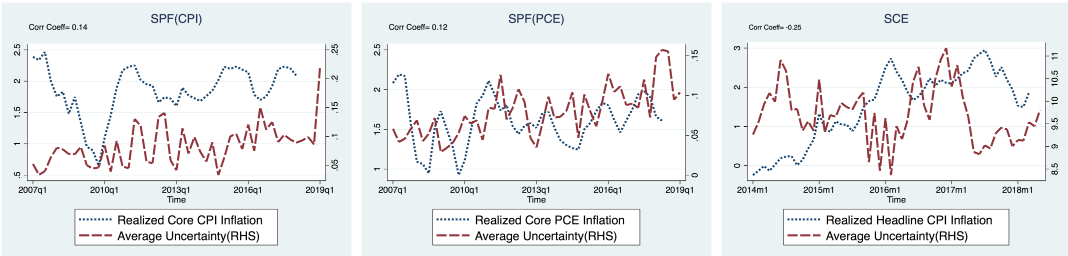
Table 9: SMM Estimates of Different Models under Stochastic Volatility: Households

SE				
Moments Used	2-Step Estimate			
	$\hat{\lambda}$	γ		
FE	0.35	0.2		
FE+Disg	0.35	0.2		
FE+Disg+Var	0.35	0.2		
NI				
Moments Used	2-Step Estimate			
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	γ	
FE	0	0.71	0.2	
FE+Disg	0	0.71	0.2	
FE+Disg+Var	0	0.71	0.2	
DE				
Moments Used	2-Step Estimate			
	$\hat{\theta}$	σ_{θ}	γ	
FE	-0.7	0.83	0.2	
FE+Disg	-0.7	0.58	0.2	
FE+Disg+Var	-0.7	0.58	0.2	
DENI				
Moments Used	2-Step Estimate			
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	γ
FE	0.8	0	0.32	0.2
FE+Disg	0.79	0	1.36	0.2
FE+Disg+Var	0.8	0	0.82	0.2

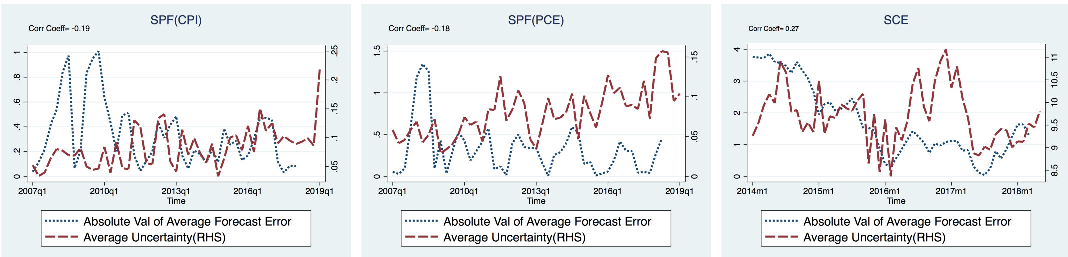
Table 10: Scoring card of different theories

Criteria	SE	NI	DE	DENI
Sensitive to moments used for estimation ?	No	No	No	No
Sensitive to assumed inflation process?	No	Yes	No	Yes
Sensitive to two-step or joint estimate?	No	Yes	No	Yes
Sensitive to the type of agents?	No	Yes	No	Yes

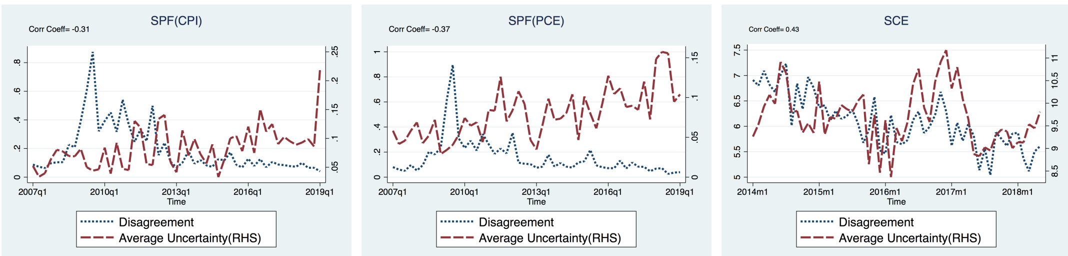
(a) Realized Inflation and Uncertainty



(b) Size of Forecast Errors and Uncertainty



(c) Disagreement and Uncertainty



Note: From left to right: SPF's forecasts of core CPI and core PCE, and SCE's forecast of headline CPI. From top to the bottom, uncertainty (red dash) versus realized inflation (blue dot) with correlation coefficient of 0.14, 0.12 and -0.25, respectively; uncertainty (red dash) versus absolute value of forecast errors (blue dot) with correlation coefficient of -0.19, -0.18, 0.27, respectively; uncertainty (red dash) versus disagreements (blue dot) with correlation coefficient of -0.31, -0.37 and 0.43,repectively. Only the Pearson tests of correlation between disagreement and uncertainty are significant for all.

Figure 1: Uncertainty and Other Moments

(a) Mean Forecasts (b) Uncertainty

Figure 2: Dispersion of Mean Forecasts and Uncertainty

(a) Mean Forecasts (b) Uncertainty

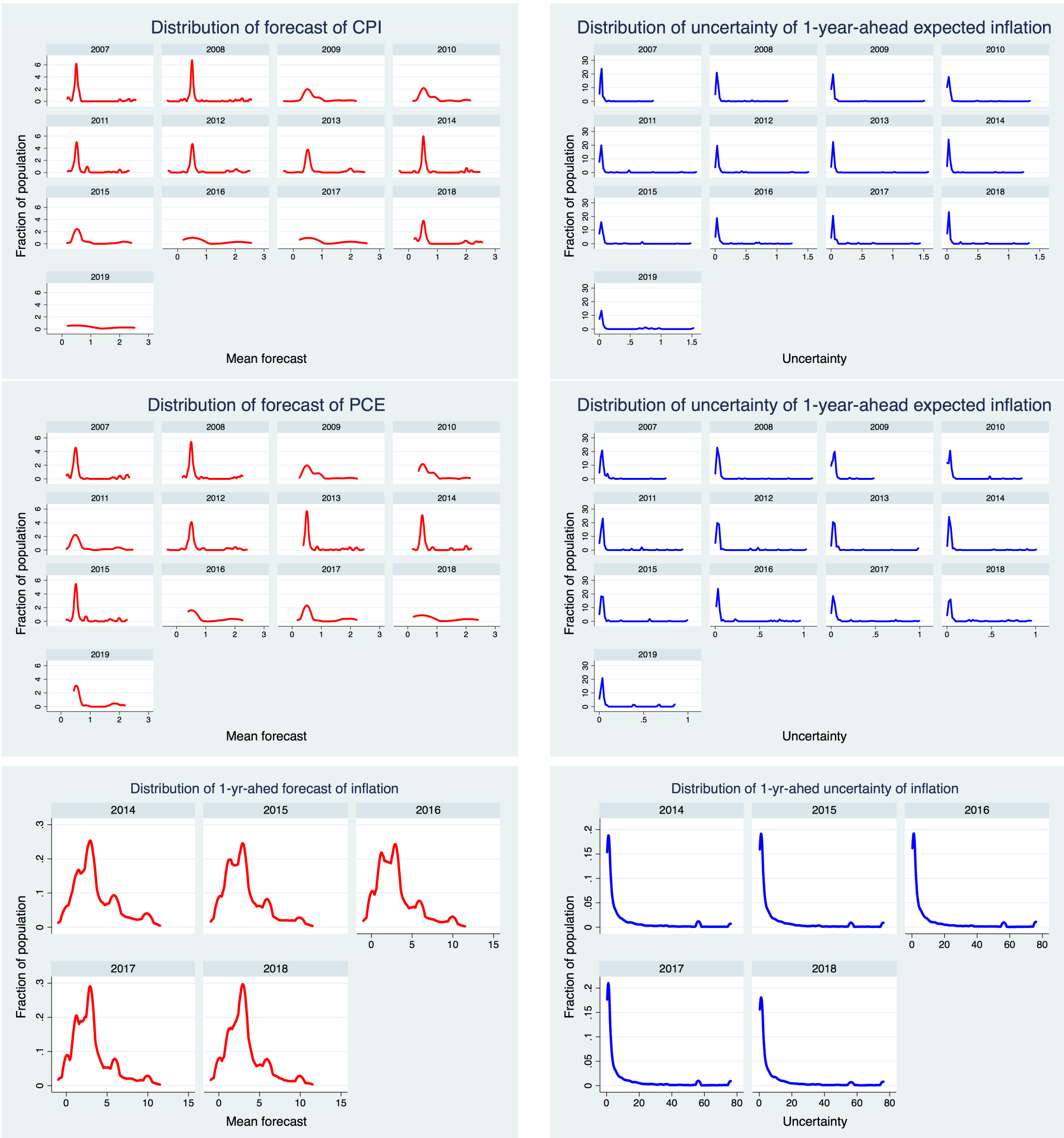
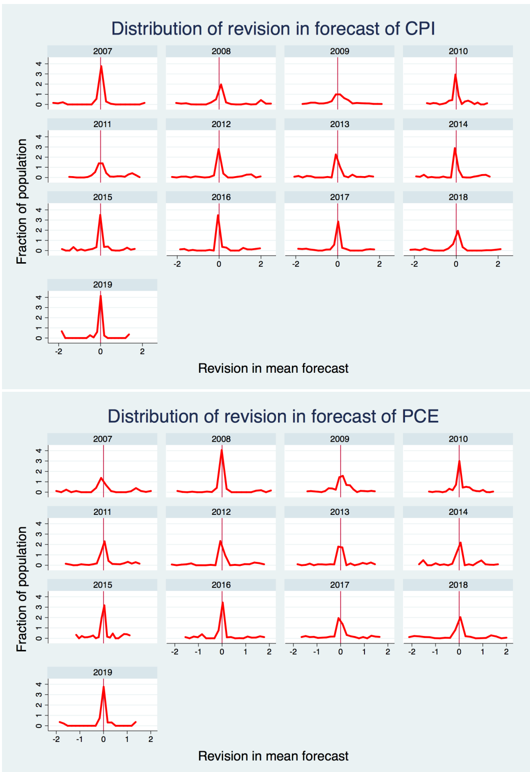


Figure 3: Distribution of Mean Forecast and Uncertainty

(a) Revision in Mean Forecasts



(b) Revision in Uncertainty

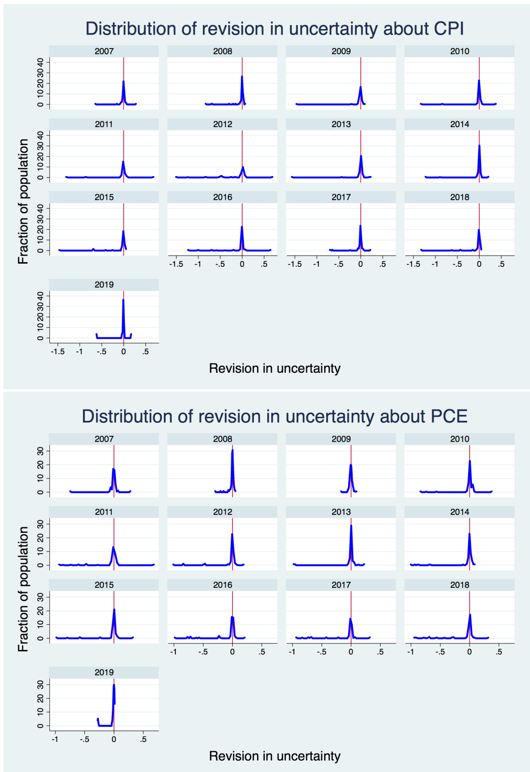


Figure 4: Distribution of Revision in Forecasts and Uncertainty

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