# Efficiency Wage Models

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#### 1 Motivation

Why is there unemloyment? If a worker is willing to take a lower wage than the prevailing rate in the market, but she cannot find the job, it implies that there is certain mechanism that prevents firms from lowering the wage. Efficiency Wage is one of such kinds of theories that raionalize this phenomenon. There are different considerations behind the notion of efficiency wage. For instance, paying a higher-than-market wage incentivizes workers to exert desirable level of efforts which is not directly observable by firms. Here efficiency wage plays an incentive role. Or ex ante, under asymmetric information of employee's quality, a higher wage attracts more productive workers to apply for the job. Here, efficiency wage arises as a screen device. Besides, efficiency wage helps keep worker's morale and build loyality. There is no single model that models all of these factors. Which factor matters the most depends on the specific contexts of the question.

#### 2 A Generic Model

In the simplest case, worker's effort is a function of wage e(W), and the production is determined by efficiency unit F(eL).

Firm maximizes profits by simultaneously choosing labor demand and wage.

$$Max_{\{L,w\}}$$
  $F(e(w)L) - wL$ 

First order conditions with respect to w and L are as below

$$F'(e(w)L)e'(w) = 1$$

$$F'(e(w)L)e(w) = w$$

Combining the two gives

$$\frac{e^w w}{e(w)} \equiv \epsilon_{e,w} = 1$$

This is a condition that is common across many efficiency wage models. It says the elasticity of effort with respect to wage should be equal to 1, namely both effort and wage change proportionally. It guarantees that in optimal the ratio of the two w/e, unit cost of effective labor, is a constant.

Notice now wage is independently determined from labor supply. If total optimal labor demand  $NL^* < L$ , there is unemloyment in equilibrium. Otherwise, wage is bidded up to clear the market.

### 3 An Extended Model

The effort may not only depend on wage paid to herself. How other jobs in the market are paid and the labor market condition as measured by unemloyment rate may also affect the effort level. Here we study such a model (Summers (1988)).

$$e(w,x) = \begin{cases} \left(\frac{w-x}{x}\right)^{\beta} & \text{if } w > x\\ 0 & \text{otherwise} \end{cases}$$

where x is an indicator of labor market condition defined as a function of unemloyment rate and market wage. b is a model parameter that captures the intensity of effect of unemloyment rate.

$$x = (1 - bu)w_a$$

Regularity assumptions require  $b > \beta$ .

Taking derivative of effort with respect to w, we get

$$e^w = \beta (\frac{w-x}{x})^{\beta-1} \frac{1}{x}$$

We know the optimal condition requires  $\epsilon_{e,w} = 1$ , that is

$$\epsilon_{e,w} = e^w w / e = \beta \frac{w}{\frac{w-x}{x}} \frac{1}{x} = 1$$

$$\beta w = w - x$$

$$(1-\beta)w = x$$

$$w = \frac{x}{(1 - \beta)}$$

In the same time, equilibrium condition requires  $w = w_a$ , that is every firm needs to pay as equal as the market wage, which gives following solution

$$u = \frac{\beta}{b} \equiv u_{EQ}$$

Notice now the unemloyment rate only depends on parameters of effort function and does not depend upon production function. This is consistent with the fact that unemployment rate does not have trend.

We can also use numerical examples to show that w responds little to the labor market conditions, in particular u. The idea is that the cost-saving is very limited from lowering wages in response to an increase in unemloyment. Here the cost is measured by cost per unit of effort. This is in line with the fact that firms do not adjust wage in response to short-run fluctuations.

## 4 Shapiro & Stiglitz(1984)

A micro-founded model of efficieny wage needs to specify the underlying mechanisms. e.g. the moral hazard problem introduced by imperfect monitoring of effort level by emloyers.

A firm and a worker. The worker chooses effort to be 0 or  $\bar{e} > 0$ , a discrete choice. Firm cannot monitor it directly, therefore, the wage cannot be based on the effort level. What the firm can do is to pay a wage that is incentive competible so that the worker does not shirk.

A worker has a reservation wage, which in the simplest case is assumed to be zero. Effort is costly, so she prefers shirking if there is no punishment. The panelty comes through a monitoring shock with probability of q. Once found shirking, she gets fired. Besides, there is an exogeneous breaking rate of the job b. Probability of finding a job is a. All are possion process. Thus memoryless, namely the probability of entering a different state does not depend on the duration in the current state. It also allows us to work with the value function at any point of time by solely focusing on the state.

In summary, a worker is faced with three states: U(unemloyed), E(Employed and not shirk) and S(employed and shirking).

The value of each state can be thought as the pdv of an asset that pays divident in each period subject to potential capital gains or losses if moving to a different state.

$$\rho V_U = a(V_E - V_U)$$

$$\rho V_S = w + (b+q)(V_U - V_S)$$

$$\rho V_E = (w - \bar{e}) + b(V_U - V_E)$$

Three equations above can be solved.

The incentive competibility condition requires  $V_S \leq V_E$ , namely the worker cannot find it better to shirk than non-shriking.

In steady state, we can also utility the flow identity to pin down the relationship between a and b. It says the inflow and outflow of employed population are equal to each other.

$$(1-u)b = ua$$

Altogether it gives the non-shirking condition (NSC), the centerpiece of the model.

$$w = \bar{e} + (\rho + \frac{b}{u})\frac{\bar{e}}{a}$$

Wage is now inversely correlated with unemployment rate. This gives an upward sloping curve with wage and employment. The higher the unemployment rate, the more discpline it imposes to workers in a job from shirking as it is more painful to lose the job. As  $u \to 0$ , the wage needs to go positive infinity to incentivize the workers.

The term  $\frac{\bar{e}}{q}$  can be thought as an agency rent, which is commonly seen in such literature. Lower proability of detection and higher cost of effort implies higher rent from shirking.

Since the wage is determined by the NSC, the only free choice variable by the firm is the amount of labor to use. Labor demand is determined by the following condition

$$F(\bar{e}L)\bar{e} = w$$

It gives a downward sloping demand curve.

The intersection of NSC and labor demand pins down the equilibrium wage and employment. Unemployment rate is positive in equilibrium.

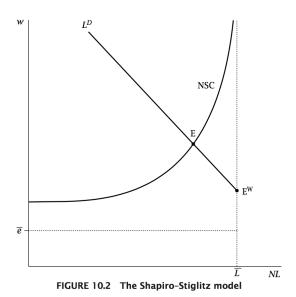


Figure 1: Source: Romer's Textbook

What factors lead to a higher unemployment rate, thus a higher wage in equilibrium? Any factors that worsens the agency problem do so, i.e. lower chacen of detection q, higher effort cost,  $\bar{e}$ , higher job-breaking rate b.

One issue with the Shapiro & Stiglitz model is that It suggest the non-shirking condition curve is quite steep at a lower rate of unemployment. It implies wage is highly responsive to labor demand change, conflicting with the observed fact that wage adjusts little in response to labor demand.

The reason why this outcome is not efficient is simple. At full employment, the equilibrium is at  $E^W$  and marginal product of labor exceeds the cost of exerting effort  $\bar{e}$ . The government could bring the economy closer to more efficient allocation by subsidizing firms in a lum-sum manner to push up the labor demand curve. In the meantime, it should be noted that any unemployment insurance arrangement will worsen the problem because what it does is to push up the NSC further up. It alleviates the pain of losing a job.