DSGE

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1 Overview

What distinguishes a full-fledged DSGE from a light-weighted version of a dynamic AS-AD is the former builds upon forward-looking optimizing decisions of individual households and firms. But most of the insights from the DSGE model stays in the non-micro-founded dynamic AS-AD framework. Here, we study such a model.

2 Model

The dynamic aggregate demand is given as below

$$Y_t = \bar{Y} - \alpha(r_t - \rho) + \epsilon_t \tag{1}$$

$\bar{Y}$ is natural output. We may assume it is invariant over time, or grows at a constant rate. r_t is the real interest rate. ρ is the natural interest rate. It is the real rate when demand is equal to natural output in the absense of shocks. Of course, ρ is simply a constant from a purely mathematical point of view. ϵ_t is the demand shock. It may simply a change in consumer sentiment. Or it comes from government fiscal policy shocks.

Higher real rate dampens the demand. It works through various kinds of channels that are left unspecified here. The first possible channel is intertemporal substitution, which is the building block of New Keyesian DSGE. Higher rate makes saving more attractive than consumption. The second chanel may be higher borrowing cost lowers firms' investment as modeled in IS-LM model.

Real interest rate is determined by Fisher equation:

$$r_t = i_t - E_t(\pi_{t+1}) \tag{2}$$

Nominal rate is set by the central bank according to Taylor Rule.

$$i_t = \rho + \pi_t + \theta_\pi(\pi_t - \pi^*) + \theta_Y(Y_t - \bar{Y}) + z_t \tag{3}$$

π^* is the target inflation by central bank, which is typically a constant.[1] v_t is a monetary policy shock that is exogeneous to current economic conditions. θ_π and θ_Y are parameters that govern how responsive the monetary policy to deviation from inflation target and output gap, respectively.

Unlike IS-LM model, the monetary policy targets at nominal rate instead of money supply. Actully, this better reflects the monetary policy practice of major economies.

The dynamic agggregate supply is essentially the short-run Phillips Curve.

$$\pi_t = E_{t-1}(\pi_t) + \phi(Y_t - \bar{Y}) + v_t \tag{4}$$

$E_{t-1}(\pi_t)$ is the expected inflation for time t. $Y_t - \bar{Y}$ is the output gap at time t. κ governs by how much the output gap translates to inflation. Any other factors that causes change in supply side of the economy is captured ξ_t. For instance, an oil shock, a labor strike, etc.

With the demand and supply plus the monetary policy rule all specified, we are near to be able to pin down the dynamics of the system. What is missing is the expectation formation mechanism. There are all kinds of theories about how $E_{t-1}(\pi_t)$ is determined. Here we follow a long tradition called Adaptive Expectation(AE). The expected inflation of next period is equal to the realized inflation in current period.

$$E_{t-1}(\pi_t) = \pi_{t-1} \tag{5}$$

Demand, Phillips Curve, Fisher equation, adaptive expectation and monetary policy rule altogether complete the model. The system is summarized by following.

- **Predetermined Variable**. π_{t-1}
- **Endogenous Variabless**. $\pi_t, Y_t, r_t, E_t(\pi_{t+1}), i_t$.
- **Exogenous Variables**. $\epsilon_t, v_t, z_t, \bar{Y}, \pi^*$.
- **Parameters**. $\rho, \alpha, \phi, \theta_\pi, \theta_Y$

What are the steady state like? First, no shocks. $\epsilon_t = v_t = z_t = 0 \quad \forall t$. Second, output equal to natural level, $Y = \bar{Y}$. Third, real interest rate equal to its natural level $r = \rho$. Fouth, inflation equal to target level: $\pi = \pi^*$. Therefore, nominal rate $i = \pi^* + \rho$.

When the economy attains its long-run equilibrium without experiencing exogenous disturbances, the output and inflation is simply where it is desired to be. What the monetary policy does is simply to set the nominal rate to reflect that long-run equilibrium. Money is neutral.

[1] One issue with the policy rule above is that in practice, the monetary policy maker cannot observe real-time inflation. So alternative specifications may use lagged inflation π_{t-1} or real-time expected inflation $E_t\pi_{t+1}$. For the purpose here, we put it aside.

If graphically presented in the space of π and Y, the dynamic equilibrium of the economy is at the intersection of a downward sloping demand curve and an upward sloping short-run supply curve. It may deviate from its long-run equilibrium, which is determined by the intersection of the same demand curve with the vertical long-run supply curve passing the level of natural output. Long-run supply curve is vertical because it is indepedent from inflation.

3 Dynamics after Shocks

Now, starting from the steady state, let us think through the response of different variables to demand, supply and policy shocks, respectively.

A positive demand shock $\epsilon_t > 0$ that lasts for only one period. At time t, it pushes up the dynamic demand curve, leading to temporarily positive output gap. Inflation picks up also. At time t, the shock is partly ofset by tightening monetary policy response, and the nominal rate increases. The response of real rate depends on the relative size of changes in nominal rate and expected inflation. Typically, we assume the nominal rate is sufficiently responsive so that the real rate increases. That is $1 + \theta_\pi > 1$, one percentage point increase inflation leads to more than one percentage point increase in nominal rate. [2]

In t+1 period, demand curve shifts downward back. Supply curve moves up due to increase in inflation expectation. Altogether, the output is lower. It gradually returns to natural level in subsequent periods.

To summarize, the impulse responses of the economy:

- Y_t picks up at the time of the shock, drops to be lower than natural level in next period, and then gradually returns to $\bar{Y}$;
- π_t picks up at the time time of the shock and gradually returns to π^*;
- $E_t(\pi_{t+1})$ picks up and gradually returns to π^*;
- i_t picks up at the time of the shock and gradually returns to $\rho + \pi^*$.
- r_t increases and gradually returns to ρ.

A negative supply shock $v_t > 0$ increases inflation. Supply curve shifts upward. Output gap is thus negative. Expected inflation for the next period picks up correspondingly. Nominal rate is set higher by monetary policy according to Taylor rule. Real rate is higher following the shock. Therefore, the demand decreases next period. In the long run, the economy is back to natural output level.

To summarize, the impulse responses of the economy in response to shock at t.

- π_t picks up and gradually returns to π^*;

[2] It is called Taylor Principle. A violation of it gives an upward sloping demand curve and unstable inflation dynamics.

- $E_t(\pi_{t+1})$ picks up and gradually returns to π^*;
- i_t is set higher and gradually returns to $\rho + \pi^*$;
- r_t picks up and gradually returns to ρ;
- Y_t drops and gradually returns to $\bar{Y}$;

The series of events above feature the so called stagflation, lower output together with higher inflation. The monetary policy in facing the shock tightens the monetary policy in order to dampen the inflation pressure, but at a cost of lower output.

A monetary policy shock $z_t > 0$ shifts dynamic demand downward as the nominal rate is set higher after the shock. Supply curve remains the same. As a result, inflation and output are both lower. Every variable gradually moves back to its steady state level afterward.