Search and Match Model

Tao Wang

April 16, 2020

1 Overview

The premise of the labor market could arrive at equilibrium through a centralized market may be unrealistic in the first place. The fact is every worker and position are heterogenous in all kinds of dimensions. Therefore, the labor market works through one-to-one matching in a decentralized manner. This provides an alternative tale of why the labor market is non-Walrasian.

With a search and match view of the labor market, one may see unemployment frictional. It is simply a transitional process toward arriving at a certain steady state of the economy.

The search and match model matches well with one notable fact of the labor market. The Beveridge curve states a negative correlation between the unemployment rate and the number of job vacancies. To put it differently, firms post more job vacancies when the total employment is high. This is one observation in the steady-state of search and match models. The number of vacancies increase with the employment rate of the economy.

Also, the search and match model predicts that the unemployment rate is invariant with long-run productivity change. This is captured by a scenario in the model where proportionate changes in labor product, the reservation wage and the cost of posting a vacancy at the same time.

At the same time, however, there is no much wage rigidity in response to short-term demand fluctuations. Higher labor demand does lead firms to increase wages in the model. In comparison, the change in unemployment is modest. This poses questions to search and match models.

2 One-sided Matching

The worker is either in employment or not. The job-finding rate is a and breaking rate is b. In steady-state, flow identity requires

$$(1-u)a = ub$$

$$u^* = \frac{a}{a+b}$$

So at any point in time with the unemployment rate being u_t, it can be seen as in the process of converting to the steady-state unemployment rate. The differential equation is written as below.

$$u_t = u^* - e^{-(a+b)t}(u^* - u_t)$$

A social planer cannot do anything about it. The dynamics are simply constrained efficient.

3 Two-sided Matching

Diamond-Mortensen-Pissarides (1994).

The model is set in continuous time. Both firms and workers are risk-neutral. A unit mass of workers in the economy.

A worker is either unemployed or in a job position. V_E and V_U represents the value of each state, separately. Being employed means getting paid by the wage of w. Being out of a job, it earns b, which can be thought of as some reservation income or unemployment insurance.

The firm chooses the number of vacancies to be posted. Posting a vacancy is free but maintaining it requires paying a cost c. Having the vacancy filled by a worker, the firm earns marginal production of labor y. V_V and V_F represent the value of the firm in two states, respectively.

Encountering does not automatically bring about a match. The number of positions to be filled is a function of the unemployment rate of U and the number of vacancies posted V. $M(t) = M(U(t), V(t))$. An increasing return of the scale of the function is called the *Thick Market* effect. A decreasing return of scale is called the *Crowding Market* effect. It is a convention to assume a matching function to be CRS.

$$M(t) = U(t)^{1-\gamma} V(t)^{\gamma}$$

Define $\frac{V(t)}{U(t)} = \theta(t)$. It indicates the looseness of the labor market. Intuitively, the fewer people looking for jobs relative to the number of vacancies, the looser the market is. Then we have

$$M(t) = U(t)m(\theta(t)) = U(t)\theta(t)^{\gamma}$$

$$Also, \quad M(t) = V(t)\frac{\theta(t)}{m(\theta(t))} = V(t)\theta(t)^{\gamma-1}$$

Job-finding rate a and vacancy-filling rate α are correspondingly defined.

$$a(t) = \frac{M(t)}{U(t)} = m(\theta(t))$$

$$\alpha(t) = \frac{M(t)}{V(t)} = \frac{m(\theta(t))}{\theta(t)}$$

One of the key assumptions is $m'(\theta) > 0$. The intuition behind it goes as followed. Posing vacancies increases V, makes θ bigger and labor market looser, leading to a larger a, the easier to find a job for an unemployed person. In the meantime, the looser the market, the smaller α, the harder for a job vacancy to be filled. As we show later, these two forces and their relative interaction plays the central role in search and match.

Besides search and match, there is an exogeneous separation rate of λ.

$$\dot{E}(t) = M(t) - \lambda E(t)$$

Let us think through the dynamics of the model. At any point in the time, there is a certain number of unemployed people. A firm decides the number of vacancies to be posted in response. Therefore, the number of matches is determined and the change of the unemployed population is determined. Then the economy moves to the next period.

Discount factor being ρ, value functions written as below.

$$\rho V_E = w + \lambda(V_U - V_E)$$
$$\rho V_U = b + a(V_E - V_U)$$
$$\rho V_V = -c + \alpha(V_F - V_V)$$
$$\rho V_F = y - c - w + \lambda(V_V - V_F)$$

Notice a and α are a function of U and V at time t, with an initial value of U_0 is given, one still needs two more equations to pin down all unknows of the system.

First, in steady-state, the flow identity holds.

$$(1 - U)\lambda = M$$

Second, the free entry of creating a job vacancy as it is zero cost.

$$V_V = 0$$

Third, w, the wage to be paid to the worker is unknown. Typically, in search and match models, the wage is determined implicitly through a Nash Bargaining process. The underlying rationale for this is that both firms and unemployed workers are better off if the match is made from the assumptions $y > c$ and $w > b$. The allocation of the total surplus of the match between two parties are not uniquely determined by the model. Specifically, we simply assume ϕ fraction of the surplus goes to the worker and $1 - \phi$ go to the firm. That is

$$(V_E - V_U)(1 - \phi) = (V_F - V_V)\phi$$

Utilizing this equation plus four value functions, we can solve the expression for the wage.

$$w = b + \frac{(a + \lambda + \gamma)\phi}{\phi a + (1 - \phi)\alpha + \lambda + \gamma}(y - b)$$

A special case of this is when $a = \alpha$, that is $1 - \phi$ fraction of the surplus goes to the worker.

$$w = b + \phi(y - b)$$

The higher a than α implies job-finding is easier than vacancy-filling, thus worker can extract more surplus than the firm.

In order to characterize the model, we first work out the value of a vacancy V_V as a function of employment $E = 1 - U$ and then find its intersection with the horizontal axis of zero. Intuitively, we would expect a downward sloping line as the lower the U, the higher the value of posting a vacancy. In one extreme case of full employment, zero matches is to be made thus the firm only pays a fixed cost c. In another extreme with zero employment, the firm gets all the surplus $y - (b + c)$.

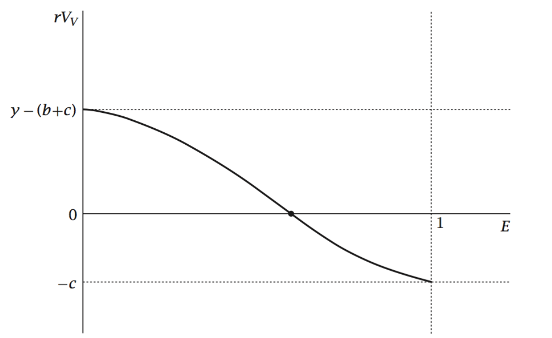

FIGURE 10.6 **The determination of equilibrium employment in the search anc matching model**

Figure 1: Source: Romer's Textbook

What factors increase the equilibrium unemployment? A higher reservation income b, a higher fixed cost c, lower production of labor y, and a higher separation rate of λ.

What is the steady-state V?

$$\lambda(1 - U) = M(V, U) = V^{\gamma} U^{1-\gamma}$$

$$V = \left(\frac{\lambda(1 - U)}{U^{1-\gamma}}\right)^{1/\gamma}$$

Notice V decreases with U, consistent with the Beveridge Curve. The intuitive reason for this is that with low employment, the number of positions to be matched to maintain constant employment is lower. This requires more vacancies to be posted.

4 Welfare Discussion

The externality caused by posting vacancies leads to inefficiency of the market outcome. A social planer may want to intervene to bring about the socially optimal level of vacancies. Here we encounter another constrained optimization problem. By constraint, we mean that the social planner cannot eliminate the vacancy maintenance cost c. Therefore, the social optimum is found to pick the V_{SP} so as to maximize the total social surplus $Ey - Vc$, where Ey the total product of employed workers and Vc are the total vacancy cost to be paid.

It turns out across different models of such kind, the social optimum requires the following simple condition (*Hosios Condition*)

$$(1 - \phi) = \gamma$$

/

γ is defined in matching function as the elasticity of match with respect to vacancies. Let us imagine that suddenly γ increases. As the job-finding rate is an increasing function of γ, it implies that each additional vacancy posted by the firm brings about greater benefit to a worker looking for a job. In the meantime, other firms may find it even harder to fill a vacancy. In order for it to post the vacancy, it has to be the case that more surplus is promised to go to the firm, requiring $1 - \phi$ to be larger.