# **Uncovering Subjective Models from Survey Expectations**

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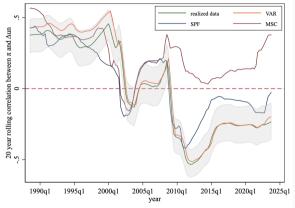
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## $\pi_t$ and $\Delta U_t$ : Actual Data v.s. Expected

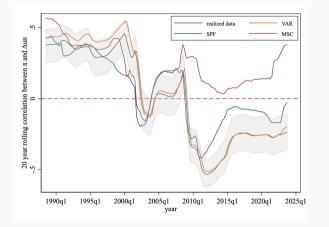


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- Households perceive  $corr(\mathbb{E}\pi,\mathbb{E}u)>0$ , different from data and professionals (Bhandari et al. (2025) and Candia et al. (2020))
- A robust pattern in survey expectations
  - All the time. Cross-section
  - Across all groups. Group
  - Not due to individual or time fixed effects.
- Growing literature with many explanations: observational equivalence in data.

- Methodology
  - 1. Test survey data patterns against model predictions under the two assumptions
    - ullet a joint sign restriction on  $\mathbb E$  correlation and between-variable serial correlation in forecast errors
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- Macro implication
  - An amplification (dampening) of supply (demand) shock responses in a textbook NK model

A Formal Test of Joint Learning

#### **Modeling framework**

- Building on the test of information rigidity in expectation formation (Coibion and Gorodnichenko, 2012; Andrade and Le Bihan, 2013)
  - A Noisy Information model with linear Gaussian noises (Lucas (1976); Woodford (2001))
  - Can be a result of Rational Inattention (Sims, 2003; Maćkowiak et al., 2018)
- Our extensions
  - Univariate -> multivariate expectation formation ("Joint learning")
  - Perceived law of motion ≠ actual law of motion ("Subjective model")
- Focus on bi-variate case:  $\pi$  and u.

#### Multivariate environment

Actual data is generated by:

$$m{L}_{t+1,t} = \underbrace{m{A}}_{ ext{Actual law of motion}} m{L}_{t,t-1} + w_{t+1,t}$$
  $w_{t+1,t} \sim N(0, \underbrace{m{Q}}_{ ext{var-cov of innovations to the state}})$ 

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### Multivariate environment + noisy information

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  $w_{t+1,t} \sim N(0, \underbrace{m{Q}}_{ ext{var-cov of innovations to the state}})$ 

Agents observe a noisy signal

$$m{s}_t = \underbrace{m{G}}_{ ext{Signal mixture}} m{L}_{t,t-1} + \eta_t$$
  $m{\eta}_t \sim m{N}(0, m{R})$  var-cov of noise on signal

# Multivariate environment + noisy information + subjective model

$$\mathbf{L}_{t+1,t} = \underbrace{\mathbf{A}}_{\mathbf{L}_{t,t-1}} + \mathbf{W}_{t+1,t} \tag{1}$$

Actual law of motion

$$w_{t+1,t} \sim N(0, \qquad \qquad Q \qquad \qquad ) \tag{2}$$

var-cov of innovations to the state

Noisy signal

$$\widehat{\boldsymbol{s}_{t}} = \underbrace{\boldsymbol{G}}_{\text{Signal mixture}} \boldsymbol{L}_{t,t-1} + \eta_{t} \tag{3}$$

$$\eta_t \sim N(0, \qquad \underbrace{R} )$$
 (4)

var-cov of noise on signal

Agent may also subjectively believe that:

$$\mathbf{L}_{t+1,t} = \underbrace{\hat{A}}_{\text{Perceived law of motion}} \mathbf{L}_{t,t-1} + w_{t+1,t}$$
(5)

## **Expectations dynamics**

Given the noisy signal and subjective model, expectation about  $oldsymbol{L}_{t+1,t}$  is

$$\underbrace{m{L}_{t+1,t|t}}_{\equiv \mathbb{E}_t(m{L}_{t+1,t})} = \underbrace{\hat{m{A}}}_{ ext{Perceived law of motion}} \left( (I - KG) m{L}_{t,t-1|t-1} + \underbrace{m{K}}_{ ext{Kalman gain}} m{s}_t \right)$$

#### Under FIRE:

- $\hat{A} = A$ , R = 0, KG = I.
- ullet o Expectation

$$L_{t+1,t|t}^{FIRE} = AL_{t,t-1}$$

ullet o Correlation between  $m{L}_{t+1,t|t}^{\mathit{FIRE}}$  is the same as the correlation of  $m{L}_{t,t-1}$ .



## Different causes of $corr(\mathbb{E}\pi, \mathbb{E}u) > 0$

$$\underbrace{ \boldsymbol{\mathcal{L}}_{t+1,t|t}}_{\text{Expectation at t regarding t}+1} = \hat{A}(\boldsymbol{\mathcal{L}}_{t,t-1|t-1} + K(\boldsymbol{s}_t - G\boldsymbol{\mathcal{L}}_{t,t-1|t-1}))$$

$$\boldsymbol{s}_t = G\boldsymbol{\mathcal{L}}_{t,t-1} + \epsilon_t$$

The positive correlation between elements in  $L_{t+1,t|t}$  stems from various possibilities:

- Information friction:
  - Non-diagonal G: correlated signals (e.g., Kamdar (2019) through R.I.).
  - Non-diagonal *R*: correlated noises (e.g., pessimistic heuristics and sentiment).

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- Subjective model:
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- These assumptions are observational equivalent in terms of expectation correlation

Resolution: Forecast error tests (Coibion and Gorodnichenko, 2012) extended to multi-variate case:

$$FE_{t+1,t|t} \equiv \boldsymbol{L}_{t+1,t} - \boldsymbol{L}_{t+1,t|t}$$

$$= \hat{A}(I - KG)FE_{t,t-1|t-1} + \underbrace{M}_{(A-\hat{A}KG-\hat{A}(I-KG))} \boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K\epsilon_{t}$$

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- Special case of FIRE:  $A = \hat{A}$  and  $KG = I \rightarrow \hat{A}(I KG) = \mathbf{0}$

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# Scenario 1: correlated signals, i.e. G is non-diagonal

$$\hat{A}(I - KG) = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \begin{pmatrix} \frac{g_2^2 \sigma_2^2 + \sigma_s^2}{m} & -\frac{g_1 g_2 \sigma_1^2}{m} \\ -\frac{g_1 g_2 \sigma_2^2}{m} & \frac{g_1^2 \sigma_1^2 + \sigma_s^2}{m} \end{pmatrix}$$

$$= \begin{pmatrix} \rho_1 \frac{g_2^2 \sigma_2^2 + \sigma_s^2}{m} & -\rho_1 \frac{g_1 g_2 \sigma_1^2}{m} \\ -\rho_2 \frac{g_1 g_2 \sigma_2^2}{m} & \rho_2 \frac{g_1^2 \sigma_1^2 + \sigma_s^2}{m} \end{pmatrix}$$
(6)

- $m = g_1^2 \sigma_1^2 + g_2^2 \sigma_2^2 + \sigma_s^2$
- $G = [g_1, g_2]$ : the vector of signals (due to "optimal signal selection")
- When signals go in the same direction,  $g_1g_2 > 0$ , the cross terms are negative.

## Scenario 2: subjective model

$$\hat{A}(I - KG) = \begin{pmatrix} \rho_1 & m_1 \\ m_2 & \rho_2 \end{pmatrix} \times \begin{pmatrix} \frac{\sigma_{1,s}^2}{\sigma_1^2 + \sigma_{1,s}^2} & 0 \\ 0 & \frac{\sigma_{2,s}^2}{\sigma_2^2 + \sigma_{2,s}^2} \end{pmatrix} \\
= \begin{pmatrix} \frac{\sigma_{1,s}^2 \rho_1}{\sigma_1^2 + \sigma_{1,s}^2} & \frac{\sigma_{2,s}^2 m_1}{\sigma_2^2 + \sigma_{2,s}^2} \\ \frac{\sigma_{1,s}^2 m_2}{\sigma_1^2 + \sigma_{1,s}^2} & \frac{\sigma_{2,s}^2 \rho_2}{\sigma_2^2 + \sigma_{2,s}^2} \end{pmatrix}$$
(7)

- $G = I_2$ : no signal correlation (can be any diagonal matrix)
- The signs of cross terms (the between-variable serial correlation of FEs) are the same as the perceived correlation

#### **Joint-learning tests for** $\pi$ **and** un

$$\begin{pmatrix}
fe_{t+1,t|t}^{\pi} \\
fe_{t+1,t|t}^{un}
\end{pmatrix} = \beta_0 + \underbrace{\begin{pmatrix}
\beta_{11} & \beta_{12} \\
\beta_{21} & \beta_{22}
\end{pmatrix}}_{\equiv \hat{A}(I-KG)} \begin{pmatrix}
fe_{t,t-1|t-1}^{\pi} \\
fe_{t,t-1|t-1}^{un}
\end{pmatrix} + \theta X_{t,t-1} + e_t \tag{8}$$

- $\beta_{12}$  and  $\beta_{21}$ : between-variable serial correlations of forecast errors
- **Predictions:** if only correlated signals but not subjective model,  $\beta_{12}$  and  $\beta_{21}$  are both negative.
- Two complications:
  - In MSC (and most household's survey),  $\mathbb{E}u$  is qualitative. We impute them following Bhandari et al. (2025). Imputation
  - Expectations are year-ahead measures, so we derive a year-ahead version of (8). Year-ahead test

#### Joint-learning tests with consensus expectations

• Joint learning: subjective model suggesting  $\pi \to un$ .

 Bottom line: information friction cannot be the only reason.

Table 1: Aggregate Test on Joint Learning, MSC v.s. SPF

	00 0			
	MSC		SPF	
	1984-2023	1990-2018	1984-2023	1990-2018
	(1)	(2)	(3)	(4)
$\beta_{11}$	0.64***	0.65***	0.79***	0.76***
	(0.080)	(0.085)	(0.064)	(0.093)
$eta_{12}$	-0.11	-0.02	0.19	-0.08
	(0.076)	(0.095)	(0.117)	(0.199)
$eta_{21}$	0.13***	0.21***	0.05	0.06
	(0.033)	(0.063)	(0.034)	(0.049)
$\beta_{22}$	0.71***	0.50***	0.63***	0.51***
	(0.044)	(0.092)	(0.060)	(0.097)
Observations	152	116	152	116

<sup>\*</sup> The first and third columns are using the full sample 1984-2023; the second and fourth columns are results for the sub-sample 1990-2018. Newey-West standard errors are reported in brackets.

**Uncovering the Subjective Model** 

### Uncovering the subjective model using an expectation-realization VAR

- ullet Previously, we used sign restrictions to test whether  $\hat{A}$  is diagonal, relying on some additional assumptions
- Here, we directly uncover  $\hat{A}$

$$\underbrace{Y_{t+1}}_{\mathbf{L}_{t,t-1}|t-1} = \underbrace{\begin{pmatrix} \hat{A}(I - KG) & \hat{A}KG \\ \mathbf{0}_{2\times 2} & A \end{pmatrix}}_{:=\Phi} \cdot Y_t + \underbrace{\begin{pmatrix} \hat{A}K & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & I_{2\times 2} \end{pmatrix}}_{F} \cdot \begin{pmatrix} \eta_t \\ w_{t+1,t} \end{pmatrix} \tag{9}$$

- Φ estimated from an (un)restricted VAR
- $\hat{A} = \Phi_{1,1} + \Phi_{1,2}$  and  $\Phi_{2,2} = A$
- Identification does not rely on K and G
- Then test  $\hat{A} = A$  elementwise

## The uncovered subjective model of households and professionals

Table 2: Estimates of Joint Learning Model (9)

	MSC, quarterly, (	Q1 1984 - Q4 2023	SPF, quarterly, Q1	1984 - Q4 2023
Parameters	Estimates	Standard Errors	Estimates	Standard Errors
4	0.836 -0.058	0.053 0.057	0.837 -0.056	0.061 0.074
Α	0.034 0.617	0.042 0.095	0.014 0.751	0.041 0.093
Â	$\begin{bmatrix} 0.741 & -0.149 \\ 0.137 & 0.831 \end{bmatrix}$	0.050     0.082       0.044     0.048	$\begin{bmatrix} 0.955 & -0.038 \\ 0.040 & 0.495 \end{bmatrix}$	$\begin{bmatrix} 0.019 & 0.016 \\ 0.035 & 0.239 \end{bmatrix}$
T-test:	test-stat	p-val	test-stat	p-val
$\hat{A}_{21} > A_{21}$	1.581	0.057	0.546	0.293

The table reports the estimates and their NW standard errors from the GMM estimation of the 4-variable VAR model. Iterative weighting matrix are used in the GMM estimation.

**Additional Evidence** 

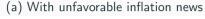
# Expectations conditional on the type of news heard

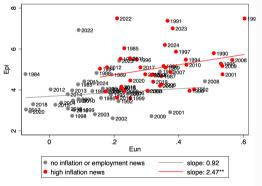
Table 3: Panel Regression with Self-reported News

$E\pi$	Eun	$E\pi$	Eun
(1)	(2)	(3)	(4)
-0.21*	-0.06***	-0.21*	-0.05***
(0.117)	(0.017)	(0.118)	(0.017)
0.43***	0.06***	0.42***	0.05***
(0.085)	(0.010)	(0.085)	(0.010)
-0.03	-0.14***	-0.01	-0.13***
(0.056)	(0.009)	(0.057)	(0.009)
0.05	0.10***	0.04	0.09***
(0.054)	(0.007)	(0.054)	(0.007)
-0.03	-0.06***	-0.01	-0.04***
(0.071)	(0.012)	(0.072)	(0.012)
0.02	0.11***	0.02	0.10***
169304	189158	169304	189158
0.673	0.677	0.673	0.681
Υ	Υ	Υ	Υ
Υ	Υ	Υ	Υ
N	N	Υ	Υ
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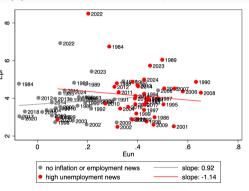
## Consensus expectations conditional on the news exposure

Figure 1: Consensus expectations conditional on news heard





#### (b) With unfavorable employment news



Notes: Scatter plot for consensus expected inflation and unemployment each year from 1984-2023. Gray dots in all panels are expectations for individuals without employment or inflation news. Left panel: red dots are expectations conditional on hearing high inflation news. Right panel: red dots are expectations conditional on hearing high unemployment news.

## Inflation-unemployment associations in newspapers

- $P(\mathbb{I}_{i,t}(\text{Joint mention}) = 1) = \Phi(\beta_0 + \sum_k \beta_k D_{k,l,t} + \beta_\pi \pi_t + \beta_u u_t)$
- Probability of news report making association between inflation and unemployment increases significantly with realized inflation.

	(1)	(2)	(3)
economy	1.07***	1.07***	1.07***
	(0.03)	(0.03)	(0.03)
fed	0.22***	0.21***	0.21***
	(0.03)	(0.03)	(0.03)
growth	0.60***	0.61***	0.61***
	(0.03)	(0.03)	(0.03)
oil price	0.24***	0.24***	0.24***
	(0.05)	(0.05)	(0.05)
recession	0.48***	0.47***	0.47***
	(0.03)	(0.03)	(0.03)
uncertainty	0.14***	0.15***	0.15***
	(0.05)	(0.05)	(0.05)
$\pi_t$		3.73***	3.62***
		(0.93)	(0.96)
$u_t$	-0.01		-0.00
	(0.01)		(0.01)
N	150465	150465	150465

Macro Implications of Subjective

Model

# Shock propagation in a textbook NK model

3-Equation NK

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + s_t$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \pi_{t+1} - \rho) + d_t$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t$$

Okun's Law

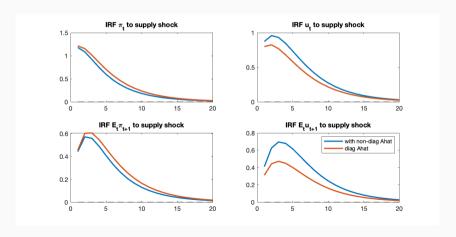
$$u_t = -\chi y_t$$

Expectation formation

$$\begin{split} L_{t+1,t|t} &= \hat{A}(I - KG)L_{t,t-1|t-1} + \hat{A}KGL_{t,t-1} + \hat{A}K\eta_t \\ L_{t+1,t|t} &\equiv \begin{pmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t u_{t+1} \end{pmatrix} \\ L_{t,t-1} &\equiv \begin{pmatrix} \pi_t \\ u_t \end{pmatrix} \end{split}$$

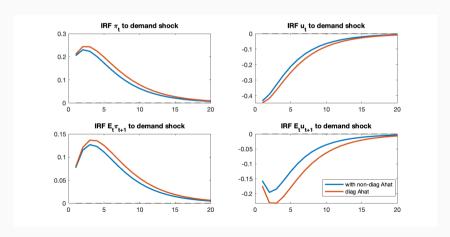
## Supply shock

Figure 2: IRF in Response to Supply Shock



#### **Demand shock**

Figure 3: IRF in Response to Demand Shock



#### **Conclusion**

- Households think about macroeconomic variables jointly
- $\mathbb{E}(\pi) \uparrow \to \mathbb{E}(un) \uparrow$
- Formal tests + structural estimation suggest:
  - HH's subjective model differs from objective one.
  - Correlated expectation is not only due to information friction.
- Asymmetric reaction to inflation and unemployment (real activity) news.
  - $\bullet$   $\pi$  news triggers associations of  $\pi$  and un in expectations
  - ... as well as newspapers' narratives
- Implications for monetary policy
  - Caution on expectation management policy may have unintended contractionary effects.
  - Dampened response to demand shocks and amplified responses to supply shocks.

# **Appendix**

#### **Cross-correlation: MSC**

Table 4: Correlation MCS: more variables

	(1)	(2)	(3)	(4)	(5)
(1) inflation	1.00	0.31***	-0.13	-0.43***	-0.51***
$(E\pi_{t+4,t})$					
(2) unemp change		1.00	-0.41***	-0.64***	-0.28***
$(E\Delta un_{t+4,t})$					
(3) interest rate change			1.00	0.40***	0.07*
$(E\Delta i_{t+4,t})$					
(4) Busi Condition change				1.00	0.77***
$(E\Delta y_{t+4,t})$					
(5) real income change					1.00
$(E\Delta w_{t+4,t})$					

<sup>\* \*\*\*</sup> means significant at 1%,\*\* means 5 % and \* means 10%, data in use are quarterly 1978q1-2018q4 from MSC.

#### **Cross-correlation: FRED**

Table 5: Correlation FRED: more variables

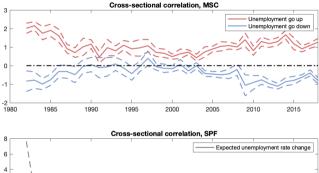
	(1)	(2)	(3)	(4)	(5)
(1) CPI	1.00	0.11	0.38***	-0.03	-0.32***
(2) <i>\Delta un</i>		1.00	-0.52***	-0.79***	-0.77***
(3) <i>∆FFR</i>			1.00	0.43***	0.26***
(4) ∆RGDP				1.00	0.79***
(5) <i>∆w</i>					1.00

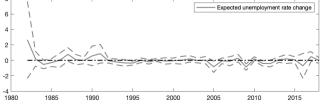
 $<sup>^*</sup>$  \*\*\* means significant at 1%,\*\* means 5 % and \* means 10%, data in use are quarterly 1978q1-2018q4 from FRED.

## Time variations of the perceived correlation in consensus expectations

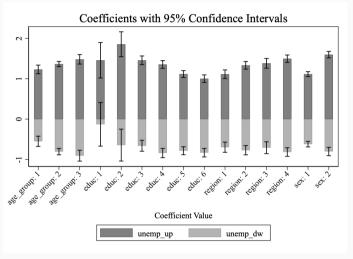
Estimate  $E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 E_{i,t} u n_{t+12,t} + \theta X_{i,t} + \epsilon_{i,t}$ . Parameter of interest is  $\beta_1$ :







#### Regression by group



Cross-sectional correlation across groups

#### Controlling for individual FE and time FE

$$E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 E_{i,t} u n_{t+12,t} + \beta_2 E_{i,t} i_{t+12,t} + \theta X_{i,t} + D_t + \mu_i + \epsilon_{i,t}$$

Table 6: FE Panel Regression

	MSC		SCE		SPF
Unemployment up	0.30***	$\hat{eta}_1$	0.012***	$\hat{eta}_1$	$-0.17^{***}$
	(0.05)		(0.002)		(0.06)
Unemployment down	-0.22***				
	(0.05)				
FE	Υ		Υ		Υ
Time dummy	Υ		Υ		Υ

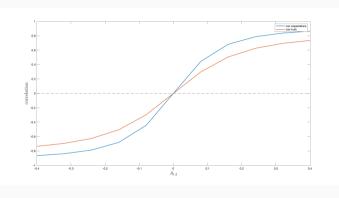
<sup>\*</sup> Controlling for individual and time-varying characteristics, individual fixed effect, and time-fixed effect. Standard errors are adjusted for heteroscedasticity and autocorrelation.

# Correlation of Expectations under FIRE:

• For our case  $\mathbf{L} = \begin{pmatrix} \pi_t \\ un_t \end{pmatrix}$  Empirical estimates of A from 1984-2023:

$$\begin{bmatrix} 0.87 & -0.05 \\ (0.05) & (0.06) \\ 0.02 & 0.67 \\ (0.06) & (0.11) \end{bmatrix}$$

 NW s.e. in brackets and BIC select 1 lag of VAR.



Correlation between expected variables and realized variables when A(1,2) changes under FIRE

#### **Joint Learning Test**

When it's not FIRE, maintain a simplification restriction:

#### Assumption 1

The variance-covariance matrix of prior  $\mathbf{L}_{t,t-1|t-1}^i$  is diagonal and common to each individual:

$$\Sigma := diag(\{\sigma_j^2\})$$

Consider different scenarios afore-mentioned:

- 1. When  $\hat{A}$  is diagonal, consider different G and R: Independent learning.
- 2. When  $\hat{A}$  non-diagonal: Joint learning.
  - ullet When R and G are diagonal.

#### Joint Learning Test I

#### Proposition 1

(Independent Learning) If  $\hat{A} = diag(\{a_i\}_{i=1}^n)$ , denote the off-diagonal elements of  $\hat{A}(I - KG)$  as  $\beta_{ij}$  with  $i \neq j$ . We have:

- (1)  $\beta_{ij} = 0$  if G and R are diagonal.
- (2)  $\beta_{ij} = \beta_{ji} = 0$  or  $\beta_{ij}\beta_{ji} > 0$  if G or R is non-diagonal.
  - Out test coefficient (2-d case):

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}$$

## Joint Learning Test I

#### Proposition 1

(Independent Learning) If  $\hat{A} = diag(\{a_i\}_{i=1}^n)$ , denote the off-diagonal elements of

- $\hat{A}(I KG)$  as  $\beta_{ij}$  with  $i \neq j$ . We have:
- (1)  $\beta_{ij} = 0$  if G and R are diagonal.
- (2)  $\beta_{ij} = \beta_{ji} = 0$  or  $\beta_{ij}\beta_{ji} > 0$  if G or R is non-diagonal.
  - Independent learning + separated signals (Coibion and Gorodnichenko, 2012; Andrade and Le Bihan, 2013): zero between-correlation, non-zero auto-correlation of F.E.

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & \beta_{12} = 0 \\ \beta_{21} = 0 & \beta_{22} \neq 0 \end{pmatrix}$$

#### Joint Learning Test I

#### Proposition 1

(Independent Learning) If  $\hat{A} = diag(\{a_i\}_{i=1}^n)$ , denote the off-diagonal elements of  $\hat{A}(I - KG)$  as  $\beta_{ij}$  with  $i \neq j$ . We have: (1)  $\beta_{ij} = 0$  if G and R are diagonal. (2)  $\beta_{ii} = \beta_{ii} = 0$  or  $\beta_{ii}\beta_{ii} > 0$  if G or R is non-diagonal.

• Independent learning + mixed signals (R.I. like in Kamdar (2019)): same signs on between-correlation of F.E.

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & \beta_{12} <> 0 \\ \beta_{21} <> 0 & \beta_{22} \neq 0 \end{pmatrix}$$

## Joint Learning Test II: non-diagonal R

#### Corollary 1

(Non-diagonal R: correlated noises) If  $\hat{A}$  and G are diagonal and  $R = \begin{pmatrix} \sigma_{1,s}^2 & \rho \\ \rho & \sigma_{2,s}^2 \end{pmatrix}$ , the off-diagonal elements of  $\hat{A}(I - KG)$  have the same signs as  $\rho$ .

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & sgn(\beta_{12}) = sgn(\rho) \\ sgn(\beta_{21}) = sgn(\rho) & \beta_{22} \neq 0 \end{pmatrix}$$





## Joint Learning Test II: non-diagonal G

#### Corollary 2

(Non-diagonal G: correlated signals) If  $\hat{A}$  is diagonal,  $R = \begin{pmatrix} \sigma_{1,s}^2 & 0 \\ 0 & \sigma_{2,s}^2 \end{pmatrix}$  is diagonal, and

$$G = \begin{pmatrix} g_1 & g_2 \\ 0 & g_4 \end{pmatrix}$$
, the off-diagonal elements of  $\hat{A}(I - KG)$  have the opposite signs as  $g_1g_2$ .

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & sgn(\beta_{12}) = -sgn(g_1g_2) \\ sgn(\beta_{21}) = -sgn(g_1g_2) & \beta_{22} \neq 0 \end{pmatrix}$$



## Joint Learning Test III

#### **Proposition 2**

(Joint Learning) If off-diagonal elements of  $\hat{A}(I - KG)$  are not both zeros and of different signs, then  $\hat{A}$  is non-diagonal, regardless whether G and R are diagonal or not.

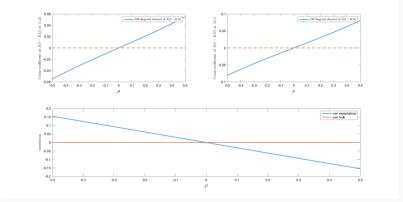
#### **Proposition 3**

(Joint Learning with separate signals) If both G and R are diagonal and  $\hat{A} = (a_{ij})_{n \times n}$  is non-diagonal, denote  $\hat{A}(I - KG) = (\beta_{ij})_{n \times n}$ . The signs of these off-diagonal elements are the same as their counterparts in  $\hat{A}$ , i.e.  $\beta_{ij}a_{ij} > 0$ .

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & sgn(\beta_{12}) = sgn(a_{12}) \\ sgn(\beta_{21}) = sgn(a_{21}) & \beta_{22} \neq 0 \end{pmatrix}$$



## **Example I:** non-diagonal R



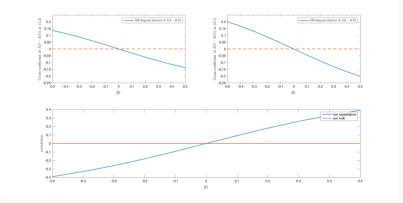
Off-diagonal elements of  $\hat{A}(I - KG)$  and correlation between expectations.

• A working example for illustration:

$$A = \hat{A} = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 7/4 & 0 \\ 0 & 2 \end{bmatrix}, \quad R = \begin{bmatrix} 1.5 & \rho \\ \rho & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1/3 \end{bmatrix},$$



#### **Example II: non-diagonal** *G*



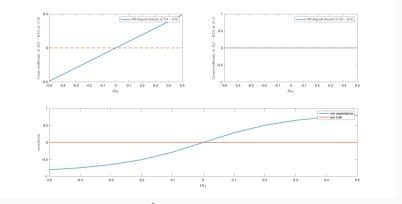
Off-diagonal elements of  $\hat{A}(I - KG)$  and correlation between expectations.

• Same working example for illustration:

$$A = \hat{A} = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 7/4 & 0 \\ 0 & 2 \end{bmatrix}, \quad R = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & g_1 \\ 0 & 1/3 \end{bmatrix},$$



# **Example III:** non-diagonal $\hat{A}$ and diagonal R and G



Off-diagonal elements of  $\hat{A}(I-KG)$  and correlation between expectations.

• Same working example as before except:

$$\hat{A} = \begin{bmatrix} 0.7 & m_1 \\ 0 & 0.9 \end{bmatrix}$$



#### **Complication I: impute** *Eun*

#### Assumption 2

At each period t, survey respondent i forms a belief  $x_{i,t}$  that indicates the change of asked variable x, this belief follows a normal distribution:

$$x_{i,t} \sim N(\mu_t, \sigma_t^2)$$

The survey respondent will respond in categorical fashion:

$$category_{i,t} = \begin{cases} increase & x_{it} > b + a \\ decrease & x_{it} < b - a \\ same & x_{it} \in [-a + b, b + a] \end{cases}$$

#### **Complication I: impute** *Eun*

We want to recover  $\mu_t$ , we can observe fraction of people responding "increase"  $(f_t^u)$  and "decreasing"  $(f_t^d)$ . From normality:

$$\sigma_t = \frac{2a}{\Phi^{-1}(1 - f_t^u) - \Phi^{-1}(f_t^d)} \tag{10}$$

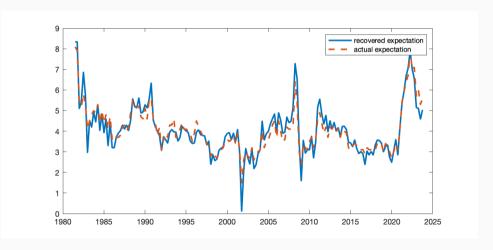
$$\mu_t = a + b - \sigma_t \Phi^{-1} (1 - f_t^u) \tag{11}$$

- Get a and b using average  $\sigma_t$  and  $\mu_t$  approximated by SPF. (Bhandari et al., 2025)
- Can test with  $E\pi_t$  data.



## **Complication I: impute** *Eun*

Figure 4: Recovered Expected Inflation v.s. Actual



Correlation between imputed and actual: 0.99. Back

## **Complication II: Year ahead Expectation**

• The baseline test is derived with quarter-to-quarter changes, whereas data on expectations are year-ahead. We can iterate the forecasting error equation forward:

$$FE_{t+4,t|t} = \hat{W}\hat{A}(I - KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (I - \hat{W}\hat{A}(I - KG)\hat{W}^{-1})\mathbf{L}_{t+3,t-1} - (I + \hat{W}\hat{A}KG)\mathbf{L}_{t,t-1} + A\mathbf{L}_{t+3,t+2} - \hat{W}\hat{A}K\eta_t + w_{t+4,t+3}$$
(12)

- With  $\hat{W} = \hat{A}^3 + \hat{A}^2 + \hat{A} + I$ .
- The test results hold true as the quarterly specification. Shown with Monte Carlo.

#### **Complication II: Year ahead test**

 ${\bf Table~13:~Simulation~Results:~FIRE~or~Independent~Learning~with~Uncorrelated~Signals}$ 

FIRE or Independent Learning: $\hat{A}=A,g_2=0,\rho=0$								
		FII	RE			Independen	t Learnii	ng
	Y-ahead	l Spec (10)	Q-ahead	d Spec (6)	Y-ahead	d Spec (10)	Q-ahea	d Spec (6
	Truth	Test	Truth	Test	Truth	Test	Truth	$\operatorname{Test}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\beta_{11}$	0	-0.01	0	0.04	0.54	0.51***	0.54	0.47***
	-	(0.03)	-	(0.09)	-	(0.09)	-	(0.09)
$\beta_{12}$	0	0.03	0	0.15	0	-0.14	0	-0.14
	-	(0.04)	-	(0.11)	-	(0.010)	-	(0.10)
$\beta_{21}$	0	0.01	0	0.10	0	-0.03	0	-0.09
	-	(0.02)	-	(0.09)	-	(0.04)	-	(0.11)
$\beta_{22}$	0	-0.00	0	0.18	0.43	0.49***	0.43	0.61***
	-	(0.05)	-	(0.12)	-	(0.07)	-	(0.11)

<sup>\* \*\*\*,\*\*,:</sup> Significance at 1%,5% and 10% level. Columns (2) and (6) are estimation results for year-ahead joint-learning test (10), and columns (4) and (8) are for quarter-ahead specification (6). Newey-West standard errors are reported in brackets.

## Complication II: Year ahead test

Table 14: Simulation Results: Independent Learning with Correlated Signals

	Independent Learning when $G$ or $R$ are non-diagonal							
		G non-d	iagonal:		R non-diagonal:			
		$m_1 = 0,  g_2 =$	= $0.5, \rho$ =	= 0		$m_1 = 0, g_2 =$	=0, ho=	-2
	Y-ahea	d spec (10)	Q-ahea	d spec (6)	Y-ahea	d spec (10)	Q-ahea	d spec (6)
	Truth	Test	Truth	Test	Truth	Test	Truth	$\operatorname{Test}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\beta_{11}$	0.57	0.56***	0.57	0.52***	0.49	0.43***	0.49	0.37***
	-	(0.05)	-	(0.08)	-	(0.05)	-	(0.09)
$\beta_{12}$	-0.14	-0.28***	-0.10	-0.26***	-0.17	-0.25***	-0.13	-0.24***
	_	(0.09)	_	(0.10)	_	(0.09)	_	(0.09)
$\beta_{21}$	-0.07	-0.10***	-0.10	-0.20**	-0.09	-0.11***	-0.12	-0.17
	_	(0.04)	_	(0.10)	_	(0.04)	_	(0.11)
$\beta_{22}$	0.40	0.46***	0.40	0.55***	0.39	0.49***	0.39	0.63***
	_	(0.07)	_	(0.11)	_	(0.07)	_	(0.11)

## **Complication II: Year ahead test**

Table 15: Simulation Results: Joint Learning

	Joint Learning: $m_1 = 0.5$ , $G$ and $R$ are diagonal						
	Year-ahe	ead spec (10)	Quarter-ahead spec (6)				
	Truth	Test	Truth	Test			
	(1)	(2)	(3)	(4)			
$\beta_{11}$	0.54	0.48***	0.54	0.44***			
	-	(0.08)	-	(0.08)			
$\beta_{12}$	0.32	0.49**	0.31	0.35***			
	-	(0.22)	-	(0.10)			
$\beta_{21}$	0	-0.02	0	-0.08			
	-	(0.04)	-	(0.09)			
$\beta_{22}$	0.43	0.54***	0.43	0.70***			
	-	(0.12)	-	(0.14)			

<sup>\* \*\*\*,\*\*,\*:</sup> Significance at 1%,5% and 10% level. Column (2)

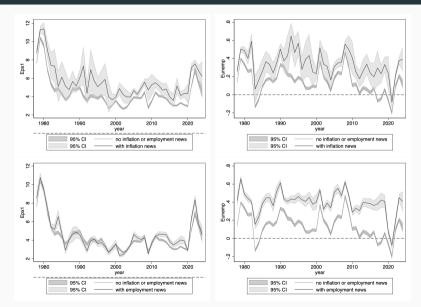
## **Joint Estimation: Alternative Sample**

	MSC, quarterly				
	Q1 1984 -	Q4 2019	Q1 1990 - Q	4 2018	
Parameters	Estimates	Standard Errors	Estimates	Standard Errors	
Α	[0.807 -0.070]	[0.059 0.114]	[0.781 -0.060]	[0.068 0.145]	
^	0.062 0.922	[0.022 0.072]	[0.059 0.930]	[0.031 0.082]	
Â	[0.663 -0.096]	[0.063 0.089]	[0.663 -0.081]	[0.080 0.094]	
Α	0.189 0.807	0.057 0.056	0.271 0.769	0.064 0.057	
T-test:	test-stat	p-val	test-stat	p-val	
$\hat{A}_{21}>A_{21}$	2.094	0.018	2.999	0.001	
		SPF, qu	ıarterly		
	Q1 1984 - Q4 20	19	Q1 1990 - Q	4 2018	
Parameters	Estimates	Standard Errors	Estimates	Standard Errors	
Α	[0.788 -0.070]	[0.070 0.100]	[0.749 -0.047]	[0.079 0.113]	
A	0.048 0.906	[0.024 0.071]	0.042 0.920	[0.030 0.077]	
Â	[0.951 0.004]	[0.018 0.041]	[0.937 -0.027]	[0.021 0.030]	
Α	[0.026 0.787]	0.016 0.041	0.026 0.806	0.031 0.044	
T-Test	test-stat	p-val	test-stat	p-val	
$\hat{A}_{21} > A_{21}$	-0.883	0.811	-0.410	0.659	

## Joint Estimation: with Feedback loop

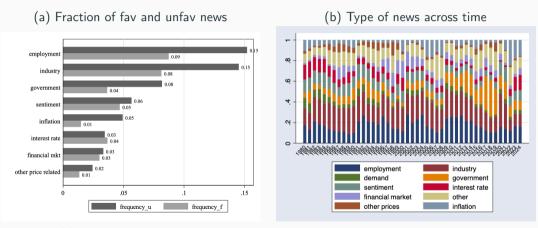
	MSC, quarterly				
	Q1 1984 -	Q4 2019	Q1 1990 - Q	4 2018	
Parameters	Estimates	Standard Errors	Estimates	Standard Errors	
Α	[ 0.863	[0.073 0.162]	[ 0.863	[0.078 0.169]	
A	$\begin{bmatrix} -0.003 & 0.751 \end{bmatrix}$	[0.042 0.074]	$\begin{bmatrix} -0.017 & 0.721 \end{bmatrix}$	0.042 0.076	
Â	[0.663 -0.096]	[0.063 0.089]	[0.663 -0.081]	[0.080 0.094]	
A	[0.189 0.807]	[0.057 0.056]	[0.271 0.769]	[0.064 0.057]	
T-test:	test-stat	p-val	test-stat	p-val	
$\hat{A}_{21} > A_{21}$	2.227	0.013	3.112	0.001	
		SPF, qı	uarterly		
	Q1 1984 - Q4 20	19	Q1 1990 - Q	4 2018	
Parameters	Estimates	Standard Errors	Estimates	Standard Errors	
Α	[0.696 -0.091]	[0.078 0.090]	[0.678 -0.062]	[0.086 0.107]	
A	0.021 0.792	[0.031 0.072]	0.019 0.785	0.034 0.089	
Â	[0.951 0.004]	[0.018 0.041]	[0.937 -0.027]	[0.021 0.030]	
A	[0.026 0.787]	[0.016 0.041]	0.026 0.806	[0.031 0.044]	
T-Test	test-stat	p-val	test-stat	p-val	
$\hat{A}_{21} > A_{21}$	0.136	0.446	0.1534	0.439	

#### $\pi$ news drives expectations across domains but $\mathit{un}$ news is domain-specific



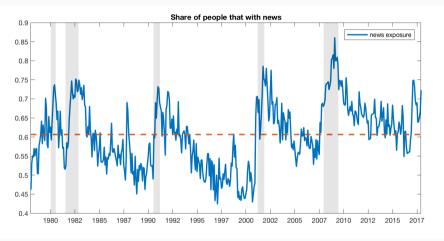
## Reported news in MSC

Figure 5: Type of news



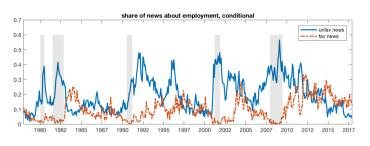
Notes: Panel (a): fractions of favorable and unfavorable news reported by individuals with news in MSC. Panel (b): shares of different types of news out of total news reported each year.

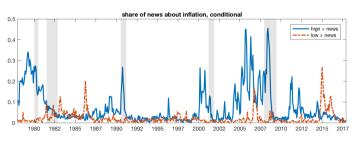
#### **News** measure



Share of people that report hearing any news across time. The dashed line represents on average 60% survey participants reported hearing about some news in the past few months.

#### **News** measure



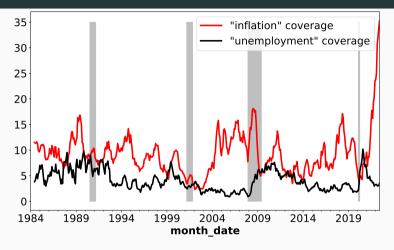


#### **Correlation conditional on news**

Table 7: Correlation Conditional on News Heard

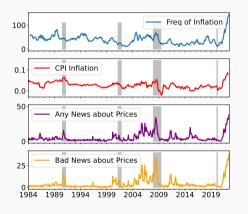
	Dependent var:	E	
		(1)	(2)
-	Eun	0.36***	0.38***
		(0.034)	(0.047)
	Inflation fav $\times Eun$	0.17	0.16
		(0.164)	(0.164)
	Inflation unfav $\times Eun$	0.36***	0.36***
		(0.117)	(0.118)
	Employment fav $\times Eun$	0.03	0.03
		(0.089)	(0.090)
	Employment unfav $\times Eun$	-0.20***	-0.16**
		(0.073)	(0.074)
	Interest rate fav $\times Eun$	-0.23**	-0.24**
		(0.104)	(0.104)
	Interest rate unfav $\times Eun$	-0.16	-0.16
		(0.114)	(0.115)
	Industry fav $\times Eun$		0.06
			(0.092)
	Industry unfav $\times Eun$		-0.23***
			(0.073)
	Demand fav $\times Eun$		-0.14
			(0.145)
	Demand unfav $\times Eun$		-0.57***
			(0.155)
	Gov fav $\times Eun$		0.08
			(0.107)
	Gov unfav $\times Eun$		0.01
			(0.079)
	Sentiment fav $\times Eun$		0.01
			(0.112)
	Sentiment unfav $\times Eun$		0.24**
			(0.113)
	Stock fav ×Eun		-0.11
			(0.085)
	Stock unfav × Eun		0.06
			(0.115)
	Other prices fav $\times Eun$		-0.01
			(0.152)
	Other prices unfav $\times Eun$		-0.16
			(0.130)
	Other real fay $\times Eun$		-0.11
			(0.168)
	Other real unfav $\times Eun$		-0.21
			(0.157)
	Wage fav ×Eun		-0.17
			(0.235)
	Wage unfav $\times Eun$		0.00
	rruge unite ADM		(0.224)
		167346	167346
-			
-	Observations R <sup>2</sup>	0.674	0.675

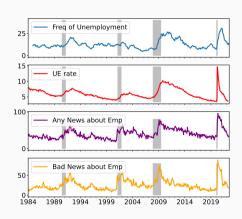
## Newspaper coverage of inflation and unemployment



The news coverage is defined as the sum of ratios of the word frequency divided by the total number of words in each article.

## News on inflation and unemployment is domain-specific





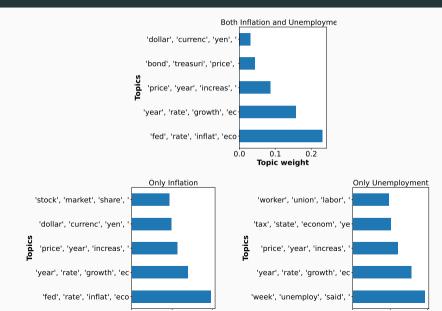
News coverage measured in the WSJ news archive.

## Inflation news is always labeled as bad news

 Table 7: News Coverage and Self-Reported News Exposure

Topic	Any News	Bad News	Good News
Inflation	0.605	0.627	-0.048
Unemployment	0.373	0.295	0.153

#### **Topics in Inflation-Unemployment Narratives**



#### **Keywords in Different Inflation-Unemployment Narratives**



- Andrade, Philippe and Hervé Le Bihan, "Inattentive professional forecasters," *Journal of Monetary Economics*, 2013, 60 (8), 967–982.
- **Bhandari, Anmol, Jaroslav Borovička, and Paul Ho**, "Survey data and subjective beliefs in business cycle models," *Review of Economic Studies*, 2025, *92* (3), 1375–1437.
- Candia, Bernardo, Olivier Coibion, and Yuriy Gorodnichenko, "Communication and the Beliefs of Economic Agents," Working Paper 27800, National Bureau of Economic Research September 2020.
- **Coibion, Olivier and Yuriy Gorodnichenko**, "What Can Survey Forecasts Tell Us about Information Rigidities?," *Journal of Political Economy*, 2012, *120* (1), 116 159.

#### References ii

- **Kamdar, Rupal**, "The Inattentive Consumer: Sentiment and Expectations," 2019 Meeting Papers 647, Society for Economic Dynamics 2019.
- **Lucas, Robert E.**, "Econometric policy evaluation: A critique," *Carnegie-Rochester Conference Series on Public Policy*, 1976, 1, 19 46.
- Maćkowiak, Bartosz, Filip Matějka, and Mirko Wiederholt, "Dynamic rational inattention: Analytical results," *Journal of Economic Theory*, 2018, *176*, 650–692.
- **Sims, Christopher A.**, "Implications of rational inattention," *Journal of Monetary Economics*, 2003, *50* (3), 665 690. Swiss National Bank/Study Center Gerzensee Conference on Monetary Policy under Incomplete Information.
- **Woodford, Michael**, "Imperfect Common Knowledge and the Effects of Monetary Policy," Working Paper 8673, National Bureau of Economic Research December 2001.