# Uncovering Subjective Models from Survey Expectations*


Chenyu Hou*
Simon Fraser University

Tao Wang
Bank of Canada



First Version: January 2020

This Version: September 2025



**Abstract**

Households perceive that macroeconomic variables move together differently from that implied by their actual realizations and sophisticated models. We use a structural test derived from a multivariate noisy-information framework and additional evidence from survey data and newspaper narratives to show that information friction alone cannot explain households' tendency to associate higher future inflation with a worse labor market outlook. We also show that the subjective model uncovered from survey data implies amplified output responses to supply shocks, but dampened output and price responses to demand shocks.



**Keywords**: Expectation Formation, Noisy Information, Survey Data, Subjective Models, News Narratives

**JEL codes:** E21, E30, E32, E71, E84



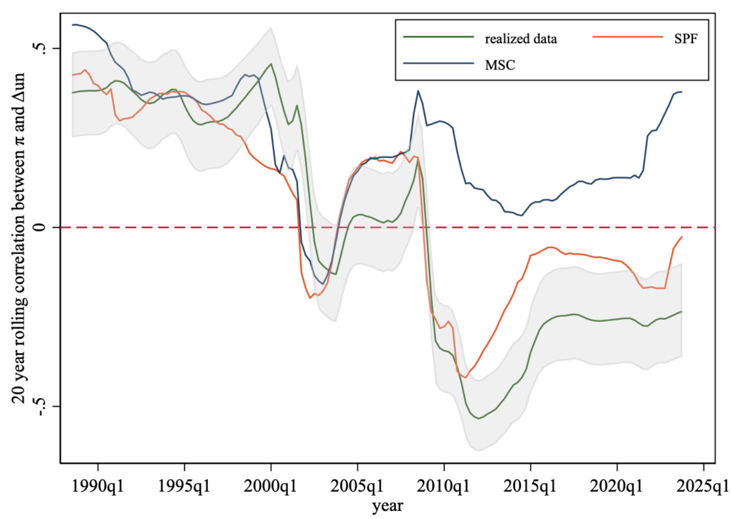
*Hou (corresponding author): chenyu_hou@sfu.ca, Wang: taowangeconomics@gmail.com.


We thank Paul Beaudry, Chris Carroll, Heng Chen, Olivier Coibion, Jesse Perla, Jonathan Adams, Laura Gáti, and participants of the seminars and conferences where this paper was presented for their invaluable comments. The views of this paper are those of the authors, instead of their institutions.

# 1 Introduction

When households expect a higher inflation rate, they also anticipate higher unemployment rates and an underperforming economy.[1] Figure 1 depicts such a pattern using the rolling-window time-series correlation between average households' inflation and unemployment expectations in the *Michigan Survey of Consumer Expectations* (MSC), that of professionals in the *Survey of Professional Forecasters* (SPF), and those of the realization of the two series.[2] Although the realized correlation between the two variables is positive before the 1990s and turns negative after 2000, as reflected more or less by professionals' forecasts in SPF, the correlation of the two expectations in MSC remains mostly positive throughout the entire sample period.[3]

Figure 1: Time Varying Correlation between Inflation and Unemployment Change




Correlation using 20-year rolling window, 1982–2024. Gray line: realized data from FRED. Blue line: expectations from the MSC. Red line: expectations from the SPF.

Such a data pattern naturally calls for an examination of how agents jointly form expectations about different macroeconomic variables. We extend the commonly used test

[1]Several contemporaneous studies, such as Bhandari *et al.* (2025); Kamdar (2019); Andre *et al.* (2022); Candia *et al.* (2020) and Han (2023), also document a similar pattern.

[2]Obtained from the Federal Reserve Bank of St. Louis (FRED). A detailed data description is included in Appendix A.1.

[3]A series of analyses reported in Appendix A.3 and A.4 confirm that such a positive correlation is seen across time and not driven by a certain group of consumers.

on information rigidity as in Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013) from a single-variable to a multi-variable environment, allowing for potentially subjective perceptions of correlation between variables. In the presence of both information rigidity and a subjective model, we characterize exactly how expectations are jointly formed rather than independently formed, and investigate the causes of such a correlation in expectations.

Two possibilities explain correlated expectations across variables: agents may believe the variables are linked-based on either sophisticated mental models or simple heuristics-or they may receive noisy signals that jointly inform both, such as a news article casually discussing inflation and macroeconomic trends. Within our noisy information framework, we offer testable predictions to distinguish models driven by information frictions from those shaped by subjective beliefs. The key insight of our test is that if expectations are formed independently under correlated information, the off-diagonal elements of the forecast error autocovariance matrix should share the same sign. Opposite signs suggest reliance on a subjective model, with the direction of correlation revealing its structure.

Another way of interpreting our test is to see it as a joint sign restriction on the contemporaneous correlations of expectations and their between-variable serial correlation of forecast errors. We show that under very general conditions, a subjective model perceiving a positive relationship between today's inflation and tomorrow's unemployment is necessary for generating the coexistence of positively correlated expectations in the survey data and a positive between-variable serial correlation in forecast errors. Not only do households expect the two variables to move in the same direction, but also overforecasting inflation today leads to overforecasting the unemployment rate tomorrow. In contrast, such a joint pattern is not evident in an alternative environment that only features incomplete information about the state of the economy, where correlated signals could also drive expectation co-movements.

With the test results, we proceed with a joint estimation in the form of a vector-autoregression (VAR) model to uncover the perceived law of motion (PLM) of the macroeconomy by households and professionals, alongside the actual law of motions (ALM) from realized macro data. The estimation enables us to directly compare the estimated PLM and ALM without relying on identifying the exact degree of information frictions and the structure of noisy signals. Our statistical tests with estimates unambiguously reject the null hypothesis of identical PLM and ALM. It confirms that households, as opposed to professionals, associate current inflation with worsening future labor markets. In addition, the direction of the subjective association between inflation and the labor market goes from the former to the latter.

One advantage of our approach to uncovering the subjective model is that it remains agnostic about the underlying drivers of specific subjective patterns. Although this paper does not intend to provide a particular micro foundation for our detected subjective models, we do show that our reduced-form representation of PLM could have a structural interpretation in a standard New Keynesian model. For instance, the positive association of current inflation with higher future unemployment rates, as captured by the off-diagonal term of the PLM matrix, encompasses several possible microfoundations that are observationally equivalent in survey data. We nevertheless view our reduced-form form of subjective model as a sufficient statistic for understanding the implications of subjective models for shock propagation.

We also show that the detected expectational pattern has an important macroeconomic implication. Once the uncovered subjective model is incorporated in a calibrated New Keynesian model (Galí, 2015), the economy's output response to a standard supply shock is amplified while the response to demand shocks is dampened. When a persistent negative supply shock leads to the initial rise in prices and drop in output, the upward change in inflation expectations induces an additional pessimistic shift in future output expectations, reducing demand and output. In contrast, when a negative demand shock hits the economy, pushing down inflation expectations, an associated improvement in the economic outlook counterbalances the negative output impact.[4]

[4] Adams and Barrett (2024) show that empirically identified sentiment shocks to inflation expectations have subsequent deflationary impacts.

Finally, we shed additional light on the detected subjective links between macroeconomic variables by turning to micro-level evidence. We first use the self-reported news exposures in the MSC as an empirical proxy for households' information set changes. We show that households sensibly revise their corresponding expectations about inflation and unemployment after hearing about the news in the respective domain. Meanwhile, it is predominantly the inflation news that also impacts expectations about other variables–unfavorable inflation news exposure is associated with expectations of worse future economic conditions across domains, including higher unemployment expectations. The association in expectations between inflation and unemployment is particularly strong among households that have heard unfavorable news about inflation. We also provide a similar type of evidence with newspaper reporting by a sample of 250,000 economic news articles published in *The Wall Street Journal* between 1984 and 2022. We show that newspaper articles are particularly likely to draw an inflation-unemployment association during episodes of high realized inflation rather than with high unemployment rates.[5]

## Related Literature

This paper builds on the empirical literature on macroeconomic expectations, which tests competing models of expectation formation using forecast errors and revisions from survey data.[6] While early studies emphasized information rigidity, more recent work highlights additional mechanisms—such as perceived persistence—that interact with informational frictions to explain expectation anomalies (Angeletos *et al.*, 2021; Ryngaert, 2018; Farmer *et al.*, 2024; Chen and Liu, 2025). Yet most of this research examines expectations about a single variable. We extend the analysis to multiple variables.[7] Our results show that, as in the univariate case, autocorrelations in forecast errors reveal information rigidity, while cross-

[5] Related to this, Chahrour *et al.* (2024) shows that unfavorable inflation news is more frequently reported than favorable news, which causes a stronger expectational response to unfavorable inflation news.

[6] Coibion and Gorodnichenko (2012), Andrade and Le Bihan (2013), Coibion and Gorodnichenko (2015), Bordalo *et al.* (2018), and Kohlhas and Walther (2021), etc.

[7] Andrade *et al.* (2016) is a notable exception, focusing on the term structure of disagreement in a multivariable setting.

variable correlations in forecast errors uncover either correlated information or perceived interdependencies in subjective models. In our framework, forecasters may hold subjective laws of motion for economic states that deviate from the true process.

We are among the few contemporaneous papers that study the positive correlation between inflation and unemployment rate in household expectations, which include Bhandari *et al.* (2025); Kamdar (2019); Candia *et al.* (2020); Andre *et al.* (2022); Han (2023); Stantcheva (2024). Our additional finding regarding this empirical pattern is that the direction of such a perceived correlation in subjective models particularly goes from inflation to unemployment rather than the other way around. Closely related is the expanding empirical evidence that most households hold negative views toward inflation(Shiller, 1997). Various hypotheses have been put forward to explain this pattern, such as the supply-over-demand view (Kamdar, 2019; Andre *et al.*, 2022; Han, 2023); ambiguity aversion (Bhandari *et al.*, 2025); neglect of macroeconomic trade-offs (Stantcheva, 2024); partisan biases (Gillitzer *et al.*, 2021); personal finance (Bolhuis *et al.*, 2024); and the erosion of real income (Hajdini *et al.*, 2022; Jain *et al.*, 2024; Stantcheva, 2024). Compared with these studies, this paper is agnostic about the relative importance of these channels. We aim at uncovering the general patterns of expectation correlations seen across all households, regardless of their heterogeneity.

More generally, this paper contributes to the literature on subjective models in macroeconomic expectation formation, particularly how expectations of different macroeconomic variables are related to each other. Andre *et al.* (2022) use survey vignettes to show that both households and experts hold heterogeneous views about how the same hypothetically exogenous macroeconomic shocks affect inflation and unemployment rates. Complementary to their paper, we adopt a different approach to detect the subjective perceptions of how macroeconomic variables are correlated with each other, relying on cross-variable restrictions in observational data.[8]

[8] We view their finding of households holding subjective views on how hypothetical shocks propagate as one possible explanation for the subjective model uncovered in our paper, although our estimation approach

The rest of the paper proceeds as follows. Section 2 derives testable implications and performs a test of joint expectation formation under the noisy information model. Section 3 jointly estimates the VAR of expected and realized inflation and unemployment, interprets the uncovered model through the lens of a canonical New Keynesian model, and shows its macroeconomic implications. Section 4 documents independent micro-level evidence on the estimated subjective model using perceived news data in the MSC, as well as provides further supporting evidence using newspaper-based narratives. Finally, Section 5 concludes.

# 2 Test of Joint Expectation Formation

In this section, we first examine different possible sources of the positive correlation between expected inflation and unemployment documented in the introduction. We do so through the lens of the noisy-information model, as in Woodford (2001) and Sims (2003). We show that in this simple framework, different hypotheses can lead to the same correlation between expectations. Consequently, we cannot distinguish between these different hypotheses using the correlation between expectations alone. To solve this problem, we show that these various explanations have different testable implications on the *serial correlations of forecast errors* for inflation and unemployment. Furthermore, the serial correlations of forecast errors are informative about whether the agent jointly or independently forms expectations.

## 2.1 Model Environment

The testable implications on forecast errors that we consider are in the spirit of those from Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013). In our model, there are multiple macroeconomic states that are not directly observable to the agent. The agent may have subjective beliefs about how these states evolve. They observe multiple noisy signals about the states that can be arbitrary combinations of these states that they try

does not rely on specifying the underlying drivers of such a subjective model.

to form beliefs about. Consider the states $\boldsymbol{L}_{t+1,t}$, which are macroeconomic variables that follow the state-space representation (1).

$$\boldsymbol{L}_{t+1,t} = A\boldsymbol{L}_{t,t-1} + w_{t+1,t} \tag{1}$$

The agent observes noisy signals on these variables, with the observational equation given by (2), where $G$ governs the signal structure.[9]

$$\boldsymbol{s}_t^i = G\boldsymbol{L}_{t,t-1} + v_t^i + \eta_t \tag{2}$$

In addition to noisy information, the agent may have a subjective model, or the Perceived Law of Motion (PLM) summarized by Equation 3, about how states evolve, in contrast to the Actual Law of Motion (ALM) summarized in Equation (1).

$$\boldsymbol{L}_{t+1,t} = \hat{A}\boldsymbol{L}_{t,t-1} + w_{t+1,t} \tag{3}$$

$\hat{A}$, namely the subjective model, may or may not be the same as $A$.[10] We show later that whether $\hat{A}$ is diagonal or not has testable implications on the serial correlations of the agent's forecast errors as well as on the correlations between expectational variables.

The signals observed contain an individual-specific noise $v_t^i$ and a time-specific one $\eta_t$, both of which follow a normal distribution with mean zero. The individual noise is independent across agents and time, and the time-specific noise is not autocorrelated and

[9] We do not consider the case where $G$ is also subjective, as in the rational inattention literature where the agents themselves can usually choose $G$. See Mafákowiak *et al.* (2018) as an example. For this reason, we assume the agents always use the correct $G$.

[10] We also assume that such PLMs are fixed, instead of being updated each period through learning, as in Milani (2007), Andrade *et al.* (2016), Gáti (2023), and Hajdini (forthcoming) among others.

independent of the structural shock $w_{t+1,t}$. Adding a time-specific noise does not change the nature of the individual's signal extraction problem. The only difference is that it allows for an imprecise signal after aggregation at each time point. To ease notations, we define $\epsilon_{i,t} := v_t^i + \eta_t$. The distribution of shocks and noises is

$$w_{t+1,t} \sim N(0, Q) \quad \epsilon_{i,t} := v_t^i + \eta_t \sim N(0, R),$$

where $Q$ and $R$ are the corresponding variance-covariance matrices.

The agent then updates their beliefs upon observing $\boldsymbol{s}_t^i$ and form expectations according to a linear Kalman Filter as described in (4), where $K$ is the Kalman Gain.[11]

$$\begin{aligned} \boldsymbol{L}_{t+1,t|t}^i &= \hat{A}\boldsymbol{L}_{t,t-1|t}^i \\ &= \hat{A}\big(\boldsymbol{L}_{t,t-1|t-1}^i + K(\boldsymbol{s}_t^i - G\boldsymbol{L}_{t,t-1|t-1}^i)\big) \end{aligned} \tag{4}$$

From equation (4), it is immediately clear that the beliefs about different macroeconomic states in $\boldsymbol{L}_{t+1,t|t}^i$ are correlated for different reasons, even if the actual states are not correlated (i.e., $A$ and $Q$ are diagonal). First, consider the case where the agent learns about different states independently (i.e., $\hat{A}$ is diagonal).[12] We call this scenario "independent learning." The beliefs are correlated if either the signals are combinations of the states (i.e., $G$ is non-diagonal) or the noises in signals are correlated (i.e., $R$ is non-diagonal). These two cases mainly consider the information frictions that can lead to correlations in expectation variables. They can also be thought of as two different formulations of pessimistic/optimistic heuristics. In the first case, the agent confuses multiple states in one signal and adjusts their beliefs on all the states while observing this signal. In the second case, if the noises are positively correlated, the agent is more likely to observe signals about states biased toward the same direction.

[11] For derivations of the standard Kalman Filter, please see Appendix B.1.

[12] This case corresponds to the common focus of expectation-formation literature on one-variable case, such as Coibion and Gorodnichenko (2012), Andrade and Le Bihan (2013), Ryngaert (2018), and many others.

Another possibility for observing correlated beliefs is that the agent has a subjective model $\hat{A}$ that is non-diagonal. The form of $\hat{A}$ represents the agent's understanding of the joint dynamics of the macroeconomic states in $\boldsymbol{L}_{t+1,t}$. We call this scenario "joint learning," as the agent believes the underlying macroeconomic states are correlated, and this is incorporated into their belief formation process. As a result, the agent adjusts their beliefs on multiple states even if they observe uncorrelated noisy signals about only one of the states.

As all of the aforementioned possibilities can give rise to the same correlation between beliefs, it is important to consider other moments from the belief data that can distinguish between these possibilities. To achieve this, we propose a test using the serial correlations of forecasting errors because they give distinct testable implications for independent learning and joint-learning models. We call this the *"joint learning test."* To derive this test, consider the forecasting error for one period ahead:

$$\begin{aligned} FE^{i}_{t+1,t|t} &\equiv \boldsymbol{L}_{t+1,t} - \boldsymbol{L}^{i}_{t+1,t|t} \\ &= \hat{A}(I-KG)FE^{i}_{t,t-1|t-1} + M\boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K\epsilon_{i,t} \end{aligned} \tag{5}$$

where $M = (A - \hat{A}KG - \hat{A}(I-KG))$. Averaging across agents $i$ at each time $t$, we get an aggregate test on forecasting errors:

$$FE_{t+1,t|t} = \hat{A}(I-KG)FE_{t,t-1|t-1} + M\boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K\eta_t \tag{6}$$

Equation (5) and Equation (6) are the basis of our *joint learning test* at the individual and consensus level, respectively.

The key parameters we focus on are the elements in $\hat{A}(I-KG)$. Considering the state vector $\boldsymbol{L}$ contains unemployment rate change and inflation, both equations can be estimated from survey data using OLS as $w_{t+1,t}$ and $\eta_t$ are independent with $FE_{t,t-1|t-1}$ and $\boldsymbol{L}_{t,t-1}$. Before we show the results from actual survey data, we discuss what the joint learning tests tell us about the different possibilities that can result in correlated expectation variables.

## 2.2 Properties of Joint Learning Test

To ease the exposition, we make the following assumptions:

**Assumption 1.** *The subjective transition matrix $\hat{A}$ has positive eigenvalues within the unit circle.*

**Assumption 2.** *The diagonal elements of the $G$ matrix are positive.*

**Assumption 3.** *The variance-covariance matrix of prior $\boldsymbol{L}^i_{t,t-1|t-1}$ is a diagonal matrix and common to each individual:*

$$\Sigma := diag(\{\sigma_i^2\}_{i=1}^n)$$

Assumption 1 suggests that the agent considers a stationary process for the unobservable states. Assumption 2 guarantees that each signal increases as the corresponding state increases.[13] Finally, Assumption 3 assumes that the agent uses priors where the two variables are not correlated with each other.[14]

Under these assumptions, expectations formed by independent learning and joint learning will lead to different properties of the coefficient matrix $\hat{A}(I - KG)$. We first consider the case of FIRE.

**Proposition 1.** *Under FIRE, e.g., $A = \hat{A}$, $G = I$ and $R \to \mathbf{0}$, the coefficient matrix $\hat{A}(I - KG) = \mathbf{0}$.*

*Proof.* See Appendix C.1. □

This proposition shows that, under FIRE, lagged forecast errors cannot predict current forecast errors. This holds even under joint expectation formation (i.e., when $A$ is non-diagonal). In other words, under FIRE, neither a variable's own past forecast errors nor

[13] This is a regularity assumption, which helps anchor our discussions about the sign restrictions regarding $\hat{A}(I - KG)$. Oppositely moved signals relative to the underlying states imply similar predictions.

[14] We do not separately consider another scenario where the *prior* beliefs of the agent perceives non-zero correlations (i.e., a non-diagonal $\Sigma$), as it is inherently similar to the case of the subjective model perceiving such a correlation (i.e., a non-diagonal $\hat{A}$.)

the past forecast errors of other variables–regardless of whether agents perceive them as correlated– can systematically predict current forecast errors.

Next, we turn to the cases with imperfect information where $R \neq \mathbf{0}$. The matrix $\hat{A}(I - KG)$ has different patterns under joint or independent learning. First, we consider the case of independent learning where $\hat{A}$ is diagonal.

**Proposition 2.** ***(Independent Learning)*** *If $\hat{A} = diag(\{a_i\}_{i=1}^n)$, denote the off-diagonal elements of $\hat{A}(I - KG)$ as $w_{ij}$ with $i \neq j$. We have:*

*(1) $w_{ij} = 0$ if $G$ and $R$ are diagonal.*

*(2) $w_{ij} = w_{ji} = 0$ or $w_{ij}w_{ji} > 0$ if $G$ or $R$ is non-diagonal.*

*Proof.* See Appendix C.2. □

Proposition 2 makes two distinct points. First, if the agent does not consider the macroeconomic states to be correlated ($\hat{A}$ is diagonal) and they observe uncorrelated, separate signals regarding each state, the expectation formation process collapses to the single-variable noisy-information model as in Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013). The forecast errors of one variable predict its future forecast errors due to information rigidity, but the forecast errors of other variables can not. Second, under independent learning, if signals on different states are mixed, the forecast errors of one state can predict future forecast errors of the other *symmetrically*. In particular, the directions of such predictability are related to how the signals are generated. For simplicity, we formalize these properties in the case with two states.

**Corollary 1.** ***(Non-diagonal $R$: correlated noises)*** *If $\hat{A}$ and $G$ are diagonal and $R = \begin{pmatrix} \sigma_{1,s}^2 & \rho \\ \rho & \sigma_{2,s}^2 \end{pmatrix}$, the off-diagonal elements of $\hat{A}(I - KG)$ have the same signs as $\rho$ if $\hat{A}$ has positive entries on the diagonal.*

*Proof.* See Appendix C.3. □

Corollary 1 shows that the forecast error of one state positively predicts the future forecast error of the other if the noises are positively correlated. The intuition is as follows. Without loss of generality, suppose the agent wants to infer the first state. When they see both signals, as they recognize the noises are positively correlated, the agent puts positive weight on the signal about the first state and negative weight on the signal about the other state to correct for the correlation in noises.[15] As a result, a positive shock to state 1 leads to positive forecast errors in both states. The forecast errors of both states are persistent due to information rigidity, so a positive forecast error in the first state predicts a positive forecast error in the second state.

Another possibility is that the signal observed combines information about both states (i.e., $G$ is non-diagonal). In this case, we consider only *triangular* $G$. This configuration is without loss of generality, as any signals with general 2 by 2 $\hat{G}$ can be reformulated into signals with triangular $G$ and they lead to the same posterior beliefs.[16]

**Corollary 2.** ***(Non-diagonal*** $G$***: correlated signals)*** *If* $\hat{A}$ *is diagonal,* $R = \begin{pmatrix} \sigma_{1,s}^2 & 0 \\ 0 & \sigma_{2,s}^2 \end{pmatrix}$ *is diagonal and* $G = \begin{pmatrix} g_1 & g_2 \\ 0 & g_4 \end{pmatrix}$*, the off-diagonal elements of* $\hat{A}(I - KG)$ *have the opposite signs as* $g_1 g_2$*.*

*Proof.* See Appendix C.4. □

To understand the intuition behind Corollary 2, consider the case where $g_1$ and $g_2$ are both positive. When the first state increases and the second state stays the same, the agent sees a positive signal 1. As they are not sure which state increases, the agent adjusts beliefs upwards on both signals. As a result, they have a positive forecast error in the first state and a negative forecast error in the second. Due to information rigidity, a positive forecast error in one state now predicts a negative forecast error in the other state in the future.

[15]We can see this from the fact that the off-diagonal elements in the Kalman Gain are both negative in this case.

[16]The formal proof is included in Appendix C.

Now we move to the case of joint learning. Note that the counter-positive argument of Proposition 2 leads to the testable implications under models of *joint expectation formation.*

**Proposition 3.** ***(Joint Learning)*** *If off-diagonal elements of $\hat{A}(I-KG)$ are not both zeros and of different signs, then $\hat{A}$ is non-diagonal, regardless of whether $G$ and $R$ are diagonal or not.*

*Proof.* This is the counter-positive of Proposition 2. □

Moreover, if we consider the case where signals are separate and not correlated, we can get more informative results about $\hat{A}$ by looking at the off-diagonal elements of $\hat{A}(I-KG)$.

**Proposition 4.** ***(Joint Learning with separate signals)*** *If both $G$ and $R$ are diagonal and $\hat{A} = \left(a_{ij}\right)_{n\times n}$ is non-diagonal, denote $\hat{A}(I-KG) = \left(w_{ij}\right)_{n\times n}$. The signs of these off-diagonal elements are the same as their counterparts in $\hat{A}$ (i.e., $w_{ij}a_{ij} > 0$).*

*Proof.* See Appendix C.5. □

This proposition shows that when the signals on multiple states are separate and noises are uncorrelated, the coefficient matrix $\hat{A}(I-KG)$ has non-zero off-diagonal elements if and only if the agent believes in a non-diagonal $\hat{A}$. The signs on the off-diagonal elements in $\hat{A}(I-KG)$ are the same as those in $\hat{A}$.

The intuition behind this proposition is also straightforward. Suppose that the first element in $\boldsymbol{L}_{t,t-1}$ is the change in the unemployment rate, and the second element is inflation. If the agent over-predicted inflation yesterday due to a noise shock to the inflation signal, they will also over-predict current inflation due to information rigidity. Such an over-prediction will create an over-prediction of unemployment today if the agent believes that higher inflation leads to a higher unemployment rate. In contrast, it will create an under-prediction of unemployment today if they believe that inflation lowers unemployment.

Finally, it is important to note that the properties of the joint learning test we describe in this section do not depend on the actual $A$ matrix at all. In other words, the joint learning test is useful to uncover the agent's subjective model $\hat{A}$ no matter what the true model ($A$) is.

### 2.3 Taking Stock

In Section 2.2, we show that the coefficient matrix $\hat{A}(I - KG)$ in the proposed joint learning test has different properties when beliefs are formed under FIRE, single-variable learning, or joint learning. It is now useful to link the results from such tests with implied correlations between belief variables under these different scenarios. We focus on the case where the hidden macroeconomic states $\boldsymbol{L}_{t+1,t}$ are inflation and change in unemployment rate. Recall the consensus mean forecast is given by the average of (4) across individuals. Define $Y_t = \begin{pmatrix} L_{t,t-1|t-1} \\ L_{t,t-1} \end{pmatrix}$ and we can write (4) and ALM (1) as the following vector autoregression (VAR) model:

$$Y_{t+1} = \underbrace{\begin{pmatrix} \hat{A}(I - KG) & \hat{A}KG \\ \mathbf{0}_{2\times 2} & A \end{pmatrix}}_{:=\Phi} \cdot Y_t + \underbrace{\begin{pmatrix} \hat{A}K & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & I_{2\times 2} \end{pmatrix}}_{F} \cdot \begin{pmatrix} \eta_t \\ w_{t+1,t} \end{pmatrix} \tag{7}$$

Then we know the stationary variance-covariance matrix is given by:

$$vec(\Sigma_L) = (I_{16} - \Phi \otimes \Phi)^{-1} vec\left(F(R + Q)F'\right) \tag{8}$$

The correlation between belief variables implied by the this covariance matrix differs depending on whether expectations are formed independently, jointly, or under FIRE. Guided by the results from the previous section, we can simply separate these different frictions into the following formulations w.l.o.g.:

$$\hat{A} = \begin{pmatrix} \cdot & m_1 \\ 0 & \cdot \end{pmatrix}, \quad G = \begin{pmatrix} \cdot & g_2 \\ 0 & \cdot \end{pmatrix}, \quad R = \begin{pmatrix} \cdot & \rho \\ \rho & \cdot \end{pmatrix}$$

Table 1: Summary of Joint Learning Test

| | Assuming actual $A$ being diagonal | | | | |
|---|---|---|---|---|---|
| Cases: | $\hat{A}$ | $G$ | $R$ | Off-diagonal elements of $\hat{A}(I-KG)$ | $Corr(E\pi, Eun)$ |
| FIRE | $= A$ | N/A | $= \mathbf{0}$ | both $= 0$ | $= Corr(\pi, un)$ |
| $\Sigma$ is Diagonal | Diag | Diag | Diag | both $= 0$ | $= 0$ |
| | Diag | Diag | $\rho > 0$ | both $> 0$ | $\gtreqless 0$ |
| | Diag | Diag | $\rho < 0$ | both $< 0$ | $\gtreqless 0$ |
| | Diag | $g_2 > 0$ | Diag | both $< 0$ | $\gtreqless 0$ |
| | Diag | $g_2 < 0$ | Diag | both $> 0$ | $\gtreqless 0$ |
| | $m_1 > 0$ | Diag | Diag | $> 0$ at (1,2), $= 0$ at (2,1) | $> 0$ |
| | $m_1 < 0$ | Diag | Diag | $< 0$ at (1,2), $= 0$ at (2,1) | $< 0$ |

Notes: The implied signs of the cross-terms in the forecast error test we proposed before, and the correlation between two macroeconomic states, for different configurations of $\hat{A}$, $G$, and $R$: $R = \begin{pmatrix} \cdot & \rho \\ \rho & \cdot \end{pmatrix}$, $G = \begin{pmatrix} \cdot & g_2 \\ 0 & \cdot \end{pmatrix}$, $\hat{A} = \begin{pmatrix} \cdot & m_1 \\ 0 & \cdot \end{pmatrix}$ We maintain the assumption as in Section 2.2 and 2.3 that $A$ and $\Sigma$ are both diagonal.

Table 1 summarizes the testable implications of these different frictions in the noisy-information model.

Unlike the properties of the joint learning test, the correlation between belief variables clearly depends on the form of $A$. We focus on the most clear-cut case where $A$ is diagonal.[17]

In Table 1, first note that under FIRE or independent learning with separate signals ($\hat{A}$, $G$, and $R$ are all diagonal), they have the same implications on the off-diagonal elements of $\hat{A}(I-KG)$ and $Corr(E\pi, Eun)$. However, under FIRE the diagonal elements of $\hat{A}(I-KG)$ would be zeroes, whereas under independent learning they would be between zero and one. More importantly, Table 1 shows that the positive correlation between expected inflation and unemployment status can come from a correlation in noises, a mix of states in the signals observed, or the agent's subjective model. The off-diagonal elements of $\hat{A}(I-KG)$ from the joint learning test offer additional moments that can help distinguish between these possible explanations. In particular, if the off-diagonal elements are estimated to have different signs,

[17] When $A$ is non-diagonal, the correlation between inflation and unemployment will be non-zero. In that case, the properties of off-diagonal elements in $\hat{A}(I-KG)$ remain the same as in Table 1. The correlation $corr(E\pi, Eun)$ will be bigger (smaller) than $corr(\pi, un)$ if the off-diagonal elements in $\hat{A}$ are bigger (smaller) than the corresponding elements in $A$.

it suggests the agent has a non-diagonal subjective model $\hat{A}$ and correlated or mixed signals *cannot* be the only reasons that lead to the correlation between expectation variables.

### 2.4 Empirical Tests on Joint Learning

Guided by Table 1, we perform the joint learning test using survey data from the MSC and the SPF. To do this, we follow (6) and simply estimate the following regressions:

$$\begin{pmatrix} fe^{\pi}_{t+1,t|t} \\ fe^{un}_{t+1,t|t} \end{pmatrix} = \boldsymbol{\beta}_0 + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} fe^{\pi}_{t,t-1|t-1} \\ fe^{un}_{t,t-1|t-1} \end{pmatrix} + \boldsymbol{\Theta} X_{t,t-1} + e_t \tag{9}$$

where $fe^{x}_{t+h,t|t}$ stands for the h-period ahead forecasting errors of variable $x$.

However, with the MSC, we do not observe $fe^{x}_{t+1,t|t}$ directly; rather, we have data on year-ahead forecast errors $fe^{x}_{t+4,t|t}$. We can then use the four-period-ahead version of equation (6):

$$\begin{aligned} FE_{t+4,t|t} &= \hat{W}\hat{A}(I-KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (I-\hat{W}\hat{A}(I-KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\ &\quad - (\hat{W}\hat{A}KG+I)\boldsymbol{L}_{t,t-1} + A\boldsymbol{L}_{t+3,t+2} + w_{t+4,t+3} - \hat{W}\hat{A}K\eta_t \end{aligned} \tag{10}$$

where $\hat{W} = I + \hat{A} + \hat{A}^2 + \hat{A}^3$, and the fact that $\hat{A}$ is stationary guarantees that $\hat{W}$ is invertible. The derivation that extends (6) to (10) is in **Appendix B.2**. More importantly, the properties of $\beta$s derived in the last section hold true for the year-ahead specification as well. To illustrate the similar performance of the proposed quarter-ahead test (6) and year-ahead test (10), we perform the proposed tests with simulated data and include these results in **Appendix D**. We can then estimate:

$$\begin{pmatrix} fe^{\pi}_{t+4,t|t} \\ fe^{un}_{t+4,t|t} \end{pmatrix} = \boldsymbol{\beta}_0 + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} fe^{\pi}_{t+3,t-1|t-1} \\ fe^{un}_{t+3,t-1|t-1} \end{pmatrix} + \boldsymbol{\Theta} X_{t+3,t-1} + e_t \tag{11}$$

The parameters of interest are $\beta_{11}, \beta_{12}, \beta_{21}$, and $\beta_{22}$. They can be estimated using OLS because, in equation (10), the two components of the error term are uncorrelated with all the regressors. The $w_{t+4,t+3}$ is an unpredictable error happening after $t+3$, thus uncorrelated with forecasting errors up to $t+3$ as well as any variable realized before $t+4$. The noise attached to public signal $\eta_t$ is realized at time $t$ and thus does not correlate with the forecast error with the information set at time $t-1$. Here we have to assume there is no feedback effect of $\eta_t$ on realized macroeconomic variables after time $t$ through general equilibrium so that $\eta_t$ is uncorrelated with any variable (except for expectational ones) realized beyond time $t$.[18]

Another complication to performing the test is that it requires the unemployment rate change to be comparable to the realized data to create forecast errors. The data in the MSC on unemployment expectation is categorical. We follow Bhandari *et al.* (2025) and Mankiw *et al.* (2004) to impute the expectation series.[19]

It is worth noting that the assumption essential to recovering unemployment expectation is that the predicted unemployment change follows a normal distribution with a constant variance across time. This assumption is particularly plausible in the framework of a noisy-information model with a stationary Kalman Filter, as the posterior distributions of forecasted variables are normally distributed and stationarity guarantees a time-invariant posterior variance.

We then estimate (11) with year-ahead forecast errors on expected inflation and expected unemployment rate change with OLS, controlling for corresponding realized variables according to (10).[20] Four coefficients in (11) are estimated. Among these, $\beta_{11}$ and $\beta_{22}$ are the typical indicators for the presence of information rigidity as in Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013). Higher values of these terms imply higher degrees of information rigidity (noisier signals). The key coefficients related to joint learning are $\beta_{12}$ and

[18]Notice $v_t^i$ disappears as we derive the consensus expectation. This is because the idiosyncratic noise has a zero mean at each time point.

[19]We discuss the imputation approach in Appendix A.5.

[20]The imputation method involves the use of SPF and uses the consensus expectation on unemployment status. Such an approach does not apply to panel data. For this reason, in the baseline analysis for the SPF and the MSC, we consider the aggregate version of the joint-learning test (10).

$\beta_{21}$. We call them *the cross-terms* of coefficients on forecast errors, the properties of which are summarized in Table 1. The goal of this exercise is to assess which model of expectation formation can be reconciled with the estimates of these four coefficients from survey data. Table 2 presents the key results with the MSC and the SPF.

Table 2: Aggregate Test on Joint Learning, MSC vs SPF

| | MSC | | | SPF | | |
|---|---|---|---|---|---|---|
| | 1984-2023 | 1981-2018 | 1990-2018 | 1984-2023 | 1981-2018 | 1990-2018 |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\beta_{11}$ | 0.64*** | 0.61*** | 0.65*** | 0.79*** | 0.74*** | 0.76*** |
| | (0.080) | (0.066) | (0.085) | (0.064) | (0.061) | (0.093) |
| $\beta_{12}$ | −0.11 | −0.14 | −0.02 | 0.19 | −0.28 | −0.08 |
| | (0.076) | (0.087) | (0.095) | (0.117) | (0.200) | (0.199) |
| $\beta_{21}$ | 0.13*** | 0.11*** | 0.21*** | 0.05 | 0.04* | 0.06 |
| | (0.033) | (0.039) | (0.063) | (0.034) | (0.024) | (0.049) |
| $\beta_{22}$ | 0.71*** | 0.60*** | 0.50*** | 0.63*** | 0.55*** | 0.51*** |
| | (0.044) | (0.079) | (0.091) | (0.060) | (0.072) | (0.097) |
| Observations | 152 | 149 | 116 | 152 | 149 | 116 |

* ***,**,*: Significance at 1%, 5%, and 10% level. Estimation results for joint-learning test (11). Columns (1)–(3) are results from the MSC and (4)–(6) are results from SPF. Columns (1) and (4) use a sample of 1984–2023, excluding the outlier of the year 2019, where the change in unemployment is around 10%. Columns (2) and (5) use samples 1981q3–2018q4 to avoid the COVID-19 period. Columns (3) and (6) use a sample from 1990–2018 to stay away from the Volker and COVID-19 periods. Newey-West standard errors are reported in brackets.

The first column of Table 2 contains estimation results using the baseline sample for 1984–2023. The estimates on $\beta_{11}$ and $\beta_{22}$ are significantly positive, meaning that the consumers form expectations with limited information. The significant estimates on $\beta_{21}$ suggest that consumers *do not* form expectations on unemployment and inflation independently, with separate signals. Moreover, the fact that $\beta_{12}$ and $\beta_{21}$ have different signs suggests that consumers are forming expectations jointly with subjective beliefs about the structural relationship between inflation and unemployment, $\hat{A}$.[21] According to Proposition 4, the agent's subjective model indicates that past inflation will lead to an increase in the unemployment rate. From Table 1, such a belief structure $\hat{A}$ can induce a positive correlation between the

[21]This follows from Proposition 3. To be clear, the test results in Table 2 suggest that $\hat{A}$ is non-diagonal, but they DO NOT exclude the possibility that $G$ and $R$ may also be non-diagonal.

two expectations.

Columns (1)–(3) in Table 2 also suggest that the pessimistic heuristics in the form of non-diagonal $R$ or $G$ cannot be the *only* reason for the positive correlation between expected inflation and unemployment status. If pessimistic heuristics are the only frictions in expectation formation, $\beta_{21}$ and $\beta_{12}$ would both be negative. These are inconsistent with the estimates in Table 2.

On the other hand, the results from columns (4)–(6) show that the professionals seem to have a different $\hat{A}$ from consumers. The significant $\beta_{11}$ and $\beta_{22}$ suggest again the presence of information rigidity. The estimates are comparable with previous studies imposing independent learning.[22] Contrary to the results with the MSC, the small and insignificant $\beta_{12}$ and $\beta_{21}$ imply that they do not believe lagged inflation will raise the future unemployment rate. These results are consistent with the finding that there is a positive correlation between expected unemployment and inflation in the MSC, whereas such a correlation does not appear in SPF. All in all, the estimates from SPF suggest that professionals are closer to independent expectation formation, or at least use a different structure $\hat{A}$ from consumers, when forming expectations.

Moreover, all the above results hold for different cuts of samples. In columns (2) and (5), we omit the COVID-19 episode and the results for both the MSC and the SPF are consistent with those in the baseline results. Recall in Figure 1 that the correlations between realized inflation and unemployment fall below zero after the 1990s.[23] Meanwhile, the correlation between expected variables in MSC stays positive. It is in this episode that the two correlations have the starkest disconnection. In columns (3) and (6), we include the estimates using a subsample from 1990–2018 for both the MSC and the SPF. The results are qualitatively in line with those using the baseline sample. Moreover, the estimated $\beta_{21}$ is twice as large, suggesting that consumers believe in a stronger response of the future unemployment rate

[22]For example, in Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013).

[23]In Figure 1, we used a 10-year rolling window and plotted the correlation against the ending date of that window. The figure suggests using realized data after the 1990s, inflation and unemployment became negatively correlated.

to current inflation.

# 3 Subjective Model: Interpretation and Caveats

The previous sections focus on the joint learning test under various assumptions according to Table 1. But there are several caveats with such a test. First, it relies on the assumption of an uncorrelated prior. Secondly, it does not assess whether the agent's perceived law of motion (PLM) aligns with the actual law of motion (ALM) (i.e., whether $\hat{A} = A$). Finally, we maintained a restriction that there is no feedback from expectations to realized inflation and unemployment. Despite being a standard assumption in the empirical literature that tests information rigidity, this assumption seems too restrictive given the recent studies showing that imperfect expectation formation has significant consequences on realized macroeconomic outcomes (e.g. Beaudry *et al.* (2025)).

To alleviate some of these concerns, we first show that our framework provides a way to directly estimate $A$ and $\hat{A}$ without imposing restrictions on the prior and signal/noise structures. We then discuss how this framework can also allow for feedback from expectations to realized inflation and unemployment, with a timing restriction on the information used in expectations. After discussing these caveats, we provide more structural interpretations of the subjective model $\hat{A}$ and the objective model $A$ by linking them with a standard New Keynesian model. Finally, we illustrate how the presence of a subjective model will affect the dynamics of inflation and unemployment in an otherwise standard New Keynesian model.

## 3.1 Uncovering the Subjective Model

We can uncover the exact shape of $A$ and $\hat{A}$ by estimating an expectation-realization VAR to match the joint dynamics of inflation, changes in the unemployment rate, and their respective expectations, as specified in Equation (7). Note that this VAR representation follows directly from the general noisy information framework and the ALM, thus not depending on the assumption of uncorrelated priors or the degree of information frictions. The estimation of yields estimates of $A$, $\hat{A}KG$, and $\hat{A}(I-KG)$. Summing the estimated $\hat{A}(I-KG) \equiv B$ and $\hat{A}G \equiv C$ yields estimates of $\hat{A}$. This is convenient in that we can directly test if $A = \hat{A}$, and the estimation of the PLM $\hat{A}$ does not rely on uncovering the exact degree of information frictions governed by $K$ and $G$.

We estimate Equation (7) with the same dataset as in our joint learning tests, using the iterative generalized method of moments (GMM) with an efficient weighting matrix. Our baseline estimation assumes no feedback loop from expectations to realized values, which means we estimate a restricted VAR such that the bottom-right 2-by-2 submatrix in $\Phi$ contains all zeros. We later relax the no-feedback assumption, which does not alter our main finding. Lastly, because quarterly observations of annualized changes are used, we use Newey-West standard errors up to four quarters when calculating the variance and covariance matrix of moment conditions. Table 3 reports the estimation results with our baseline sample 1984q1–2023q4.[24]

We primarily focus on the estimates of the off-diagonal terms of $\hat{A}$, which reveal the perceived between-variable serial correlation between inflation and unemployment rate changes. Although according to the ALM, $A_{2,1}$, the realized unemployment rate change's response to the lagged inflation rate, is 0.033 and is insignificantly different from 0, the perceived response is as big as 0.16 with a standard error of 0.04. This result is consistent with the evidence thus far that current inflation leads to more pessimistic labor market expectations. To decide if the difference is statistically different from zero, we perform a statistical test of GMM estimates under a null hypothesis of $A_{2,1} \geq B_{2,1} + C_{2,1} \equiv \hat{A}_{2,1}$. The null hypothesis is easily rejected at the 5% significance level.

In contrast, the estimation of professional forecasts in the same sample confirms different PLM patterns from those of households. In particular, the professionals' subjective model perceives little impact of current inflation on future unemployment rate changes ($\hat{A}_{2,1} =$

[24]Estimation results based on alternative samples, 1984-2019 or 1990-2018, yield identical conclusions (Table 14).

Table 3: Estimates of Joint Learning Model (7)

| **MSC, quarterly, Q1 1984–Q1 2023** | | |
|---|---|---|
| Parameters | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.852 & -0.053 \\ 0.033 & 0.616 \end{bmatrix}$ | $\begin{bmatrix} 0.051 & 0.056 \\ 0.044 & 0.095 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.741 & -0.151 \\ 0.161 & 0.837 \end{bmatrix}$ | $\begin{bmatrix} 0.057 & 0.082 \\ 0.045 & 0.048 \end{bmatrix}$ |
| T-test: | test-stat | p-val |
| $\hat{A}_{21} \leq A_{21}$ | 1.905 | 0.028 |
| **SPF, quarterly, Q1 1984–Q1 2023** | | |
| Parameters | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.837 & -0.056 \\ 0.014 & 0.751 \end{bmatrix}$ | $\begin{bmatrix} 0.061 & 0.074 \\ 0.041 & 0.093 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.955 & -0.038 \\ 0.040 & 0.495 \end{bmatrix}$ | $\begin{bmatrix} 0.019 & 0.016 \\ 0.035 & 0.239 \end{bmatrix}$ |
| T-Test | test-stat | p-val |
| $\hat{A}_{21} \leq A_{21}$ | 0.546 | 0.293 |

The table reports the estimates and their Newey-West standard errors from the GMM estimation of the four-variable VAR model. An iterative weighting matrix is used in the GMM estimation. The standard errors are based on the variance-covariance matrix of model estimates. Since $\hat{A}$ is the element-wise sum of directly estimated $B$ and $C$, the element-wise variance-covariance matrix of $B$ and $C$ are used to calculate the standard errors of $\hat{A}$ estimates.

$0.04, s.e. = 0.035$). This is not significantly different from the actual impacts of inflation on unemployment ($A_{2,1} = 0.014, s.e. = 0.041$). It is also worth noting that in both MSC and SPF cases, estimates of $A_{2,1}$ are not significantly different from zero.

Besides the between-variable correlation, PLM also differs from ALM in terms of the persistence of the inflation rate. Households, on average, underperceive the persistence of inflation. In contrast, the professionals' subjective model overly perceives the persistence of inflation. This is consistent with Ryngaert (2018), who finds an overperception of persistence in SPF inflation forecasts.[25]

[25] However, it is important to note that in a structural model, the persistence of inflation and the unemployment rate are endogenous. We discuss these in section 3.2 where we link the PLM and ALM with parameters in a standard New Keynesian Model.

In light of these results, it is worth emphasizing that both our estimated PLM and ALMs, as captured by $A$ and $\hat{A}$, are in reduced form. We view this as a strength of our approach in that the estimated $\hat{A}$ sufficiently captures households' perceived correlations between the two variables, regardless of whether they stem from naive reasoning, sophisticated inference, or misperceptions of certain parts of the propagation mechanisms in the macroeconomy. Therefore, the fact that $\hat{A}$ and $A$ differ is sufficient for us to conclude that PLM differs from ALM. With that said, however, in Section 3.2, we provide a more structural interpretation of such PLM and ALM through the lens of a canonical New Keynesian model.

## 3.2 Interpreting the Results via a Standard NK model

In this subsection, we show that the $\hat{A}$-implied perceived law of motion can be linked with subjective beliefs about structural parameters from a simple three-equation New Keynesian Model as in Galí (2015). To stay close to our expectation formation model, we first invoke Okun's Law and link the output gap $y_t$ with the unemployment rate, $u_t = -\chi y_t$. The model can be written as:

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\kappa}{\chi} u_t + s_t \tag{12}$$

$$u_t = E_t u_{t+1} + \frac{\chi}{\sigma}(i_t - E_t \pi_{t+1}) - \chi d_t \tag{13}$$

$$i_t = \phi_\pi \pi_t - \frac{\phi_y}{\chi} y_t, \tag{14}$$

where $i_t$ is the deviation of nominal interest rate from its long-run mean, $s_t$ and $d_t$ are supply and demand shocks that follow persistent AR(1) processes with:

$$\begin{pmatrix} d_t \\ s_t \end{pmatrix} = \underbrace{\begin{pmatrix} \rho_d & 0 \\ 0 & \rho_s \end{pmatrix}}_{\equiv \Gamma} \begin{pmatrix} d_{t-1} \\ s_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^d \\ \varepsilon_t^s \end{pmatrix} \tag{15}$$

The agents can form expectations of $L_{t,t-1} \equiv \begin{pmatrix} \pi_t \\ u_t \end{pmatrix}$ according to Equations (12)-(14) with subjective parameters $\{\hat{\beta}, \hat{\kappa}, \hat{\rho}_d, \hat{\rho}_s, \hat{\chi}, \hat{\sigma}, \hat{\phi}_\pi, \hat{\phi}_y\}$. They recognize that this model adopts a state-space representation where:[26]

$$L_{t,t-1} = \underbrace{\begin{pmatrix} \frac{\hat{\sigma}\hat{\kappa}}{\hat{\Delta}_d} & \frac{\hat{\sigma}(1-\hat{\rho}_s)+\hat{\phi}_y}{\hat{\Delta}_s} \\ -\frac{\hat{\sigma}\hat{\chi}(1-\hat{\beta}\hat{\rho}_d)}{\hat{\Delta}_d} & \frac{\hat{\chi}(\hat{\phi}_\pi-\hat{\rho}_s)}{\hat{\Delta}_s} \end{pmatrix}}_{\equiv\hat{\Psi}} \begin{pmatrix} d_t \\ s_t \end{pmatrix} = \begin{pmatrix} \hat{\Psi}_{\pi d} & \hat{\Psi}_{\pi s} \\ \hat{\Psi}_{ud} & \hat{\Psi}_{us} \end{pmatrix} \begin{pmatrix} d_t \\ s_t \end{pmatrix} \tag{16}$$

This leads to the following VAR(1) representation which coincides with the PLM (3), and the agents use this PLM to solve the signal extraction problem and form expectations as described in Section 2.[27]

$$L_{t+1,t} = \underbrace{\hat{\Psi}\hat{\Gamma}\hat{\Psi}^{-1}}_{\equiv\hat{A}} L_{t,t-1} + w_{t+1,t} \tag{17}$$

Following similar steps, the ALM can be written as:

$$L_{t+1,t} = \underbrace{\Psi\Gamma\Psi^{-1}}_{\equiv A} L_{t,t-1} + w_{t+1,t} \tag{18}$$

From (17) and (18), it is clear that the subjective model $\hat{A}$ can be thought of as a reduced-form representation of the subjective structural parameters of the agent. The differences between $\hat{A}$ and $A$, may arise from a range of reasons that are hard to differentiate. To see this clearly, we express $\hat{A}$ explicitly as follows.

[26] See Appendix F for detailed derivations.

[27] Note that one implicit assumption to formulate (17) is that the agents are sophisticated enough to understand the model (12)-(14) but they do not realize the expectations formed under noisy information will affect observed signals. This is the same type of bounded rationality considered in Beaudry *et al.* (2025).

$$\hat{A} \equiv \hat{\Psi}\hat{\Gamma}\hat{\Psi}^{-1} = \frac{1}{\hat{\Psi}_{\pi d}\hat{\Psi}_{us} - \hat{\Psi}_{\pi s}\hat{\Psi}_{ud}} \begin{bmatrix} \hat{\rho}_d\hat{\Psi}_{\pi d}\hat{\Psi}_{us} - \hat{\rho}_s\hat{\Psi}_{\pi s}\hat{\Psi}_{ud} & (\hat{\rho}_s - \hat{\rho}_d)\hat{\Psi}_{\pi d}\hat{\Psi}_{\pi s} \\ (\hat{\rho}_d - \hat{\rho}_s)\hat{\Psi}_{ud}\hat{\Psi}_{us} & \hat{\rho}_s\hat{\Psi}_{\pi d}\hat{\Psi}_{us} - \hat{\rho}_d\hat{\Psi}_{\pi s}\hat{\Psi}_{ud} \end{bmatrix} \quad (19)$$

Suppose households correctly understand that demand and supply shocks push inflation and unemployment in opposite directions, i.e., $\hat{\Psi}_{ud} < 0$ while $\hat{\Psi}_{us}, \hat{\Psi}_{\pi d}, \hat{\Psi}_{\pi s} > 0$. A finding of $\hat{A}_{21} > 0$ implies that they believe $\hat{\rho}_s > \hat{\rho}_d$. In other words, they perceive supply shocks as more persistent and as the dominant force shaping the correlation between inflation and unemployment. It is also possible, however, that households subjectively misperceive specific mechanisms of the model, such as the monetary policy reaction function. In this case, $\hat{A}_{21} > A_{21}$ may reflect an overestimation of the central bank's responsiveness to inflation, $\hat{\phi}_\pi$. This interpretation is consistent with the evidence in Carvalho and Nechio (2014); Dräger *et al.* (2016); Bauer *et al.* (2024), who document that perceptions of monetary policy are both time-varying and subjective. Meanwhile, the fact that $\hat{A}_{12}$ is not significantly different from zero may suggest that households perceive the Phillips Curve slope $\hat{\kappa}$ to be close to zero, which would imply $\hat{\Psi}_{\pi d} \approx 0$.[28] These alternative explanations highlight different micro-foundations that could underlie the subjective model we detect.

## 3.3 Allowing Feedback from Expectations to States

Up to now, we have maintained the standard restriction that there is no feedback from expectations to realized inflation and unemployment as in Ryngaert (2018); Coibion and Gorodnichenko (2012); Andrade and Le Bihan (2013). When expectations are simultaneously determined with the realized states, our test based on Equation (6) cannot be estimated with OLS as the noise $\eta_t$ is correlated with forecast errors $FE_{t,t-1|t-1}$. Moreover, the joint

[28] In general, the source of the subjective $\hat{A}$ matrix can either come from perceived shock persistence, or the perceived impact matrix $\hat{\Psi}$, which reflects households' beliefs about how demand and supply shocks affect inflation and unemployment. An example is Andre *et al.* (2022), who show that, for instance, households often perceive inflation to rise in response to a negative demand shock such as monetary policy tightening, i.e., $\hat{\Psi}_{\pi d} < 0$.

estimation (7) is not the correct specification anymore because now states $L_{t,t-1}$ will also depend on beliefs $L_{t+1|t}$.

To alleviate this problem, we have to impose an explicit timing restriction to avoid the simultaneity issue. We assume that in the structural model (12)-(14), states $L_{t,t-1}$ is affected only by expectation formed with past information, $L_{t,t-1|t-1}$. Under this assumption, the noise $\eta_t$ is no longer correlated with state $L_{t,t-1}$, which makes the original test (6) valid under OLS. Moreover, we show in Appendix F.1 that the joint dynamic equation (7) can be modified as:

$$Y_{t+1} = \underbrace{\begin{pmatrix} \hat{A}(I-KG) & \hat{A}KG \\ \mathbf{\Theta} & A \end{pmatrix}}_{:=\Phi} Y_t + F \begin{pmatrix} \eta_t \\ w_{t+1,t} \end{pmatrix} \tag{20}$$

where again $Y_t = \begin{pmatrix} L_{t,t-1|t-1} \\ L_{t,t-1} \end{pmatrix}$. We can now estimate (20) allowing feedback from $L_{t,t-1|t-1}$ to $L_{t,t-1}$. Table 4 shows the results from our baseline sample. Our key results qualitatively hold: the estimate of $\hat{A}_{21}$ from MSC is significantly bigger than 0, whereas the point estimate of $A_{21}$ is close to 0 and insignificant. All in all, allowing feedback from expectations to realized states won't change the estimates on $A$ materially.[29] In Appendix E, we show that these results are robust to different sub-samples.

### 3.4 Shock Propagation Under a Subjective Model

How does the uncovered subjective model affect the dynamics of realized inflation and unemployment rate in general equilibrium? To answer this, we can again consider the simple three-equation New Keynesian Model from (12)-(14), assuming both $E_t\pi_{t+1}$ and $E_t u_{t+1}$ are

[29] The test that $\hat{A}_{21} \leq A_{21}$ is marginally rejected in this case but only because the s.e. on the estimate of $A_{21}$ becomes twice as large as before.

Table 4: Estimates of Joint Learning Model (20) with feedback loops

| **MSC, quarterly, Q1 1984–Q1 2023** | | |
|---|---|---|
| Parameters | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.902 & 0.00 \\ 0.034 & 0.506 \end{bmatrix}$ | $\begin{bmatrix} 0.080 & 0.031 \\ 0.084 & 0.085 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.741 & -0.151 \\ 0.161 & 0.837 \end{bmatrix}$ | $\begin{bmatrix} 0.057 & 0.082 \\ 0.045 & 0.048 \end{bmatrix}$ |
| T-test: | test-stat | p-val |
| $\hat{A}_{21} \leq A_{21}$ | 1.115 | 0.132 |
| **SPF, quarterly, Q1 1984–Q1 2023** | | |
| Parameters | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.821 & -0.047 \\ 0.014 & 0.735 \end{bmatrix}$ | $\begin{bmatrix} 0.078 & 0.059 \\ 0.034 & 0.065 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.955 & -0.038 \\ 0.040 & 0.495 \end{bmatrix}$ | $\begin{bmatrix} 0.019 & 0.016 \\ 0.035 & 0.239 \end{bmatrix}$ |
| T-Test | test-stat | p-val |
| $\hat{A}_{21} \leq A_{21}$ | 0.580 | 0.281 |

The table reports the estimates and their Newey-West standard errors from the GMM estimation of the four-variable VAR model. An iterative weighting matrix is used in the GMM estimation. The standard errors are based on the variance-covariance matrix of model estimates. Since $\hat{A}$ is the element-wise sum of directly estimated $B$ and $C$, the element-wise variance-covariance matrix of $B$ and $C$ are used to calculate the standard errors of $\hat{A}$ estimates.

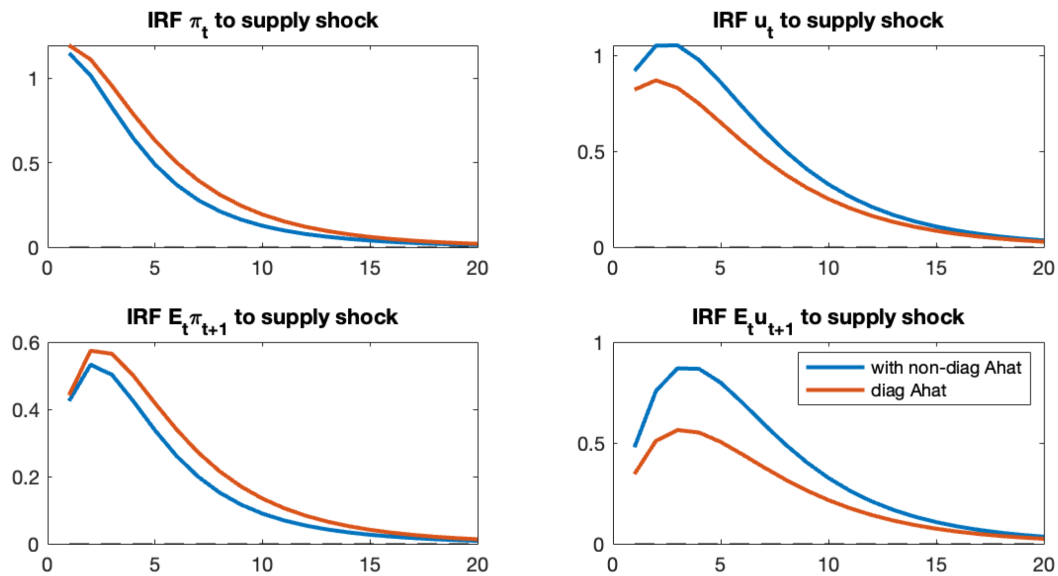
formed by households using the subjective model $\hat{A}$:

$$L_{t+1,t|t} = \hat{A}(I - KG)L_{t,t-1|t-1} + \hat{A}KGL_{t,t-1} + \hat{A}K\eta_t, \tag{21}$$

again with $L_{t+1,t|t} \equiv \begin{pmatrix} E_t\pi_{t+1} \\ E_t u_{t+1} \end{pmatrix}$ and $L_{t,t-1} \equiv \begin{pmatrix} \pi_t \\ u_t \end{pmatrix}$.

**Calibration:** The micro-founded model that gives (12)–(14) features households with intertemporal elasticity $\sigma$ and a Frisch elasticity of labor supply $\psi$. There is a continuum of monopolistic competitive firms with $1-\theta$ probability to adjust prices every period. As a result, the slope of the Phillips curve is $\kappa = \lambda(\sigma + \frac{\psi+\alpha}{1-\alpha})$, where $1-\alpha$ is the share of labor

Figure 2: IRF in Response to Supply Shock




input in the firm's production function and $\lambda$ is the coefficient on marginal cost. It follows that $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$, with $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

We follow the baseline calibration in Galí (2015) by setting $\beta = 0.99$, $\sigma = 1$, $\psi = 1$, $\alpha = 1/3$, $\epsilon = 6$, and $\theta = 2/3$, which implies an average price duration of three quarters. The policy parameters in the Taylor rule are $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$, which guarantee the determinacy of the equilibrium. Finally, we use $\chi = 0.43$ from Ball *et al.* (2017). The persistence of supply and demand shocks are set to be 0.8 with unit standard deviations.

The purpose of this exercise is to illustrate how the subjective belief that inflation leads to unemployment influences the dynamics of realized inflation and unemployment. To this end, we first examine the impulse response functions (IRFs) to supply and demand shocks in our baseline model, where $\hat{A} = \begin{pmatrix} 0.74 & 0 \\ 0.16 & 0.84 \end{pmatrix}$—a structure chosen to align with the lower triangular matrix estimated in Table 3. We then compare these IRFs to those obtained when $\hat{A}$ has zero off-diagonal elements.

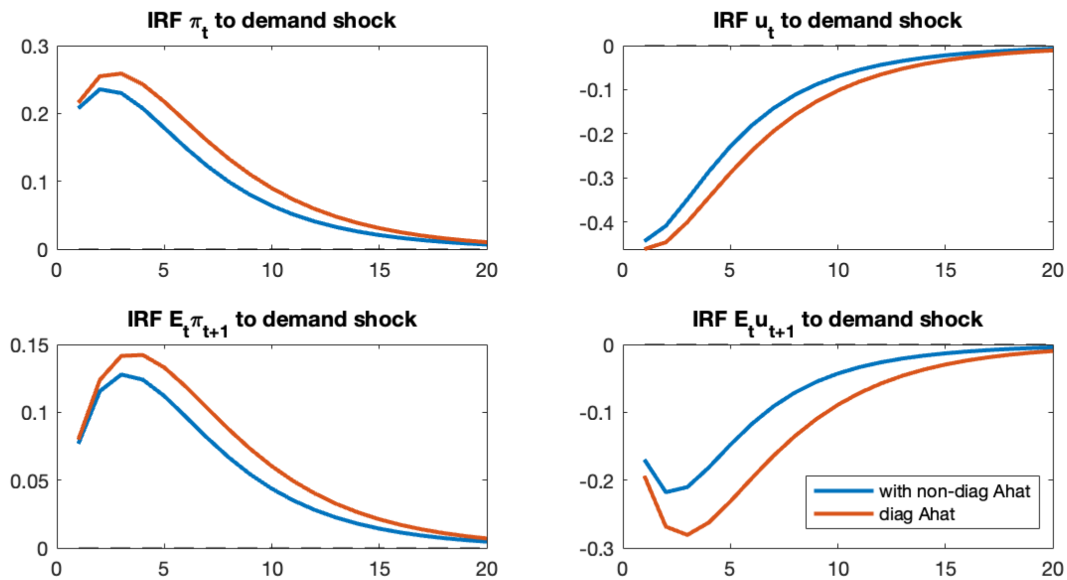
Figure 2 shows the responses of inflation, the unemployment rate, and their corresponding expectations to a 1% increase in the supply shock. The blue lines represent the IRFs under the baseline model, where the agent holds a subjective belief that past inflation leads to higher unemployment. The red lines correspond to the case where $\hat{A}$ is diagonal, implying no such perceived link between inflation and unemployment.

Consistent with the standard New Keynesian (NK) model, both inflation and unemployment rise following a positive supply shock. However, the responses in our framework are more persistent due to noisy information. Comparing the two models, the key difference lies in the behavior of unemployment: It rises more sharply in the baseline model. This is because households, believing that higher inflation signals deteriorating economic conditions, reduce their demand more aggressively than in the diagonal $\hat{A}$ case. This additional contractionary force also slightly dampens inflation through the New Keynesian Phillips Curve (NKPC) channel.

In contrast, responses to a positive demand shock reveal a different pattern, as depicted in Figure 3. In the baseline model, higher inflation leads households to perceive the economy as less overheated compared to the diagonal $\hat{A}$ case. This perception induces a moderating effect on both expectations of inflation and unemployment, which dampens the responses of these expected and realized variables both on impact and in subsequent periods.

Overall, when households form expectations based on a subjective model in which past inflation is believed to lead to higher unemployment, policymakers face a more challenging trade-off between inflation and unemployment following a supply shock. In contrast, this belief can help stabilize both inflation and unemployment in response to demand shocks. During episodes characterized by significant supply-side disruptions, policymakers may incur greater welfare losses if they fail to account for the subjective models underpinning household expectations.

Figure 3: IRF in Response to Demand Shock




# 4 Additional Evidence: Inflation News and Expectations

## 4.1 Self-reported News Exposure

The goal of this section is to supplement structural evidence presented thus far with micro survey evidence regarding the pattern of $\hat{A}$, by directly controlling the information set of individuals (i.e., $s_t^i$ in the model environment). Of course, it would be impossible to fully capture signals that enter the information set of the agents. It is therefore best to interpret our measures as empirical proxies for them.

Our empirical proxies for $s_{i,t}$ are the self-reported exposures to macroeconomic news in the MSC.[30] The survey question asks what kind of news the respondent has heard in the last few months. We classify these answers into 10 categories, including news about prices and labor markets. We treat the former as inflation news and the latter as unemployment news.[31]

[30]The idea of using the MSC's news exposure as a proxy of the respondents' information set goes back to Doms and Morin (2004), Pfajfar and Santoro (2013), and Lamla and Maag (2012).

[31]The descriptions of the question and the variable are included in Appendix G.1. Table 16 in Appendix G.1 summarizes what types of news are included in each category.

We estimate a panel regression of inflation expectations and unemployment-rate–rate expectations[32] on indicators of receiving different types of news, controlling for individual and time fixed effects. This allows us to assess how exposure to distinct news types shapes expectations. Table 5 shows that hearing news about high (low) inflation raises (lowers) reported expected inflation by about 0.43% (0.21%) and increases the probability of believing unemployment will rise (fall) by 2.5%. By contrast, employment news significantly affects unemployment expectations but has no detectable impact on inflation expectations. These patterns persist in columns (3) and (4), where all news types enter simultaneously. A further notable result is that, unlike unemployment expectations, inflation expectations rarely respond to news originating outside the inflation domain. These results suggest that while news often has domain-specific impacts on expectations, only inflation news affects expectations across domains.

We further investigate whether the positive association between expected inflation and unemployment varies with the type of news exposure. Specifically, we estimate a panel regression of expected inflation on expected unemployment, news indicators, and their interactions. The interaction terms allow us to test whether the relationship between inflation and unemployment expectations depends on the type of news respondents report.

In column (1) of Table 6, we include only inflation, employment, and interest rate news indicators. The baseline correlation between expected inflation and unemployment is 0.36, doubling for individuals hearing inflation-up news and falling sharply with unfavorable employment or favorable interest rate news. Column (2), which adds all news categories from Table 16, shows a similar baseline of 0.38. The correlation is significantly lower with news on real activity (employment, industries, demand) but much higher with inflation news, consistent with our subjective model interpretation and the evidence in Bhandari *et al.* (2025).

How much does the news–conditional individual correlation help explain the correlation in consensus expectations? Figure 4 plots annual means of consensus inflation and unemploy-

[32]The unemployment expectation variable equals 1 if the respondent predicts an increase, 0 if unchanged, and $-1$ if a decrease.

Table 5: Fixed-effect Panel Regression with Self-reported News

| Expectation on: | Inflation | Likelihood Unemployment Increase | Inflation | Likelihood Unemployment Increase |
|---|---|---|---|---|
| news on: | (1) | (2) | (3) | (4) |
| Inflation fav | -0.21* | -0.06*** | -0.21* | -0.05*** |
| | (0.117) | (0.017) | (0.118) | (0.017) |
| Inflation unfav | 0.43*** | 0.06*** | 0.42*** | 0.05*** |
| | (0.085) | (0.010) | (0.085) | (0.010) |
| Employment fav | -0.03 | -0.14*** | -0.01 | -0.13*** |
| | (0.056) | (0.009) | (0.057) | (0.009) |
| Employment unfav | 0.05 | 0.10*** | 0.04 | 0.09*** |
| | (0.054) | (0.007) | (0.054) | (0.007) |
| Interest rate fav | -0.03 | -0.06*** | -0.01 | -0.04*** |
| | (0.071) | (0.012) | (0.072) | (0.012) |
| Interest rate unfav | 0.02 | 0.11*** | 0.02 | 0.10*** |
| | (0.081) | (0.012) | (0.081) | (0.012) |
| Industry fav | | | -0.20*** | -0.10*** |
| | | | (0.059) | (0.008) |
| Industry unfav | | | 0.11** | 0.08*** |
| | | | (0.053) | (0.006) |
| Demand fav | | | -0.16 | -0.09*** |
| | | | (0.104) | (0.014) |
| Demand unfav | | | -0.04 | 0.07*** |
| | | | (0.111) | (0.013) |
| Gov fav | | | -0.12 | -0.09*** |
| | | | (0.077) | (0.012) |
| Gov unfav | | | 0.21*** | 0.10*** |
| | | | (0.058) | (0.008) |
| Sentiment fav | | | -0.12* | -0.12*** |
| | | | (0.069) | (0.010) |
| Sentiment unfav | | | 0.09 | 0.07*** |
| | | | (0.078) | (0.009) |
| Stock fav | | | -0.07 | -0.07*** |
| | | | (0.059) | (0.011) |
| Stock unfav | | | 0.05 | 0.07*** |
| | | | (0.077) | (0.011) |
| Other prices fav | | | -0.22** | -0.04*** |
| | | | (0.102) | (0.016) |
| Other prices unfav | | | 0.04 | 0.04*** |
| | | | (0.087) | (0.013) |
| Other real fav | | | -0.02 | -0.07*** |
| | | | (0.108) | (0.019) |
| Other real unfav | | | 0.22* | 0.04*** |
| | | | (0.117) | (0.013) |
| Wage fav | | | 0.03 | -0.03 |
| | | | (0.158) | (0.024) |
| Wage unfav | | | -0.09 | 0.07*** |
| | | | (0.149) | (0.016) |
| Observations | 169304 | 189158 | 169304 | 189158 |
| $R^2$ | 0.673 | 0.677 | 0.673 | 0.681 |
| Time F.E. | Y | Y | Y | Y |
| Individual F.E. | Y | Y | Y | Y |

* ***,**,*: Significance at 1%, 5%, and 10% level. Results come from fixed-effect panel regressions of expectations on different dummies of self-reported news. Columns (1) and (3) use expected inflation as the dependent variable; columns (2) and (4) use the categorical variable of the expected unemployment rate to increase/stay the same/decrease as the dependent variable. The results control for individual fixed effects and time-fixed effects. Standard errors are adjusted for heteroscedasticity and autocorrelation.

Table 6: Correlation Conditional on News Heard

| Dependent var: | $E\pi$ | |
|---|---|---|
| | (1) | (2) |
| $Eun$ | 0.36*** | 0.38*** |
| | (0.034) | (0.047) |
| Inflation fav $\times Eun$ | 0.17 | 0.16 |
| | (0.164) | (0.164) |
| Inflation unfav $\times Eun$ | 0.36*** | 0.36*** |
| | (0.117) | (0.118) |
| Employment fav $\times Eun$ | 0.03 | 0.03 |
| | (0.089) | (0.090) |
| Employment unfav $\times Eun$ | -0.20*** | -0.16** |
| | (0.073) | (0.074) |
| Interest rate fav $\times Eun$ | -0.23** | -0.24** |
| | (0.104) | (0.104) |
| Interest rate unfav $\times Eun$ | -0.16 | -0.16 |
| | (0.114) | (0.115) |
| Industry fav $\times Eun$ | | 0.06 |
| | | (0.092) |
| Industry unfav $\times Eun$ | | -0.23*** |
| | | (0.073) |
| Demand fav $\times Eun$ | | -0.14 |
| | | (0.145) |
| Demand unfav $\times Eun$ | | -0.57*** |
| | | (0.155) |
| Gov fav $\times Eun$ | | 0.08 |
| | | (0.107) |
| Gov unfav $\times Eun$ | | 0.01 |
| | | (0.079) |
| Sentiment fav $\times Eun$ | | 0.01 |
| | | (0.112) |
| Sentiment unfav $\times Eun$ | | 0.24** |
| | | (0.113) |
| Stock fav $\times Eun$ | | -0.11 |
| | | (0.085) |
| Stock unfav $\times Eun$ | | 0.06 |
| | | (0.115) |
| Other prices fav $\times Eun$ | | -0.01 |
| | | (0.152) |
| Other prices unfav $\times Eun$ | | -0.16 |
| | | (0.130) |
| Other real fav $\times Eun$ | | -0.11 |
| | | (0.168) |
| Other real unfav $\times Eun$ | | -0.21 |
| | | (0.157) |
| Wage fav $\times Eun$ | | -0.17 |
| | | (0.235) |
| Wage unfav $\times Eun$ | | 0.00 |
| | | (0.224) |
| Observations | 167346 | 167346 |
| $R^2$ | 0.674 | 0.675 |
| Time F.E. | Y | Y |
| Individual F.E. | Y | Y |

* ***,**,*: Significance at 1%, 5%, and 10% level. The results control for individual fixed effects and time-fixed effects. Standard errors are adjusted for heteroscedasticity and autocorrelation.

ment expectations for 1984–2023, conditional on hearing inflation or unemployment news.[33] In panels (a) and (b), red dots denote expectations conditional on unfavorable inflation or unemployment news, while gray dots represent those without such news.

Several patterns emerge. First, individuals who hear high inflation news expect both higher inflation and higher unemployment than those who report no news. By contrast, individuals who hear high unemployment news raise only their unemployment expectations, with little change in inflation expectations. Second, the correlation between consensus expectations is strongly positive for those hearing high inflation news, but low and insignificant for those without news, and negative for those exposed to unfavorable employment news. Finally, favorable news—whether on inflation or employment—lowers both expected inflation and unemployment. In these cases, the correlation is not statistically different from that among individuals who report no news.

Overall, these results, consistent with Table 6, suggest that the correlation in consensus expectations is driven primarily by individuals who hear high inflation news, while it weakens or disappears for those hearing negative employment news.

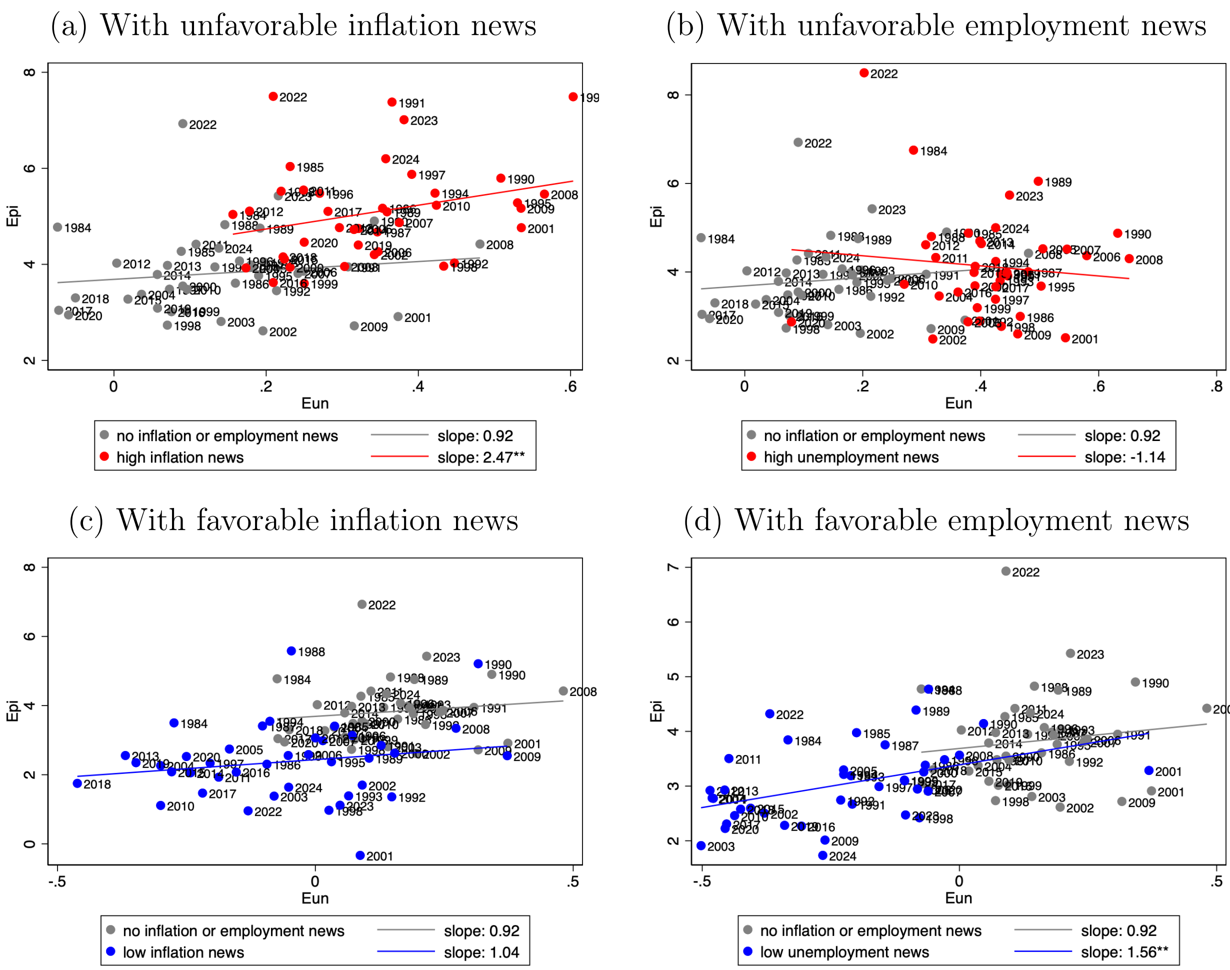
## 4.2 Inflation-Unemployment Narratives in Newspapers

The mass news media have been considered as one of the important sources of information for households to learn about the state of the macroeconomy.[34]. In this section, we hypothesize that news media's reporting on the macroeconomy not only provides signals ($s_t$) about the state of the economy in respective domains, but also prompts readers to think about certain subjective models through a narrative ($\hat{A}$). It is also possible that news coverage causes selective recall of certain mental models of the economy, a mechanism documented by Andre *et al.* (2022).

We measure news coverage of inflation, unemployment, and their joint mentions using a

[33]We exclude 2020–2021 because the unemployment spike during COVID-19 makes them extreme outliers.
[34]See evidence from Carroll (2003), Doms and Morin (2004), Larsen *et al.* (2021), and Chahrour *et al.* (2024).

Figure 4: Consensus Expectations Conditional on News Heard




*Notes: Scatterplot for consensus expected inflation and unemployment each year for 1984–2023. Gray dots in all panels are expectations for individuals who do not hear employment or inflation news. Top left panel: red dots are expectations conditional on hearing high inflation news. Top right panel: red dots are expectations conditional on hearing high unemployment news. Bottom left panel: blue dots are expectations conditional on hearing low inflation news. Bottom right panel: blue dots are expectations conditional on hearing low unemployment news.*

historical archive of 150,000 articles published in *The Wall Street Journal* (WSJ) between January 1984 and June 2022.[35] Topic assignment relies on keyword counts that flag whether an article discusses terms such as "inflation," "unemployment," "Fed," "growth," "oil prices," "recession," or "uncertainty."[36]

An article is coded as drawing an association between inflation and unemployment if both terms appear in its main text. We then estimate Probit regressions to examine factors associated with an article's likelihood of making such an association. Regressors include the article-specific topic dummies as well as realized inflation ($\pi_t$) and unemployment ($u_t$) rates. Columns 1–3 of Table 7 report the results. We are essentially asking: in what context and under macroeconomic conditions do newspapers jointly talk about inflation and the unemployment rate?

The link between unemployment and inflation is most likely to appear in articles that also reference terms such as "Fed," "growth," "economy," "recession," or "uncertainty." Columns (1)–(3) restrict the analysis to realized unemployment, realized inflation, and both variables, respectively. Across these specifications, a higher inflation rate $\pi_t$ consistently increases the probability that an article mentions both inflation and unemployment, while the unemployment rate itself has no effect. In other words, higher inflation not only drives more coverage of inflation but also amplifies the extent to which news articles frame inflation and unemployment together.

To summarize, our additional evidence based on self-reported news exposure and newspapers' narratives further highlights that the direction of the subjective model goes from inflation to unemployment rate, instead of the other way around. Inflation news is special among all types of economic news. It not only serves as a signal about the state of inflation, but also a salient cue of subjective models that associate inflation with the labor market and

[35] We focus on the WSJ because of its emphasis on U.S. economic and financial news. Articles are drawn from repeated random samples of 25,000, covering about 25% of all WSJ articles in this period. We exclude those focused on non-U.S. news or unrelated topics (e.g., sports, culture).

[36] In the Appendix, we report results using Latent Dirichlet Allocation (LDA) topic modeling following Bybee *et al.* (2020). Unlike keyword counts, LDA represents topics as clusters of co-occurring words. See also Bybee *et al.* (2020) and Macaulay and Song (2022).

Table 7: Drivers of Inflation-Unemployment Association

| | (1) | (2) | (3) |
|---|---|---|---|
| economy | 1.07*** | 1.07*** | 1.07*** |
| | (0.03) | (0.03) | (0.03) |
| fed | 0.22*** | 0.21*** | 0.21*** |
| | (0.03) | (0.03) | (0.03) |
| growth | 0.60*** | 0.61*** | 0.61*** |
| | (0.03) | (0.03) | (0.03) |
| oil price | 0.24*** | 0.24*** | 0.24*** |
| | (0.05) | (0.05) | (0.05) |
| recession | 0.48*** | 0.47*** | 0.47*** |
| | (0.03) | (0.03) | (0.03) |
| uncertainty | 0.14*** | 0.15*** | 0.15*** |
| | (0.05) | (0.05) | (0.05) |
| $\pi_t$ | | 3.73*** | 3.62*** |
| | | (0.93) | (0.96) |
| $u_t$ | -0.01 | | -0.00 |
| | (0.01) | | (0.01) |
| N | 150465 | 150465 | 150465 |

** $p<0.001$, ** $p<0.01$ and * $p<0.05$.
The table reports results from Probit regressions with the dependent variable being the dummy indicating if an article mentions both "inflation" and "unemployment" in the texts. Regressors are dummy variables to indicate if the particular keyword, e.g., "growth", is mentioned in the article. $\pi_t$ and $u_t$ are the inflation and unemployment rates at time $t$, the date of publication of the article.

broader economy. In our model of expectation formation in Section 2, this may take the form of additional dependence of the subjective model $\hat{A}$ on the signals $s_{i,t}$. We leave the formal formulation of this idea for future research.

# 5 Conclusion

Several studies have documented that households tend to unconditionally associate current and future inflation with a worse economic outlook and labor market, the so-called "stagflation view" or "supply view" of the economy.[37] We study the theoretically relevant mechanisms that can generate such belief patterns and conclude that information friction alone is insufficient. Rather, it reflects households' subjective views regarding how macroeconomic variables move together.

By extending the single-variable noisy information model to a multivariable setting, we derive a pair of sign restrictions on the correlation of expectations and the serial correlation between forecast errors of different macroeconomic variables. This restriction informs a test against data that helps differentiate expectation patterns due to only information friction versus those due to subjective models. Our claim is further supported by self-reported news exposure in the survey and narratives in newspapers.

We also illustrate that the presence of the uncovered subjective model of "inflation means bad economy" alters the propagation mechanisms of macroeconomic shocks. In particular, it amplifies the output and price responses to a supply shock but dampens those to a demand shock. This poses a more stark trade-off faced by central banks in response to adverse supply shocks. It also questions the effectiveness of demand management policies, especially through increasing inflation expectations. Our findings speak to the macro implications of the emerging micro causal evidence that suggests household consumption responses to inflation expectations are often negative, due to expected real-income erosion or the precautionary responses to uncertainty.[38]

# References


Adams, J. J. and Barrett, P. (2024). Shocks to inflation expectations. *Review of Economic Dynamics*, **54**, 101234.

Andrade, P., Crump, R. K., Eusepi, S. and Moench, E. (2016). Fundamental disagreement. *Journal of Monetary Economics*, **83**, 106–128.


[37] See Bhandari *et al.* (2025); Kamdar (2019); Andre *et al.* (2022); Candia *et al.* (2020); Han (2023).

[38] See Candia *et al.* (2020); Jain *et al.* (2024); Stantcheva (2024); Georgarakos *et al.* (2024).

— and Le Bihan, H. (2013). Inattentive professional forecasters. *Journal of Monetary Economics*, **60** (8), 967–982.

Andre, P., Pizzinelli, C., Roth, C. and Wohlfart, J. (2022). Subjective models of the macroeconomy: Evidence from experts and representative samples. *The Review of Economic Studies*, **89** (6), 2958–2991.

Angeletos, G.-M., Huo, Z. and Sastry, K. A. (2021). Imperfect macroeconomic expectations: Evidence and theory. *NBER Macroeconomics Annual*, **35** (1), 1–86.

Armantier, O., Topa, G., Van der Klaauw, W. and Zafar, B. (2016). An overview of the survey of consumer expectations.

Bachmann, R., Berg, T. O. and Sims, E. R. (2015). Inflation expectations and readiness to spend: Cross-sectional evidence. *American Economic Journal: Economic Policy*, **7** (1), 1–35.

Ball, L., Leigh, D. and Prakash, L. (2017). Okun's law: Fit at 50? *Journal of Money, Credit and Banking*, **49** (7), 1413–1441.

Barsky, R. B. and Sims, E. R. (2012). Information, animal spirits, and the meaning of innovations in consumer confidence. *American Economic Review*, **102** (4), 1343–77.

Bauer, M. D., Pflueger, C. E. and Sunderam, A. (2024). Perceptions about monetary policy. *The Quarterly Journal of Economics*, **139** (4), 2227–2278.

Beaudry, P., Hou, C. and Portier, F. (2025). The dominant role of expectations and broad-based supply shocks in driving inflation. *NBER Macroeconomics Annual*, **39** (1), 235–276.

Bhandari, A., Borovička, J. and Ho, P. (2025). Survey data and subjective beliefs in business cycle models. *Review of Economic Studies*, **92** (3), 1375–1437.

Bolhuis, M. A., Cramer, J. N., Schulz, K. O. and Summers, L. H. (2024). *The Cost of Money is Part of the Cost of Living: New Evidence on the Consumer Sentiment Anomaly*. Tech. rep., National Bureau of Economic Research.

Bordalo, P., Gennaioli, N. and Shleifer, A. (2018). Diagnostic expectations and credit cycles. *The Journal of Finance*, **73** (1), 199–227.

Burke, M. and Ozdagli, A. (2013). *Household inflation expectations and consumer spending: evidence from panel data*. Working Papers 13-25, Federal Reserve Bank of Boston.

Bybee, L., Kelly, B. T., Manela, A. and Xiu, D. (2020). *The structure of economic news*. Tech. rep., National Bureau of Economic Research.

Candia, B., Coibion, O. and Gorodnichenko, Y. (2020). *Communication and the Beliefs of Economic Agents*. Working Paper 27800, National Bureau of Economic Research.

Carroll, C. D. (2003). Macroeconomic expectations of households and professional forecasters. *The Quarterly Journal of Economics*, **118** (1), 269–298.

Carvalho, C. and Nechio, F. (2014). Do people understand monetary policy? *Journal of Monetary Economics*, **66**, 108–123.

Chahrour, R., Shapiro, A. H. and Wilson, D. J. (2024). News selection and household inflation expectations. Federal Reserve Bank of San Francisco.

Chen, H. and Liu, Y. (2025). Expectation and confusion: Evidence and theory. *Available at SSRN 5343753*.

Coibion, O. and Gorodnichenko, Y. (2012). What can survey forecasts tell us about information rigidities? *Journal of Political Economy*, **120** (1), 116 – 159.

— and — (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review*, **105** (8), 2644–78.

Doms, M. and Morin, N. (2004). Consumer sentiment and the media. *FRBSF Economic Letter*.

Dräger, L., Lamla, M. J. and Pfajfar, D. (2016). Are survey expectations theory-consistent? the role of central bank communication and news. *European Economic Review*, **85**, 84–111.

Eggertsson, G. and Woodford, M. (2003). The zero bound on interest rates and optimal monetary policy. *Brookings Papers on Economic Activity*, **34** (1), 139–235.

Farmer, L. E., Nakamura, E. and Steinsson, J. (2024). Learning about the long run. *Journal of Political Economy*, **132** (10), 3334–3377.

Ferreira, C. and Pica, S. (2024). Households' subjective expectations: disagreement, common drivers and reaction to monetary policy. *Documentos de Trabajo/Banco de España, 2445*.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.

Gáti, L. (2023). Monetary policy & anchored expectations,Äîan endogenous gain learning model. *Journal of Monetary Economics*, **140**, S37–S47.

Georgarakos, D., Gorodnichenko, Y., Coibion, O. and Kenny, G. (2024). *The Causal Effects of Inflation Uncertainty on Households' Beliefs and Actions*. Tech. rep., National Bureau of Economic Research.

Gillitzer, C., Prasad, N. and Robinson, T. (2021). Political attitudes and inflation expectations: Evidence and implications. *Journal of Money, Credit and Banking*, **53** (4), 605–634.

Hajdini, I. (forthcoming). Mis-specified forecasts and myopia in an estimated new keynesian model. *American Economic Journal: Macroeconomics*.

—, Knotek, E. S., Leer, J., Pedemonte, M., Rich, R. W. and Schoenle, R. (2022). Low passthrough from inflation expectations to income growth expectations: why people dislike inflation.

Han, Z. (2023). Asymmetric information and misaligned inflation expectations. *Journal of Monetary Economics*, p. 103529.

Hou, C. (2021). *Hou, Chenyu, Learning and Subjective Expectation Formation: A Recurrent Neural Network Approach.* Tech. rep., available at SSRN: https://ssrn.com/abstract=3728129.

Jain, M., Kostyshyna, O. and Zhang, X. (2024). How do people view wage and price inflation? *Journal of Monetary Economics*, **145**, 103552.

Kamdar, R. (2019). *The Inattentive Consumer: Sentiment and Expectations*. 2019 Meeting Papers 647, Society for Economic Dynamics.

Kohlhas, A. N. and Walther, A. (2021). Asymmetric attention. *American Economic Review*, **111** (9), 2879–2925.

Lamla, M. J. and Maag, T. (2012). The role of media for inflation forecast disagreement of households and professional forecasters. *Journal of Money, Credit and Banking*, **44** (7), 1325–1350.

Larsen, V. H., Thorsrud, L. A. and Zhulanova, J. (2021). News-driven inflation expectations and information rigidities. *Journal of Monetary Economics*, **117**, 507–520.

Lucas, R. E. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, **1**, 19 – 46.

Macaulay, A. and Song, W. (2022). Narrative-driven fluctuations in sentiment: Evidence linking traditional and social media. *Available at SSRN 4150087.*

Malmendier, U. and Nagel, S. (2015). Learning from Inflation Experiences. *The Quarterly Journal of Economics*, **131** (1), 53–87.

Mankiw, N. G., Reis, R. and Wolfers, J. (2004). Disagreement about inflation expectations. In *NBER Macroeconomics Annual 2003, Volume 18*, National Bureau of Economic Research, Inc, pp. 209–270.

MaḟÁkowiak, B., MatfÕjka, F. and Wiederholt, M. (2018). Dynamic rational inattention: Analytical results. *Journal of Economic Theory*, **176**, 650–692.

Milani, F. (2007). Expectations, learning and macroeconomic persistence. *Journal of monetary Economics*, **54** (7), 2065–2082.

Pfajfar, D. and Santoro, E. (2013). News on inflation and the epidemiology of inflation expectations. *Journal of Money, Credit and Banking*, **45** (6), 1045–1067.

Reis, R. (2022). Losing the inflation anchor. *Brookings Papers on Economic Activity*, **2021** (2), 307–379.

Ryngaert, J. (2018). What do (and don't) forecasters know about us inflation? *Journal of Money, Credit and Banking*.

Shiller, R. J. (1997). Why do people dislike inflation? In *Reducing inflation: Motivation and strategy*, University of Chicago Press, pp. 13–70.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics*, **50** (3), 665 – 690, swiss National Bank/Study Center Gerzensee Conference on Monetary Policy under Incomplete Information.

Stantcheva, S. (2024). *Why do we dislike inflation?* Tech. rep., National Bureau of Economic Research.

Woodford, M. (2001). *Imperfect Common Knowledge and the Effects of Monetary Policy*. Working Paper 8673, National Bureau of Economic Research.

# A Data Appendix

## A.1 Data Description

**SCE:** The Survey of Consumer Expectations is run by the New York Fed, starting in June 2013, and is available monthly.[39] We use the median year-ahead inflation expectation as a proxy for expected inflation and the expected chance that the unemployment rate will increase in 12 months as a proxy for expected unemployment rate change.

**MSC:** The monthly component of the Michigan Survey of Consumers is available starting in 1978.[40] We use the expected price change in one year as a proxy for expected inflation, and the question about whether the unemployment rate will go up, go down, or stay the same as a proxy for expected unemployment rate change.

**FRED:** We use year-to-year Headline CPI (CPIAUCSL) as a measure of realized inflation and year-to-year change of unemployment rate (UNRATE) as a measure of changes in unemployment status.

**SPF:** We use the series on CPI inflation rate (CPI) from the Survey of Professional Forecasters as a measure of expected inflation. We use the series on the civilian unemployment rate as a measure of the expected unemployment rate. To make it comparable with consumer surveys, we compute the expected year-ahead change in unemployment rate from this series.

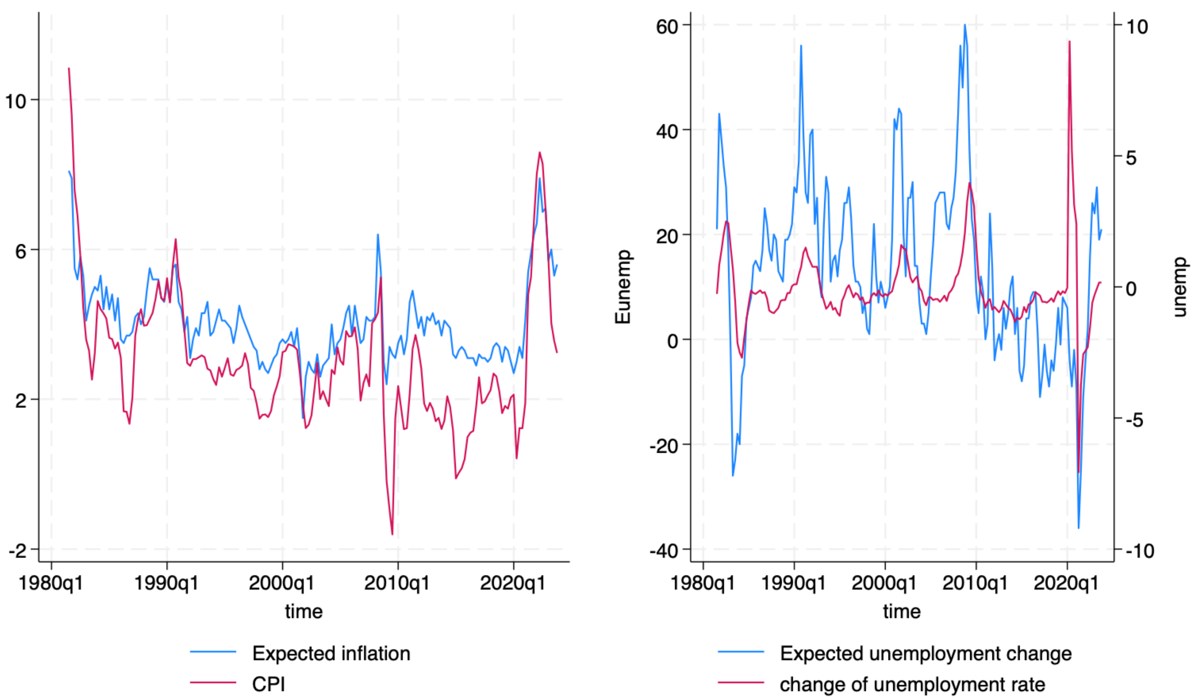
## A.2 Aggregate Survey Forecast and Real-time Data

To illustrate the difference between the survey expectation and realized data, Figure 5 plots raw data on average expectation from the MSC with realized data for inflation and unemployment rate change. All real-time series have changed from a year ago, as the corresponding

[39] For details of SCE, see Armantier *et al.* (2016).
[40] Quarterly data starts earlier, from 1960, but many dimensions are missing.

expectation series are one-year forward forecasts. The abnormal spikes in unemployment rate changes correspond to the COVID-19 pandemic.

Figure 5: Actual and Expected Inflation and Unemployment




Survey expectation from the MSC against the realized data. All macro data are changes from a year ago, and survey expectations are one-year forward forecasts. Unemployment expectation is aggregated from categorical data. A positive number means more people believe unemployment will increase in the future.

## A.3 Time Series Evidence

We first report the simultaneous correlation between consensus expectations on inflation and unemployment from the MSC, the SPF, and realized data. All the expectation variables are the average of individual expectations within the quarter.[41]

[41] In the MSC, expectation data is available monthly. We use quarterly data to keep the MSC at the same frequency as the SPF. The use of monthly data does not change our results qualitatively.

Table 8: Correlations between Expected/Actual Inflation and Unemployment

| Sample | MSC | SPF | FRED |
|---|---|---|---|
| 1984-2023 | $0.14^{*}$ | $-0.03$ | $-0.32^{***}$ |
| 1981-2018 | $0.16^{**}$ | $0.03$ | $0.00$ |
| 1990-2018 | $0.27^{***}$ | $0.05$ | $-0.08$ |

* **** means significant at 1%,** means 5% and * means 10%, indicating significance level of Pearson Correlation. In the sample 1984–2023, we exclude the COVID year 2021.*

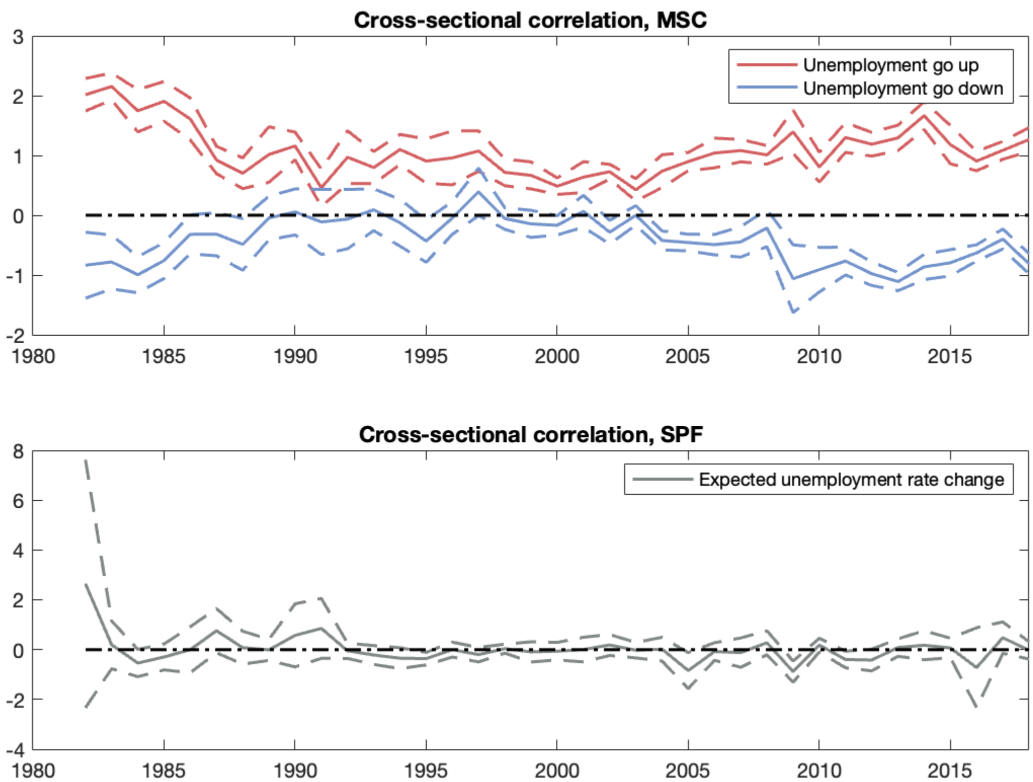
Table 8 summarizes the Pearson correlation between (expected) inflation and unemployment change in different samples that we considered in our empirical analysis. Throughout the different samples, the correlation between these expected variables in household surveys is significantly positive, different from that in the SPF and actual data.

## A.4 Evidence from Individual-level Cross-correlation

There are potentially many possible explanations for the observed positive correlation between consensus expectations. One possibility is that waves of pessimism and optimism move the average unemployment and inflation beliefs in the same direction. Furthermore, as seen in Figure 1, the time-series correlation heavily depends on the presence of aggregate shocks.

To rule out these possibilities, we examine whether individual respondents in household surveys make a similar association. This will help us understand whether the patterns in aggregate-level data have a micro-level foundation or are mainly due to the aggregation process. Previous research suggests that the properties of consensus expectations may differ from those of individual expectations.[42] Figure 6 shows the estimated correlation from the cross-sectional regression in each year.

Figure 6: Time-varying Correlation between Inflation and Unemployment Change




The top panel reports estimates $\beta_1$ from: $E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 U_{t+12,t} + \theta\mu_i + D_t + \epsilon_{i,t}$, where $U_{t+12,t}$ stands for two dummy variables. This indicates the MSC consumer believes the unemployment rate will go up or down in the next 12 months. The bottom panel reports the estimates $\beta_1$ from: $E_{i,t}\pi_{t+4,t} = \beta_0 + \beta_1 E_{i,t}un_{t+4,t} + \theta\mu_i + D_t + \epsilon_{i,t}$, where $E_{i,t}un_{t+4,t}$ stands for the expected change of unemployment rate from SPF. The MSC data is monthly, and the SPF data is quarterly. The dashed lines show the 10% confidence interval.

The top panel of Figure 6 uses data from the MSC. In this survey, respondents are asked whether they think unemployment will go up, stay the same, or go down a year from now. The two lines show the differences in inflation expectations relative to consumers who believe unemployment will stay the same for each year. The figure suggests that households' beliefs about inflation are again positively associated with their beliefs about unemployment change.

[42]For instance, Coibion and Gorodnichenko (2015) suggests the predictability of forecast errors from forecast revisions is an emerging property of aggregation across individuals and may not be seen at the individual level; Bordalo *et al.* (2018) documents over-reaction of inflation expectation to new information on the individual level, contrary to under-reaction typically found with consensus expectations.

Such a positive relation is significant and relatively stable across time.

The bottom panel of Figure 6 is the cross-sectional correlation between expected inflation and unemployment rate change in the SPF. Contrary to consumers, professionals do not associate inflation with the unemployment rate when forming their beliefs.

Could this correlation be driven by a specific group of individuals? For example, if there are groups of pessimistic individuals, they will always form worse-than-average unemployment expectations together with higher-than-average inflation expectations. This creates a positive association in the cross-sectional analysis above. We use the panel dataset in the MSC and the SPF to control for individual fixed effects as well as time-fixed effects:

$$E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 E_{i,t}un_{t+12,t} + \beta_2 E_{i,t}i_{t+12,t} + \theta X_{i,t} + D_t + \mu_i + \epsilon_{i,t} \tag{22}$$

Again, because in MSC, the expected unemployment change is a categorical variable, $\beta_1$ in (22) contains coefficients when expected unemployment goes up or down. $X_{i,t}$ includes controls such as expectations on other subjects and social-economic status, $\mu_i$ and $D_t$ stand for individual and time-fixed effects, respectively. Because the panel dataset from MSC contains fewer observations and only keeps the participants for two waves of surveys six months apart, we also report the results from the same regression using panel data from SCE.[43]

[43] When using MSC, the expected unemployment and interest rate change are categorical variables, and we construct dummies that stand for an increase or decrease for each of these variables. In SCE, those variables are reported as percentage points for the likelihood of the corresponding variable increasing.

Table 9: FE Panel Regression

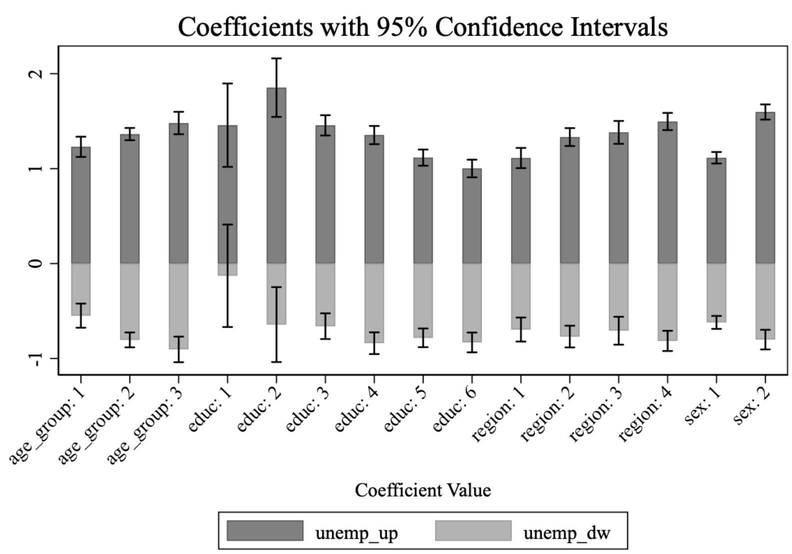
| | MSC | | SCE | | SPF |
|---|---|---|---|---|---|
| Unemployment up | $0.30^{***}$ | $\hat{\beta}_1$ | $0.012^{***}$ | $\hat{\beta}_1$ | $-0.17^{***}$ |
| | (0.05) | | (0.002) | | (0.06) |
| Unemployment down | $-0.22^{***}$ | | | | |
| | (0.05) | | | | |
| FE | Y | | Y | | Y |
| Time dummy | Y | | Y | | Y |

* ***,**,*: Significance at 1%, 5%, and 10% level. Estimation results for specification (22) controlling for individual and time-varying characteristics, individual fixed effect, and time-fixed effect. Standard errors are adjusted for heteroscedasticity and autocorrelation.

Table 9 column 1 shows that for the MSC, an agent who expects the unemployment rate to go up will predict inflation to be 0.3% higher on average than one who believes unemployment to be stable, and 0.52% higher than one who believes the unemployment rate will fall. Meanwhile, the standard deviation of expected inflation across this episode is 1.17%, and the standard deviation of CPI is around 2.19%. These results are comparable to those from Kamdar (2019), where the author estimates a similar fixed-effect model but only on the correlation between expected inflation and unemployment change, without controlling for other expectational variables. The estimates shown in column 2 from the SCE are consistent with those from the MSC: If the consumer expects a 22% higher chance (which is the standard deviation of the variable) the unemployment rate will increase in 12 months, they will also expect inflation to be 0.22% higher. It is worth noting that controlling for individual and time-fixed effects means the positive correlation between unemployment and inflation is not due to a common time-varying bias, which should have been captured by the time-fixed effect. It is also not due to the effect of "pessimistic individuals," which is taken out by individual fixed effects. Finally, in contrast with consumers' expectations, column 3 shows that there is a negative correlation between expected inflation and the change in the unemployment rate. On average, a 1% increase in the expected unemployment rate is associated with a 0.17% fall in expected inflation for professionals. This again coincides with the message from the aggregate correlation that professionals believe in a different relationship between future inflation and unemployment movements compared with consumers.

We also verify that the regression results are robust across subgroups of the MSC, as shown in Figure 7. The figure reports the same regression coefficients as in Figure 6, but separately for each demographic group. Across age, education, region, and gender, households consistently report higher inflation expectations, together with expectations of rising unemployment. This indicates that the correlation pattern in household expectations is not specific to any particular subgroup.

Figure 7: Correlation between expected inflation and unemployment change by different groups



## A.5 Recover Survey Mean from Categorical Data

From the cross-sectional dataset of the MSC, we can acquire information on the fraction of respondents with different answers. Denote $f_t^u$ as fraction of responses that are "increase" and $f_t^d$ as "decrease." Assume for each period of $t$, individuals form a cross-section of answers about the change of the asked subject (unemployment rate or business condition and price). And assume this measure follows a normal distribution with mean $\mu_t$ and variance $\sigma_t^2$.

**Assumption 4.** *At each period $t$, survey respondent $i$ forms a belief $x_{i,t}$ that indicates the change of the asked variable $x$. This belief follows a normal distribution:*

$$x_{i,t} \sim N(\mu_t, \sigma_t^2)$$

Then, suppose the agents have a common scale in answering the categorical question. If $x_{i,t}$ is close to some level $b$, then they will consider the subject will barely change; if $x_{i,t}$ is much bigger than $b$, they will answer "increase". Otherwise, they will answer "decrease

.

$$category_{i,t} = \begin{cases} increase & x_{it} > b + a \\ decrease & x_{it} < b - a \\ same & x_{it} \in [-a + b, b + a] \end{cases}$$

Then the fraction of answer "increase," denoted as $f_t^u$, and "decrease," denoted $f_t^d$, will directly follow from normality:

$$f_t^d = \Phi\left(\frac{b - a - \mu_t}{\sigma_t}\right) \tag{23}$$

$$f_t^u = 1 - \Phi\left(\frac{a + b - \mu_t}{\sigma_t}\right) \tag{24}$$

The item we want to recover is $\mu_t$, which is the corresponding average change of the asked subject a year from now. This can be computed using:

$$\sigma_t = \frac{2a}{\Phi^{-1}(1 - f_t^u) - \Phi^{-1}(f_t^d)} \tag{25}$$

$$\mu_t = a + b - \sigma_t \Phi^{-1}(1 - f_t^u) \tag{26}$$

From (25) and (26), computing the average across time gives us:

$$\hat{\sigma} = 1/T \sum_t^T \sigma_t = 1/T \sum_t^T \frac{2a}{\Phi^{-1}(1 - f_t^u) - \Phi^{-1}(f_t^d)} \tag{27}$$

$$\hat{\mu} = 1/T \sum_t^T \mu_t = 1/T(a + b - \sigma_t \Phi^{-1}(1 - f_t^u)) \tag{28}$$

As in the MSC, there is no information on $\hat{\sigma}$ and $\hat{\mu}$, so we use the time-series mean of the data from SPF on comparable questions to approximate those from the MSC.[44] Following Bhandari *et al.* (2025), we assume the ratio of the time-series average between inflation expectation and other expectations in the MSC equals its counterpart in the SPF:

**Assumption 5.** *For the variable $x$ asked in the survey:*

$$\hat{\sigma}_x^{MCS} = \frac{1/T \sum_t^T \sigma_{E\pi,t}^{MCS}}{1/T \sum_t^T \sigma_{E\pi,t}^{SPF}} \times 1/T \sum_t^T \sigma_{x,t}^{MCS}$$

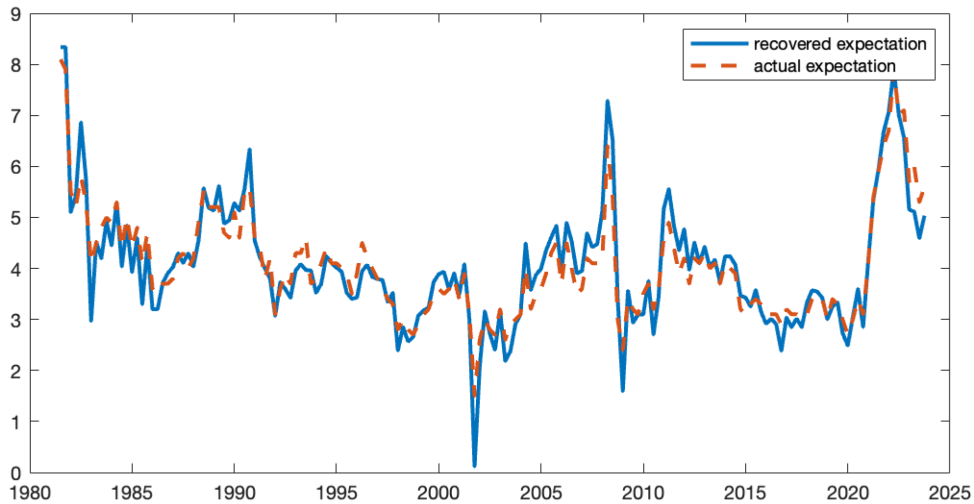
*And*

$$\hat{\mu}_x^{MCS} = \frac{1/T \sum_t^T \mu_{E\pi,t}^{MCS}}{1/T \sum_t^T \mu_{E\pi,t}^{SPF}} \times 1/T \sum_t^T \mu_{x,t}^{MCS}$$

Then from (27) and (28) and Assumption 5, we can back out $a$ and $b$, and with (26) we can recover $\mu_{x,t}$ for the expectational variable $x$.

[44] For unemployment rate change, we use the average difference between projected unemployment rate at $t+3$ and the historical data at $t-1$, which is the last information available to the economist. For real GDP growth, we use the real GDP growth projection for the next four quarters after $t-1$.

Figure 8: Recovered Expected Inflation vs. Actual




**Recovered series:** To test whether the above method is plausible, we use the proposed method to recover $\mu_{\pi,t}$ and compare it with the actual average of expected inflation from the MSC. Figure 8 plots the recovered mean and the actual mean.

Figure 8 shows that the recovered data is actually quite close to the actual mean expectation, with a correlation of 0.95. Figure 9 shows the recovered data on expected unemployment change compared to actual data.

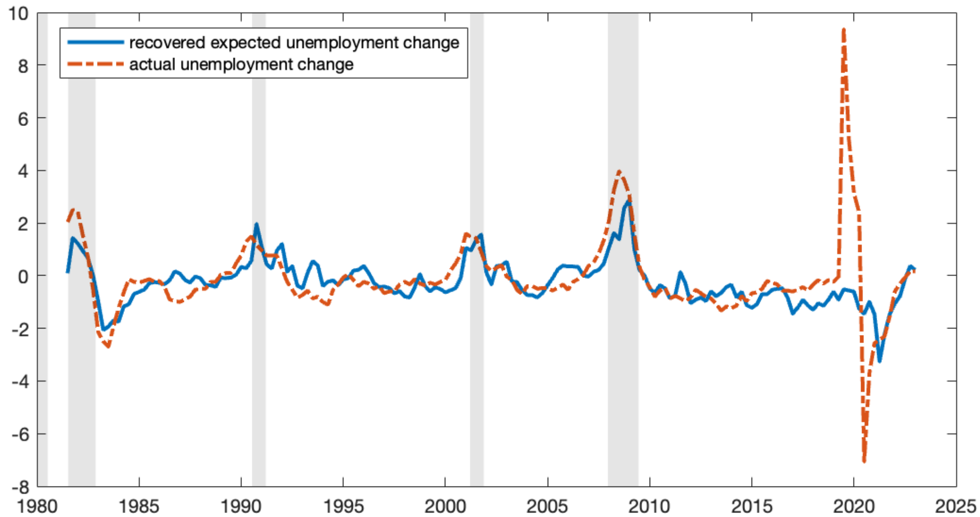
# B Derivation of Noisy Information Model

## B.1 Basic Stationary Kalman Filter

Consider the ALM and observational equation as in (1) and (2), where $w_{t+1,t}$, $v_t^i$, and $\eta_t$ are independent and normally distributed:

$$w_{t+1,t} \sim N(\mathbf{0}, Q) \quad v_t^i \sim N(\mathbf{0}, R_1) \quad \eta_t \sim N(\mathbf{0}, R_1)$$

Figure 9: Recovered Expected Unemployment Change vs. Actual




Data from 1981q3 to 2023q4 due to availability of quarterly SPF on CPI inflation.

Consistent with the main-text, we denote $R = R_1 + R_2$, and the perceived value of $\boldsymbol{L}_{t,t-1}$ for individual $i$ at time $t$ as $\boldsymbol{L}^i_{t,t-1|t}$. The filtering process is:

$$\boldsymbol{L}^i_{t,t-1|t} = \hat{A}\boldsymbol{L}^i_{t,t-1|t} = \boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}^i_t - G\boldsymbol{L}^i_{t,t-1|t-1}) \tag{29}$$

The Kalman Filter is given by:

$$K = \Sigma G'(G\Sigma G' + R)^{-1}$$

$$\Sigma_p = \hat{A}\Sigma\hat{A}' - \hat{A}K_t G\Sigma\hat{A}' + Q,$$

where $\Sigma$ is the covariance matrix of priors as defined in assumption 2 and $\Sigma_p$ is the covariance matrix of posteriors.[45] Then the expectation is given by:

$$\boldsymbol{L}^i_{t+1,t|t} = \hat{A}\big(\boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}^i_t - G\boldsymbol{L}^i_{t,t-1|t-1})\big)$$

[45] Given common beliefs on $\hat{A}$ and $G$, it can be shown that prior and posterior covariance matrices converge.

## B.2 Derivation of Year-ahead Forecasting Error Rule

Consider the year-ahead consensus forecast $\boldsymbol{L}^c_{t+4,t|t}$ and year-ahead realization $\boldsymbol{L}_{t+4,t}$. Using ALM (1), we have:

$$\boldsymbol{L}_{t+4,t} \equiv \sum_{j=1}^{4} \boldsymbol{L}_{t+j,t+j-1} = A\boldsymbol{L}_{t+3,t-1} + \sum_{j=1}^{4} w_{t+j,t+j-1} \tag{30}$$

Similar to equation (4), the year-ahead consensus expectation is:

$$\boldsymbol{L}^c_{t+4,t|t} = (\hat{A}^3 + \hat{A}^2 + \hat{A} + I)[\hat{A}(I - KG)\boldsymbol{L}^c_{t,t-1|t-1} + \hat{A}KG\boldsymbol{L}_{t,t-1} + \hat{A}K\eta_t] \tag{31}$$

Meanwhile from (29) and ALM we know:

$$\boldsymbol{L}^c_{t+3,t-1|t-1} = \sum_{j=0}^{3} \boldsymbol{L}^c_{t+j,t+j-1|t-1} = (\hat{A}^3 + \hat{A}^2 + \hat{A} + I)\boldsymbol{L}^c_{t,t-1|t-1}$$

Denote $\hat{W} = (\hat{A}^3 + \hat{A}^2 + \hat{A} + I)$ and the stationarity of $\hat{A}$ guarantees $\hat{W}$ is invertible. Plug the above equation into (31) we have:

$$\boldsymbol{L}^c_{t+4,t|t} = \hat{W}[\hat{A}(I - KG)\hat{W}^{-1}\boldsymbol{L}^c_{t+3,t-1|t-1} + \hat{A}KG\boldsymbol{L}_{t,t-1} + \hat{A}K\eta_t]$$

Now write the forecasting error $FE_{t+4,t|t}$ as defined:

$$
\begin{aligned}
FE_{t+4,t|t} \equiv \boldsymbol{L}_{t+4,t} - \boldsymbol{L}^c_{t+4,t|t} &= A\boldsymbol{L}_{t+3,t-1} + \sum_{j=1}^{4} w_{t+j,t+j-1} - \boldsymbol{L}^c_{t+4,t|t} \\
&= \hat{W}\hat{A}(I-KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (A - \hat{W}\hat{A}(I-KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- \hat{W}\hat{A}KG\boldsymbol{L}_{t,t-1} - \hat{W}\hat{A}K\eta_t + \sum_{j=1}^{4} w_{t+j,t+j-1} \\
&= \hat{W}\hat{A}(I-KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (A - \hat{W}\hat{A}(I-KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- \hat{W}\hat{A}KG\boldsymbol{L}_{t,t-1} + \boldsymbol{L}_{t+3,t} - A\boldsymbol{L}_{t+2,t-1} - \hat{W}\hat{A}K\eta_t + w_{t+4,t+3} \\
&= \hat{W}\hat{A}(I-KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (I - \hat{W}\hat{A}(I-KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- (I + \hat{W}\hat{A}KG)\boldsymbol{L}_{t,t-1} + A\boldsymbol{L}_{t+3,t+2} - \hat{W}\hat{A}K\eta_t + w_{t+4,t+3} \qquad (32)
\end{aligned}
$$

The last equation follows from the fact:

$$\boldsymbol{L}_{t+3,t-1} = \boldsymbol{L}_{t+3,t+2} + \boldsymbol{L}_{t+2,t+1} + \boldsymbol{L}_{t+1,t} + \boldsymbol{L}_{t,t-1} = \boldsymbol{L}_{t+2,t-1} + \boldsymbol{L}_{t+3,t+2}$$

# C Proofs

## C.1 Proposition 1

*Proof.* The Kalman Gain in this case:

$$K = \Sigma G'(G\Sigma G' + R)^{-1} = I \quad \Rightarrow \quad \hat{A}(I-KG) = \mathbf{0}$$

□

## C.2 Proposition 2

*Proof.* (1) From Kalman Filter:

$$KG = \Sigma G'(G\Sigma G' + R)^{-1}G$$

If both $G$ and $R$ are diagonal, $KG$ will be diagonal and $\hat{A}(I - KG)$ is diagonal.

(2) Define

$$V = G'(G\Sigma G' + R)^{-1}G$$

As both $\Sigma$ and $R$ are symmetric and positive semi-definite, $G$ is non-singular and it follows that $G\Sigma G' + R$ is invertible and symmetric. We can immediately see that $V$ is symmetric. Denote $V := \left(v_{ij}\right)_{n\times n}$, we have:

$$KG = \Sigma V = \left(\sigma_i^2 v_{ij}\right)_{n\times n}$$

The off-diagonal elements of the coefficient matrix, $w_{ij}$, is given by:

$$w_{ij} = -a_i \sigma_i^2 v_{ij}$$

As $v_{ij} = v_{ji}$ for any $i \neq j$, it is obvious that either $w_{ij} = w_{ji} = 0$ if $v_{ij} = 0$, or $w_{ij}w_{ji} = a_i a_j \sigma_i^2 \sigma_j^2 v_{ij}^2 > 0$ if $v_{ij} \neq 0$. □

## C.3 Corollary 1

*Proof.* Denote $G = \begin{pmatrix} g_1 & \rho \\ \rho & g_4 \end{pmatrix}$ and $\Omega = (G\Sigma G' + R)$, we have:

$$KG = \Sigma G'\Omega^{-1}G = \begin{pmatrix} g_1\sigma_1^2 & 0 \\ 0 & g_4\sigma_2^2 \end{pmatrix} \frac{1}{det(\Omega)} \begin{pmatrix} \sigma_{2,s}^2 & -\rho \\ -\rho & \sigma_{1,s}^2 \end{pmatrix} \begin{pmatrix} g_1 & \rho \\ \rho & g_4 \end{pmatrix}$$

The off-diagonal elements are $-\frac{1}{det(\Omega)}\rho g_1 g_4 \sigma_1^2$ and $-\frac{1}{det(\Omega)}\rho g_1 g_4 \sigma_2^2$. As $\Omega$ is positive definite, the off-diagonal elements of $\hat{A}(I-KG)$ have the same signs as $\rho$ if $\hat{A}$ have positive entries on the diagonal. $\square$

## C.4 Corollary 2

**Lemma 1.** *Consider 2-dimensional $\boldsymbol{L}_{t,t-1}$, 2 by 2 $G$, signals generated by $s_t = G\boldsymbol{L}_{t,t-1} + \eta_t$ with $G = \begin{pmatrix} g_1 & g_2 \\ g_3 & g_4 \end{pmatrix}$, and $\eta_t$ independent normal. $\exists\ \tilde{G}$ triangular and $\tilde{\eta}_t$ independent normal such that $\tilde{s}_t = \tilde{G}\boldsymbol{L}_{t,t-1} + \tilde{\eta}_t$ and $\mathbb{E}[\boldsymbol{L}_{t,t-1}|s_t] = \mathbb{E}[\boldsymbol{L}_{t,t-1}|\tilde{s}_t]$.*

*Proof.* Denote the noise $\eta_t \sim N\left(0, \begin{pmatrix} \sigma_{s,1}^2 & 0 \\ 0 & \sigma_{s,2}^2 \end{pmatrix}\right)$. Consider $\Gamma = \begin{pmatrix} \frac{\sigma_{s,2}^2 g_1}{\sigma_{s,1}^2 g_3} & 1 \\ -\frac{g_3}{g_1} & 1 \end{pmatrix}$ and the new signals:

$$\tilde{s}_t = \Gamma G \boldsymbol{L}_{t,t-1} + \Gamma \eta_t$$

Define $\tilde{G} \equiv \Gamma\eta_t$ and $\tilde{\eta}_t \equiv \Gamma\eta_t$. It is easy to verify that $\tilde{\eta}_t$ is independent normal and $\tilde{G}$ has only one non-zero off-diagonal element. Denote the Kalman gain of the original signals as $K$ and the new signals as $\tilde{K}$. It is straightforward that:

$$\begin{aligned}
\tilde{K}\Gamma G &= \Sigma G'\Gamma'\Big(\Gamma(G\Sigma G' + R)\Gamma'\Big)^{-1}\Gamma G \\
&= \Sigma G'\Gamma'(\Gamma')^{-1}(G\Sigma G' + R)^{-1}\Gamma^{-1}\Gamma G \\
&= \Sigma G'(G\Sigma G' + R)^{-1} G = KG
\end{aligned}$$

The second equality holds as $\Gamma$ is invertible. For the same reason, $K = \tilde{K}\Gamma$. Then we have:

$$\begin{aligned}
\mathbb{E}[\boldsymbol{L}_{t,t-1}|s_t] &= \hat{A}\big((I-KG)\boldsymbol{L}_{t,t-1|t-1} + Ks_t\big) \\
&= \hat{A}\big((I-\tilde{K}\tilde{G})\boldsymbol{L}_{t,t-1|t-1} + \tilde{K}\tilde{s}_t\big) \\
&= \mathbb{E}[\boldsymbol{L}_{t,t-1}|\tilde{s}_t]
\end{aligned}$$

$\square$

Here we prove the corollary with the general $G$:

**Corollary 3.** ***(Non-diagonal** $G$**)*** *If $\hat{A}$ is diagonal, $R = \begin{pmatrix} \sigma_{1,s}^2 & 0 \\ 0 & \sigma_{2,s}^2 \end{pmatrix}$ is diagonal, and $G = \begin{pmatrix} g_1 & g_2 \\ g_3 & g_4 \end{pmatrix}$, the off-diagonal elements of $\hat{A}(I-KG)$ have signs depending on $g_1g_2\sigma_{2,s}^2+g_3g_4\sigma_{1,s}^2$.*

*Proof.* Again, denote $\Omega = G\Sigma G' + R = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where:

$$\begin{cases} a = g_1^2\sigma_1^2 + g_2^2\sigma_2^2 + \sigma_{1,s}^2 \\ b = g_1g_3\sigma_1^2 + g_2g_4\sigma_2^2 \\ c = g_1g_3\sigma_1^2 + g_2g_4\sigma_2^2 \\ d = g_3^2\sigma_1^2 + g_4^2\sigma_2^2 + \sigma_{2,s}^2 \end{cases}$$

Denote the matrix $KG := \frac{1}{det(\Omega)} \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$. The off-diagonal elements of $\hat{A}(I - KG)$ depend on the signs of $x_2$ and $x_3$. It is easy to show:

$$\begin{cases} x_2 = \sigma_1^2(g_1g_2d - g_2g_3c - g_1g_4b + g_3g_4a) = \sigma_1^2(g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2) \\ x_3 = \sigma_2^2(g_1g_2d - g_1g_4c - g_3g_2b + g_3g_4a) = \sigma_2^2(g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2) \end{cases}$$

As $det(\Omega) > 0$, if the diagonal elements of $\hat{A}$ are both positive, the off-diagonal elements of $\hat{A}(I - KG)$ are:

$$\begin{cases} \text{negative} & \text{if } g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2 > 0 \\ \text{positive} & \text{if } g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2 < 0 \end{cases}$$

$\square$

The proof of Corollary 2 follows directly from Lemma 1 and Corollary 3.

### C.5 Proposition 4

*Proof.* If both $G$ and $R$ are diagonal, $KG = \Sigma G'(G\Sigma G' + R)^{-1}G$ is also diagonal. Denote $G = diag(\{g_i\}_{i=1}^n)$ and $R = diag(\{\sigma_{s,i}^2\})$. The matrix $KG$ is also diagonal:

$$KG = \Sigma G'(G\Sigma G' + R)^{-1}G = diag\left(\left\{\frac{g_i^2\sigma_i^2}{g_i^2\sigma_i^2 + \sigma_{s,i}^2}\right\}\right)$$

with diagonal elements $0 < \frac{g_i^2\sigma_i^2}{g_i^2\sigma_i^2+\sigma_{s,i}^2} < 1$. It follows immediately that:

$$w_{ij} = a_{ij}\frac{\sigma_{s,j}^2}{g_j^2\sigma_j^2 + \sigma_{s,j}^2}$$

Consequently, $w_{ij}$ has the same sign as $a_{ij}$. □

## D Monte Carlo Simulation

We consider the different learning structures discussed in Table 1 and simulate expectation data according to the noisy information model from (1) and (3) with sample sizes similar to the survey data used in Section 2.4. We then perform our joint learning test with year-ahead forecast as in (10), or with quarter-ahead forecast as in (6). This comparison is to show the test with year-ahead forecasts has similar performance to the one using quarter-ahead forecasts. Table 10 summarizes the parameters we use for simulation.

Table 10: Parameters for Simulation

| Fixed Parameters | | |
|---|---|---|
| Variable | Value | Description |
| $Q := \begin{pmatrix} \sigma_{1,t}^2 & 0 \\ 0 & \sigma_{2,t}^2 \end{pmatrix}$ | $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ | Cov matrix of shocks |
| $\Sigma := \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}$ | $\begin{pmatrix} 2 & 0 \\ 0 & 2.5 \end{pmatrix}$ | Cov matrix of prior |
| $A := \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$ | $\begin{pmatrix} 0.9 & 0 \\ 0 & 0.7 \end{pmatrix}$ | Structural parameters from ALM |
| $T$ | 152 | time-series sample size |
| Model-specific Parameters | | |
| $\hat{A} := \begin{pmatrix} \rho_1 & m_1 \\ 0 & \rho_2 \end{pmatrix}$ | $\begin{pmatrix} 0.9 & m_1 \\ 0 & 0.7 \end{pmatrix}$ | Structural parameters from PLM |
| $G = \begin{pmatrix} g_1 & g_2 \\ 0 & g_4 \end{pmatrix}$ | $\begin{pmatrix} 1 & g_2 \\ 0 & 1 \end{pmatrix}$ | Signal generating matrix |
| $R := \begin{pmatrix} \sigma_{1,s}^2 & \rho \\ \rho & \sigma_{2,s}^2 \end{pmatrix}$ | $\begin{pmatrix} 3 & \rho \\ \rho & 4 \end{pmatrix}$ | Cov matrix of noises |

As in Table 1, we consider five different cases: (1) FIRE; (2) Independent Learning with noisy but uncorrelated signals; (3) Independent Learning with mixture of states (i.e., $G$ is non-diagonal); (4) Independent Learning with correlated noise (i.e., $R$ is non-diagonal); and (5) Joint Learning with $\hat{A}$ being non-diagonal. In Table 11, we show the results with the first two cases. In both cases, $\hat{A} = A$ and $G = I$. The difference is that under FIRE, $\sigma_{1,s} = \sigma_{2,s} = 0$.

Table 11: Simulation Results: FIRE or Independent Learning with Uncorrelated Signals

| FIRE or Independent Learning: $\hat{A} = A$, $g_2 = 0$, $\rho = 0$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | FIRE | | | | Independent Learning | | | |
| | Y-ahead Spec (10) | | Q-ahead Spec (6) | | Y-ahead Spec (10) | | Q-ahead Spec (6) | |
| | Truth | Test | Truth | Test | Truth | Test | Truth | Test |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $\beta_{11}$ | 0 | $-0.01$ | 0 | 0.04 | 0.54 | $0.51^{***}$ | 0.54 | $0.47^{***}$ |
| | - | (0.03) | - | (0.09) | - | (0.09) | - | (0.09) |
| $\beta_{12}$ | 0 | 0.03 | 0 | 0.15 | 0 | $-0.14$ | 0 | $-0.14$ |
| | - | (0.04) | - | (0.11) | - | (0.010) | - | (0.10) |
| $\beta_{21}$ | 0 | 0.01 | 0 | 0.10 | 0 | $-0.03$ | 0 | $-0.09$ |
| | - | (0.02) | - | (0.09) | - | (0.04) | - | (0.11) |
| $\beta_{22}$ | 0 | $-0.00$ | 0 | 0.18 | 0.43 | $0.49^{***}$ | 0.43 | $0.61^{***}$ |
| | - | (0.05) | - | (0.12) | - | (0.07) | - | (0.11) |

* ***,**,*: Significance at 1%, 5%, and 10% level. Columns (2) and (6) are estimation results for one-year-ahead joint-learning test (10), and columns (4) and (8) are for the quarter-ahead specification (6). Newey-West standard errors are reported in brackets.

The results in Table 11 show the clear differences in test results under FIRE and Independent learning. For all specifications considered, if the expectation is formed under FIRE, all the $\beta$s will be insignificantly different from zero. Meanwhile, if expectations are formed independently but with information friction, only estimates on $\beta_{11}$ and $\beta_{22}$ are significantly positive. The estimates on $\beta_{21}$ and $\beta_{12}$ will be insignificant.

Table 12: Simulation Results: Independent Learning with Correlated Signals

| | Independent Learning when $G$ or $R$ are non-diagonal | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | G non-diagonal: $m_1 = 0,\ g_2 = 0.5,\ \rho = 0$ | | | | R non-diagonal: $m_1 = 0,\ g_2 = 0,\ \rho = -2$ | | | |
| | Y-ahead spec (10) | | Q-ahead spec (6) | | Y-ahead spec (10) | | Q-ahead spec (6) | |
| | Truth | Test | Truth | Test | Truth | Test | Truth | Test |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $\beta_{11}$ | 0.57 | 0.56*** | 0.57 | 0.52*** | 0.49 | 0.43*** | 0.49 | 0.37*** |
| | – | (0.05) | – | (0.08) | – | (0.05) | – | (0.09) |
| $\beta_{12}$ | -0.14 | −0.28*** | -0.10 | −0.26*** | -0.17 | −0.25*** | -0.13 | −0.24*** |
| | – | (0.09) | – | (0.10) | – | (0.09) | – | (0.09) |
| $\beta_{21}$ | -0.07 | −0.10*** | -0.10 | −0.20** | -0.09 | −0.11*** | -0.12 | −0.17 |
| | – | (0.04) | – | (0.10) | – | (0.04) | – | (0.11) |
| $\beta_{22}$ | 0.40 | 0.46*** | 0.40 | 0.55*** | 0.39 | 0.49*** | 0.39 | 0.63*** |
| | – | (0.07) | – | (0.11) | – | (0.07) | – | (0.11) |

* ***,**,*: Significance at 1%, 5%, and 10% level. Columns (2) and (6) are estimation results for year-ahead joint-learning test (10), and columns (4) and (8) are for quarter-ahead specification (6). Newey-West standard errors are reported in brackets.

Table 12 shows the results if beliefs are formed under independent learning with noisy signals that are correlated. We consider two different cases of correlated signals: either $G$ is non-diagonal or $R$ is non-diagonal. In particular, we consider either $g_2 = 0.5$ or $\rho = -2$. According to Corollary 2 and 1, in both these two scenarios $\beta_{12}$ and $\beta_{21}$ will be negative. Both regressions with (6) and (10) perform well to uncover such a pattern.

We then consider the test results under joint learning when $\hat{A}$ is non-diagonal and signals are uncorrelated. In Table 13, we report the test results from simulated data for both year-ahead specification (6) and quarter-ahead specification (10). Both test results are in line with the predictions from Proposition 4.

Table 13: Simulation Results: Joint Learning

| | Joint Learning: $m_1 = 0.5$, $G$ and $R$ are diagonal | | | |
|---|---|---|---|---|
| | Year-ahead spec (10) | | Quarter-ahead spec (6) | |
| | Truth | Test | Truth | Test |
| | (1) | (2) | (3) | (4) |
| $\beta_{11}$ | 0.54 | 0.48*** | 0.54 | 0.44*** |
| | - | (0.08) | - | (0.08) |
| $\beta_{12}$ | 0.32 | 0.49** | 0.31 | 0.35*** |
| | - | (0.22) | - | (0.10) |
| $\beta_{21}$ | 0 | $-0.02$ | 0 | $-0.08$ |
| | - | (0.04) | - | (0.09) |
| $\beta_{22}$ | 0.43 | 0.54*** | 0.43 | 0.70*** |
| | - | (0.12) | - | (0.14) |

* ***,**,*: Significance at 1%, 5%, and 10% level. Column (2) contains estimation results for year-ahead joint-learning test (10), and column (4) is for quarter-ahead specification (6). Newey-West standard errors are reported in brackets.

All in all, the test results using simulated data are consistent with the theoretical predictions. The performance of tests using year-ahead forecast error or quarter-ahead forecast error is similar throughout the different scenarios we considered.

# E Estimation: Robustness

Table 14 estimates the parameters with an alternative data sample to those used in Table 3. It yields very similar estimates. Furthermore, our benchmark estimation assumes no feedback loop from expectations to realized data. For robustness, we estimate an unrestricted version of the VAR model dropping such an assumption, and report the results in Table 15.

Table 14: Estimates of Joint Learning Model (7): Alternative Sample

| | **MSC, quarterly** | | | |
|---|---|---|---|---|
| | **Q1 1984-Q4 2019** | | **Q1 1990-Q4 2018** | |
| Parameters | Estimates | Standard Errors | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.807 & -0.070 \\ 0.062 & 0.922 \end{bmatrix}$ | $\begin{bmatrix} 0.059 & 0.114 \\ 0.022 & 0.072 \end{bmatrix}$ | $\begin{bmatrix} 0.781 & -0.060 \\ 0.059 & 0.930 \end{bmatrix}$ | $\begin{bmatrix} 0.068 & 0.145 \\ 0.031 & 0.082 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.663 & -0.096 \\ 0.189 & 0.807 \end{bmatrix}$ | $\begin{bmatrix} 0.063 & 0.089 \\ 0.057 & 0.056 \end{bmatrix}$ | $\begin{bmatrix} 0.663 & -0.081 \\ 0.271 & 0.769 \end{bmatrix}$ | $\begin{bmatrix} 0.080 & 0.094 \\ 0.064 & 0.057 \end{bmatrix}$ |
| T-test: | test-stat | p-val | test-stat | p-val |
| $\hat{A}_{21} \leq A_{21}$ | 2.094 | 0.018 | 2.999 | 0.001 |
| | **SPF, quarterly** | | | |
| | **Q1 1984-Q4 2019** | | **Q1 1990-Q4 2018** | |
| Parameters | Estimates | Standard Errors | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.788 & -0.070 \\ 0.048 & 0.906 \end{bmatrix}$ | $\begin{bmatrix} 0.070 & 0.100 \\ 0.024 & 0.071 \end{bmatrix}$ | $\begin{bmatrix} 0.749 & -0.047 \\ 0.042 & 0.920 \end{bmatrix}$ | $\begin{bmatrix} 0.079 & 0.113 \\ 0.030 & 0.077 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.951 & 0.004 \\ 0.026 & 0.787 \end{bmatrix}$ | $\begin{bmatrix} 0.018 & 0.041 \\ 0.016 & 0.041 \end{bmatrix}$ | $\begin{bmatrix} 0.937 & -0.027 \\ 0.026 & 0.806 \end{bmatrix}$ | $\begin{bmatrix} 0.021 & 0.030 \\ 0.031 & 0.044 \end{bmatrix}$ |
| T-Test | test-stat | p-val | test-stat | p-val |
| $\hat{A}_{21} \leq A_{21}$ | -0.883 | 0.811 | -0.410 | 0.659 |

The table reports the estimates and their Newey-West standard errors from the GMM estimation of the four-variable VAR model. An iterative weighting matrix is used in the GMM estimation. The standard errors are based on the variance-covariance matrix of model estimates. Since $\hat{A}$ is the element-wise sum of directly estimated $B$ and $C$, the element-wise variance-covariance matrix of $B$ and $C$ is used to calculate the standard errors of $\hat{A}$ estimates.

# F Derivations of NK Model

We derive the State Space Representation (17) and (18) from the standard New Keynesian model. To simplify the exposition, we solve the ALM using the true parameters (without "hats"). The PLM can be derived analogously. Consider the simple three-equation NK model

Table 15: Estimates of Joint Learning Model (20): with Feedback Loop

| | MSC, quarterly | | | |
|---|---|---|---|---|
| | **Q1 1984-Q4 2019** | | **Q1 1990-Q4 2018** | |
| Parameters | Estimates | Standard Errors | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.863 & 0.021 \\ -0.003 & 0.751 \end{bmatrix}$ | $\begin{bmatrix} 0.073 & 0.162 \\ 0.042 & 0.074 \end{bmatrix}$ | $\begin{bmatrix} 0.863 & 0.051 \\ -0.017 & 0.721 \end{bmatrix}$ | $\begin{bmatrix} 0.078 & 0.169 \\ 0.042 & 0.076 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.663 & -0.096 \\ 0.189 & 0.807 \end{bmatrix}$ | $\begin{bmatrix} 0.063 & 0.089 \\ 0.057 & 0.056 \end{bmatrix}$ | $\begin{bmatrix} 0.663 & -0.081 \\ 0.271 & 0.769 \end{bmatrix}$ | $\begin{bmatrix} 0.080 & 0.094 \\ 0.064 & 0.057 \end{bmatrix}$ |
| T-test: | test-stat | p-val | test-stat | p-val |
| $\hat{A}_{21} \leq A_{21}$ | 2.227 | 0.013 | 3.112 | 0.001 |
| | **SPF, quarterly** | | | |
| | **Q1 1984-Q4 2019** | | **Q1 1990-Q4 2018** | |
| Parameters | Estimates | Standard Errors | Estimates | Standard Errors |
| $A$ | $\begin{bmatrix} 0.696 & -0.091 \\ 0.021 & 0.792 \end{bmatrix}$ | $\begin{bmatrix} 0.078 & 0.090 \\ 0.031 & 0.072 \end{bmatrix}$ | $\begin{bmatrix} 0.678 & -0.062 \\ 0.019 & 0.785 \end{bmatrix}$ | $\begin{bmatrix} 0.086 & 0.107 \\ 0.034 & 0.089 \end{bmatrix}$ |
| $\hat{A}$ | $\begin{bmatrix} 0.951 & 0.004 \\ 0.026 & 0.787 \end{bmatrix}$ | $\begin{bmatrix} 0.018 & 0.041 \\ 0.016 & 0.041 \end{bmatrix}$ | $\begin{bmatrix} 0.937 & -0.027 \\ 0.026 & 0.806 \end{bmatrix}$ | $\begin{bmatrix} 0.021 & 0.030 \\ 0.031 & 0.044 \end{bmatrix}$ |
| T-Test | test-stat | p-val | test-stat | p-val |
| $\hat{A}_{21} \leq A_{21}$ | 0.136 | 0.446 | 0.1534 | 0.439 |

The table reports the estimates and their standard errors from the GMM estimation of the unrestricted four-variable VAR model. Iterative weighting matrix is used in the GMM estimation. The standard errors are based on the variance-covariance matrix of model estimates. Since $\hat{A}$ is the element-wise sum of directly estimated $B$ and $C$, the element-wise variance-covariance matrix of $B$ and $C$ is used to calculate the standard errors of $\hat{A}$ estimates.

as in Galí (2015):

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa y_t + s_t \tag{NKPC}$$

$$y_t = E_t[y_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}]) + d_t \tag{IS Curve}$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t \tag{Monetary Policy Rule}$$

where $y_t$ is the output gap, $i_t$ is the nominal interest rate deviation from its long-run mean. $d_t$ and $s_t$ are demand and supply shocks respectively. We introduce the unemployment rate,

$u_t$, linked to the output gap by Okun's Law: $u_t = -\chi y_t$. We substitute $y_t = -u_t/\chi$ into the model equations and get:

**The NKPC**

$$\pi_t = \beta E_t[\pi_{t+1}] - \frac{\kappa}{\chi} u_t + s_t \tag{33}$$

**The IS Curve**

$$u_t = E_t[u_{t+1}] + \frac{\chi}{\sigma}(i_t - E_t[\pi_{t+1}]) - \chi d_t \tag{34}$$

**The Monetary Policy Rule**

$$i_t = \phi_\pi \pi_t - \frac{\phi_y}{\chi} u_t \tag{35}$$

Substitute the policy rule into the IS curve to eliminate $i_t$:

$$\begin{aligned} u_t &= E_t[u_{t+1}] + \frac{\chi}{\sigma}\left(\left[\rho + \phi_\pi \pi_t - \frac{\phi_y}{\chi} u_t\right] - E_t[\pi_{t+1}] - \rho\right) - \chi d_t \\ &= E_t[u_{t+1}] + \frac{\chi \phi_\pi}{\sigma} \pi_t - \frac{\phi_y}{\sigma} u_t - \frac{\chi}{\sigma} E_t[\pi_{t+1}] - \chi d_t \end{aligned}$$

Grouping the current endogenous variables on the left side gives:

$$\left(1 + \frac{\phi_y}{\sigma}\right) u_t - \frac{\chi \phi_\pi}{\sigma} \pi_t = E_t[u_{t+1}] - \frac{\chi}{\sigma} E_t[\pi_{t+1}] - \chi d_t \tag{36}$$

The two-equation system is:

$$\pi_t = \beta E_t[\pi_{t+1}] - \frac{\kappa}{\chi} u_t + s_t \tag{37}$$

$$\left(\frac{\sigma + \phi_y}{\sigma}\right) u_t - \frac{\chi \phi_\pi}{\sigma} \pi_t = E_t[u_{t+1}] - \frac{\chi}{\sigma} E_t[\pi_{t+1}] - \chi d_t \tag{38}$$

We then solve $L_{t,t-1} = \begin{pmatrix} \pi_t \\ u_t \end{pmatrix}$ via undetermined coefficients. We posit a linear solution:

$$u_t = \Psi_{ud} d_t + \Psi_{us} s_t$$
$$\pi_t = \Psi_{\pi d} d_t + \Psi_{\pi s} s_t$$

Substituting these into the above two equations and equating coefficients for each shock yields two systems of equations for the $\Psi$ coefficients. After rearranging, we get:

**System for $d_t$ coefficients:**

$$\begin{cases} (1-\beta\rho_d)\Psi_{\pi d} + \frac{\kappa}{\chi}\Psi_{ud} & = 0 \\ (\sigma(1-\rho_d) + \phi_y)\Psi_{ud} - \chi(\phi_\pi - \rho_d)\Psi_{\pi d} & = -\sigma\chi \\ (1-\beta\rho_s)\Psi_{\pi s} + \frac{\kappa}{\chi}\Psi_{us} & = 1 \\ (\sigma(1-\rho_s) + \phi_y)\Psi_{us} - \chi(\phi_\pi - \rho_s)\Psi_{\pi s} & = 0 \end{cases}$$

Solving these systems gives the explicit values for the $\Psi$ coefficients in terms of the model's structural parameters:

$$\begin{cases} \Psi_{ud} & = -\frac{\sigma\chi(1-\beta\rho_d)}{\Delta_d} \\ \Psi_{\pi d} & = \frac{\sigma\kappa}{\Delta_d} \\ \Psi_{\pi s} & = \frac{\sigma(1-\rho_s)+\phi_y}{\Delta_s} \\ \Psi_{us} & = \frac{\chi(\phi_\pi-\rho_s)}{\Delta_s} \end{cases}$$

with

$$\begin{cases} \Delta_d & = (\sigma(1-\rho_d) + \phi_y)(1-\beta\rho_d) + \kappa(\phi_\pi - \rho_d) \\ \Delta_s & = (\sigma(1-\rho_s)(1-\beta\rho_s) + \phi_y) + \kappa(\phi_\pi - \rho_s) \end{cases}$$

These lead to the solution of $L_{t,t-1}$ as a function of shocks described in equation (16).

**The State-Space Representation** Finally to obtain the state-space representation, let $X_t = [d_t, s_t]'$, and $\Psi$ be the $2 \times 2$ matrix of coefficients.

$$Z_t = \Psi X_t, \quad \text{where} \quad \Psi = \begin{bmatrix} \Psi_{\pi d} & \Psi_{\pi s} \\ \Psi_{ud} & \Psi_{us} \end{bmatrix}$$

Recall that the shocks follow a VAR(1) process $X_t = \Gamma X_{t-1} + \varepsilon_t$, where $\Gamma = \text{diag}(\rho_d, \rho_s)$. Then it leads to (18):

$$\begin{aligned} L_{t,t-1} &= \Psi(\Gamma X_{t-1} + \varepsilon_t) \\ &= \Psi\Gamma(\Psi^{-1} L_{t-1,t-2}) + \Psi\varepsilon_t \\ &= A L_{t-1,t-2} + w_{t,t-1} \end{aligned}$$

with $A \equiv \Psi\Gamma\Psi^{-1}$. Following similar rationale, $\hat{A} \equiv \hat{\Psi}\hat{\Gamma}\hat{\Psi}^{-1}$.

## F.1 Derivation of (20)

We now derive the joint dynamics equation (20) of realized and expected inflation and unemployment, $Y_t = \begin{pmatrix} L_{t,t-1|t-1} \\ L_{t,t-1} \end{pmatrix}$, with the timing restriction that $L_{t,t-1}$ is affected only by expectation formed with past information, $L_{t,t-1|t-1}$. Under this restriction, the structural equations (37) and (38) become:

$$L_{t,t-1} = \underbrace{\begin{pmatrix} \beta & 0 \\ -\frac{\chi}{\sigma} & 1 \end{pmatrix}}_{\equiv B} L_{t,t-1|t-1} + \underbrace{\begin{pmatrix} 0 & -\frac{\kappa}{\chi} \\ \frac{\chi}{\sigma}\phi_\pi & -\frac{\phi_y}{\sigma} \end{pmatrix}}_{\equiv C} L_{t,t-1} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -\chi \end{pmatrix}}_{\equiv D} \underbrace{\begin{pmatrix} s_t \\ d_t \end{pmatrix}}_{\equiv X_t} \tag{39}$$

Recall the expectation is formed by:

$$L_{t+1,t|t} = \hat{A}(I - KG)L_{t,t-1|t-1} + \hat{A}KGL_{t,t-1} + \hat{A}K\eta_t \tag{40}$$

Take $t+1$ version of (39) and write:

$$
\begin{aligned}
(I-C)L_{t+1,t} =& BL_{t+1,t|t} + DX_{t+1} && (41)\\
=& B\Big(\hat{A}(I-KG)L_{t,t-1|t-1} + \hat{A}KGL_{t,t-1} + \hat{A}K\eta_t\Big) + D\Gamma X_t + D\varepsilon_{t+1} && (42)\\
=& B\hat{A}(I-KG)L_{t,t-1|t-1} + B\hat{A}KGL_{t,t-1} + B\hat{A}K\eta_t\\
&+ D\Gamma D^{-1}\Big((I-C)L_{t,t-1} - BL_{t,t-1|t-1}\Big) + D\varepsilon_{t+1} && (43)
\end{aligned}
$$

This gives:

$$
Y_{t+1} = \underbrace{\begin{pmatrix} \hat{A}(I-KG) & \hat{A}KG \\ \mathbf{\Theta} & A \end{pmatrix}}_{:=\Phi} Y_t + F \begin{pmatrix} \eta_t \\ \tilde{w}_{t+1,t} \end{pmatrix} \tag{44}
$$

where

$$
\begin{cases}
A & = (I-C)^{-1}\Big(B\hat{A}KG + D\Gamma D^{-1}(I-C)\Big)\\
\Theta & = (I-C)^{-1}\Big(B\hat{A}(I-KG) - D\Gamma D^{-1}B\Big)
\end{cases}
$$

# G News Measure from MSC

## G.1 Description

In the MSC, there is a question asking about news heard recently about business conditions:

*A6. During the last few months, have you heard of any favorable or unfavorable changes in business conditions?*

*A6a. What did you hear?*

The news reported in this question should be considered as self-reported information. It may contain both public and private information heard by the survey respondents. The

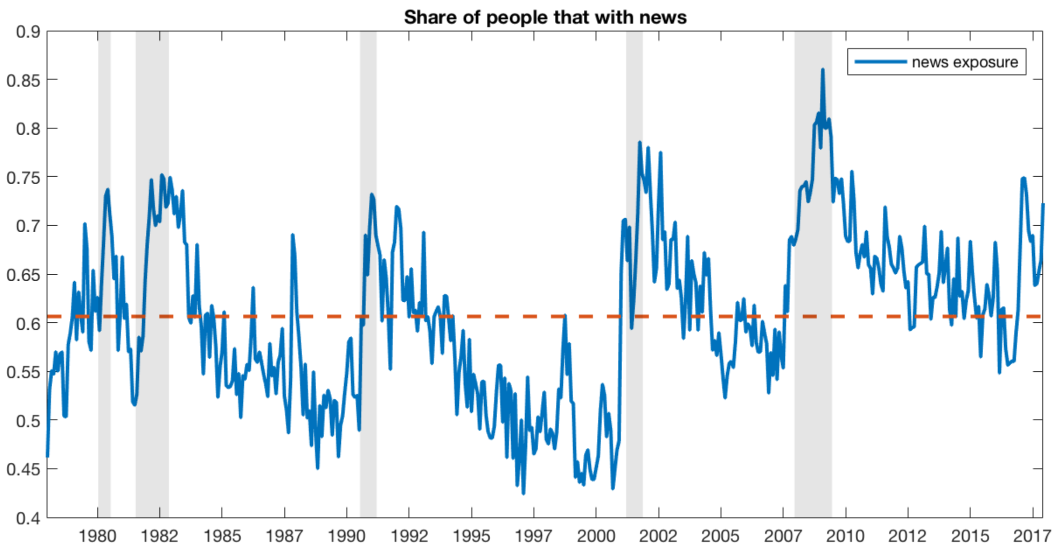
MSC categorizes the content of news described by the respondents is categorized into 80 different categories. We further summarize these categories into 10 different types of news, as described in Table 16. In Figure 10 we plot the share of survey respondents that report hearing any news. Figure 11 depicts the fraction of agents hearing news about unemployment and inflation conditional on hearing any news.

Table 16: Types of News Reported

| **Categories Defined** | **News description in the MSC** | |
|---|---|---|
| | Favorable | Unfavorable |
| Employment | Employ is high, plenty of jobs | Drop in employ, less overtime |
| | Other references to employ and purch power (fav) | Other references to employ and purch power (unfav) |
| Industry | Opening of plants, factories, stores | Closing of plants, factories, stores |
| | Improvements in specific industries | Decline in specific industries |
| | Farm situation good, crops good | Farm situation is bad, low farm prices, drought |
| Inflation | Lower/stable prices, less inflation | Prices falling, deflation |
| Interest rate | Easier money, credit easy to get, low int rates | Tight money, int rates high |
| Demand | Consumer/auto demand high | Consumer/auto demand low |
| | Population increase, more people to buy | Population increase, immigration |
| Government | Elections, admin, Congress, President (fav) | Elections, admin, Congress, President (unfav) |
| | More military spending, more war/tensions (fav) | More military spending, more war/tensions (unfav) |
| | Less military spending, few tensions (fav) | Less military spending, few tensions (unfav) |
| | etc. | etc. |
| Sentiment/Unclear | Better race relations, less crime | Bad race relations; more crime |
| | Times/business is good in the coming year | Times are bad now and won't change in next year |
| | Economy more stable, optimism | Economy in general less stable, lack of confidence |
| | etc. | etc. |
| Financial Market | Stock market, rise in price of stocks | Stock market decline |
| Other Real Activities | Low debts, higher savings/assets, invest up | High(er) debts, lower savings/assets |
| | Production increasing, GNP is up | Production decreasing, GNP down |
| Other Price Related | Profits high/rising | Profits high, too high |
| | Balance of payments, dollar devalue | Balance of payments, dollar devalue |
| | Price or wage controls (fav) | Price or wage controls (unfav) |
| | etc. | etc. |

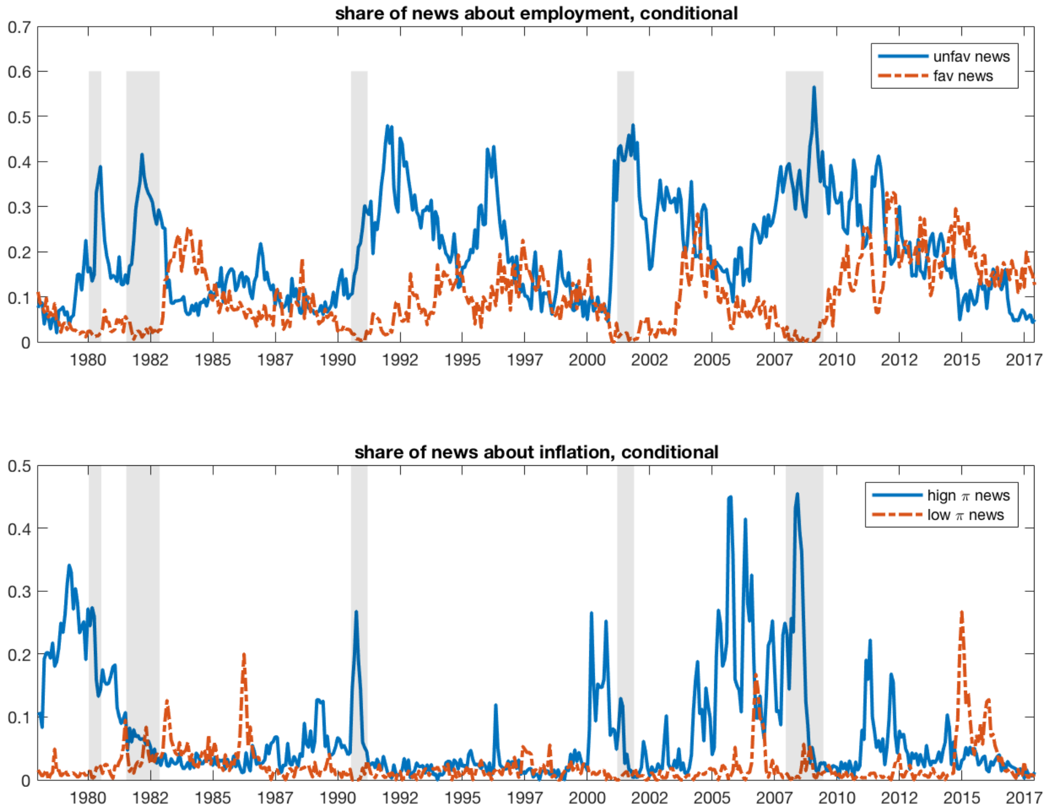
*Notes: The descriptions of news are documented by the Michigan Survey of Consumers. We reclassified them according to these descriptions.*

Figure 10: Share of People Who Report Hearing News




Share of people who report hearing any news across time. The dashed line represents on average 60% of survey participants report hearing about some news in the past few months.

Figure 11: Share of People Who Report Hearing News on Inflation and Employment




Share of people who report hearing news on employment or inflation, conditional on hearing any news. In the top panel, the blue line is the fraction with unfavorable news on employment, and the red dashed line is the fraction with favorable news. In the bottom panel, the blue line is the fraction with news on higher inflation.

On average, more than 60% of agents report they have heard some news about the economy, and the fraction is co-moving with the business cycle, peaking in each recession. This news about unemployment and inflation accounts for more than 40% on average, peaking at about 80% in the recent recession. Figure 12 summarizes the types of news they report, conditional on reporting having heard about some news. Panel (b) of Figure 12 plots the shares of different types of news out of the total news reported in each year. We see that most of the news is clearly labeled to be related to some specific economic aspect. The news with unclear labels is categorized as "sentiment"[46] and only accounts for around 11% of news

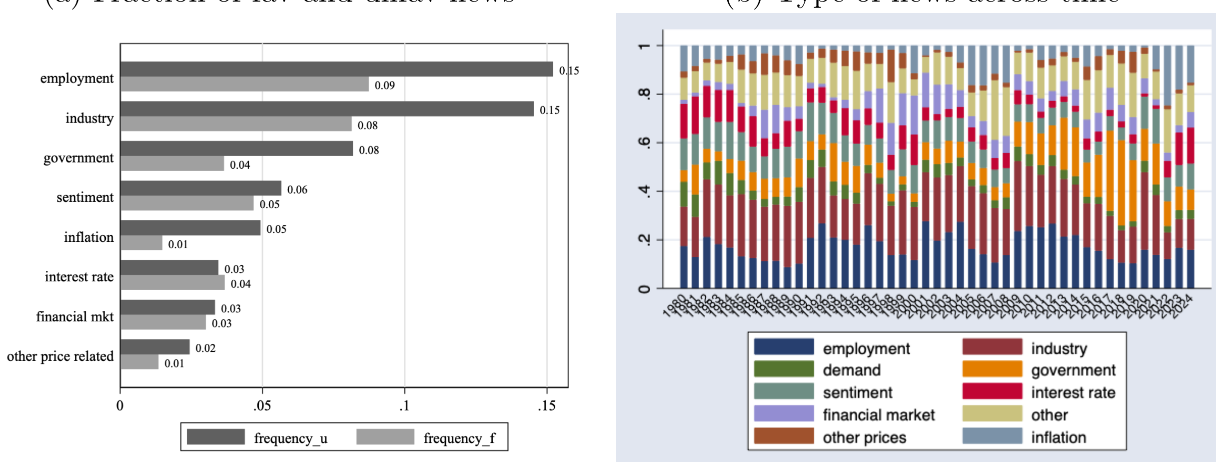
[46]See Table 16.

reported.

The MSC labels the reported news as "favorable" or "unfavorable" according to the description of the news. There is an asymmetry in tones of news: the blue curve is almost always above the red one, which suggests agents report hearing bad news more often than good news. In Figure 12, panel (a) shows the fraction of favorable and unfavorable news reported by the survey respondents who have heard news. News on industry, employment, government, and inflation account for 60% of the news reported. Among these types, the respondents report much more unfavorable than favorable news. News on industry, employment, and demand are major categories related to real activities in the economy.

Figure 12: Type of News

(a) Fraction of fav and unfav news (b) Type of news across time




*Notes: Panel (a): fractions of favorable and unfavorable news reported by individuals with news in the MSC. Panel (b): shares of different types of news out of total news reported each year.*

# H Additional Evidence from Newspapers

We define the news coverage of a particular topic (e.g., inflation) as the sum of the frequencies of the term "inflation" mentioned as a share of the total number of words within each article. Over the sample period, the time series of the news coverage of inflation and unemployment are highly correlated with their respective self-reported news exposure in the

MSC. In particular, the correlation coefficient between the news measure and the share of MSC households that report having heard *any* news about prices is 0.6. The correlation regarding unemployment news is around 0.37 (see Table 17). Note that here *any* news is measured by gross exposure: the total fraction who have heard some either good or bad news (see Figure 13).

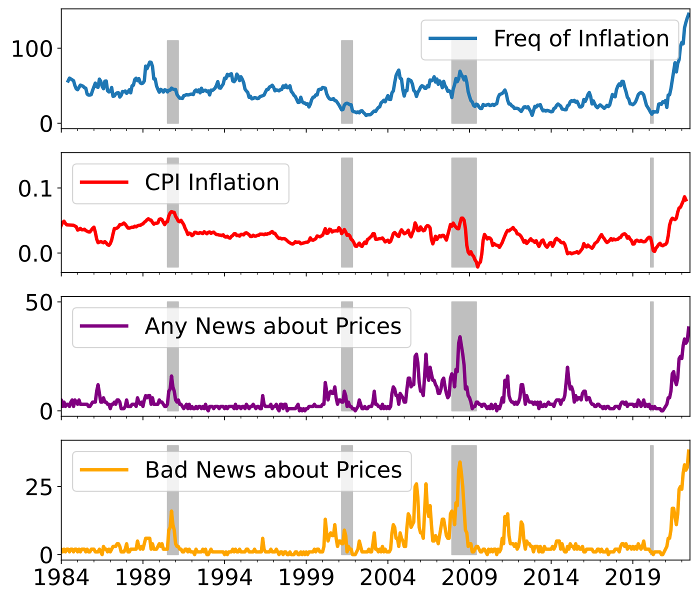
The news coverage is often domain-specific. Over the sample period, the time variations of news coverage of inflation and unemployment exhibit patterns of their own and do not simultaneously move. The correlation coefficients between two measures of news coverage are close to zero across various measures. This suggests that there is a sufficient amount of newspaper discussions separately devoted to unemployment and inflation, echoing our finding earlier that common signals cannot be the common factor that drives the correlations between unemployment and inflation expectations.

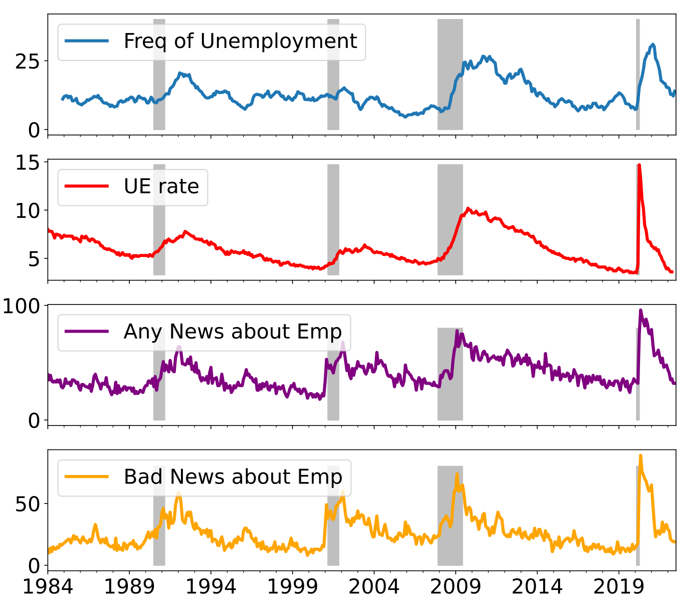
But there are differences between the two types of news. Unlike unemployment news, inflation news coverage is mostly labeled as unfavorable. This can be seen from the fact that the high correlation between news coverage and self-reported exposure to any news on inflation is entirely driven by the share of agents who "have heard about *unfavorable* news about prices." The correlation between self-reported negative exposure and news coverage is almost equal to that of the gross measure. In contrast, news coverage of unemployment is less correlated with exposure to either positive or negative news alone than gross exposure (see Table 17). This suggests that although labor market news coverage is likely to be either favorable or unfavorable from households' point of view, inflation news coverage is more likely to be associated with a negative connotation.

Table 17: News Coverage and Self-Reported News Exposure

| Topic | Any News | Bad News | Good News |
|---|---|---|---|
| Inflation | 0.605 | 0.627 | -0.048 |
| Unemployment | 0.373 | 0.295 | 0.153 |

Figure 13: News Coverage, Self-reported News Exposure, and Macroeconomic Realizations




This plots news coverage measured in the WSJ sample, realized inflation and unemployment rates, and two self-reported news exposures in the MSC.

### The inflation-unemployment association is seen in different narratives

We define the news coverage of a particular topic (e.g., inflation) as the sum of the frequencies of the term "inflation" mentioned as a share of the total number of words within each article. Over the sample period, the time series of the news coverage of inflation and unemployment are highly correlated with their respective self-reported news exposure in the MSC. In particular, the correlation coefficient between the news measure and the share of MSC households that report having heard *any* news about prices is 0.6. The correlation regarding unemployment news is around 0.37 (see Table 17). Note that here *any* news is measured by gross exposure: the total fraction who have heard some either good or bad news (see Figure 13).

The news coverage is often domain-specific. Over the sample period, the time variations of news coverage of inflation and unemployment exhibit patterns of their own and do not simultaneously move. The correlation coefficients between two measures of news coverage are close to zero across various measures. This suggests that there is a sufficient amount of newspaper discussions separately devoted to unemployment and inflation, echoing our finding earlier that common signals cannot be the common factor that drives the correlations between unemployment and inflation expectations.

But there are differences between the two types of news. Unlike unemployment news, inflation news coverage is mostly labeled as unfavorable. This can be seen from the fact that the high correlation between news coverage and self-reported exposure to any news on inflation is entirely driven by the share of agents who "have heard about *unfavorable* news about prices." The correlation between self-reported negative exposure and news coverage is almost equal to that of the gross measure. In contrast, news coverage of unemployment is less correlated with exposure to either positive or negative news alone than gross exposure (see Table 17). This suggests that although labor market news coverage is likely to be either favorable or unfavorable from households' point of view, inflation news coverage is more likely to be associated with a negative connotation. Since the association between unemployment and inflation is not driven by common signals in the newspaper, we inspect, instead, if such an association is driven by different subjective models or narratives in news discourses. We identify a narrative as a correlation between different topics that are *within* a news article.

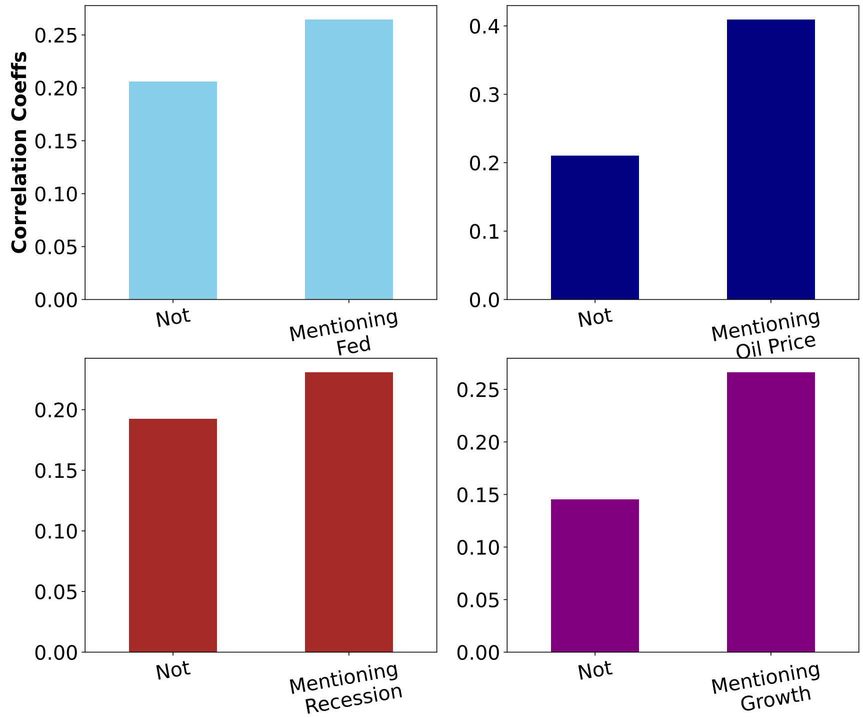
To get some context, consider monetary policy as one example of a topic. It is indicated by an article mentioning the keyword "Fed," or by having a positive weight of a topic consisting of a list of keywords that can be interpreted as primarily related to the monetary policy (e.g.,"Fed," "Rate," "Inflation," "Economy," etc.). With these measures, we can examine if a particular article discussing monetary policy is more likely to draw connections between unemployment and inflation than other articles. We are not trying to identify causal links or directional correlations made in news articles. Instead, we treat the correlation between the frequencies of mentioning both terms as an indication of an article associating the two variables according to some model. Our goal is then to identify the topics prevailing in inflation-unemployment narratives and if such an association is more common in certain narratives than in others.

Throughout the entire sample, the correlation between the frequencies of mentioning "inflation" and "unemployment" within each article is 0.2. This indicates that economic news articles tend to associate the two variables/concepts in economic discussions. Note that this is different from the zero correlation across time between the news coverage of unemployment and inflation.

We also find that there is a wide range of contexts in which articles make an association between inflation and unemployment. Figure 14 shows that conditional on mentioning any one of the keywords such as "Fed," "Oil price," "growth," and "recession," economic news has higher correlation coefficients between the frequencies of jointly discussing inflation and unemployment.

Going beyond simple word counts, Figure 15 plots the most common LDA topics, ranked by their weights, in articles mentioning both inflation and unemployment and mentioning either topic alone. The articles that jointly mention both words and inflation-only articles

Figure 14: Associations between "Inflation" and "Unemployment" by Topic




This bar chart shows the correlation coefficients between frequencies of mentioning "inflation" and "unemployment" by all articles, conditional on mentioning four other keywords.

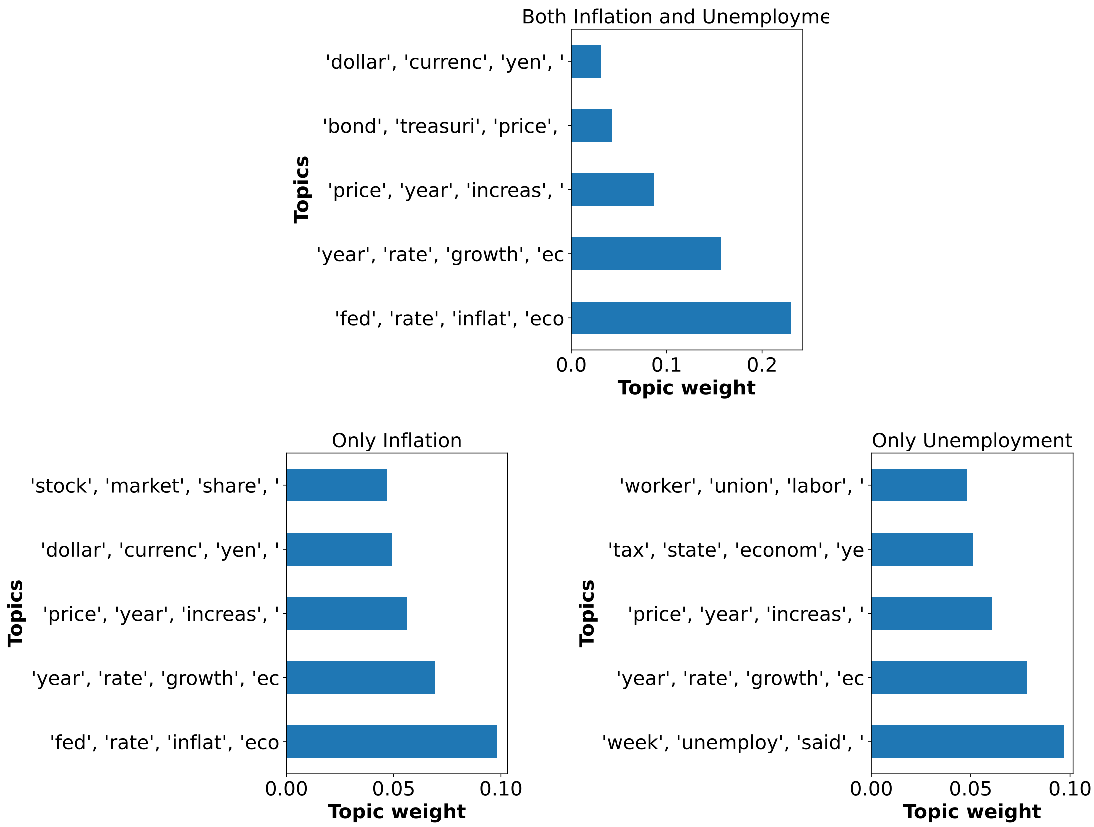
largely overlap in the common topics, such as monetary policy, economic growth, prices, and exchange rates. In contrast, the most common topics in unemployment-only articles are not the same. For instance, unemployment, tax policy, and union topics are all specific to unemployment news.

**Negative sentiment cannot be the common driver of expectation correlation**

One alternative explanation for the correlated inflation and unemployment expectations is a broadly defined negative sentiment. Based on measures of overall and topic-specific sentiment using newspaper texts, we find no direct support for this hypothesis. In particular, we show that the average sentiment score of articles that mention both inflation and unemployment is uncorrelated with the tendency of economic articles to associate the two within articles.

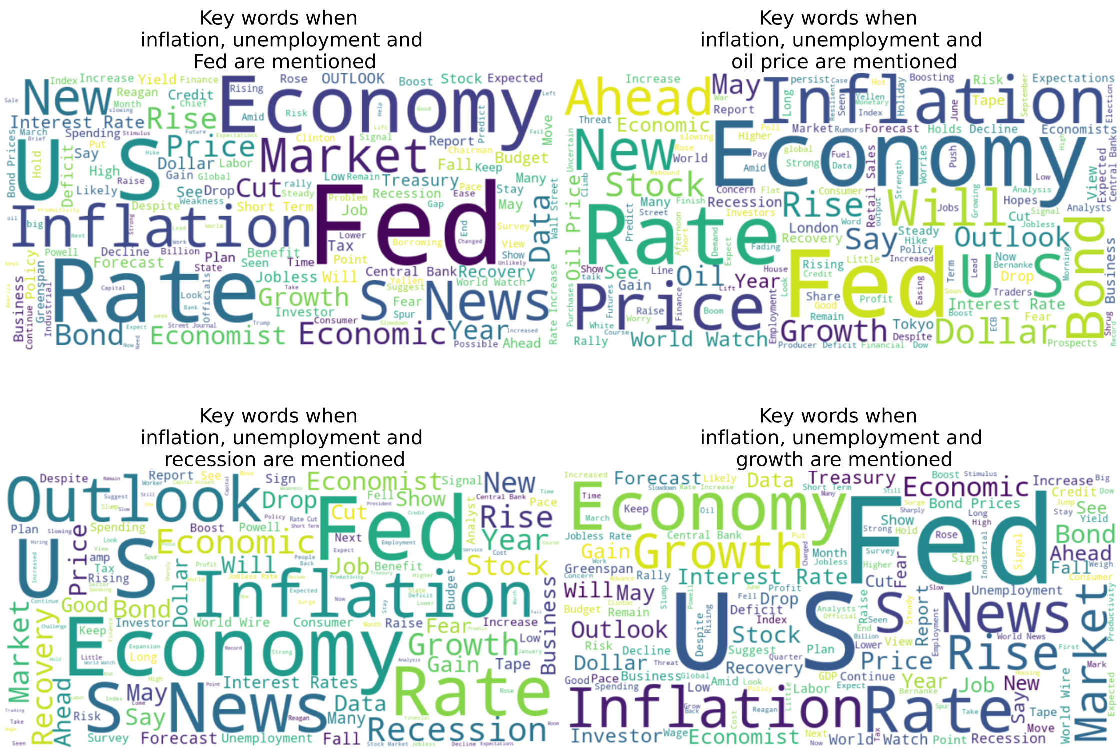
Figure 17 shows the time series of within-article correlation between coverage of un-

Figure 15: Topics in Inflation-Unemployment Narratives



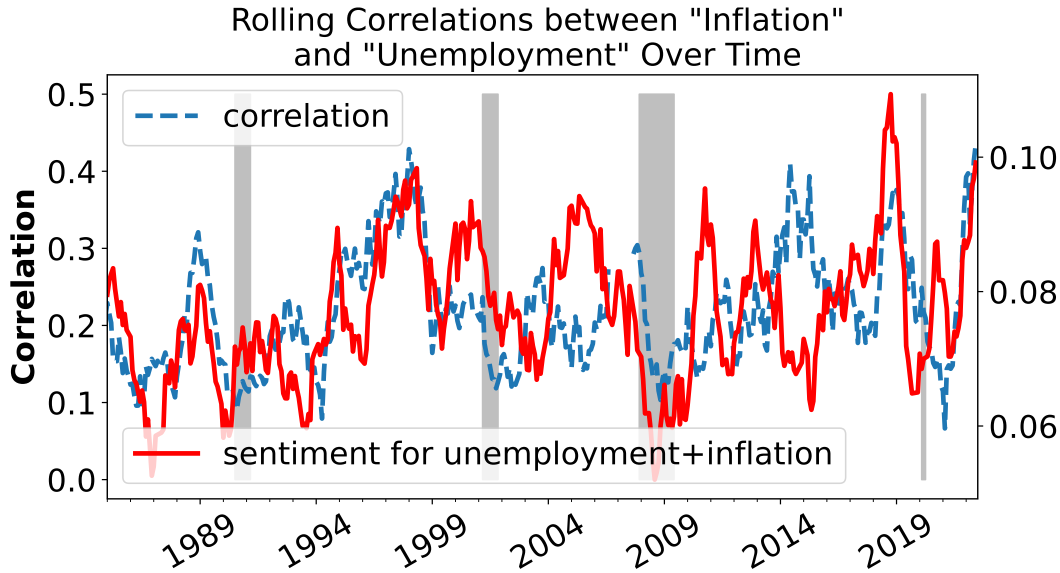

The bar charts plot the top five topics identified by the topic model in articles that mention both inflation and unemployment and those that only mention inflation or unemployment. Topic weights are between 0–1.

Figure 16: Key Words in Different Inflation-Unemployment Narratives




The figure plots the 100 most frequently used words in news articles that mention inflation, unemployment, and one of the four economic topics: Fed, oil price, recession, and growth, respectively.

Figure 17: Sentiment in Inflation-Unemployment News




On the left axis is the average within-article correlation coefficients between frequencies of "inflation" and "unemployment" for a rolling window of two years. In the right axis is the average sentiment score of articles mentioning both terms.

employment and inflation in rolling windows and the measured sentiment of articles that mention both unemployment and inflation. The correlation between the two is weakly positive. It suggests that negative sentiment, as measured in inflation-unemployment news, cannot be the only driver of the inflation-unemployment association.