

# Uncovering Subjective Models from Survey Expectations

---

Chenyu (Sev) Hou <sup>1</sup>   Tao Wang <sup>2</sup>

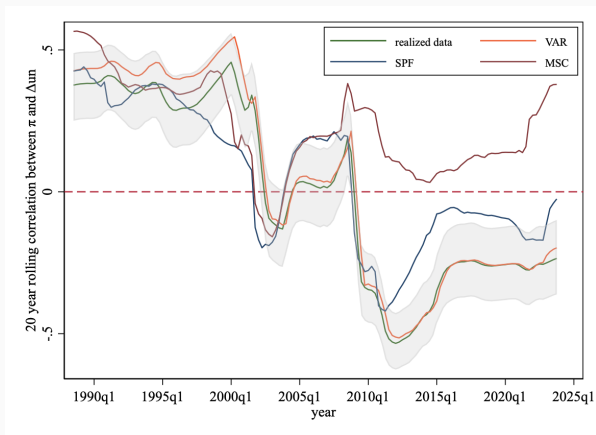
December 16, 2025

EDMM 2025, University of Sydney

<sup>1</sup>Simon Fraser University

<sup>2</sup>Bank of Canada

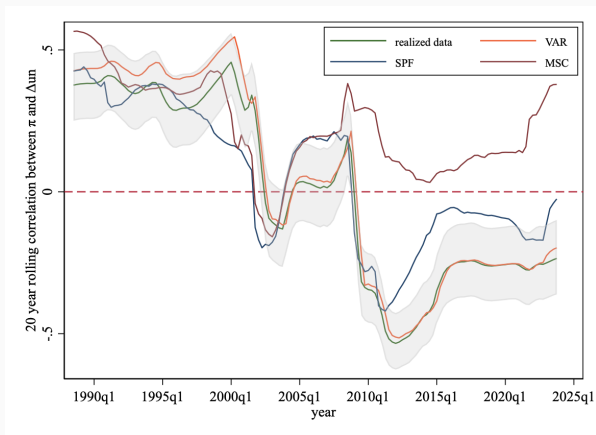
## $\pi_t$ and $\Delta U_t$ : Actual Data v.s. Expected



- Households perceive  $\text{corr}(\pi, u) > 0$ , different from data and professionals (Bhandari et al. (2025) and Candia et al. (2020))

Correlation using 20-year rolling window with 90% CI, 1969-2023. Data from FRED, SPF, and Michigan Survey of Consumers (MSC).

# $\pi_t$ and $\Delta U_t$ : Actual Data v.s. Expected



Correlation using 20-year rolling window with 90% CI, 1969-2023. Data from FRED, SPF, and Michigan Survey of Consumers (MSC).

- Households perceive  $\text{corr}(\pi, u) > 0$ , different from data and professionals (Bhandari et al. (2025) and Candia et al. (2020))
- A robust pattern in survey expectations
  - All the time. Cross-section
  - Across all groups. Group
  - Not due to individual or time fixed effects. Panel
- Growing literature with many explanations: **observational equivalence in data**.

Uses survey expectations to distinguish models generating the expectation comovement:

**incomplete information** vs. **subjective model**

## 1. Identification via the cross-predictability of forecast errors (fe)

- Key moment: the off-diagonals of the auto-correlation of fe
- **Incomplete information**: correlated signals  $\rightarrow$  symmetric forecast-error spillovers
- **Subjective model**: allows for asymmetric spillovers in forecast errors
- Seen in the **data**: one-way spillovers from  $\pi$  to  $u$

# This Paper

Uses survey expectations to distinguish models generating the expectation comovement:

**incomplete information** vs. **subjective model**

## 1. Identification via the cross-predictability of forecast errors (fe)

- Key moment: the off-diagonals of the auto-correlation of fe
- **Incomplete information**: correlated signals  $\rightarrow$  symmetric forecast-error spillovers
- **Subjective model**: allows for asymmetric spillovers in forecast errors
- Seen in the **data**: one-way spillovers from  $\pi$  to  $u$

## 2. Uncovering the subjective model

- Estimate an expectation–realization VAR
- **Reject** equality between subjective and objective models
- Admits different underlying mechanisms

# This Paper

Uses survey expectations to distinguish models generating the expectation comovement:

**incomplete information** vs. **subjective model**

## 1. Identification via the cross-predictability of forecast errors (fe)

- Key moment: the off-diagonals of the auto-correlation of fe
- **Incomplete information**: correlated signals  $\rightarrow$  symmetric forecast-error spillovers
- **Subjective model**: allows for asymmetric spillovers in forecast errors
- Seen in the **data**: one-way spillovers from  $\pi$  to  $u$

## 2. Uncovering the subjective model

- Estimate an expectation–realization VAR
- **Reject** equality between subjective and objective models
- Admits different underlying mechanisms

## 3. Implications for shock propagation

- Subjective beliefs alter textbook NK responses
- Supply shocks: amplified output responses
- Demand shocks: dampened output and price responses

# This Paper

Uses survey expectations to distinguish models generating the expectation comovement:

**incomplete information** vs. **subjective model**

## 1. Identification via the cross-predictability of forecast errors (fe)

- Key moment: the off-diagonals of the auto-correlation of fe
- **Incomplete information**: correlated signals  $\rightarrow$  symmetric forecast-error spillovers
- **Subjective model**: allows for asymmetric spillovers in forecast errors
- Seen in the **data**: one-way spillovers from  $\pi$  to  $u$

## 2. Uncovering the subjective model

- Estimate an expectation–realization VAR
- **Reject** equality between subjective and objective models
- Admits different underlying mechanisms

## 3. Implications for shock propagation

- Subjective beliefs alter textbook NK responses
- Supply shocks: amplified output responses
- Demand shocks: dampened output and price responses

## 4. Additional evidence based on surveyed news exposure and newspaper narratives

# A Formal Test of Joint Learning

---



- Building on the test of information rigidity in expectation formation (Coibion and Gorodnichenko, 2012; Andrade and Le Bihan, 2013)
  - A Noisy Information model with linear Gaussian noises (Lucas (1976); Woodford (2001))
  - Can be a result of Rational Inattention (Sims, 2003; Maćkowiak et al., 2018)
- Our extensions
  - **Univariate**→**multivariate** expectation formation (“Joint learning”)
    - Focus on bi-variate case:  $\pi$  and  $u$ .

# Modeling framework

- Building on the test of information rigidity in expectation formation (Coibion and Gorodnichenko, 2012; Andrade and Le Bihan, 2013)
  - A Noisy Information model with linear Gaussian noises (Lucas (1976); Woodford (2001))
  - Can be a result of Rational Inattention (Sims, 2003; Maćkowiak et al., 2018)
- Our extensions
  - **Univariate**→**multivariate** expectation formation (“Joint learning”)
    - Focus on bi-variate case:  $\pi$  and  $u$ .
  - **Perceived** law of motion  $\neq$  **actual** law of motion (“**Subjective model**”)
    - PLMs here are exogenous/fixed, no “adaptive” learning. (Milani, 2007; Andrade et al., 2016; Gáti, 2023; Hajdini et al., 2022)
    - PLMs here are “reduced-form”, directly on endogenous states instead of on the shock propagation mechanisms (Andre et al., 2022)

Actual data is generated by:

$$L_{t+1,t} = \underbrace{A}_{\text{Actual law of motion}} L_{t,t-1} + w_{t+1,t}$$
$$w_{t+1,t} \sim N(0, \underbrace{Q}_{\text{var-cov of innovations to the state}})$$

# Multivariate environment + **noisy information**

$$\begin{aligned} L_{t+1,t} &= \underbrace{A}_{\text{Actual law of motion}} L_{t,t-1} + w_{t+1,t} \\ w_{t+1,t} &\sim N(0, \underbrace{Q}_{\text{var-cov of innovations to the state}}) \end{aligned}$$

Agents observe a noisy signal

$$\begin{aligned} s_t &= \underbrace{G}_{\text{Signal mixture}} L_{t,t-1} + \eta_t \\ \eta_t &\sim N(0, \underbrace{R}_{\text{var-cov of noise on signal}}) \end{aligned}$$

# Multivariate environment + noisy information + subjective model

$$\begin{aligned} \mathbf{L}_{t+1,t} &= \underbrace{A}_{\text{Actual law of motion}} \mathbf{L}_{t,t-1} + \mathbf{w}_{t+1,t} \\ \mathbf{w}_{t+1,t} &\sim N(0, \underbrace{Q}_{\text{var-cov of innovations to the state}}) \\ \text{Noisy signal} \quad \underbrace{\mathbf{s}_t} &= \underbrace{G}_{\text{Signal mixture}} \mathbf{L}_{t,t-1} + \boldsymbol{\eta}_t \\ \boldsymbol{\eta}_t &\sim N(0, \underbrace{R}_{\text{var-cov of noise on signal}}) \end{aligned}$$

Agent may also **subjectively** believe that:

$$\begin{aligned} \mathbf{L}_{t+1,t} &= \underbrace{\hat{A}}_{\text{Perceived law of motion}} \mathbf{L}_{t,t-1} + \mathbf{w}_{t+1,t} \end{aligned}$$

# Expectations dynamics

Given the noisy signal and subjective model, expectation about  $L_{t+1,t}$  is

$$\underbrace{L_{t+1,t|t}}_{\equiv \mathbb{E}_t(L_{t+1,t})} = \underbrace{\hat{A}}_{\text{Perceived law of motion}} \left( (I - KG)L_{t,t-1|t-1} + \underbrace{K}_{\text{Kalman gain}} s_t \right)$$

Under **FIRE**:

- $\hat{A} = A$ ,  $R = \mathbf{0}$ ,  $KG = I$ .
- $\rightarrow$  Expectation

$$L_{t+1,t|t}^{FIRE} = AL_{t,t-1}$$

- $\rightarrow$  Correlation between  $L_{t+1,t|t}^{FIRE}$  is the same as the correlation of  $L_{t,t-1}$ . FIRE

## Different causes of $\text{Cov}(\pi, u) > 0$

$$\underbrace{L_{t+1,t|t}}_{\text{Expectation at } t \text{ regarding } t+1} = \hat{A}(I - KG)L_{t,t-1|t-1} + \hat{A}Ks_t$$
$$s_t = GL_{t,t-1} + \epsilon_t$$

The positive correlation between elements in  $L_{t+1,t|t}$  stems from various possibilities:

- **Information friction:**
  - Non-diagonal  $G$ : correlated signals (e.g., Kamdar (2019) through R.I.).
  - Non-diagonal  $R$ : correlated noises (e.g., pessimistic heuristics and sentiment).

## Different causes of $\text{Cov}(\pi, u) > 0$

$$\underbrace{L_{t+1,t|t}}_{\text{Expectation at } t \text{ regarding } t+1} = \hat{A}(I - KG)L_{t,t-1|t-1} + \hat{A}Ks_t$$
$$s_t = GL_{t,t-1} + \epsilon_t$$

The positive correlation between elements in  $L_{t+1,t|t}$  stems from various possibilities:

- **Information friction:**
  - Non-diagonal  $G$ : correlated signals (e.g., Kamdar (2019) through R.I.).
  - Non-diagonal  $R$ : correlated noises (e.g., pessimistic heuristics and sentiment).
- **Subjective model:**
  - Non-diagonal  $\hat{A}$ :  $\hat{A}$  has positive off-diagonal elements.



## Different causes of $\text{Cov}(\pi, u) > 0$

$$\underbrace{L_{t+1,t|t}}_{\text{Expectation at } t \text{ regarding } t+1} = \hat{A}(I - KG)L_{t,t-1|t-1} + \hat{A}Ks_t$$
$$s_t = GL_{t,t-1} + \epsilon_t$$

The positive correlation between elements in  $L_{t+1,t|t}$  stems from various possibilities:

- **Information friction:**
  - Non-diagonal  $G$ : correlated signals (e.g., Kamdar (2019) through R.I.).
  - Non-diagonal  $R$ : correlated noises (e.g., pessimistic heuristics and sentiment).
- **Subjective model:**
  - Non-diagonal  $\hat{A}$ :  $\hat{A}$  has positive off-diagonal elements.
- These assumptions are **observationally equivalent** in terms of  $\mathbb{E}$  correlation

## Serial correlations of forecast errors (FE)

Resolution: Forecast error tests (Coibion and Gorodnichenko, 2012) extended to multi-variate case:

$$\begin{aligned} FE_{t+1,t|t} &\equiv \mathbf{L}_{t+1,t} - \mathbf{L}_{t+1,t|t} \\ &= \hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG})FE_{t,t-1|t-1} + \underbrace{\mathbf{M}}_{(\mathbf{A} - \hat{\mathbf{A}}\mathbf{KG} - \hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG}))} \mathbf{L}_{t,t-1} + \mathbf{w}_{t+1,t} - \hat{\mathbf{A}}\mathbf{K}\epsilon_t \end{aligned}$$

# Serial correlations of forecast errors (FE)

Resolution: Forecast error tests (Coibion and Gorodnichenko, 2012) extended to multi-variate case:

$$\begin{aligned}
 FE_{t+1,t|t} &\equiv \mathbf{L}_{t+1,t} - \mathbf{L}_{t+1,t|t} \\
 &= \hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG})FE_{t,t-1|t-1} + \underbrace{\mathbf{M}}_{(\mathbf{A} - \hat{\mathbf{A}}\mathbf{KG} - \hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG}))} \mathbf{L}_{t,t-1} + \mathbf{w}_{t+1,t} - \hat{\mathbf{A}}\mathbf{K}\epsilon_t
 \end{aligned}$$

- **Diagonal terms** of  $\hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG})$ : auto-correlation (Coibion and Gorodnichenko, 2012)
  - Non-zero diagonals indicate information rigidity

# Serial correlations of forecast errors (FE)

Resolution: Forecast error tests (Coibion and Gorodnichenko, 2012) extended to multi-variate case:

$$\begin{aligned} FE_{t+1,t|t} &\equiv \mathbf{L}_{t+1,t} - \mathbf{L}_{t+1,t|t} \\ &= \hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG})FE_{t,t-1|t-1} + \underbrace{\mathbf{M}}_{(\mathbf{A} - \hat{\mathbf{A}}\mathbf{KG} - \hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG}))} \mathbf{L}_{t,t-1} + \mathbf{w}_{t+1,t} - \hat{\mathbf{A}}\mathbf{K}\epsilon_t \end{aligned}$$

- **Diagonal terms** of  $\hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG})$ : auto-correlation (Coibion and Gorodnichenko, 2012)
  - Non-zero diagonals indicate information rigidity
- **Off-diagonal terms**: between-correlation (our focus)
  - their signs depend on off-diagonals of  $\hat{\mathbf{A}}$  and  $\mathbf{G}$

# Serial correlations of forecast errors (FE)

Resolution: Forecast error tests (Coibion and Gorodnichenko, 2012) extended to multi-variate case:

$$\begin{aligned} FE_{t+1,t|t} &\equiv \mathbf{L}_{t+1,t} - \mathbf{L}_{t+1,t|t} \\ &= \hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG})FE_{t,t-1|t-1} + \underbrace{\mathbf{M}}_{(\mathbf{A} - \hat{\mathbf{A}}\mathbf{KG} - \hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG}))} \mathbf{L}_{t,t-1} + \mathbf{w}_{t+1,t} - \hat{\mathbf{A}}\mathbf{K}\epsilon_t \end{aligned}$$

- **Diagonal terms** of  $\hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG})$ : auto-correlation (Coibion and Gorodnichenko, 2012)
  - Non-zero diagonals indicate information rigidity
- **Off-diagonal terms**: between-correlation (our focus)
  - their signs depend on off-diagonals of  $\hat{\mathbf{A}}$  and  $\mathbf{G}$
- Special case of FIRE:  $\mathbf{A} = \hat{\mathbf{A}}$  and  $\mathbf{KG} = \mathbf{I} \rightarrow \hat{\mathbf{A}}(\mathbf{I} - \mathbf{KG}) = \mathbf{0}$

## Scenario 1: correlated signals, i.e. $G$ is non-diagonal

$$\begin{aligned}\hat{A}(I - KG) &= \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \begin{pmatrix} \frac{g_2^2 \sigma_2^2 + \sigma_s^2}{m} & -\frac{g_1 g_2 \sigma_1^2}{m} \\ -\frac{g_1 g_2 \sigma_2^2}{m} & \frac{g_1^2 \sigma_1^2 + \sigma_s^2}{m} \end{pmatrix} \\ &= \begin{pmatrix} \rho_1 \frac{g_2^2 \sigma_2^2 + \sigma_s^2}{m} & -\rho_1 \frac{g_1 g_2 \sigma_1^2}{m} \\ -\rho_2 \frac{g_1 g_2 \sigma_2^2}{m} & \rho_2 \frac{g_1^2 \sigma_1^2 + \sigma_s^2}{m} \end{pmatrix}\end{aligned}$$

- $m = g_1^2 \sigma_1^2 + g_2^2 \sigma_2^2 + \sigma_s^2$
- $G = [g_1, g_2]$ : the vector of signals (due to “optimal signal selection”)
- **Symmetric signs** of off-diagonal terms
- When signals go in the same direction,  $g_1 g_2 > 0$ , the **cross terms** are **negative**

## Scenario 2: subjective model

$$\begin{aligned}\hat{A}(I - KG) &= \begin{pmatrix} \rho_1 & m_1 \\ m_2 & \rho_2 \end{pmatrix} \times \begin{pmatrix} \frac{\sigma_{1,s}^2}{\sigma_1^2 + \sigma_{1,s}^2} & 0 \\ 0 & \frac{\sigma_{2,s}^2}{\sigma_2^2 + \sigma_{2,s}^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\sigma_{1,s}^2 \rho_1}{\sigma_1^2 + \sigma_{1,s}^2} & \frac{\sigma_{2,s}^2 m_1}{\sigma_2^2 + \sigma_{2,s}^2} \\ \frac{\sigma_{1,s}^2 m_2}{\sigma_1^2 + \sigma_{1,s}^2} & \frac{\sigma_{2,s}^2 \rho_2}{\sigma_2^2 + \sigma_{2,s}^2} \end{pmatrix} \quad (1)\end{aligned}$$

- $G = I_2$ : no signal correlation (can be any diagonal matrix)
- The signs of **cross terms** (the between-variable serial correlation of FEs) are the same as the **perceived correlation**, and **can be different**

## Joint-learning tests for $\pi$ and $un$

$$\begin{pmatrix} fe_{t+1,t|t}^{\pi} \\ fe_{t+1,t|t}^{un} \end{pmatrix} = \beta_0 + \underbrace{\begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}}_{\equiv \hat{A}(I-KG)} \begin{pmatrix} fe_{t,t-1|t-1}^{\pi} \\ fe_{t,t-1|t-1}^{un} \end{pmatrix} + \theta X_{t,t-1} + e_t \quad (2)$$

- $\beta_{12}$  and  $\beta_{21}$ : between-variable serial correlations of forecast errors
- **Prediction:** if only correlated signals,  $\beta_{12}$  and  $\beta_{21}$  are both negative.
- Two complications:
  - In MSC (and most household surveys),  $\mathbb{E}u$  is qualitative. We impute them following Bhandari et al. (2025). Imputation
  - Expectations are year-ahead measures, so we derive a year-ahead version of (2). Year-ahead test



# Joint-learning tests with consensus expectations

- Joint learning: **subjective model suggesting**  $\pi \rightarrow un$ .
- Bottom line: **information friction cannot be the only reason.**

**Table 1:** Aggregate Test on Joint Learning, MSC v.s. SPF

	MSC		SPF	
	1984-2023 (1)	1990-2018 (2)	1984-2023 (3)	1990-2018 (4)
$\beta_{11}$	0.64*** (0.080)	0.65*** (0.085)	0.79*** (0.064)	0.76*** (0.093)
$\beta_{12}$	-0.11 (0.076)	-0.02 (0.095)	0.19 (0.117)	-0.08 (0.199)
$\beta_{21}$	<b>0.13***</b> (0.033)	<b>0.21***</b> (0.063)	0.05 (0.034)	0.06 (0.049)
$\beta_{22}$	0.71*** (0.044)	0.50*** (0.092)	0.63*** (0.060)	0.51*** (0.097)
Observations	152	116	152	116

\* The first and third columns are using the full sample 1984-2023; the second and fourth columns are results for the sub-sample 1990-2018. Newey-West standard errors are reported in brackets.

# Uncovering the Subjective Model

---

# Uncovering the **subjective model** using an expectation-realization VAR

- Previously, we used sign restrictions to test whether  $\hat{A}$  is diagonal, relying on some additional assumptions
- Here, we directly uncover  $\hat{A}$

$$\underbrace{Y_{t+1}}_{\equiv \begin{pmatrix} L_{t,t-1|t-1} \\ L_{t,t-1} \end{pmatrix}} = \underbrace{\begin{pmatrix} \hat{A}(I - KG) & \hat{A}KG \\ \mathbf{0}_{2 \times 2} & A \end{pmatrix}}_{:=\Phi} \cdot Y_t + \underbrace{\begin{pmatrix} \hat{A}K & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & I_{2 \times 2} \end{pmatrix}}_F \cdot \begin{pmatrix} \eta_t \\ w_{t+1,t} \end{pmatrix} \quad (3)$$

- $\Phi$  estimated from an (un)restricted VAR
- $\hat{A} = \Phi_{1,1} + \Phi_{1,2}$  and  $\Phi_{2,2} = A$
- Identification does not rely on  $K$  and  $G$
- Then test  $\hat{A} = A$  elementwise

# The uncovered subjective model of households and professionals

**Table 2:** Estimates of Joint Learning Model (3)

Parameters	MSC, quarterly, Q1 1984 - Q4 2023				SPF, quarterly, Q1 1984 - Q4 2023			
	Estimates		Standard Errors		Estimates		Standard Errors	
$A$	0.836 0.034	-0.058 0.617	0.053 0.042	0.057 0.095	0.837 0.014	-0.056 0.751	0.061 0.041	0.074 0.093
$\hat{A}$	0.741 0.137	-0.149 0.831	0.050 0.044	0.082 0.048	0.955 0.040	-0.038 0.495	0.019 0.035	0.016 0.239
T-test: $\hat{A}_{21} > A_{21}$	test-stat 1.581		p-val 0.057		test-stat 0.546		p-val 0.293	

The table reports the estimates and their NW standard errors from the GMM estimation of the 4-variable VAR model. Iterative weighting matrices are used in the GMM estimation.

# Interpretation of the subjective model through an NK structure

- We uncover the subjective model in reduced-form
- But we can interpret such a PLM through a structural lens

NK model

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\kappa}{\chi} u_t + s_t$$

$$u_t = E_t u_{t+1} + \frac{\chi}{\sigma} (i_t - E_t \pi_{t+1}) - \chi d_t$$

$$i_t = \phi_\pi \pi_t - \frac{\phi_y}{\chi} y_t$$

# Interpretation of the subjective model through an NK structure

- We uncover the subjective model in reduced-form
- But we can interpret such a PLM through a structural lens

NK model

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\kappa}{\chi} u_t + s_t$$

$$u_t = E_t u_{t+1} + \frac{\chi}{\sigma} (i_t - E_t \pi_{t+1}) - \chi d_t$$

$$i_t = \phi_\pi \pi_t - \frac{\phi_y}{\chi} y_t$$

Structural shocks

$$\begin{pmatrix} d_t \\ s_t \end{pmatrix} = \underbrace{\begin{pmatrix} \rho_d & 0 \\ 0 & \rho_s \end{pmatrix}}_{\equiv \Gamma} \begin{pmatrix} d_{t-1} \\ s_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^d \\ \varepsilon_t^s \end{pmatrix}$$

# Interpretation of the subjective model through an NK structure

- We uncover the subjective model in reduced-form
- But we can interpret such a PLM through a structural lens

NK model

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} - \frac{\kappa}{\chi} u_t + s_t \\ u_t &= E_t u_{t+1} + \frac{\chi}{\sigma} (i_t - E_t \pi_{t+1}) - \chi d_t \\ i_t &= \phi_\pi \pi_t - \frac{\phi_y}{\chi} y_t\end{aligned}$$

Structural shocks

$$\begin{pmatrix} d_t \\ s_t \end{pmatrix} = \underbrace{\begin{pmatrix} \rho_d & 0 \\ 0 & \rho_s \end{pmatrix}}_{\equiv \Gamma} \begin{pmatrix} d_{t-1} \\ s_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^d \\ \varepsilon_t^s \end{pmatrix}$$

Impact matrix

$$L_{t,t-1} = \underbrace{\begin{pmatrix} \Psi_{\pi d} & \Psi_{\pi s} \\ \Psi_{ud} & \Psi_{us} \end{pmatrix}}_{\equiv \Psi} \begin{pmatrix} d_t \\ s_t \end{pmatrix}$$

# Interpretation of the subjective model through an NK structure

- We uncover the subjective model in reduced-form
- But we can interpret such a PLM through a structural lens

NK model

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} - \frac{\kappa}{\chi} u_t + s_t \\ u_t &= E_t u_{t+1} + \frac{\chi}{\sigma} (i_t - E_t \pi_{t+1}) - \chi d_t \\ i_t &= \phi_\pi \pi_t - \frac{\phi_y}{\chi} y_t\end{aligned}$$

Structural shocks

$$\begin{pmatrix} d_t \\ s_t \end{pmatrix} = \underbrace{\begin{pmatrix} \rho_d & 0 \\ 0 & \rho_s \end{pmatrix}}_{\equiv \Gamma} \begin{pmatrix} d_{t-1} \\ s_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^d \\ \varepsilon_t^s \end{pmatrix}$$

Impact matrix

$$L_{t,t-1} = \underbrace{\begin{pmatrix} \Psi_{\pi d} & \Psi_{\pi s} \\ \Psi_{ud} & \Psi_{us} \end{pmatrix}}_{\equiv \Psi} \begin{pmatrix} d_t \\ s_t \end{pmatrix}$$

Actual law of motion

$$L_{t+1,t} = \underbrace{\Psi \Gamma \Psi^{-1}}_{\equiv A} L_{t,t-1} + w_{t+1,t}$$



# Interpretation of the subjective model through an NK structure

- We uncover the subjective model in reduced-form
- But we can interpret such a PLM through a structural lens

NK model

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} - \frac{\kappa}{\chi} u_t + s_t \\ u_t &= E_t u_{t+1} + \frac{\chi}{\sigma} (i_t - E_t \pi_{t+1}) - \chi d_t \\ i_t &= \phi_\pi \pi_t - \frac{\phi_y}{\chi} y_t\end{aligned}$$

Structural shocks

$$\begin{pmatrix} d_t \\ s_t \end{pmatrix} = \underbrace{\begin{pmatrix} \rho_d & 0 \\ 0 & \rho_s \end{pmatrix}}_{\equiv \Gamma} \begin{pmatrix} d_{t-1} \\ s_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^d \\ \varepsilon_t^s \end{pmatrix}$$

Impact matrix

$$L_{t,t-1} = \underbrace{\begin{pmatrix} \Psi_{\pi d} & \Psi_{\pi s} \\ \Psi_{ud} & \Psi_{us} \end{pmatrix}}_{\equiv \Psi} \begin{pmatrix} d_t \\ s_t \end{pmatrix}$$

Actual law of motion

$$L_{t+1,t} = \underbrace{\Psi \Gamma \Psi^{-1}}_{\equiv A} L_{t,t-1} + w_{t+1,t}$$

Perceived law of motion

$$L_{t+1,t} = \underbrace{\hat{\Psi} \hat{\Gamma} \hat{\Psi}^{-1}}_{\equiv \hat{A}} L_{t,t-1} + w_{t+1,t}$$

# Possible mechanisms of a subjective $\hat{A}$

$$\hat{A} \equiv \hat{\Psi} \hat{\Gamma} \hat{\Psi}^{-1} = \frac{1}{\hat{\Psi}_{\pi d} \hat{\Psi}_{us} - \hat{\Psi}_{\pi s} \hat{\Psi}_{ud}} \begin{bmatrix} \hat{\rho}_d \hat{\Psi}_{\pi d} \hat{\Psi}_{us} - \hat{\rho}_s \hat{\Psi}_{\pi s} \hat{\Psi}_{ud} & (\hat{\rho}_s - \hat{\rho}_d) \hat{\Psi}_{\pi d} \hat{\Psi}_{\pi s} \\ (\hat{\rho}_d - \hat{\rho}_s) \hat{\Psi}_{ud} \hat{\Psi}_{us} & \hat{\rho}_s \hat{\Psi}_{\pi d} \hat{\Psi}_{us} - \hat{\rho}_d \hat{\Psi}_{\pi s} \hat{\Psi}_{ud} \end{bmatrix}$$

- Suppose HH understands demand and supply shocks push unemployment rate in opposite directions,  $\hat{A}_{2,1}$  is positive when  $\hat{\rho}_s > \hat{\rho}_d$
- $\hat{A}_{2,1} > A_{2,1}$  when  $\hat{\phi}_\pi > \phi_\pi$ , i.e., HH perceives strong MP reaction to inflation
- $\hat{A}_{1,2} = 0$  possibly because  $\hat{\kappa} = 0$ , i.e., subjectively perceive a flat Phillips Curve

Bottom line: different micro-founded mechanisms yield observationally equivalent  $\hat{A}$

# Shock propagation in a textbook NK model

## 3-Equation NK

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + s_t$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \pi_{t+1} - \rho) + d_t$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t$$

## Okun's Law

$$u_t = -\chi y_t$$

## Expectation formation

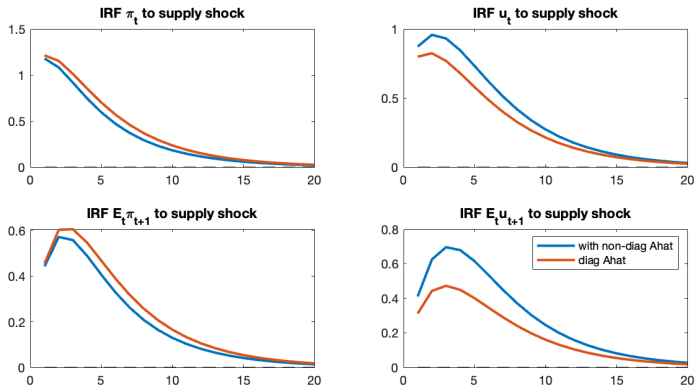
$$L_{t+1,t|t} = \hat{A}(I - KG)L_{t,t-1|t-1} + \hat{A}KGL_{t,t-1} + \hat{A}K\eta_t$$

$$L_{t+1,t|t} \equiv \begin{pmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t u_{t+1} \end{pmatrix}$$

$$L_{t,t-1} \equiv \begin{pmatrix} \pi_t \\ u_t \end{pmatrix}$$

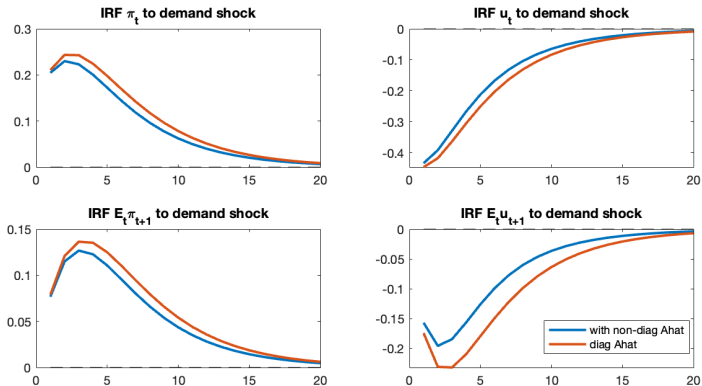
# Subjective model amplifies labor market responses to a supply shock

Figure 1: IRF in Response to Supply Shock



# Subjective model dampens price and labor market responses to demand shock

Figure 2: IRF in Response to Demand Shock



## **Additional Evidence**

---

# Expectations conditional on the type of news heard

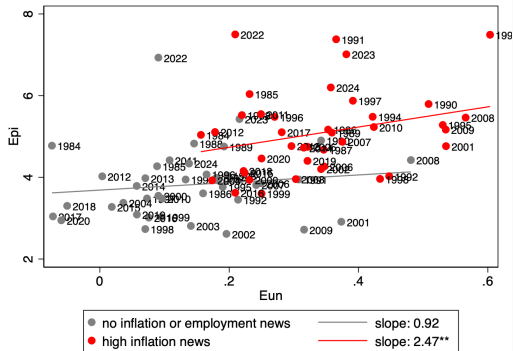
**Table 3:** Panel Regression with Self-reported News

Expectation on: news on:	$E\pi$ (1)	$Eun$ (2)	$E\pi$ (3)	$Eun$ (4)
Inflation fav	-0.21* (0.117)	-0.06*** (0.017)	-0.21* (0.118)	-0.05*** (0.017)
Inflation unfav	0.43*** (0.085)	0.06*** (0.010)	0.42*** (0.085)	0.05*** (0.010)
Employment fav	-0.03 (0.056)	-0.14*** (0.009)	-0.01 (0.057)	-0.13*** (0.009)
Employment unfav	0.05 (0.054)	0.10*** (0.007)	0.04 (0.054)	0.09*** (0.007)
Interest rate fav	-0.03 (0.071)	-0.06*** (0.012)	-0.01 (0.072)	-0.04*** (0.012)
Interest rate unfav	0.02	0.11***	0.02	0.10***
Observations	169304	189158	169304	189158
$R^2$	0.673	0.677	0.673	0.681
Time F.E.	Y	Y	Y	Y
Individual F.E.	Y	Y	Y	Y
Full set news control	N	N	Y	Y

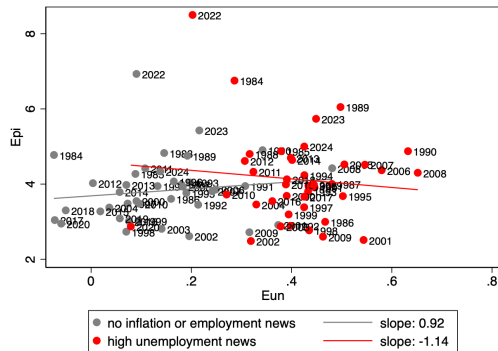
# Consensus expectations conditional on the news exposure

Figure 3: Consensus expectations conditional on news heard

(a) With unfavorable inflation news



(b) With unfavorable employment news



Notes: Scatter plot for consensus expected inflation and unemployment each year from 1984–2023. Gray dots in all panels are expectations for individuals without employment or inflation news. Left panel: red dots are expectations conditional on hearing high inflation news. Right panel: red dots are expectations conditional on hearing high unemployment news.



# Inflation-unemployment associations in newspapers

- $P(\mathbb{I}_{i,t}(\text{Joint mention}) = 1) = \Phi(\beta_0 + \sum_k \beta_k D_{k,l,t} + \beta_\pi \pi_t + \beta_u u_t)$
- Probability of news report making an association between inflation and unemployment increases significantly with realized inflation.

	(1)	(2)	(3)
economy	1.07*** (0.03)	1.07*** (0.03)	1.07*** (0.03)
fed	0.22*** (0.03)	0.21*** (0.03)	0.21*** (0.03)
growth	0.60*** (0.03)	0.61*** (0.03)	0.61*** (0.03)
oil price	0.24*** (0.05)	0.24*** (0.05)	0.24*** (0.05)
recession	0.48*** (0.03)	0.47*** (0.03)	0.47*** (0.03)
uncertainty	0.14*** (0.05)	0.15*** (0.05)	0.15*** (0.05)
$\pi_t$		<b>3.73***</b> (0.93)	<b>3.62***</b> (0.96)
$u_t$	-0.01 (0.01)		-0.00 (0.01)
N	150465	150465	150465

# Conclusion

- We highlight the importance of modeling  $\mathbb{E}s$  across variables
  - e.g., HHs always believe inflation and unemployment rates rise together, unlike actual data
- We use the cross-variable predictability of forecast errors to show
  - $\rightarrow$  correlated expectation stems from a **subjective model** instead of **incomplete information**
- We also **directly estimate** the perceived law of motion from the survey
  - $\rightarrow$  statistical tests reject the equality of **PLM** and **ALM**
- We show the uncovered subjective model can be used to discipline  $\mathbb{E}s$  in standard models
  - $\rightarrow$  the stagflation beliefs change shock propagation: amplified output responses to supply shocks and dampened output and price responses to demand shocks.

# Appendix

---

# Cross-correlation: MSC

**Table 4:** Correlation MCS: more variables

	(1)	(2)	(3)	(4)	(5)
(1) inflation ( $E\pi_{t+4,t}$ )	1.00	0.31***	-0.13	-0.43***	-0.51***
(2) unemp change ( $E\Delta un_{t+4,t}$ )		1.00	-0.41***	-0.64***	-0.28***
(3) interest rate change ( $E\Delta i_{t+4,t}$ )			1.00	0.40***	0.07*
(4) Busi Condition change ( $E\Delta y_{t+4,t}$ )				1.00	0.77***
(5) real income change ( $E\Delta w_{t+4,t}$ )					1.00

\* \*\*\* means significant at 1%, \*\* means 5 % and \* means 10%, data in use are quarterly 1978q1-2018q4 from MSC.

## Cross-correlation: FRED

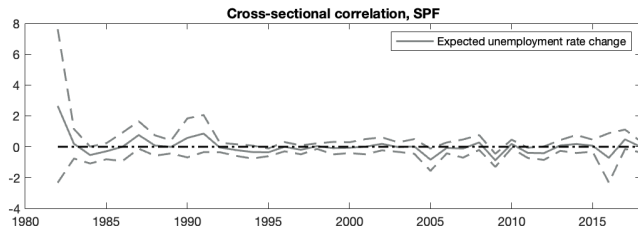
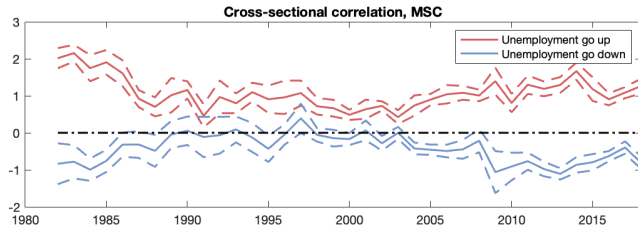
**Table 5:** Correlation FRED: more variables

	(1)	(2)	(3)	(4)	(5)
(1) CPI	1.00	0.11	0.38***	-0.03	-0.32***
(2) $\Delta un$		1.00	-0.52***	-0.79***	-0.77***
(3) $\Delta FFR$			1.00	0.43***	0.26***
(4) $\Delta RGDP$				1.00	0.79***
(5) $\Delta w$					1.00

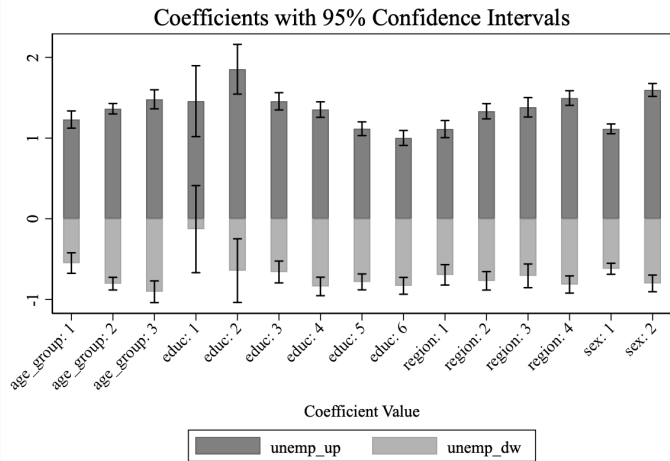
\* \*\*\* means significant at 1%, \*\* means 5 % and \* means 10%, data in use are quarterly 1978q1-2018q4 from FRED.

# Time variations of the perceived correlation in consensus expectations

Estimate  $E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 E_{i,t}un_{t+12,t} + \theta X_{i,t} + \epsilon_{i,t}$ . Parameter of interest is  $\beta_1$ : [Back](#)



# Regression by group



Cross-sectional correlation across groups

## Controlling for individual FE and time FE

$$E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 E_{i,t}un_{t+12,t} + \beta_2 E_{i,t}i_{t+12,t} + \theta X_{i,t} + D_t + \mu_i + \epsilon_{i,t}$$

**Table 6:** FE Panel Regression

	MSC		SCE		SPF
Unemployment up	0.30*** (0.05)	$\hat{\beta}_1$	0.012*** (0.002)	$\hat{\beta}_1$	-0.17*** (0.06)
Unemployment down	-0.22*** (0.05)				
FE	Y		Y		Y
Time dummy	Y		Y		Y

\* Controlling for individual and time-varying characteristics, individual fixed effect, and time-fixed effect. Standard errors are adjusted for heteroscedasticity and autocorrelation.



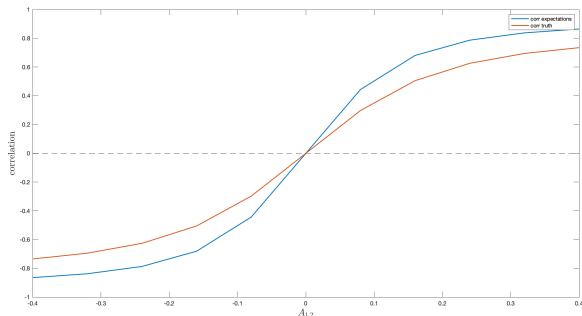
# FIRE

## Correlation of Expectations under FIRE:

- For our case  $\mathbf{L} = \begin{pmatrix} \pi_t \\ un_t \end{pmatrix}$  Empirical estimates of  $A$  from 1984-2023:

$$\begin{bmatrix} 0.87 & -0.05 \\ (0.05) & (0.06) \\ 0.02 & 0.67 \\ (0.06) & (0.11) \end{bmatrix}$$

- NW s.e. in brackets and BIC select 1 lag of VAR.



Correlation between expected variables and realized variables when  $A(1,2)$  changes under FIRE

# Joint Learning Test

When it's not FIRE, maintain a simplification restriction:

## Assumption 1

*The variance-covariance matrix of prior  $\mathbf{L}_{t,t-1|t-1}^i$  is diagonal and common to each individual:*

$$\Sigma := \text{diag}(\{\sigma_j^2\})$$

Consider different scenarios afore-mentioned:

1. When  $\hat{A}$  is diagonal, consider different  $G$  and  $R$ : **Independent learning**.
2. When  $\hat{A}$  non-diagonal: **Joint learning**.
  - When  $R$  and  $G$  are diagonal.

## Proposition 1

**(Independent Learning)** If  $\hat{A} = \text{diag}(\{a_i\}_{i=1}^n)$ , denote the off-diagonal elements of  $\hat{A}(I - KG)$  as  $\beta_{ij}$  with  $i \neq j$ . We have:

- (1)  $\beta_{ij} = 0$  if  $G$  and  $R$  are diagonal.
- (2)  $\beta_{ij} = \beta_{ji} = 0$  or  $\beta_{ij}\beta_{ji} > 0$  if  $G$  or  $R$  is non-diagonal.

- Our test coefficient (2-d case):

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}$$

## Proposition 1

**(Independent Learning)** If  $\hat{A} = \text{diag}(\{a_i\}_{i=1}^n)$ , denote the off-diagonal elements of  $\hat{A}(I - KG)$  as  $\beta_{ij}$  with  $i \neq j$ . We have:

- (1)  $\beta_{ij} = 0$  if  $G$  and  $R$  are diagonal.
- (2)  $\beta_{ij} = \beta_{ji} = 0$  or  $\beta_{ij}\beta_{ji} > 0$  if  $G$  or  $R$  is non-diagonal.

- Independent learning + separated signals (Coibion and Gorodnichenko, 2012; Andrade and Le Bihan, 2013): **zero between-correlation**, **non-zero auto-correlation** of F.E.

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & \beta_{12} = 0 \\ \beta_{21} = 0 & \beta_{22} \neq 0 \end{pmatrix}$$

## Proposition 1

**(Independent Learning)** If  $\hat{A} = \text{diag}(\{a_i\}_{i=1}^n)$ , denote the off-diagonal elements of  $\hat{A}(I - KG)$  as  $\beta_{ij}$  with  $i \neq j$ . We have:

- (1)  $\beta_{ij} = 0$  if  $G$  and  $R$  are diagonal.
- (2)  $\beta_{ij} = \beta_{ji} = 0$  or  $\beta_{ij}\beta_{ji} > 0$  if  $G$  or  $R$  is non-diagonal.

- Independent learning + mixed signals (R.I. like in Kamdar (2019)): **same signs on between-correlation** of F.E.

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & \beta_{12} <> 0 \\ \beta_{21} <> 0 & \beta_{22} \neq 0 \end{pmatrix}$$

## Joint Learning Test II: non-diagonal $R$

### Corollary 1

**(Non-diagonal  $R$ : correlated noises)** If  $\hat{A}$  and  $G$  are diagonal and  $R = \begin{pmatrix} \sigma_{1,s}^2 & \rho \\ \rho & \sigma_{2,s}^2 \end{pmatrix}$ , the off-diagonal elements of  $\hat{A}(I - KG)$  have the **same** signs as  $\rho$ .

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & \text{sgn}(\beta_{12}) = \text{sgn}(\rho) \\ \text{sgn}(\beta_{21}) = \text{sgn}(\rho) & \beta_{22} \neq 0 \end{pmatrix}$$

Illustration

Back

## Joint Learning Test II: non-diagonal $G$

### Corollary 2

**(Non-diagonal  $G$ : correlated signals)** If  $\hat{A}$  is diagonal,  $R = \begin{pmatrix} \sigma_{1,s}^2 & 0 \\ 0 & \sigma_{2,s}^2 \end{pmatrix}$  is diagonal, and  $G = \begin{pmatrix} g_1 & g_2 \\ 0 & g_4 \end{pmatrix}$ , the off-diagonal elements of  $\hat{A}(I - KG)$  have the *opposite* signs as  $g_1g_2$ .

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & \text{sgn}(\beta_{12}) = -\text{sgn}(g_1g_2) \\ \text{sgn}(\beta_{21}) = -\text{sgn}(g_1g_2) & \beta_{22} \neq 0 \end{pmatrix}$$

Illustration

Back

# Joint Learning Test III

## Proposition 2

**(Joint Learning)** If off-diagonal elements of  $\hat{A}(I - KG)$  are not both zeros and of different signs, then  $\hat{A}$  is non-diagonal, regardless whether  $G$  and  $R$  are diagonal or not.

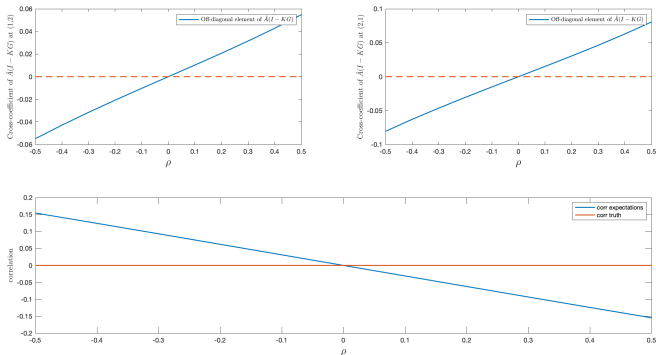
## Proposition 3

**(Joint Learning with separate signals)** If both  $G$  and  $R$  are diagonal and  $\hat{A} = (a_{ij})_{n \times n}$  is non-diagonal, denote  $\hat{A}(I - KG) = (\beta_{ij})_{n \times n}$ . The signs of these off-diagonal elements are the same as their counterparts in  $\hat{A}$ , i.e.  $\beta_{ij}a_{ij} > 0$ .

$$\hat{A}(I - KG) = \begin{pmatrix} \beta_{11} \neq 0 & \text{sgn}(\beta_{12}) = \text{sgn}(a_{12}) \\ \text{sgn}(\beta_{21}) = \text{sgn}(a_{21}) & \beta_{22} \neq 0 \end{pmatrix}$$



# Example I: non-diagonal $R$

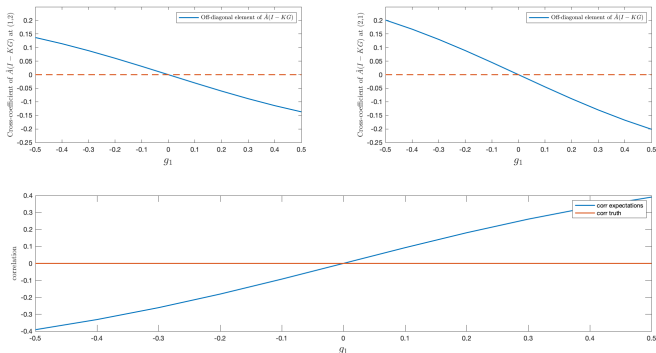


Off-diagonal elements of  $\hat{A}(I - KG)$  and correlation between expectations.

- A working example for illustration:

$$A = \hat{A} = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 7/4 & 0 \\ 0 & 2 \end{bmatrix}, \quad R = \begin{bmatrix} 1.5 & \rho \\ \rho & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1/3 \end{bmatrix},$$

## Example II: non-diagonal $G$

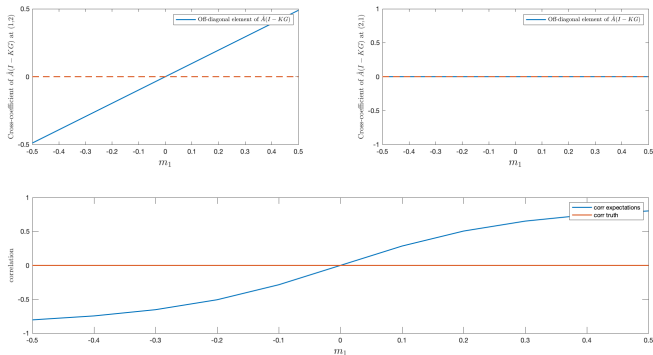


Off-diagonal elements of  $\hat{A}(I - KG)$  and correlation between expectations.

- Same working example for illustration:

$$A = \hat{A} = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 7/4 & 0 \\ 0 & 2 \end{bmatrix}, \quad R = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & g_1 \\ 0 & 1/3 \end{bmatrix},$$

## Example III: non-diagonal $\hat{A}$ and diagonal $R$ and $G$



Off-diagonal elements of  $\hat{A}(I - KG)$  and correlation between expectations.

- Same working example as before except:

$$\hat{A} = \begin{bmatrix} 0.7 & m_1 \\ 0 & 0.9 \end{bmatrix}$$

## Complication I: impute $E_{un}$

### Assumption 2

*At each period  $t$ , survey respondent  $i$  forms a belief  $x_{i,t}$  that indicates the change of asked variable  $x$ , this belief follows a normal distribution:*

$$x_{i,t} \sim N(\mu_t, \sigma_t^2)$$

The survey respondent will respond in categorical fashion:

$$category_{i,t} = \begin{cases} \text{increase} & x_{it} > b + a \\ \text{decrease} & x_{it} < b - a \\ \text{same} & x_{it} \in [-a + b, b + a] \end{cases}$$

## Complication I: impute $E\pi_t$

We want to recover  $\mu_t$ , we can observe fraction of people responding “increase” ( $f_t^u$ ) and “decreasing” ( $f_t^d$ ). From normality:

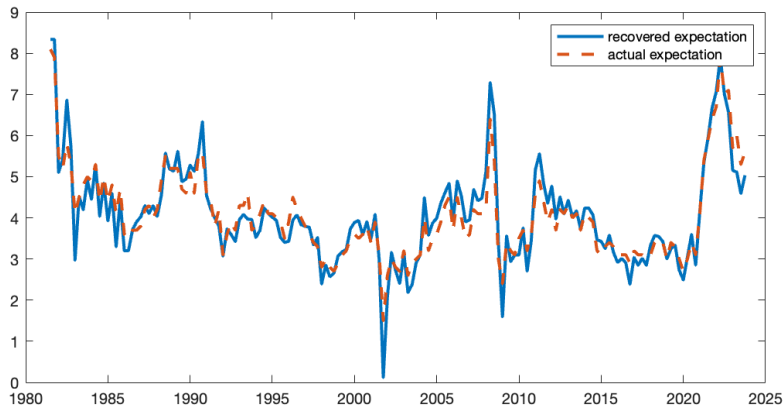
$$\sigma_t = \frac{2a}{\Phi^{-1}(1 - f_t^u) - \Phi^{-1}(f_t^d)} \quad (4)$$

$$\mu_t = a + b - \sigma_t \Phi^{-1}(1 - f_t^u) \quad (5)$$

- Get  $a$  and  $b$  using average  $\sigma_t$  and  $\mu_t$  approximated by SPF. (Bhandari et al., 2025)
- Can test with  $E\pi_t$  data.

## Complication 1: impute $E_{un}$

Figure 4: Recovered Expected Inflation vs. Actual



Correlation between imputed and actual: 0.99. [Back](#)

## Complication II: Year ahead Expectation

- The baseline test is derived with quarter-to-quarter changes, whereas data on expectations are year-ahead. We can iterate the forecasting error equation forward:

$$\begin{aligned} FE_{t+4,t|t} = & \hat{W}\hat{A}(I - KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (I - \hat{W}\hat{A}(I - KG)\hat{W}^{-1})\mathbf{L}_{t+3,t-1} \\ & - (I + \hat{W}\hat{A}KG)\mathbf{L}_{t,t-1} + A\mathbf{L}_{t+3,t+2} - \hat{W}\hat{A}K\eta_t + w_{t+4,t+3} \end{aligned} \quad (6)$$

- With  $\hat{W} = \hat{A}^3 + \hat{A}^2 + \hat{A} + I$ .
- The test results hold as the quarterly specification. Shown with Monte Carlo.

## Complication II: Year ahead test

Table 13: Simulation Results: FIRE or Independent Learning with Uncorrelated Signals

FIRE or Independent Learning: $\hat{A} = A, g_2 = 0, \rho = 0$								
	FIRE				Independent Learning			
	Y-ahead Spec (10)		Q-ahead Spec (6)		Y-ahead Spec (10)		Q-ahead Spec (6)	
	Truth	Test	Truth	Test	Truth	Test	Truth	Test
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\beta_{11}$	0	-0.01	0	0.04	0.54	0.51***	0.54	0.47***
	-	(0.03)	-	(0.09)	-	(0.09)	-	(0.09)
$\beta_{12}$	0	0.03	0	0.15	0	-0.14	0	-0.14
	-	(0.04)	-	(0.11)	-	(0.010)	-	(0.10)
$\beta_{21}$	0	0.01	0	0.10	0	-0.03	0	-0.09
	-	(0.02)	-	(0.09)	-	(0.04)	-	(0.11)
$\beta_{22}$	0	-0.00	0	0.18	0.43	0.49***	0.43	0.61***
	-	(0.05)	-	(0.12)	-	(0.07)	-	(0.11)

\* \*\*\*, \*\*, \*: Significance at 1%, 5% and 10% level. Columns (2) and (6) are estimation results for year-ahead joint-learning test (10), and columns (4) and (8) are for quarter-ahead specification (6). Newey-West standard errors are reported in brackets.



## Complication II: Year ahead test

Table 14: Simulation Results: Independent Learning with Correlated Signals

Independent Learning when $G$ or $R$ are non-diagonal								
	G non-diagonal:				R non-diagonal:			
	$m_1 = 0, g_2 = 0.5, \rho = 0$				$m_1 = 0, g_2 = 0, \rho = -2$			
	Y-ahead spec (10)		Q-ahead spec (6)		Y-ahead spec (10)		Q-ahead spec (6)	
	Truth	Test	Truth	Test	Truth	Test	Truth	Test
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\beta_{11}$	0.57	0.56***	0.57	0.52***	0.49	0.43***	0.49	0.37***
	–	(0.05)	–	(0.08)	–	(0.05)	–	(0.09)
$\beta_{12}$	-0.14	-0.28***	-0.10	-0.26***	-0.17	-0.25***	-0.13	-0.24***
	–	(0.09)	–	(0.10)	–	(0.09)	–	(0.09)
$\beta_{21}$	-0.07	-0.10***	-0.10	-0.20**	-0.09	-0.11***	-0.12	-0.17
	–	(0.04)	–	(0.10)	–	(0.04)	–	(0.11)
$\beta_{22}$	0.40	0.46***	0.40	0.55***	0.39	0.49***	0.39	0.63***
	–	(0.07)	–	(0.11)	–	(0.07)	–	(0.11)

## Complication II: Year ahead test

Table 15: Simulation Results: Joint Learning

Joint Learning: $m_1 = 0.5$ , $G$ and $R$ are diagonal				
	Year-ahead spec (10)		Quarter-ahead spec (6)	
	Truth	Test	Truth	Test
	(1)	(2)	(3)	(4)
$\beta_{11}$	0.54	0.48***	0.54	0.44***
	-	(0.08)	-	(0.08)
$\beta_{12}$	0.32	0.49**	0.31	0.35***
	-	(0.22)	-	(0.10)
$\beta_{21}$	0	-0.02	0	-0.08
	-	(0.04)	-	(0.09)
$\beta_{22}$	0.43	0.54***	0.43	0.70***
	-	(0.12)	-	(0.14)

\* \*\*\*, \*\*, \*: Significance at 1%, 5% and 10% level. Column (2)

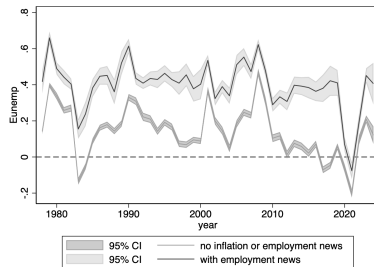
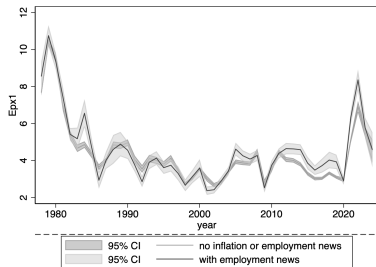
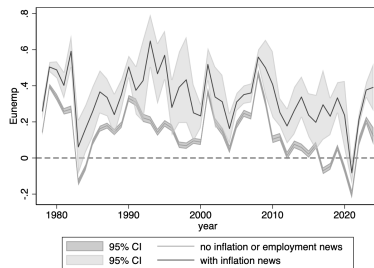
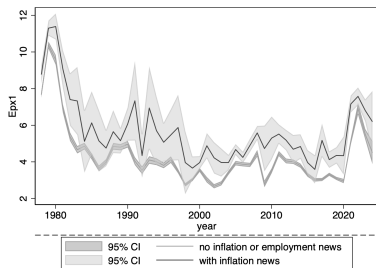
# Joint Estimation: Alternative Sample

MSC, quarterly									
Q1 1984 - Q4 2019					Q1 1990 - Q4 2018				
Parameters	Estimates		Standard Errors		Estimates		Standard Errors		
$A$	0.807	-0.070	0.059	0.114	0.781	-0.060	0.068	0.145	
	0.062	0.922	0.022	0.072	0.059	0.930	0.031	0.082	
$\hat{A}$	0.663	-0.096	0.063	0.089	0.663	-0.081	0.080	0.094	
	0.189	0.807	0.057	0.056	0.271	0.769	0.064	0.057	
T-test: $\hat{A}_{21} > A_{21}$	test-stat 2.094		p-val 0.018		test-stat 2.999		p-val 0.001		
SPF, quarterly									
Q1 1984 - Q4 2019					Q1 1990 - Q4 2018				
Parameters	Estimates		Standard Errors		Estimates		Standard Errors		
$A$	0.788	-0.070	0.070	0.100	0.749	-0.047	0.079	0.113	
	0.048	0.906	0.024	0.071	0.042	0.920	0.030	0.077	
$\hat{A}$	0.951	0.004	0.018	0.041	0.937	-0.027	0.021	0.030	
	0.026	0.787	0.016	0.041	0.026	0.806	0.031	0.044	
T-Test $\hat{A}_{21} > A_{21}$	test-stat -0.883		p-val 0.811		test-stat -0.410		p-val 0.659		

# Joint Estimation: with Feedback loop

MSC, quarterly								
Q1 1984 - Q4 2019					Q1 1990 - Q4 2018			
Parameters	Estimates		Standard Errors		Estimates		Standard Errors	
$A$	$\begin{bmatrix} 0.863 & 0.021 \\ -0.003 & 0.751 \end{bmatrix}$	$\begin{bmatrix} 0.073 & 0.162 \\ 0.042 & 0.074 \end{bmatrix}$	$\begin{bmatrix} 0.863 & 0.051 \\ -0.017 & 0.721 \end{bmatrix}$	$\begin{bmatrix} 0.078 & 0.169 \\ 0.042 & 0.076 \end{bmatrix}$				
$\hat{A}$	$\begin{bmatrix} 0.663 & -0.096 \\ 0.189 & 0.807 \end{bmatrix}$	$\begin{bmatrix} 0.063 & 0.089 \\ 0.057 & 0.056 \end{bmatrix}$	$\begin{bmatrix} 0.663 & -0.081 \\ 0.271 & 0.769 \end{bmatrix}$	$\begin{bmatrix} 0.080 & 0.094 \\ 0.064 & 0.057 \end{bmatrix}$				
T-test: $\hat{A}_{21} > A_{21}$	test-stat 2.227	p-val 0.013	test-stat 3.112	p-val 0.001				
SPF, quarterly								
Q1 1984 - Q4 2019					Q1 1990 - Q4 2018			
Parameters	Estimates		Standard Errors		Estimates		Standard Errors	
$A$	$\begin{bmatrix} 0.696 & -0.091 \\ 0.021 & 0.792 \end{bmatrix}$	$\begin{bmatrix} 0.078 & 0.090 \\ 0.031 & 0.072 \end{bmatrix}$	$\begin{bmatrix} 0.678 & -0.062 \\ 0.019 & 0.785 \end{bmatrix}$	$\begin{bmatrix} 0.086 & 0.107 \\ 0.034 & 0.089 \end{bmatrix}$				
$\hat{A}$	$\begin{bmatrix} 0.951 & 0.004 \\ 0.026 & 0.787 \end{bmatrix}$	$\begin{bmatrix} 0.018 & 0.041 \\ 0.016 & 0.041 \end{bmatrix}$	$\begin{bmatrix} 0.937 & -0.027 \\ 0.026 & 0.806 \end{bmatrix}$	$\begin{bmatrix} 0.021 & 0.030 \\ 0.031 & 0.044 \end{bmatrix}$				
T-Test $\hat{A}_{21} > A_{21}$	test-stat 0.136	p-val 0.446	test-stat 0.1534	p-val 0.439				

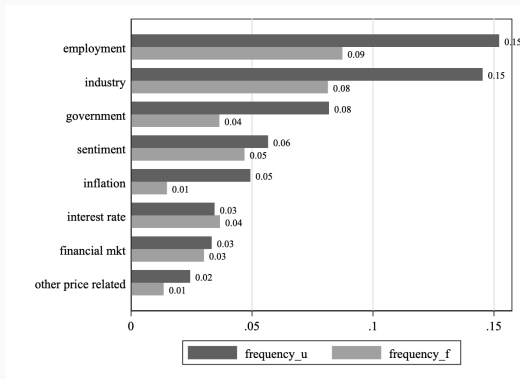
# $\pi$ news drives expectations across domains but $un$ news is domain-specific



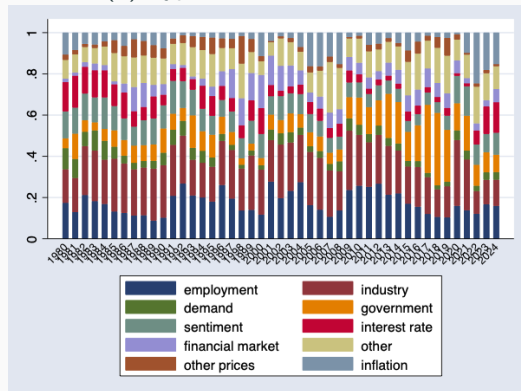
# Reported news in MSC

Figure 5: Type of news

(a) Fraction of fav and unfav news

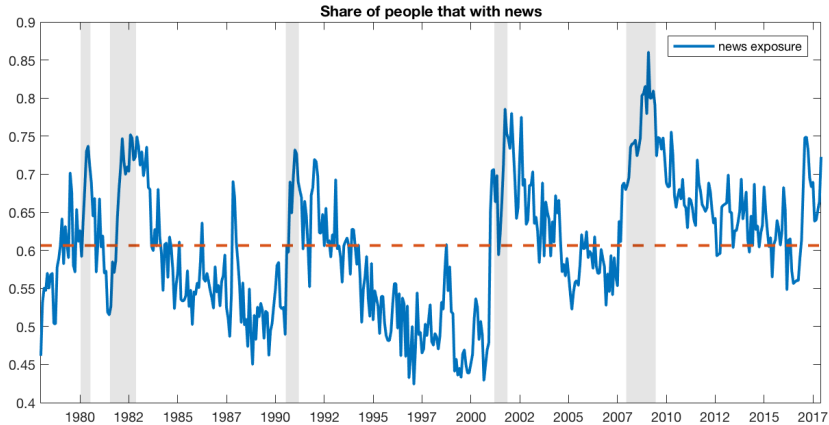


(b) Type of news across time



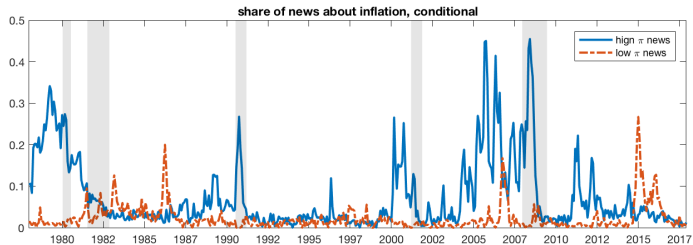
Notes: Panel (a): fractions of favorable and unfavorable news reported by individuals with news in MSC. Panel (b): shares of different types of news out of total news reported each year.

# News measure



Share of people who report hearing any news across time. The dashed line indicates that, on average, 60% of survey participants reported hearing about some news in the past few months.

# News measure



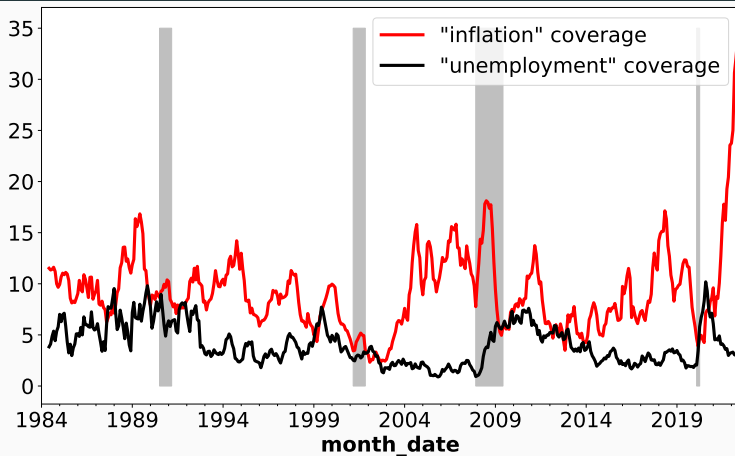


# Correlation conditional on news

Table 7: Correlation Conditional on News Heard

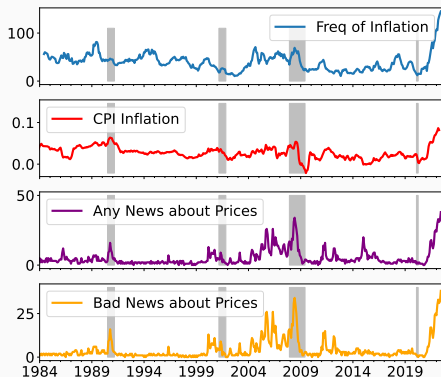
Dependent var:	$E\pi$	
	(1)	(2)
$Eun$	0.36*** (0.034)	0.38*** (0.047)
Inflation fav $\times Eun$	0.17 (0.164)	0.16 (0.164)
Inflation unfav $\times Eun$	0.36*** (0.117)	0.36*** (0.118)
Employment fav $\times Eun$	0.03 (0.089)	0.03 (0.090)
Employment unfav $\times Eun$	-0.20*** (0.073)	-0.16** (0.074)
Interest rate fav $\times Eun$	-0.23** (0.104)	-0.24** (0.104)
Interest rate unfav $\times Eun$	-0.16 (0.114)	-0.16 (0.115)
Industry fav $\times Eun$		0.06 (0.092)
Industry unfav $\times Eun$		-0.23*** (0.073)
Demand fav $\times Eun$		-0.14 (0.145)
Demand unfav $\times Eun$		-0.57*** (0.155)
Gov fav $\times Eun$		0.08 (0.107)
Gov unfav $\times Eun$		0.01 (0.079)
Sentiment fav $\times Eun$		0.01 (0.112)
Sentiment unfav $\times Eun$		0.24** (0.113)
Stock fav $\times Eun$		-0.11 (0.085)
Stock unfav $\times Eun$		0.06 (0.115)
Other prices fav $\times Eun$		-0.01 (0.152)
Other prices unfav $\times Eun$		-0.16 (0.130)
Other real fav $\times Eun$		-0.11 (0.168)
Other real unfav $\times Eun$		-0.21 (0.157)
Wage fav $\times Eun$		-0.17 (0.235)
Wage unfav $\times Eun$		0.00 (0.224)
Observations	167346	167346
$R^2$	0.674	0.675
Time F.E.	Y	Y

# Newspaper coverage of inflation and unemployment



The news coverage is defined as the sum of ratios of the word frequency divided by the total number of words in each article.

# News on inflation and unemployment is domain-specific



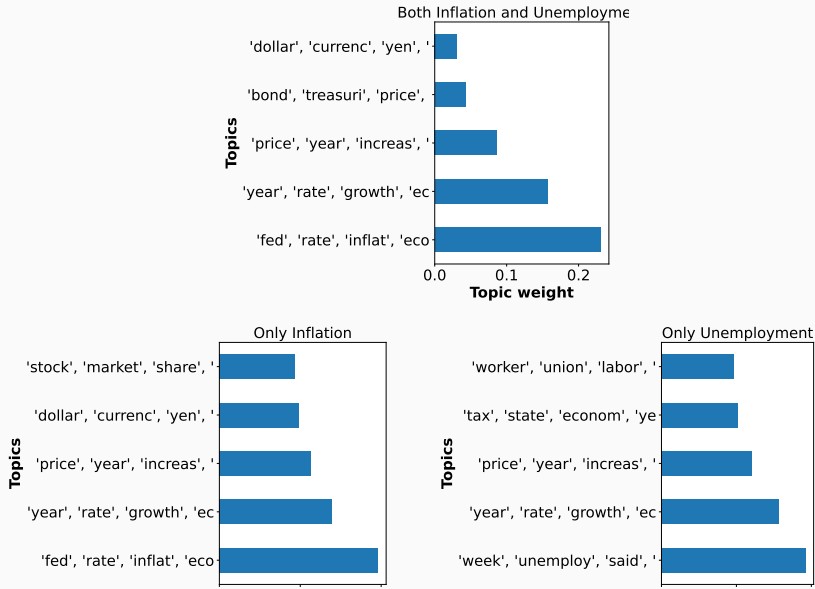
News coverage measured in the WSJ news archive.

## Inflation news is always labeled as bad news

**Table 7:** News Coverage and Self-Reported News Exposure

Topic	Any News	Bad News	Good News
Inflation	0.605	0.627	-0.048
Unemployment	0.373	0.295	0.153

# Topics in Inflation-Unemployment Narratives



## Keywords in Different Inflation-Unemployment Narratives

Key words when  
inflation, unemployment and  
Fed are mentioned



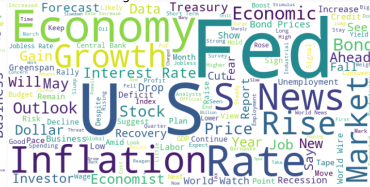
Key words when  
inflation, unemployment and  
oil price are mentioned



Key words when  
inflation, unemployment and  
recession are mentioned



Key words when  
inflation, unemployment and  
growth are mentioned



## References

---

**Andrade, Philippe and Hervé Le Bihan**, “Inattentive professional forecasters,” *Journal of Monetary Economics*, 2013, 60 (8), 967–982.

—, **Richard K Crump, Stefano Eusepi, and Emanuel Moench**, “Fundamental disagreement,” *Journal of Monetary Economics*, 2016, 83, 106–128.

**Andre, Peter, Carlo Pizzinelli, Christopher Roth, and Johannes Wohlfart**, “Subjective models of the macroeconomy: Evidence from experts and representative samples,” *The Review of Economic Studies*, 2022, 89 (6), 2958–2991.

**Bhandari, Anmol, Jaroslav Borovička, and Paul Ho**, “Survey data and subjective beliefs in business cycle models,” *Review of Economic Studies*, 2025, 92 (3), 1375–1437.

- Candia, Bernardo, Olivier Coibion, and Yuriy Gorodnichenko**, “Communication and the Beliefs of Economic Agents,” Working Paper 27800, National Bureau of Economic Research September 2020.
- Coibion, Olivier and Yuriy Gorodnichenko**, “What Can Survey Forecasts Tell Us about Information Rigidities?,” *Journal of Political Economy*, 2012, 120 (1), 116 – 159.
- Gáti, Laura**, “Monetary policy & anchored expectations—An endogenous gain learning model,” *Journal of Monetary Economics*, 2023, 140, S37–S47.
- Hajdini, Ina, Edward S Knotek, John Leer, Mathieu Pedemonte, Robert W Rich, and Raphael Schoenle**, “Low passthrough from inflation expectations to income growth expectations: why people dislike inflation,” 2022.
- Kamdar, Rupal**, “The Inattentive Consumer: Sentiment and Expectations,” 2019 Meeting Papers 647, Society for Economic Dynamics 2019.



- Lucas, Robert E.**, “Econometric policy evaluation: A critique,” *Carnegie-Rochester Conference Series on Public Policy*, 1976, 1, 19 – 46.
- Maćkowiak, Bartosz, Filip Matějka, and Mirko Wiederholt**, “Dynamic rational inattention: Analytical results,” *Journal of Economic Theory*, 2018, 176, 650–692.
- Milani, Fabio**, “Expectations, learning and macroeconomic persistence,” *Journal of monetary Economics*, 2007, 54 (7), 2065–2082.
- Sims, Christopher A.**, “Implications of rational inattention,” *Journal of Monetary Economics*, 2003, 50 (3), 665 – 690. Swiss National Bank/Study Center Gerzensee Conference on Monetary Policy under Incomplete Information.
- Woodford, Michael**, “Imperfect Common Knowledge and the Effects of Monetary Policy,” Working Paper 8673, National Bureau of Economic Research December 2001.