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Scaled consensus of second-order multiagent systems via distributed adaptive control

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Abstract
This article investigates scaled consensus of second-order multiagent systems (MASs) via distributed adaptive control. Two distributed adaptive scaled consensus protocols are proposed. First, to solve the dynamic scaled consensus problem, based on the neighbors' position and velocity information, two time-varying coupling gains are introduced into the node-based distributed adaptive protocol. Second, static scaled consensus is discussed under another distributed adaptive algorithm. It is shown proposed scaled consensus protocols are flexible, which are able to solve standard consensus, group consensus, and bipartite consensus problems with specific scale values. Finally, two examples of robot formation with nine agents are given to verify the effectiveness of the theoretical analysis.

KEYWORDS
distributed adaptive protocol, scaled consensus, second-order multiagent systems

1 | INTRODUCTION

Distributed cooperative control of multiagent systems (MASs) has been intensively studied in the field of automatic control. Its potential applications include coordination of multiple robots, wireless sensor networks, and spacecraft formation. In particular, consensus is an important problem to make all agents connected via networks converge to a common value under some distributed algorithms by using local information.[1-5]

In the literature related to the consensus problem, first-order MASs have gained much attention at the beginning. In Reference 2, Vicsek et al. proposed a simple discrete-time model to study the emergence of self-ordered motion in groups of mobile particles. Later, Jadbabaie et al. made an explanation for Vicsek model by using tools from graph theory.[3] A systematic framework of consensus problems was formally introduced in Reference 4. As for mobile autonomous agents, position and velocity have the clear physical meaning, the second-order consensus problem has been well studied, which is more challenging than the first-order one. For example, a framework to solve the consensus problem of second-order MASs was proposed in Reference 6. In Reference 7, necessary and sufficient conditions were given to achieve consensus of second-order MASs with continuous control. Later, second-order consensus in multiagent dynamical systems with sampled data was studied in References 8,9. Second-order multiagent systems with time-varying delays were considered in Reference 10. H_∞ synchronization of second-order coupled oscillators was investigated in Reference 11, in which positive effects of delays on master-slave synchronization were analyzed. Consensus of general linear systems was also studied in Reference 12.

One common feature in the above mentioned literature[7-10] is that the coupling strength is related to the second small eigenvalue of the Laplacian matrix. Then, each agent should have the ability to get the information about the whole communication topology to determine its coupling weight. These so-called distributed consensus algorithms in References 7-10 cannot be applied in a fully distributed way. To design a fully distributed consensus algorithm, adaptive control was introduced in References 13-17. A local adaptive algorithm was proposed in Reference 13 to study synchronization of networked nonlinear oscillators. Inspired by Reference 13, another adaptive consensus algorithm was designed in Reference 14 for consensus of MASs with Lipchitz nonlinear dynamics or general linear dynamics. In References 15 and 16, two different types of distributed adaptive consensus protocols: node-based and edge-based distributed adaptive consensus algorithms were proposed. The node-based one involved the design of the adaptive law for each node, while the other assigned a time-varying adaptive gain for each link in the communication topology.

Note that the standard consensus problem, studied in References 13-18 requires that all agents achieve consensus with the same consensus value. However, in some real-world scenarios, agents in a network may have to be separated into different groups, such as localization of clustered space robots, compartmental mass-action systems, and water distribution. As an extension of the standard consensus, group (cluster) consensus has been extensively studied recently in References 19-21. One limitation of cluster consensus is that there is no clear relationship among each group. While in some special missions, for example, cooperative control of multiple UAVs in the war, the relationship, or proportions of vehicles in space and on the ground should be clearly known. To overcome the limitation, scaled consensus was introduced in Reference 22. Scaled consensus means that all the agents in a network convergence to different groups with assigned proportions. In Reference 23, scaled consensus with bounded disturbances for first-order MASs in a fixed network topology was studied, and three different protocols were proposed to make agents achieve scaled consensus, finite-time consensus, and fixed-time consensus, respectively. The extension to switching networks was done in Reference 24. Scaled consensus in a group of first-order discrete-time MASs with time delays was considered in Reference 25. In Reference 26, scaled consensus of two groups of second-order agents with or without delays were discussed. As pointed in Reference 26, scaled consensus of second-order systems is more complex than that in first-order systems discussed in References 22-25. Until now, there are only few studies on this direction. This article aims to establish an analytical framework to investigate the impact of different scales in multiagent systems on consensus.

Scaled consensus of second-order multiagent systems via distributed adaptive control is investigated in this article. Different from cluster consensus in References 19-21 and common consensus in References 15 and 16, the scaled consensus problem discussed in this article is to make sure that agents in the network are divided into different clusters with assigned proportions by designing adaptive protocols involving each agent's scale information. The scale information will bring much difficulties for the design of consensus protocols, especially in the situation that some of scale values are negative. How to use the scale information to move agents into a scaled consensus motion is challenging. A fully distributed scaled consensus protocol for second-order MASs is proposed in this article based on the node-based type of distributed adaptive control. It will demonstrate that the scale information should be used both in scaled consensus protocol and dynamic adaptive laws. Moreover, the sign function is used to ensure scaled consensus can be achieved when some scale values are negative. In summary, the main contribution of this article can be summarized as follows:

1. Two types of node-based adaptive scaled consensus protocols are proposed to make agents achieve static consensus, that is, each agent's velocity converges to zero and its position converges to some fixed value with assigned proportions, and dynamic consensus with nonzero velocities with different scales, respectively.
2. Different from adaptive consensus protocols in References 15 and 16, two different coupling gains are proposed and the corresponding adjusting laws with the scale information are designed.
3. Sufficient conditions for scaled consensus of second-order MASs are derived and two examples are given to verify the effectiveness of proposed adaptive scaled consensus protocols.

Notations: Throughout this article, $\mathbf{1}_N$ denotes the column vector with all N elements being one, and I_N denotes the $N \times N$ identity matrix. For $a \in \mathbb{R}$, $|a|$ represents the absolute value. $A \otimes B$ denotes the Kronecker product of matrices A and B. A semipositive definite matrix A is denoted by $A \geq 0$.

2 | MODEL DESCRIPTION AND PRELIMINARIES

2.1 | Algebraic graph theory

Graph theory has been widely used to model the communication topology of MASs. The communication topology is denoted by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is a nonempty finite set of nodes. Ordered pairs of distinct nodes are represented by a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and the weighted adjacency matrix is denoted by $\mathcal{A} = (a_{ij})_{N\times N}$ with non-negative entries. Node i denotes the ith agent, and an order pair $\{j, i\}$ denotes an edge in $\mathcal{V}$. $\{i,j\} \in \mathcal{E}$ represents the jth agent can obtain the ith agent's information. A graph is said to be undirected if $\{i,j\} \in \mathcal{E}$ and $\{j, i\} \in \mathcal{E}$. It is said that the undirected communication graph $\mathcal{G}$ is connected when there exists a path among any distinct two nodes. $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$ denotes the set of neighbors of the ith agent. It is assumed that $a_{ij} > 0 \Leftrightarrow j \in \mathcal{N}_i$, otherwise $a_{ij} = 0$. The Laplacian matrix L of the communication topology $\mathcal{G}$ is defined as $l_{ij} = -a_{ij}, i \neq j$ and $l_{ii} = \sum_{j\in\mathcal{N}_i}^{N} a_{ij}$. Eigenvalues of L are denoted by $\lambda_i \in \mathbb{C}, i = 1, 2, \dots, N$. Assume that $0 = |\lambda_1| \le |\lambda_2| \le \dots \le |\lambda_N|$. For undirected graphs, L is asymmetric and $\lambda_i \in \mathbb{R}, i = 1, 2, \dots, N$

The following are some useful lemmas, which are necessary in the later sections.

Lemma 1 (4). *The second small eigenvalue $\lambda_2(L)$ of the Laplacian matrix L satisfies $\lambda_2(L) = \min_{x^T \mathbf{1}_N = 0, x \neq 0_N} \frac{x^T L x}{x^T x}$ when the communication graph $\mathcal{G}$ is undirected.*

Lemma 2 (15).

(1) For an undirected connected graph $\mathcal{G}$, the matrix LL is also semipositive definite and satisfies $0 = \lambda_1(LL) < \lambda_2(LL) \le \dots \le \lambda_N(LL)$, $\lambda_i(LL), i = 1, 2, \dots, N$ denotes the eigenvalues of the matrix LL.

(2) The second small eigenvalue $\lambda_2(LL)$ of the matrix LL satisfies $\lambda_2^2(L) = \lambda_2(LL) = \min_{x^T \mathbf{1}_N = 0, x \neq 0_N} \frac{x^T LL x}{x^T x}$.

2.2 | Model description

In this article, a second-order MAS consisting of N agents in an undirected network is considered. The dynamics of each agent is described by

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t), \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $v_i(t) \in \mathbb{R}^n$, and $u_i(t) \in \mathbb{R}^n$ $(i = 1, 2, \dots, N)$ present the position states, velocity states, and the control input of ith agent, respectively.

The objective of this article is to design distributed adaptive scaled consensus protocols, wherein each agent's states reach the prescribed ratios with other agents on its own scale but do not come to a common quantity.

Definition 1. Scaled consensus of the second-order MAS (1) is said to be achieved with the scale vector $\omega = (\omega_1, \omega_2, \dots, \omega_N)$, where ω_i is a nonzero constant, if for any initial conditions $x_i(0)$, $v_i(0)$ such that

$$\lim_{t\to\infty} (\omega_1 x_1(t) - \omega_j x_j(t)) = 0$$

and

$$\lim_{t\to\infty} (\omega_1 v_1(t) - \omega_j v_j(t)) = 0$$

for $i = 1, 2, \dots, N$, $j = 2, \dots, N$.

3 | MAIN RESULTS

In this section, two fully distributed adaptive scaled consensus protocols are proposed to solve the static and dynamic consensus problems, respectively.

3.1 Dynamic scaled consensus protocol with two adaptive gains

To achieve dynamic scaled consensus, the controller of the MAS (1) with the scale information and two different adaptive coupling gains is designed as

$$\begin{aligned} u_i(t) = \alpha c_i(t)\operatorname{sign}(\omega_i)\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i) \\ + \beta d_i(t)\operatorname{sign}(\omega_i)\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j v_j - \omega_i v_i) \end{aligned} \tag{2}$$

with

$$\begin{aligned} \dot{c}_i(t) = \alpha\gamma_i\,|\omega_i|\Bigg\{\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i)\right)^T\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i)\right) \\ + \left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i)\right)^T\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j v_j - \omega_i v_i)\right)\Bigg\} \end{aligned} \tag{3}$$

and

$$\begin{aligned} \dot{d}_i(t) = \beta\kappa_i\,|\omega_i|\Bigg\{\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j v_j - \omega_i v_i)\right)^T\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j v_j - \omega_i v_i)\right) \\ + \left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i)\right)^T\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j v_j - \omega_i v_i)\right)\Bigg\}, \end{aligned} \tag{4}$$

where α and β are coupling gains. γ_i and κ_i are positive constants, $c_i(t)$ is the time-varying adaptive coupling gains of the ith agent associated with the position information and $d_i(t)$ is the time-varying adaptive coupling gains of the ith agent associated with the velocity information. ω_i is the scale value of the ith agent and $\operatorname{sign}(\cdot)$ is the sign function, which is described by

$$\operatorname{sign}(\tau) = \begin{cases} -1, & \tau < 0 \\ 0, & \tau = 0 \\ 1, & \tau > 0 \end{cases}.$$

Let $\overline{x} = \frac{1}{N}\sum_{k=1}^{N}\omega_k x_k$ and $\overline{v} = \frac{1}{N}\sum_{k=1}^{N}\omega_k v_k$ denote the scaled average values of agents. Define error states between each agent to the scaled average values as $\tilde{x}_i = \omega_i x_i - \overline{x}$, $\tilde{v}_i = \omega_i v_i - \overline{v}$. Then, one has

$$\begin{aligned} \dot{\tilde{x}}_i(t) &= \tilde{v}_i(t), \\ \dot{\tilde{v}}_i(t) &= \omega_i\dot{v}_i - \frac{1}{N}\sum_{k=1}^{N}\omega_k\dot{v}_k \\ &= \omega_i\alpha c_i(t)\operatorname{sign}(\omega_i)\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i) + \omega_i\beta d_i(t)\operatorname{sign}(\omega_i)\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j v_j - \omega_i v_i) \\ &\quad - \frac{\alpha}{N}\sum_{k=1}^{N}\omega_k c_k(t)\operatorname{sign}(\omega_k)\sum_{j\in\mathcal{N}_k}^{N} a_{kj}(\omega_j x_j - \omega_k x_k) \end{aligned}$$

$$
\begin{aligned}
&-\frac{\beta}{N}\sum_{k=1}^{N}\omega_k d_k(t)\mathrm{sign}(\omega_k)\sum_{j\in\mathcal{N}_k}^{N}a_{kj}(\omega_j v_j-\omega_k v_k)\\
&=\alpha c_i(t)\,|\omega_i|\sum_{j\in\mathcal{N}_i}^{N}a_{ij}(\tilde{x}_j-\tilde{x}_i)+\beta d_i(t)\,|\omega_i|\sum_{j\in\mathcal{N}_i}^{N}a_{ij}(\tilde{v}_j-\tilde{v}_i)\\
&-\frac{\alpha}{N}\sum_{k=1}^{N}c_k(t)\,|\omega_k|\sum_{j\in\mathcal{N}_k}^{N}a_{kj}(\tilde{x}_j-\tilde{x}_k)-\frac{\beta}{N}\sum_{k=1}^{N}d_k(t)\,|\omega_k|\sum_{j\in\mathcal{N}_k}^{N}a_{kj}(\tilde{v}_j-\tilde{v}_k)\\
&=-\alpha c_i(t)\,|\omega_i|\sum_{j=1}^{N}l_{ij}\tilde{x}_j-\beta d_i(t)\,|\omega_i|\sum_{j=1}^{N}l_{ij}\tilde{v}_j\\
&+\frac{\alpha}{N}\sum_{k=1}^{N}c_k(t)\,|\omega_k|\sum_{j=1}^{N}l_{kj}\tilde{x}_j+\frac{\beta}{N}\sum_{k=1}^{N}d_k(t)\,|\omega_k|\sum_{j=1}^{N}l_{kj}\tilde{v}_j.
\end{aligned}
\tag{5}
$$

Let $C(t)=\mathrm{diag}\{c_1(t)|\omega_1|,c_2(t)|\omega_2|,\ \dots\ ,c_N(t)|\omega_N|\}$ and $D(t)=\mathrm{diag}\{d_1(t)\ |\omega_1|,d_2(t)|\omega_2|,\ \dots\ ,d_N(t)|\omega_N|\}$, $e(t)=(\tilde{x}^T(t),\tilde{v}^T(t))^T$, $\tilde{x}(t)=(\tilde{x}_1^T(t),\tilde{x}_2^T(t),\ \dots\ ,\tilde{x}_N^T(t))^T$ and $\tilde{v}(t)=(\tilde{v}_1^T(t),\tilde{v}_2^T(t),\ \dots\ ,\tilde{v}_N^T(t))^T$. Then, the error system (5) can be rewritten into the matrix form as

$$
\dot{e}(t)=(\Phi\otimes I_n)\,e(t)+F(\tilde{x},\tilde{v}),
\tag{6}
$$

where

$$
\Phi=\begin{pmatrix}0_N & I_N\\ -\alpha C(t)L & -\beta D(t)L\end{pmatrix},
$$

$$
F(\tilde{x},\tilde{v})=\begin{pmatrix}0_{Nn}\\ \mathbf{1}_N\otimes f(\tilde{x},\tilde{v})\end{pmatrix},
$$

and

$$
\begin{aligned}
f(\tilde{x},\tilde{v})=&\frac{\alpha}{N}\sum_{k=1}^{N}c_k(t)\,|\omega_k|\sum_{j=1}^{N}l_{kj}\tilde{x}_j\\
&+\frac{\beta}{N}\sum_{k=1}^{N}d_k(t)\,|\omega_k|\sum_{j=1}^{N}l_{kj}\tilde{v}_j.
\end{aligned}
$$

Theorem 1. *Suppose that the undirected communication topology $\mathcal{G}$ is connected. Scaled consensus of the second-order MAS (1) is said to be achieved under the distributed adaptive protocol (2)–(4) if $\alpha\beta\neq 0$*

Proof. Choose the following Lyapunov function candidate as

$$
V(t)=V_1(t)+V_2(t),
\tag{7}
$$

where

$$
V_1(t)=\frac{1}{2}e^T(t)\left(\begin{pmatrix}\mu LL & L\\ L & L\end{pmatrix}\otimes I_n\right)e(t),
\tag{8}
$$

and

$$
V_2(t)=\sum_{i=1}^{N}\frac{1}{2\gamma_i}(c_i(t)-\hat{c}_i)^2+\sum_{i=1}^{N}\frac{1}{2\kappa_i}(d_i(t)-\hat{d}_i)^2.
\tag{9}
$$

Here, μ is a sufficiently large constant. $\hat{c}_i$ and $\hat{d}_i$ are constants to be determined.

It will be shown that $V(t) \geq 0$ and $V(t)=0$ only when $e(t)=0$, $c_i(t)=\hat{c}_i$ and $d_i(t)=\hat{d}_i$. From Lemmas 1 to 2, one has

$$V(t) \geq \frac{1}{2}e^T(t)\left(\begin{pmatrix} \mu\lambda_2(LL)I_N & L \\ L & \lambda_2(L)I_N \end{pmatrix} \otimes I_n\right)e(t).$$

According to the Schur complement lemma,[27] $\begin{pmatrix} \mu\lambda_2(LL) & L \\ L & \lambda_2(L) \end{pmatrix} > 0$ is equivalent to $\lambda_2(L) > 0$ and $\mu\lambda_2(LL)I_N - \frac{1}{\lambda_2(L)}LL > 0$. When μ is sufficient large, then the two inequalities can be satisfied, that is, $V(t) \geq 0$ and $V(t)=0$ only when $e(t)=0$, $c_i(t)=\hat{c}_i$, and $d_i(t)=\hat{d}_i$, $i=1,2,\ldots,N$. It is worth pointing out that when $e(t)=(m,m,\ldots,m)^T$, where m is an arbitrary constant, one also gets $V(t)=0$ as $L\mathbf{1}_N=0$. But, by simply analysis, it can get $m=0$.

Next, taking the time derivative of $V_1(t)$ along the trajectory of (6) yields

$$\begin{aligned}
\dot{V}_1(t) &= e^T(t)\left(\left(\begin{pmatrix} -\alpha LC(t)L & \mu LL - \beta LD(t)L \\ -\alpha LC(t)L & L - \beta LD(t)L \end{pmatrix} \otimes I_n\right)e(t) \right. \\
&\quad \left. + \begin{pmatrix} L\mathbf{1}_N \otimes f(\tilde{x},\tilde{v}) \\ L\mathbf{1}_N \otimes f(\tilde{x},\tilde{v}) \end{pmatrix}\right) \\
&= -\alpha\widetilde{x}^T((LC(t)L) \otimes I_n)\widetilde{x} + \widetilde{v}^T((L - \beta LD(t)L) \otimes I_n)\widetilde{v} \\
&\quad + \widetilde{x}^T((\mu LL - \alpha LC(t)L - \beta LD(t)L) \otimes I_n)\widetilde{v} \\
&\quad + (\widetilde{x}^T + \widetilde{v}^T) \times (L\mathbf{1}_N \otimes f(\tilde{x},\tilde{v})).
\end{aligned} \tag{10}$$

Taking the derivative of $V_2(t)$ along the trajectories of (3) and (4), one has

$$\begin{aligned}
\dot{V}_2(t) &= \sum_{i=1}^{N}\frac{1}{\gamma_i}(c_i(t)-\hat{c}_i)\dot{c}_i(t) + \sum_{i=1}^{N}\frac{1}{\kappa_i}(d_i(t)-\hat{d}_i)\dot{d}_i(t) \\
&= \sum_{i=1}^{N}(c_i(t)-\hat{c}_i)\,|\omega_i|\left|\alpha\left(\sum_{j=1}^{N}l_{ij}\tilde{x}_j\right)^T\left(\sum_{j=1}^{N}l_{ij}\tilde{x}_j\right)\right| \\
&\quad + \sum_{i=1}^{N}(c_i(t)-\hat{c}_i)\,|\omega_i|\left|\alpha\left(\sum_{j=1}^{N}l_{ij}\tilde{x}_j\right)^T\left(\sum_{j=1}^{N}l_{ij}\tilde{v}_j\right)\right| \\
&\quad + \sum_{i=1}^{N}(d_i(t)-\hat{d}_i)\,|\omega_i|\left|\beta\left(\sum_{j=1}^{N}l_{ij}\tilde{v}_j\right)^T\left(\sum_{j=1}^{N}l_{ij}\tilde{v}_j\right)\right| \\
&\quad + \sum_{i=1}^{N}(d_i(t)-\hat{d}_i)\,|\omega_i|\left|\beta\left(\sum_{j=1}^{N}l_{ij}\tilde{x}_j\right)^T\left(\sum_{j=1}^{N}l_{ij}\tilde{v}_j\right)\right| \\
&= \alpha\widetilde{x}^T((LC(t)L) \otimes I_n)\widetilde{x} - \alpha\widetilde{x}^T((L\widetilde{C}L) \otimes I_n)\widetilde{x} \\
&\quad + \alpha\widetilde{x}^T((LC(t)L) \otimes I_n)\widetilde{v} - \alpha\widetilde{x}^T((L\widetilde{C}L) \otimes I_n)\widetilde{v} \\
&\quad + \beta\widetilde{v}^T((LD(t)L) \otimes I_n)\widetilde{v} - \beta\widetilde{v}^T((L\widetilde{D}L) \otimes I_n)\widetilde{v} \\
&\quad + \beta\widetilde{x}^T((LD(t)L) \otimes I_n)\widetilde{v} - \beta\widetilde{x}^T((L\widetilde{D}L) \otimes I_n)\widetilde{v}
\end{aligned} \tag{11}$$

where $\widehat{C} = \mathrm{diag}\{\hat{c}_1|\omega_1|, \hat{c}_2|\omega_2|, \ldots, \hat{c}_N|\omega_N|\}$ and $\widehat{D} = \mathrm{diag}\{\hat{d}_1|\omega_1|, \hat{d}_2|\omega_2|, \ldots, \hat{d}_N|\omega_N|\}$.

Combining (10) and (11) and noting that $L\mathbf{1}_N=0$, it follows

$$\begin{aligned}
\dot{V}(t) &= -\alpha\widetilde{x}^T((L\widehat{C}L) \otimes I_n)\widetilde{x} + \widetilde{v}^T((L - \beta L\widehat{D}L) \otimes I_n)\widetilde{v} \\
&\quad + \widetilde{x}^T((\mu LL - \alpha L\widehat{C}L - \beta L\widehat{D}L) \otimes I_n)\widetilde{v}
\end{aligned}$$

Let $\hat{c}_i\,|\omega_i| = c^*$, $\hat{d}_i\,|\omega_i| = d^*$ and $\mu = \alpha c^* + \beta d^*$, then one obtains

$$\begin{aligned}\dot{V}(t) &= -\alpha c^* \widetilde{x}^T((LL)\otimes I_n)\widetilde{x} + \widetilde{v}^T((L-\beta d^* LL)\otimes I_n)\widetilde{v} \\ &\le -\alpha c^* \lambda_2(LL)\widetilde{x}^T\widetilde{x} - (\beta d^* \lambda_2(LL) - \lambda_N(L))\widetilde{v}^T\widetilde{v}\end{aligned}$$

Choose sufficiently large c^*, d^* when $\alpha > 0$, $\beta > 0$ and sufficiently small c^*, d^* when $\alpha < 0$, $\beta < 0$, such that αc^* $\lambda_2(LL) > 1$ and $\beta d^* \lambda_2(LL) - \lambda_N(L) > 1$. Then, one gets

$$\dot{V}(t) \le -\widetilde{x}^T\widetilde{x} - \widetilde{v}^T\widetilde{v}. \tag{12}$$

From (12), it follows that only when $\widetilde{x}(t) = 0$ and $\widetilde{v}(t) = 0$, it has $\dot{V}(t) = 0$. Then, according to the LaSalle's Invariance principle,[28] $e(t) = 0$ as $t \to \infty$. That is, scaled consensus of the second-order MAS (1) is achieved under the fully distributed scaled consensus protocol (2)–(4). This completes the proof. ∎

Remark 1. Scaled consensus can be regarded as a generalization of standard consensus,[13-16] group (cluster) consensus,[19-21] and bipartite consensus.[29,30] More specifically, when all agents have the same scale, that is, $\omega_1 = \omega_2 = \dots = \omega_N$, the scaled consensus problem becomes the standard consensus problem. Scaled consensus can also solve the problem of bipartite consensus when each agent's scale is selected from $\{1, -1\}$. Different from cluster consensus,[19-21] in which agents are divided into different groups with no clear relationship, MAS (1) achieves scaled consensus under the scaled consensus protocol (2), in which agents are not only divided into different groups but also with assigned proportions.

Remark 2. Scaled consensus of MASs has been investigated in References 22-25. These studies provide effective methods to deal with the scaled consensus problem. However, the main focus is on first-order MASs. As second-order MASs are much more challenging than first-order ones, this article makes an attempt to establish an analytical framework to investigate the impact of different scales in MASs on consensus. Only the linear system is considered in this article. In practice, nonlinear dynamics is unavoidable in many cases. Results can be extended for certain systems with the Lipschitz-type nonlinearity. Moreover, the networked setting may give rise to some security issues suffering from cyberattacks, as discussed in Reference 31, secure scaled consensus of nonlinear MASs will be studied in the near future.

Remark 3. Unlike results in References 6, 7 and 32, each agent should have the ability to know the global communication topology to calculate coupling gains. Scaled consensus of the second-order MAS (1) is achieved under adaptive laws given in (3) and (4) for all connected communication topologies. The distributed adaptive algorithm proposed in this article is in a fully distributed way without the global topology information.

Remark 4. Compare with Reference 15, in which a distributed consensus protocol with one adaptive gain was studied, the distributed scaled consensus protocol in this article has two different adaptive laws associated with the position and the velocity, respectively. Therefore, the scaled consensus protocol (2) is more flexible. Gains influence the convergence rate of adaptive laws (3) and (4). From Equation (12), one can find that the larger α and β will accelerate the consensus process. In addition, standard consensus was considered in Reference 15. While under the scaled consensus protocol (2), the second-order MAS (1) is able to achieve standard or scaled consensus.

3.2 | Static scaled consensus protocol with one adaptive gain

In Subsection 3.1, a distributed adaptive scaled consensus algorithm is proposed, which ensure agents' position states and nonzero velocity states convergence to different values with assigned proportions. But in some practical applications, such as, static formation control,[6,33] it may require that each agent achieves a prior unknown location with fixed position and zero velocity. To solve this problem, a distributed adaptive scaled consensus protocol with only one adaptive gain is designed as follows

$$u_i(t) = \alpha h_i(t)\mathrm{sign}(\omega_i)\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i) - \beta v_i, \tag{13}$$

where

$$\dot{h}_i(t) = \alpha\gamma_i\,|\omega_i|\left\{\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i)\right)^T\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i)\right)\right.$$
$$\left.+\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i)\right)^T\left(\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j v_j - \omega_i v_i)\right)\right\}. \tag{14}$$

Here, α and β are coupling gains. γ_i are positive constants, $h_i(t)$ is the time-varying adaptive control strength. With the proposed controller (13), the dynamics of the error states $\tilde{x}$ and $\tilde{v}$ follows

$$\begin{aligned}
\dot{\tilde{x}}_i(t) &= \tilde{v}_i(t),\\
\dot{\tilde{v}}_i(t) &= \omega_i\dot{v}_i - \frac{1}{N}\sum_{k=1}^{N}\omega_k\dot{v}_k\\
&= \omega_i\alpha h_i(t)\mathrm{sign}(\omega_i)\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j x_j - \omega_i x_i) - \beta\omega_i v_i\\
&\quad - \frac{\alpha}{N}\sum_{k=1}^{N}\omega_k h_k(t)\mathrm{sign}(\omega_k)\sum_{j\in\mathcal{N}_k}^{N} a_{kj}(\omega_j x_j - \omega_k x_k) + \frac{\beta}{N}\sum_{k=1}^{N}\omega_k v_k\\
&= \alpha h_i(t)\,|\omega_i|\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\tilde{x}_j - \tilde{x}_i) - \beta\widetilde{v}_i - \frac{\alpha}{N}\sum_{k=1}^{N} h_k(t)\,|\omega_k|\sum_{j\in\mathcal{N}_k}^{N} a_{kj}(\tilde{x}_j - \tilde{x}_k)\\
&= -\alpha h_i(t)\,|\omega_i|\sum_{j=1}^{N} l_{ij}\tilde{x}_j - \beta\widetilde{v}_i + \frac{\alpha}{N}\sum_{k=1}^{N} h_k(t)\,|\omega_k|\sum_{j=1}^{N} l_{kj}\tilde{x}_j.
\end{aligned} \tag{15}$$

Let $H(t) = \mathrm{diag}\{h_1(t)|\omega_1|, h_2(t)|\omega_2|, \ldots, h_N(t)|\omega_N|\}$. Rewriting (15) into the matrix form yields

$$\dot{e}(t) = \left(\widetilde{\Phi}\otimes I_n\right)e(t) + \hat{F}(\widetilde{x},\widetilde{v}), \tag{16}$$

where

$$\widetilde{\Phi} = \begin{pmatrix} 0_N & I_N\\ -\alpha H(t)L & -\beta I_N\end{pmatrix},$$
$$\hat{F}(\widetilde{x},\widetilde{v}) = \begin{pmatrix} 0_{Nn}\\ \mathbf{1}_N\otimes\hat{f}(\tilde{x},\tilde{v})\end{pmatrix},$$

and

$$\hat{f}(\tilde{x},\tilde{v}) = \frac{\alpha}{N}\sum_{k=1}^{N} h_k(t)\,|\omega_k|\sum_{j=1}^{N} l_{kj}\tilde{x}_j.$$

Theorem 2. *Suppose that the communication topology $\mathcal{G}$ is undirected and connected. Under the distributed adaptive protocol (13) and (14), the second-order MAS (1) achieves scaled consensus if $\alpha \neq 0$, $\beta > 1$*

Proof. Choose the following Lyapunov function candidate as

$$\widetilde{V}(t) = \frac{1}{2}e^T(t)\left(\begin{pmatrix}\beta L & L\\ L & L\end{pmatrix}\otimes I_n\right)e^T(t)$$
$$+\sum_{i=1}^{N}\frac{1}{2\gamma_i}(h_i(t) - \hat{h}_i)^2 \tag{17}$$

where $\hat{h}_i$ $(i = 1, 2, \ldots, N)$ are constants to be determined.

Since

$$\begin{aligned}\widetilde{V}(t) &= \frac{1}{2}(\beta\tilde{x}^T(L\otimes I_n)\tilde{x} + \tilde{v}^T(L\otimes I_n)\tilde{v} + 2\tilde{x}^T(L\otimes I_n)\tilde{v})\\ &= \frac{1}{2}(\beta-1)\tilde{x}^T(L\otimes I_n)\tilde{x} + \frac{1}{2}((P\otimes I_n)(\tilde{x}+\tilde{v}))^T((P\otimes I_n)(\tilde{x}+\tilde{v}))\\ &\geq 0,\end{aligned}$$

where $L = P^TP$. As $\beta > 1$, $\widetilde{V}(t) \geq 0$ and $\widetilde{V}(t) = 0$ if only when $e(t) = 0$ and $h_i(t) = \hat{h}_i$.

The time derivative of $\widetilde{V}(t)$ along the trajectory of (16) has:

$$\begin{aligned}\dot{\widetilde{V}}(t) &= e^T(t)\left(\left(\left(\begin{matrix}-\alpha LH(t)L & 0\\ -\alpha LH(t)L & (1-\beta)L\end{matrix}\right)\otimes I_n\right)e(t)\right.\\ &\quad\left.+\left(\begin{matrix}L\mathbf{1}_N\otimes\hat{f}(\overline{x}^T,\overline{v}^T)\\ L\mathbf{1}_N\otimes\hat{f}(\overline{x}^T,\overline{v}^T)\end{matrix}\right)\right) + \sum_{i=1}^{N}\frac{(h_i(t)-\hat{h}_i)}{\gamma_i}\dot{h}_i(t)\\ &= -\alpha\widetilde{x}^T((LH(t)L)\otimes I_n)\widetilde{x} + \widetilde{v}^T((L-\beta L)\otimes I_n)\widetilde{v}\\ &\quad -\alpha\widetilde{x}^T((LH(t)L)\otimes I_n)\widetilde{v} + \sum_{i=1}^{N}\frac{(h_i(t)-\hat{h}_i)}{\gamma_i}\dot{h}_i(t)\end{aligned}$$

Note that

$$\begin{aligned}\sum_{i=1}^{N}\frac{(h_i(t)-\hat{h}_i)}{\gamma_i}\dot{h}_i(t) &= \alpha\widetilde{x}^T((LH(t)L)\otimes I_n)\widetilde{x} + \alpha\widetilde{x}^T((LH(t)L)\otimes I_n)\widetilde{v}\\ &\quad -\alpha\widetilde{x}^T((L\hat{H}L)\otimes I_n)\widetilde{x} - \alpha\widetilde{x}^T((L\hat{H}L)\otimes I_n)\widetilde{v}\end{aligned}$$

where $\hat{H} = \mathrm{diag}\{\hat{h}_1|\omega_1|, \hat{h}_2|\omega_2|, \ldots, \hat{h}_N|\omega_N|\}$. Let $\hat{h}_i(t)|\omega_i| = h^*$. One has

$$\begin{aligned}\dot{\widetilde{V}}(t) &= -\alpha\widetilde{x}^T((L\hat{H}L)\otimes I_n)\widetilde{x} + \widetilde{v}^T((L-\beta L)\otimes I_n)\widetilde{v}\\ &\quad -\alpha\widetilde{x}^T((L\hat{H}L)\otimes I_n)\widetilde{v}\\ &= -\alpha h^*\widetilde{x}^T((LL)\otimes I_n)\widetilde{x} - \widetilde{v}^T((\beta L - L)\otimes I_n)\widetilde{v}\\ &\quad -\alpha h^*\widetilde{x}^T((LL)\otimes I_n)\widetilde{v}\\ &= -\frac{1}{2}\alpha h^*\widetilde{x}^T((LL)\otimes I_n)\widetilde{x} - \widetilde{v}^T\left(\left(\beta L - L - \frac{1}{2}\alpha h^*(LL)\right)\otimes I_n\right)\widetilde{v}\\ &\quad -\frac{1}{2}\alpha h^*(L\widetilde{x}+L\widetilde{v})^T(L\widetilde{x}+L\widetilde{v})\end{aligned} \tag{18}$$

Choose an appropriate value such that $(\beta-1)\lambda_2(L) - \frac{1}{2}\alpha h^*\lambda_N(LL) > 0$. Then, one has

$$\dot{\widetilde{V}}(t) \leq 0. \tag{19}$$

According to (18), it can be seen that $\dot{\widetilde{V}}(t) = 0$ if and only if $\widetilde{x}(t) = 0$ and $\widetilde{v}(t) = 0$. Then, according to LaSalle's Invariance principle,[28] it follows that as $t \to \infty$, $e(t) = 0$, that is, scaled consensus of the second-order MAS (1) can be achieved under the adaptive scaled protocol (13). This completes the proof. ∎

Remark 5. In Reference 26, the problem of scaled group consensus was well studied in two groups of agents. Constant feedback gains were used in the design of static scaled consensus protocols. While in this article, two adaptive protocols are proposed to solve the static and dynamic scaled consensus problems, respectively. In addition, the scale ratio between two groups is required to be positive in Reference 26, which means it cannot solve the bipartite consensus problem. While, in this article, bipartite consensus can be solved by setting the agent scale values as 1 or −1.

4 | NUMERICAL SIMULATIONS

In this section, two examples are given to illustrate that scaled consensus of the second-order MAS (1) can be achieved under the proposed scaled consensus protocols.

Consider a network consisting of nine autonomous mobile robots[15] and each robot's dynamics is described by

$$\begin{cases} \dot{x}_i(t) = \zeta_i(t)\cos(\varsigma_i(t)) \\ \dot{y}_i(t) = \zeta_i(t)\sin(\varsigma_i(t)) \\ \dot{\varsigma}_i(t) = \varphi_i(t), \end{cases} \tag{20}$$

where $x_i(t)$ and $y_i(t)$ denote the Cartesian coordinates of the center of mass of the ith robot, respectively. $\varphi_i(t)$ and $\zeta_i(t) \in \mathbb{R}$ denote the angular velocity and linear velocity, respectively. $\varsigma_i(t)$ denotes the heading angle of ith. Let $v_{xi}(t) = \zeta_i(t)\cos(\varsigma_i(t))$ be the linear velocity component along the X axis and $v_{yi}(t) = \zeta_i(t)\sin(\varsigma_i(t))$ represents the linear velocity component along the Y axis. Then, using dynamic feedback linearization,[15] the unicycle model (20) can be transformed into the system (1) with $n = 2$:

$$\begin{aligned} \dot{x}_i(t) &= v_{xi}(t), \quad \dot{v}_{xi}(t) = u_{xi}(t), \\ \dot{y}_i(t) &= v_{yi}(t), \quad \dot{v}_{yi}(t) = u_{yi}(t) \end{aligned}$$

where $u_{xi}(t)$ and $u_{yi}(t)$ are the control inputs along X and Y axes, respectively.

Assume nine robots are divided into three groups as shown in Figure 1. Each group has the same scale value. Robot 1 to robot 3 have the same scale (i.e., $\omega_1 = \omega_2 = \omega_3 = 1$). Robot 4 to robot 6 have the same scale (i.e., $\omega_4 = \omega_5 = \omega_6 = 2$). Robot 7 to robot 9 have the same scale (i.e., $\omega_7 = \omega_8 = \omega_9 = -1$).

4.1 | Example 1. Dynamic scaled consensus protocol with two different adaptive gains

In this example, scale consensus with three triangle scaled formations will be illustrated. To achieve this specified formation shape, distributed adaptive protocols (2) is modified as follows

$$\begin{aligned} u_i(t) = {} & \alpha c_i(t)\mathrm{sign}(\omega_i)\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j(x_j-\delta_j)-\omega_i(x_i-\delta_i)) \\ & + \beta d_i(t)\mathrm{sign}(\omega_i)\sum_{j\in\mathcal{N}_i}^{N} a_{ij}(\omega_j v_j - \omega_i v_i). \end{aligned} \tag{21}$$

where c_i, d_i are defined in the same way as (3) and (4), δ_i, $i = 1, 2, \ldots, N$, are constants.

In the distributed adaptive protocol (21), only the neighbors' states information is used. Choose $\alpha = 2$, $\beta = 1$, $\gamma_1 = \gamma_2 = \ldots = \gamma_9 = 0.1$, and $\kappa_1 = \kappa_2 = \ldots = \kappa_9 = 0.4$. Initial conditions of agents are randomly selected in $[-50, 50]$ and

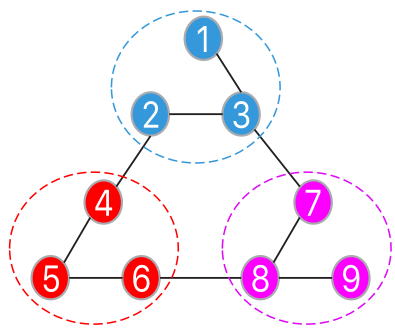

FIGURE 1 The communication graph [Colour figure can be viewed at wileyonlinelibrary.com]

adaptive coupling gains c_{xi}, c_{yi}, d_{xi}, and d_{yi} are randomly selected in $[-10, 10]$. $\delta = [\delta_1^T, \delta_2^T, \delta_3^T, \delta_4^T, \delta_5^T, \delta_6^T, \delta_7^T, \delta_8^T, \delta_9^T]^T = 8 * \begin{pmatrix} 0 & 1 & 2 & 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & \sqrt{3} & 0 & 0 & \sqrt{3} & 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$.

The evolution of each robot's position and velocity is shown in Figures 2 and 3, respectively. It shows that nine robots are divided into three groups with triangle formations. Robot 1, robot 2, and robot 3 converge to Group 1. Group 2 includes robot 4, robot 5, and robot 6. Robot 7, robot 8, and robot 9 converge to Group 3. Robots in different groups have different velocity and different orientations. Moreover, there exists fixed ratios among three groups, that is, $x_1 = 2x_4 = -x_7$, $x_2 = 2x_5 = -x_8$, $x_3 = 2x_6 = -x_9$, $y_1 = 2y_4 = -y_7$, $y_2 = 2y_5 = -y_8$, $y_3 = 2y_6 = -y_9$, $v_{x1} = v_{x2} = v_{x3} = 2v_{x4} = 2v_{x5} = 2v_{x6} = -v_{x7} = -v_{x8} = -v_{x9}$, and $v_{y1} = v_{y2} = v_{y3} = 2v_{y4} = 2v_{y5} = 2v_{y6} = -v_{y7} = -v_{y8} = -v_{y9}$. Adaptive coupling gains of nine robots are depicted in Figures 4 and 5. It can be seen that each adaptive gain converges to some finite values. Formation control of the second-order MAS (1) is successfully achieved under the distributed adaptive protocol (21), that is, scaled consensus of the second-order MASs (20) can be achieved under the fully distributed adaptive protocol (2).

4.2 | Example 2. Static scaled consensus protocol with one adaptive gain

The distributed adaptive protocol (13) is modified as follows

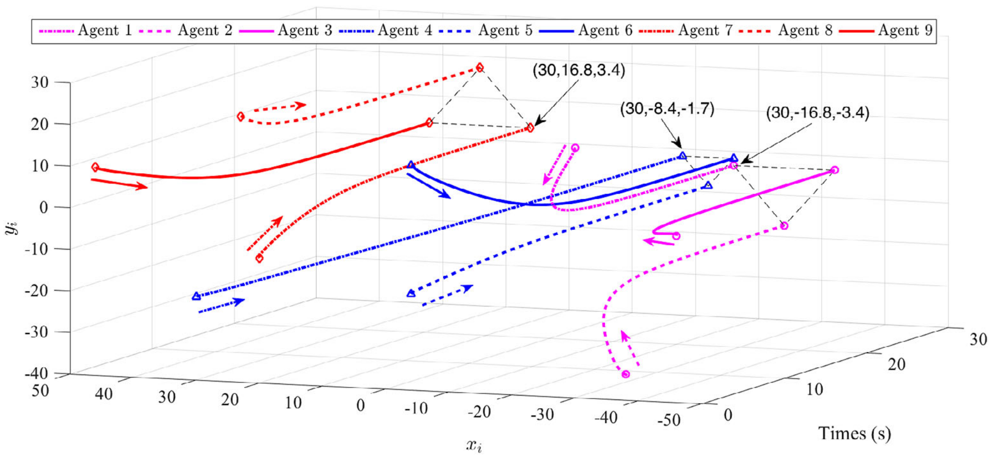

FIGURE 2 The trajectories of x_i and y_i in Example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

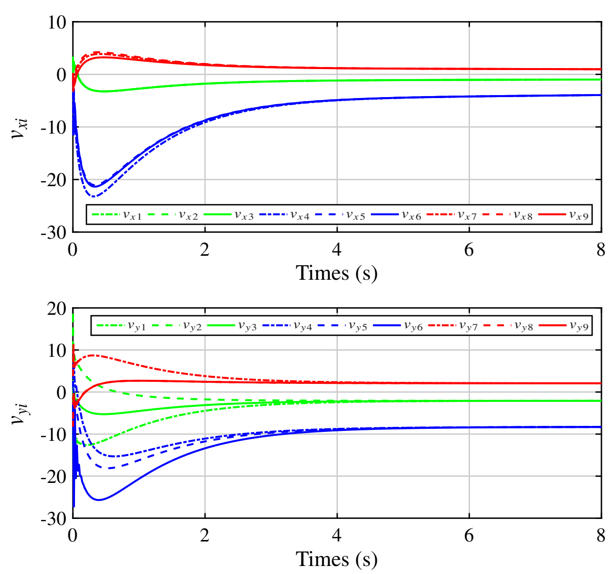

FIGURE 3 The trajectories of v_{xi} and v_{yi} in Example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

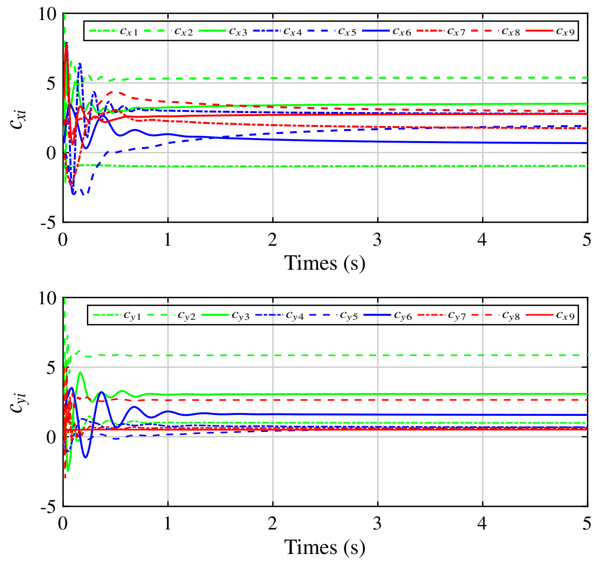

FIGURE 4 The trajectories of c_{xi} and c_{yi} in Example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

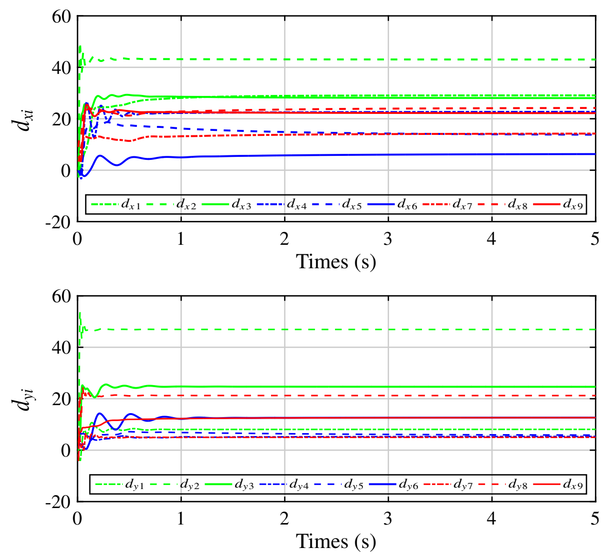

FIGURE 5 The trajectories of d_{xi} and d_{yi} in Example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

$$u_i(t) = \alpha h_i(t)\mathrm{sign}(\omega_i)\sum_{j\in\mathcal{N}_i}^{N} a_{ij}((\omega_j(x_j-\delta_j)-\omega_i(x_i-\delta_i)) - \beta v_i, \tag{22}$$

where h_i is defined in the same way as (14) and δ_i, $i = 1, 2, \ldots, N$, are constants as Example 1.

Choose $\alpha = 4$, $\beta = 4$, $\gamma_1, \gamma_2, \ldots, \gamma_9$ are random selected from $[0, 1]$. Initial conditions of agents are randomly selected in $[-50, 50]$ and adaptive coupling gains c_{xi}, c_{yi} are randomly selected in $[-10, 10]$.

From Theorem 2, scaled consensus can be achieved. Position states of nine robots are given in Figure 6 with $x_1 = 2x_4 = -x_7$, $x_2 = 2x_5 = -x_8$, $x_3 = 2x_6 = -x_9$, $y_1 = 2y_4 = -y_7$, $y_2 = 2y_5 = -y_8$, and $y_3 = 2y_6 = -y_9$. Velocity states are shown in Figure 7. It is shown that when scaled consensus is achieved, each robot's velocity converges to zero and all robots stop moving. Adaptive coupling gains of nine robots are depicted in Figure 8.

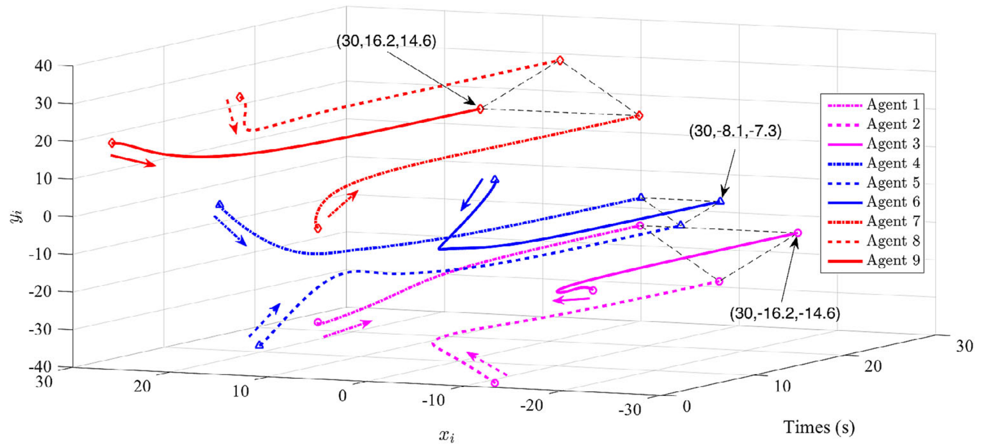

FIGURE 6 The trajectories of x_i and y_i in Example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

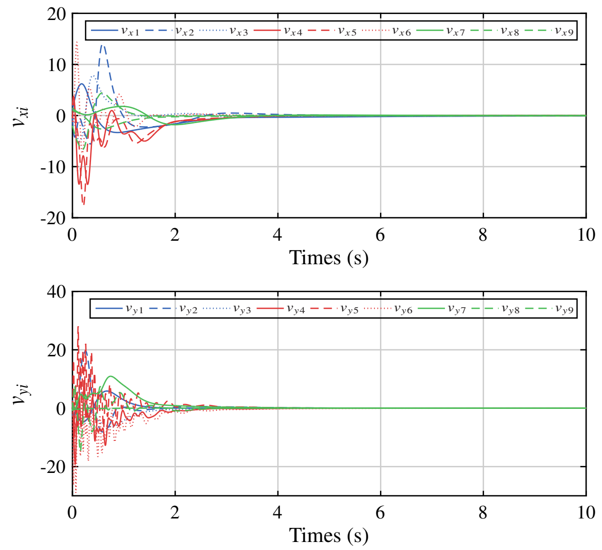

FIGURE 7 The trajectories of v_{xi} and v_{yi} in Example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

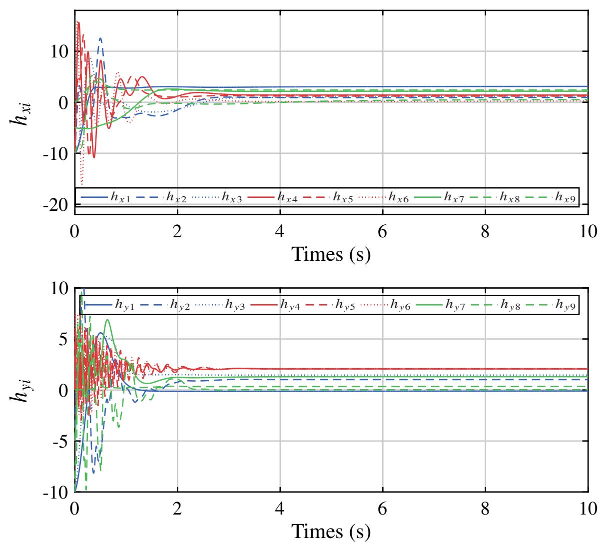

FIGURE 8 The trajectories of h_{xi} and h_{yi} in Example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

5 | CONCLUSION

In this article, the scaled consensus problem of second-order MASs has been studied. Two scaled adaptive consensus algorithms have been proposed to solve static and dynamic consensus problems, respectively. First, sufficient condition is derived for dynamic scaled consensus. Then static scaled consensus with zero velocity is analyzed. Finally, two numerical simulations are given to verify the effectiveness of the theoretical results. It is shown that with the proposed scaled protocols, agents can achieve consensus with assigned proportions without using global topology information.

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CONFLICT OF INTEREST

The authors declare no potential conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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REFERENCES

1. Song Q, Liu F, Vasilakos A. Semi-global and global containment control of multi-agent systems with second-order dynamics and input saturation. *Int J R Nonlinear Control*. 2016;26(16):3460-3480.
2. Vicsek T, Czirók A, Ben-Jacob E, Shochet O. Novel type of phase transition in a system of self-driven particles. *Phys Rev Lett*. 1995;75:1226.
3. Jadbabaie A, Lin J, Morse A. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Trans Autom Control*. 2003;48:988-1001.
4. Olfati-Saber R, Murray R. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans Autom Control*. 2004;49:1520-1533.
5. He W, Luo T, Tang Y, Du W, Tian Y, Qian F. Secure communication based on quantized synchronization of chaotic neural networks under an event-triggered strategy. *IEEE Trans Neural Netw Learn Syst*. 2020;31(9):3334-3345.
6. Ren W, Atkins E. Distributed multi-vehicle coordinated control via local information exchange. *Int J R Nonlinear Control*. 2007;17:1002-1033.
7. Yu W, Chen G, Cao M. Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems. *Automatica*. 2010;46:1089-1095.
8. Yu W, Zheng W, Chen G, Ren W, Cao J. Second-order consensus in multi-agent dynamical systems with sampled position data. *Automatica*. 2011;47:1496-1503.
9. Liu Z, Yu X, Guan Z, Hu B, Li C. Pulse-modulated intermittent control in consensus of multi-agent systems. *IEEE Trans Syst Man Cybern Syst*. 2017;47:783-793.
10. Zhu W, Cheng D. Leader-following consensus of second-order agents with multiple time-varying delays. *Automatica*. 2010;46:1994-1999.
11. Yang Y, He W, Han Q-L, Chen P. H∞ synchronization of networked master-slave oscillators with delayed position data: the positive effects of network-induced delays. *IEEE Trans Cybern*. 2019;49(12):4090-4102.
12. He W, Xu B, Han Q-L, Qian F. Adaptive consensus control of linear multi-agent systems with dynamic event-triggered strategies. *IEEE Trans Cybern*. 2020;50(7):2996-3008.
13. De Lellis P, Bernardo D, Garofalo F. Synchronization of complex networks through local adaptive coupling. *Chaos Interdiscipl J Nonlinear Sci*. 2008;18:037110.
14. Li Z, Ren W, Liu X, Fu M. Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols. *IEEE Trans Autom Control*. 2013;58:1786-1791.
15. Yu W, Ren W, Zheng X, Chen G, Lü J. Distributed control gains design for consensus in multi-agent systems with second-order nonlinear dynamics. *Automatica*. 2013;49:2107-2115.
16. Yu Z, Huang D, Jiang H, Hu C. Consensus of second-order multi-agent systems with nonlinear dynamics via edge-based distributed adaptive protocols. *J Frankl Inst*. 2016;353:4821-4844.
17. Zhao L, Jia Y, Yu J. Adaptive finite-time bipartite consensus for second-order multi-agent systems with antagonistic interactions. *Syst Control Lett*. 2017;102:22-31.
18. He W, Qian F, Lam J, Chen G, Han Q-L, Kurths J. Quasi-synchronization of heterogeneous dynamic networks via distributed impulsive control: error estimation, optimization and design. *Automatica*. 2015;62:249-262.

19. Xia W, Cao M. Clustering in diffusively coupled networks. *Automatica*. 2011;47:2395-2405.
20. Feng Y, Xu S, Zhang B. Group consensus control for double integrator dynamic multiagent systems with fixed communication topology. *Int J R Nonlinear Control*. 2014;24:532-547.
21. Xia H, Huang T, Shao J, Yu J. Group consensus of multi-agent systems with communication delays. *Neurocomputing*. 2016;171:1666-1673.
22. Roy S. Scaled consensus. *Automatica*. 2015;51:259-262.
23. Meng D, Jia Y. Robust consensus algorithms for multiscale coordination control of multivehicle systems with disturbances. *IEEE Trans Ind Electron*. 2016;63:1107-1119.
24. Meng D, Jia Y. Scaled consensus problems on switching networks. *IEEE Trans Autom Control*. 2016;61:1664-1669.
25. Hou B, Sun F, Li H, Chen Y, Liu G. Scaled cluster consensus of discrete-time multi-agent systems with general directed topologies. *Int J Syst Sci*. 2016;47:3839-3845.
26. Yu J, Shi Y. Scaled group consensus in multiagent systems with first/second-order continuous dynamics. *IEEE Trans Cybern*. 2018;48(8):2259-2271.
27. Boyd S, Ghaoui L, Feron E, Balakrishnan V. *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA: SIAM; 1994.
28. Krstic M, Kanellakopoulos I, Kanellakopoulos P. Nonlinear and adaptive control. *Lect Notes Contr Inf*. 1995;5:4475-4480.
29. Altafini C. Consensus problems on networks with antagonistic interactions. *IEEE Trans Autom Control*. 2013;58:935-946.
30. Valcher M, Misra P. On the consensus and bipartite consensus in high-order multi-agent dynamical systems with antagonistic interactions. *Syst Control Lett*. 2014;66:94-103.
31. He W, Qian F, Han Q-L, Chen G. Almost sure stability of nonlinear systems under random and impulsive sequential attacks. *IEEE Trans Autom Control*. 2020;65(9):3879-3886.
32. Li J, Li J. Adaptive iterative learning control for coordination of second-order multi-agent systems. *Int J R Nonlinear Control*. 2014;24:3282-3299.
33. Li X, Liu C, Liu F. Scaled group consensus of delayed second-order multi-agent systems. *In Proc 7th World Cong Int Control Autom (WCICA)*. 2016;2163-2168.

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