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# Robust Second-Order Consensus Using a Fixed-Time Convergent Sliding Surface in Multiagent Systems

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***Abstract*—Faster convergence is always sought in many applications. Designing fixed-time control has recently gained much attention since, for this type of control structure, the convergence time of the states does not depend on initial conditions, unlike other control methods providing faster convergence. This paper proposes a new distributed algorithm for second-order consensus in multiagent systems by using a full-order fixed-time convergent sliding surface. The stability analysis is performed using the Lyapunov function and bi-homogenous property. Moreover, the proposed control is smooth and free from any singularity. The robustness of the proposed scheme is verified both in the presence of Lipschitz disturbances and uncertainties in the network. The proposed method is compared with a state-of-the-art method to show the effectiveness.**

***Index Terms*—Consensus, fixed-time convergence, multiagent systems, robustness, sliding mode control (SMC).**

## I. Introduction

COOPERATIVE control in multiagent systems has found its applications in many areas, such as power grids, unmanned air vehicles [1], neural sensory networks, formation control, biological systems, etc. and has become a most popular topic of research. In order to reach a global objective, the important idea for cooperative control in multiagent systems is to design the distributed controllers on each agent by utilizing its local neighboring information [2], [3]. Usually, under a distributed local protocol, the agents can work cooperatively to attain a global goal. The main idea of cooperation means that the agents in multiagent systems not only share the information with their neighbors locally but also attempt to reach an agreement to a certain degree [4]. Typical collective global behaviors under cooperative control have various names in the literature, such as consensus, synchronization, flocking, or swarming [5]–[7]. The consensus problem basically focuses on how a group of autonomous agents can reach an agreement on position, velocity, or any other physical quantity of interest. First-order consensus has been widely discussed in the past decade, where the consensus within any network can be achieved with a switching topology if the network is properly connected oftentimes as the network evolves with time. In fact, in many practical applications, agents are controlled by both position and velocity states. Therefore, currently, much research has focused on the study of consensus in multiagent systems with second-order dynamics. In recent years, many interesting results have been reported in developing the second-order consensus in multiagent systems [8]–[11]. However, in all of these methods, the consensus achieved using the linear controllers is mainly asymptotic. However, in many practical applications, finite-time consensus is more suitable for more accuracy and a better convergence rate. Recently, the finite-time consensus in multiagent systems has received much attention [12]–[19]. However, in all of these finite-time consensus approaches, the convergence time to achieve the consensus still depends on the initial condition, that is, the farther the initial conditions are from equilibrium, the greater the convergence time.

Some practical applications need strict constraints on time response due to security reasons or to enhance the productiveness. For example, a missile or any aerial launch vehicle can be hugely affected by a strong wind gust deviating it from the desired trajectory yielding a large amount of initial tracking error. This motivated the researchers to focus on developing such controllers where the convergence time does not depend on initial conditions and a well-defined theoretical analysis is present in the literature about the so-called fixed-time convergence and thereafter fixed-time consensus [20]–[23]. In all of these fixed time consensus methods, only a first-order consensus has been considered, that is, only convergence in the position error is achieved. Recently, some of the works have reported toward achieving the second-order consensus in fixed time [24]–[28]. However, in most cases, only a second-order dynamics is considered for each agent. It is also important to have continuous robust control in order to realize its practicality. Most of the control strategies to obtain fixed-time consensus as discussed earlier are discontinuous in nature and, hence, impose significant questions about their practicality. In [29], a fixed-time consensus strategy is proposed by Zuo *et al.* for the multiagent systems having higher-order

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integrator dynamics with discontinuous control. Moreover, the analysis of the control cost along with the convergence time is necessary while discussing fixed-time convergence which was missing in earlier cases.

Physical plants are often affected by various disturbances and uncertainties. The performance of linear controllers deteriorates in the presence of such anomalies and there is always a need to design robust controllers for such plants, where the nature of the disturbance is not actually known. Sliding mode control (SMC) is a nonlinear robust control strategy and easier to implement, where the control input is actually discontinuous. This, in fact, is a demerit in many mechanical plants, where this discontinuity leads to high-frequency switching that is undesirable. Therefore, recent research in the SMC community deals with designing continuous robust control strategies. It is worth mentioning that the convergence achieved in SMC during sliding can be either asymptotic or in finite time [30], depending on the selection of the surface. Furthermore, it mainly depends on the initial conditions of the states.

The main contribution of this paper lies in proposing a novel decoupled distributed continuous SMC providing fixed-time second-order consensus for multiagent systems. The important feature of the proposed method is that it can be generalized to arbitrary-order agent dynamics. An interesting fact about the proposed formulation is that the states of the agents are coupled on the sliding surface. However, the sliding-mode states are decoupled out of this surface, which makes the design of SMC independent of the extent of coupling between the agents. The proposed surface will provide fixed-time consensus, that is, independency to the initial condition. The robustness of the proposed controller is demonstrated in the presence of Lipschitz disturbance in agent dynamics and uncertainties in the network structure. Moreover, the efficacy of the proposed scheme is compared with one of the state-of-the-art techniques and the average control effort is found to be comparatively less in the proposed scheme.

The rest of this paper is organized as follows. Some basic definitions and notations are described in Section II. In Section III, some preliminary concepts about graph theory are discussed. The second-order dynamics for each agent is described in Section IV along with the problem formulation. Section V talks about a novel fixed-time convergent full-order sliding surface and the fixed-time convergence is proven via bi-homogeneity. In Section VI, the control is formulated to ensure that the trajectories reach the proposed surface in finite time. Simulation results are shown along with a thorough comparison with the existing method in Section VII. Section VIII focuses on the concluding remarks and future direction of this paper.

## II. Notations and Definitions

In this paper the following notations are used.

$\text{sig}(\cdot) : \mathbb{R}^k \to \mathbb{R}^k$ is an odd function and can be written as

$$\text{sig}(z)^\alpha = \left[|z_1|^\alpha \text{sgn}(z_1) \;\; |z_2|^\alpha \text{sgn}(z_2) \;\; \cdots \;\; |z_n|^\alpha \text{sgn}(z_n)\right]^T \quad (1)$$

with $\alpha > 0$, where sgn is the signum function and can be defined as

$$\text{sgn}(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0. \end{cases}$$

Consider the following autonomous nonlinear system:

$$\dot{\sigma}(t) = f(\sigma(t)), \sigma(0) = \sigma_0 \quad (2)$$

where $\sigma \in \mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function such that $f(0) = 0$, that is, the origin $\sigma = 0$ is an equilibrium point (2). The following standard definitions are introduced.

*Definition 1 (Global Finite-Time Stability):* The equilibrium point of the system (2) is *globally finite-time stable* if it is globally asymptotically stable and any solution $\sigma(t, \sigma_0)$ of that system reaches the equilibrium point at some finite time, that is, $\forall t \geq T(\sigma_0) : \sigma(t, \sigma_0) = 0$, where $T : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$.

*Definition 2 (Fixed-Time Stability [31], [32]):* The equilibrium point of the system (2) is *fixed-time stable* if it is globally finite-time stable and the settling time is bounded, that is, $\exists\, T_{\max} > 0 : \forall \sigma_0 \in \mathbb{R}^n$ and $T(\sigma_0) \leq T_{\max}$.

In the case of fixed-time stable equilibrium, if the time of convergence is bounded by some finite-time $T_{\max}$ irrespective of any initial condition, it is also bounded by $\lambda T_{\max}$ for all $\lambda \geq 1$, where $\lambda \in \mathbb{R}$.

## III. Preliminaries

Graph theory plays a very important role in the study of multiagent systems. Some basic concepts and results about graph theory are introduced briefly in the following paragraph.

The adjacency matrix $\mathcal{A}(\mathcal{G}) = [a_{ij}]$ of an undirected and unweighted graph $\mathcal{G}$ is defined as $a_{ij} = 1$ if there is a link between nodes $i$ and $j$, else $a_{ij} = 0$. This will generate a symmetric matrix. $D(\mathcal{G}) = [\gamma_{ij}]$ is used to represent the degree matrix of $\mathcal{G}$ that is an $N \times N$ diagonal matrix, with the positive scalar $\gamma_{ii}$ being the degree of node $i$. The Laplacian of $\mathcal{G}$ is defined as $G(\mathcal{G}) = D(\mathcal{G}) - \mathcal{A}(\mathcal{G})$. The Laplacian matrix is a zero-row sum matrix and, thus, its smallest eigenvalue is 0 and the corresponding eigenvector is 1. $G(\mathcal{G})$ is a rank-deficient matrix and positive semidefinite always. The Laplacian matrix $G = (g_{ij})_{N\times N}$ can be denoted as $g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}, g_{ij} = -a_{ij}, i \neq j$ and satisfies the diffusion property, that is, $\sum_{j=1}^{N} g_{ij} = 0$.

## IV. Problem Formulation

The multiagent system under consideration is

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = f(x_i, v_i, t) + d_i(x_i, v_i, t) + b_i(x_i, v_i, t)u_i(t) \end{cases} \quad (3)$$

where $x_i \in \Re^n$ and $v_i \in \Re^n$ $i = 1, 2, \ldots, N$. $d_i(x_i, v_i, t) \in \Re^n$ refers to any kind of parametric uncertainty or external added disturbance with $b_i(x_i, v_i, t) \in \Re \neq 0$. Both $f(x_i, v_i, t)$ and $b_i(x_i, v_i, t)$ are two smooth known nonlinear functions of $x_i$, $v_i$, and $t$. The main objective is to design a control $u_i$ for each agent which can drive the states to the consensus in fixed time irrespective of any initial conditions. The control objectives are described as

$$\lim_{t\to T} ||x_i(t) - x_j(t)|| = 0$$
$$||x_i(t) - x_j(t)|| = 0, \quad \forall t \geq T$$

and

$$\lim_{t\to T} ||v_i(t) - v_j(t)|| = 0$$
$$||v_i(t) - v_j(t)|| = 0, \quad \forall t \geq T$$

where $T$ is the convergence time and finite. Moreover, $T$ is independent of the initial conditions.

*Assumption 1:* The disturbances/uncertainties acted on the above systems are considered to be Lipschitz in nature, i.e.,

$$||d_i(x_i, v_i, t)||_\infty \leq p_i$$
$$||\dot{d}_i(x_i, v_i, t)||_\infty \leq q_i$$

where $p_i, q_i > 0$ are known constants with $||\cdot||_\infty$ being the infinite norm of any vector.

*Lemma 1:* Let us consider an $n$-dimensional system

$$\dot{y} = g(y) \tag{4}$$

where $y = [y_1, y_2, \ldots, y_n]^T$ and $g(y) = [g_1(y), g_1(y), \ldots, g_n(y)]^T$. If the system (4) has homogenous degree $\gamma_1$ with dilation $(a_1, a_2, \ldots, a_n)$ and homogenous degree $\gamma_2$ with dilation $(b_1, b_2, \ldots, b_n)$ and with a nonlinear continuous function $g$, then $y = 0$ is the asymptotic equilibrium of the system. If homogeneity degrees $\gamma_1 < 0$ in 0-limit and $\gamma_2 > 0$ in $\infty$-limit, then the system is said to be bihomogenous and by satisfying such property, the equilibrium point is considered to be globally fixed-time stable [31]–[33].

## V. Full Order Fixed-Time Sliding Surface

The first task is to design a stable fixed-time converging sliding surface as

$$\sigma_i = \ddot{x}_i - \sum_{j=1}^{N} a_{ij}\Big[\varphi_1\big(\mathrm{sig}(x_j - x_i)^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}(x_j - x_i)^{\beta_1}\Big) + \varphi_3\big(\mathrm{sig}(v_j - v_i)^{\alpha_2}\big) + \varphi_4 \mathrm{sig}(v_j - v_i)^{\beta_2}\Big)\Big] \tag{5}$$

where $0 < \alpha_1, \alpha_2 < 1$ and $\beta_1, \beta_2 > 1$ with $\alpha_2 = (2\alpha_1/[1+\alpha_1])$ and $\beta_2 = (2\beta_1/[1+\beta_1])$. $\varphi_1$, $\varphi_2$, $\varphi_3$, and $\varphi_4$ are four continuous odd functions satisfying $y_i^T \varphi_k(y_i) > 0$ if $y_i \neq 0$ and $\varphi_k(y_i) = c_{ki} y_i + \mathcal{O}(y_i)$ around $y_i = 0$; where $\mathcal{O}$ is the complexity function. Moreover, $\varphi_k(y_i) = [\varphi_k(y_{i1}), \varphi_k(y_{i2}), \ldots, \varphi_k(y_{in})]^T$ if $y_i = [y_{i1}, y_{i2}, \ldots, y_{in}]^T$ with $y_i \in \mathbb{R}^n$.

*Theorem 1:* If the states of the system (3) can reach the designed sliding surface (5), then second-order consensus can be achieved in fixed time.

*Proof:* When sliding is achieved, $\sigma_i = 0$. Therefore, the system dynamics given in (3) can be represented as

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = \hat{u}_i, \quad \text{for } i = 1, \ldots, N \end{cases} \tag{6}$$

where $\hat{u}_i = \sum_{j=1}^N a_{ij}[\varphi_1(\mathrm{sig}(x_j - x_i)^{\alpha_1}) + \varphi_2(\mathrm{sig}(x_j - x_i)^{\beta_1}) + \varphi_3(\mathrm{sig}(v_j - v_i)^{\alpha_2}) + \varphi_4(\mathrm{sig}(v_j - v_i)^{\beta_2})]$. The above dynamics (6) can be equivalently represented as the fixed-time convergence of error dynamics of the system as follows:

$$\begin{cases} \dot{e}_{xi} = e_{vi} \\ \dot{e}_{vi} = \hat{u}_i - \check{u}, \quad \text{for } i = 1, \ldots, N \end{cases} \tag{7}$$

where $e_{xi} = [e_{xi1}, e_{xi2}, \ldots, e_{xin}]^T = x_i - (1/N)\sum_{j=1}^N x_j$, $e_{vi} = [e_{vi1}, e_{vi2}, \ldots, e_{vin}]^T = v_i - (1/N)\sum_{j=1}^N v_j$ and $\check{u} = (1/N)\sum_{j=1}^N \hat{u}_j$. The equivalent control $\hat{u}_i$ can be written in terms of error dynamics as

$$\hat{u}_i = \sum_{j=1}^{N} a_{ij}\Big[\varphi_1\big(\mathrm{sig}(e_{xj} - e_{xi})^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}(e_{xj} - e_{xi})^{\beta_1}\Big) + \varphi_3\big(\mathrm{sig}(e_{vj} - e_{vi})^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}(e_{vj} - e_{vi})^{\beta_2}\Big)\Big]. \tag{8}$$

As $\varphi_1$, $\varphi_2$, $\varphi_3$, and $\varphi_4$ are odd functions

$$\check{u}_i = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\Big[\varphi_1\big(\mathrm{sig}(x_j - x_i)^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}(x_j - x_i)^{\beta_1}\Big) + \varphi_3\big(\mathrm{sig}(v_j - v_i)^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}(v_j - v_i)^{\beta_2}\Big)\Big] = 0. \tag{9}$$

Let us define a Lyapunov function as

$$V = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{n}\int_0^{e_{xik}-e_{xjk}} a_{ij}\big[\varphi_1\big(\mathrm{sig}(z)^{\alpha_1}\big) + \varphi_2\big(\mathrm{sig}(z)^{\beta_1}\big)\big]dz + \frac{1}{2}\sum_{i=1}^{N} e_{vi}^T e_{vi}. \tag{10}$$

Taking the derivative of the above function, we have

$$\dot{V} = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} e_{vi}^T\Big[\varphi_1\big(\mathrm{sig}(e_{xi} - e_{xj})^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}(e_{xi} - e_{xj})^{\beta_1}\Big)\Big] + \sum_{i=1}^{N} e_{vi}^T \dot{e}_{vi}. \tag{11}$$

The solution to the above differential equation (11) with a nondifferentiable continuous right-hand side can be understood in Filippov sense [34]

$$\dot{V} = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} e_{vi}^T\Big[\varphi_1\big(\mathrm{sig}(e_{xi} - e_{xj})^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}(e_{xi} - e_{xj})^{\beta_1}\Big)\Big] + \sum_{i=1}^{N} e_{vi}^T \sum_{j=1}^{N} a_{ij}\Big[\varphi_1\big(\mathrm{sig}(e_{xj} - e_{xi})^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}(e_{xj} - e_{xi})^{\beta_1}\Big) + \varphi_3\big(\mathrm{sig}(e_{vj} - e_{vi})^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}(e_{vj} - e_{vi})^{\beta_2}\Big)\Big]. \tag{12}$$

Since $\varphi_1$ and $\varphi_2$ are odd functions, $\varphi_1(\mathrm{sig}(e_{xi} - e_{xj})^{\alpha_1}) = -\varphi_1(\mathrm{sig}(e_{xj} - e_{xi})^{\alpha_1})$ and $\varphi_2(\mathrm{sig}(e_{xi} - e_{xj})^{\beta_1}) = -\varphi_2(\mathrm{sig}(e_{xj} -$

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$e_{xi})^{\beta_1}$). Therefore, (12) can be rewritten as

$$\dot{V} = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{vi}^{T}[\varphi_3(\mathrm{sig}(e_{vj}-e_{vi})^{\alpha_2}) + \varphi_4(\mathrm{sig}(e_{vj}-e_{vi})^{\beta_2})]. \tag{13}$$

It can also be seen that since $a_{ij} = a_{ji}$, (13) can be further simplified as

$$\dot{V} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(a_{ij}+a_{ji})e_{vi}^{T}\Big[\varphi_3\big(\mathrm{sig}\big(e_{vj}-e_{vi}\big)^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}\big(e_{vj}-e_{vi}\big)^{\beta_2}\Big)\Big] \tag{14}$$

$$\dot{V} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\big(e_{vi}-e_{vj}\big)^{T}a_{ij}\Big[\varphi_3\big(\mathrm{sig}\big(e_{vj}-e_{vi}\big)^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}\big(e_{vj}-e_{vi}\big)^{\beta_2}\Big)\Big] \tag{15}$$

$$\dot{V} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{n}\big(e_{vik}-e_{vjk}\big)^{T}a_{ij}\Big[\varphi_3\big(\mathrm{sig}\big(e_{vjk}-e_{vik}\big)^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}\big(e_{vjk}-e_{vik}\big)^{\beta_2}\Big)\Big]. \tag{16}$$

Since $\varphi_3$ and $\varphi_4$ are odd functions, we have

1) $\forall e_{vik}-e_{vjk} \neq 0$, $(e_{vik}-e_{vjk})\varphi_3(\mathrm{sig}(e_{vjk}-e_{vik})^{\alpha_2}) = -|e_{vjk}-e_{vik}|^{\alpha_2+1} < 0$ and $(e_{vik}-e_{vjk})\varphi_4(\mathrm{sig}(e_{vjk}-e_{vik})^{\beta_2}) = -|e_{vjk}-e_{vik}|^{\beta_2+1} < 0$;
2) when $e_{vik}-e_{vjk} = 0$, $(e_{vik}-e_{vjk})\varphi_3(\mathrm{sig}(e_{vjk}-e_{vik})^{\alpha_2}) < 0$ and $(e_{vik}-e_{vjk})\varphi_4(\mathrm{sig}(e_{vjk}-e_{vik})^{\beta_2}) < 0$.

Therefore, $\dot{V} < 0$ is always satisfied. It can be seen from (16) that if $\dot{V} = 0$, then $e_{vi} = e_{vj}, \forall j \neq i$. When $e_{vi} = e_{vj}$, the equivalent control becomes

$$\hat{u}_i = \sum_{j=1}^{N} a_{ij}\Big[\varphi_1\big(\mathrm{sig}\big(e_{xj}-e_{xi}\big)^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}\big(e_{xj}-e_{xi}\big)^{\beta_1}\Big)\Big]. \tag{17}$$

As the connectivity graph is undirected ($a_{ij} = a_{ji}$), we have $\sum_{i=1}^{N}\hat{u}_i = 0$. That leads to $\sum_{i=1}^{N} e_{xi}^{T}\sum_{j=1}^{N} a_{ij}[\varphi_1(\mathrm{sig}(e_{xj}-e_{xi})^{\alpha_1}) + \varphi_2(\mathrm{sig}(e_{xj}-e_{xi})^{\beta_1})] = 0$, which, in turn, leads to $(1/2)\sum_{i=1}^{N}(e_{xi}-e_{xj})^{T}\sum_{j=1}^{N} a_{ij}[\varphi_1(\mathrm{sig}(e_{xj}-e_{xi})^{\alpha_1}) + \varphi_2(\mathrm{sig}(e_{xj}-e_{xi})^{\beta_1})] = 0$.

As $e_{xi}-e_{xj} \to 0$, $e_{vi}-e_{vj} \to 0$ as $\dot{V} < 0$ when $t \to \infty$; therefore, $x_i - x_j \to 0$, $v_i - v_j \to 0$ as $t \to \infty$ $\forall i, j = 1, 2, \ldots N$.

The system (6) with variables $[(x_1)^T \cdots (x_N)^T, (v_1)^T \cdots (v_N)^T]^T$ has a homogenous degree of $\gamma_1 = \alpha_1 - 1 < 0$ with dilation $(2, \ldots, 2,\ 1+\alpha_1, \ldots, 1+\alpha_1)$ and has a homogenous degree of $\gamma_2 = \beta_1 - 1 > 0$ with dilation $(2, \ldots, 2,\ 1+\beta_1, \ldots, 1+\beta_1)$ since system (7) is globally asymptotically stable and locally bihomogeneous. Therefore, according to Lemma 1, the above system is globally fixed-time stable and the second-order consensus can be achieved in fixed time on the designed sliding surface (5). This completes the proof. ■

*Remark 1:* The main reason for adding some terms in the surface (5) with a power greater than one is to make the convergence faster when the initial conditions are far from the origin. Therefore, when initial conditions of the states are far away from the origin, these terms with a power greater than one will be dominating and when the states converge toward the origin, the terms with a power less than one will be dominating. Hence, the convergence is achieved in fixed time [35], [36].

## VI. Design of Control

The control for system (3) is designed to be of form

$$u_i(t) = b_i^{-1}(x_i, v_i, t)[u_{ni}(t) + u_{ei}(t)] \tag{18}$$

where

$$u_{ei}(t) = -f(x_i, v_i, t) + \sum_{j=1}^{N} a_{ij}\Big[\varphi_1\big(\mathrm{sig}\big(x_j-x_i\big)^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}\big(x_j-x_i\big)^{\beta_1}\Big) + \varphi_3\big(\mathrm{sig}\big(v_j-v_i\big)^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}\big(v_j-v_i\big)^{\beta_2}\Big)\Big] \tag{19}$$

and

$$\dot{u}_{ni}(t) + m_i u_{ni}(t) = -k_i \mathrm{sgn}(\sigma_i(t)) \tag{20}$$

where $m_i > 0$, $k_i > 0$ with $k_i > q_i + p_i m_i + \zeta_i$, $\zeta_i > 0$.

*Theorem 2:* Under Assumption 1, the proposed control (18) can bring the states of the multiagent system (3) to reach the sliding surface in finite time, leading to achieve second-order consensus in fixed time along the surface (5).

*Proof:* We will analyze the following proof mainly during the reaching phase.

Substituting the control as given by (18) in the surface dynamics (5) for the multiagent system as described by (3), we have

$$\sigma_i = \ddot{x}_i - \sum_{j=1}^{N} a_{ij}\Big[\varphi_1\big(\mathrm{sig}\big(x_j-x_i\big)^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}\big(x_j-x_i\big)^{\beta_1}\Big) + \varphi_3\big(\mathrm{sig}\big(v_j-v_i\big)^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}\big(v_j-v_i\big)^{\beta_2}\Big)\Big]$$

$$\sigma_i = f(x_i, v_i, t) + d_i(x_i, v_i, t) + b_i(x_i, v_i, t)u_i(t) - \sum_{j=1}^{N} a_{ij}\Big[\varphi_1\big(\mathrm{sig}\big(x_j-x_i\big)^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}\big(x_j-x_i\big)^{\beta_1}\Big) + \varphi_3\big(\mathrm{sig}\big(v_j-v_i\big)^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}\big(v_j-v_i\big)^{\beta_2}\Big)\Big]$$

$$\sigma_i = d_i(x_i, v_i, t) - m_i u_{ni}(t) - k_i \mathrm{sgn}(\sigma_i(t)). \tag{21}$$

Now taking the first derivative of (21)

$$\dot{\sigma}_i = \dot{d}_i(x_i, v_i, t) + \dot{u}_{ni}(t). \tag{22}$$

Substituting $u_{ni}(t)$ in (22)

$$\dot{\sigma}_i = \dot{d}_i(x_i, v_i, t) - m_i u_{ni}(t) - k_i \mathrm{sgn}(\sigma_i(t)). \tag{23}$$

It can be clearly seen that

$$\dot{u}_{ni}(t) + m_i u_{ni}(t) = -k_i \mathrm{sgn}(d_i(x_i, v_i, t) + u_{ni}(t)) \tag{24}$$

which can be further written as

$$\frac{d}{dt}(u_{ni} + d_i) + m_i(u_{ni} + d_i) = -k_i \mathrm{sgn}(\sigma_i) + \dot{d}_i + m_i d_i \tag{25}$$

which can be written as

$$\frac{d}{dt}\left[e^{m_i t}(u_{ni}+d_i)\right] = e^{m_i t}[-k_i \mathrm{sgn}(u_{ni}+d_i) + \dot{d}_i + m_i d_i]. \tag{26}$$

When $u_{ni} + d_i > 0$, it can be seen from (26) that $(d/dt)[e^{m_i t}(u_{ni}+d_i)] < 0$ as $k_i > q_i + p_i m_i + \zeta_i$ and $\mathrm{sgn}(u_{ni} + d_i) = 1$. Therefore, $\sigma_i$ will start to decay toward $\sigma_i = 0$, that is, toward the sliding surface. Similarly, when $u_{ni} + d_i < 0$, it can be again verified from (26) that $(d/dt)[e^{m_i t}(u_{ni}+d_i)] > 0$ as $k_i > q_i + p_i m_i + \zeta_i$ and $\mathrm{sgn}(u_{ni} + d_i) = -1$. Therefore, $\sigma_i$ will again start to decay toward $\sigma_i = 0$. The most important observation here is that whenever $\sigma_i$, that is, $u_{ni}+d_i$ is nonzero, the control dynamics (18) forces $\sigma_i$ dynamics to converge toward zero. Now, it is important to prove that the convergence is achieved in finite time. Let us define a Lyapunov function as

$$V = \frac{1}{2}\sum_{i=1}^{n} \sigma_i^T(t)\sigma_i(t). \tag{27}$$

Taking the first derivative of (27)

$$\dot{V} = \sum_{i=1}^{n} \sigma_i^T(t)\dot{\sigma}_i(t). \tag{28}$$

From (23), it can be verified that

$$\sigma_i^T(t)\dot{\sigma}_i(t) = \sigma_i^T(t)\dot{d}_i(t) - \sigma_i^T(t)m_i u_{ni}(t) - k_i|\sigma_i(t)| \tag{29}$$

which further can be simplified as

$$\begin{aligned}\sigma_i^T(t)\dot{\sigma}_i(t) \le &\left[\sigma_i^T(t)\dot{d}_i(t) - q_i|\sigma_i(t)|\right] \\ &+ \left[-\sigma_i^T(t)m_i u_{ni}(t) + m_i p_i|\sigma_i(t)|\right] \\ &- (k_i - q_i - p_i m_i)|\sigma_i(t)|.\end{aligned} \tag{30}$$

Substituting (30) in (28), we have

$$\dot{V} \le -\sum_{i=1}^{n} \zeta_i|\sigma_i(t)|, \text{ as } k_i > q_i + p_i m_i + \zeta_i \tag{31}$$

$$\dot{V} \le -\zeta_{\min}\sum_{i=1}^{n}\sum_{j=1}^{n} |\sigma_{ij}(t)| \tag{32}$$

$$\le -\zeta_{\min}\sqrt{V}. \tag{33}$$

Therefore, for the system as described by (3), convergence to the sliding surface $\sigma_i$ can be achieved in finite time.

It can be clearly seen that if we choose

$$\dot{u}_{ni}(t) = -k_i \mathrm{sgn}(\sigma_i(t)) \tag{34}$$

where $k_i > q_i + \zeta_i$, $\zeta_i > 0$, then Theorem 2 can be easily proved in an usual way. Adding an extra term $m_i u_{ni}(t)$ makes the control be a low-pass filter which helps to smoothen out the effects of disturbances and makes the convergence faster. However, from (23), it can be seen that a negative feedback of control $u_{ni}$ is not a good idea in actual practise. Therefore, if we just modify the dynamics of $u_{ni}$ as

$$\dot{u}_{ni}(t) = -m_i\sigma_i(t) - k_i \mathrm{sgn}(\sigma_i(t)) \tag{35}$$

where $m_i > 0$, $k_i > 0$ with $k_i > q_i + \zeta_i$, $\zeta_i > 0$, then with the above control, the analysis will become simpler as described below.

Let us analyze the Lyapunov function

$$\dot{V} = \sum_{i=1}^{n} \sigma_i^T(t)\dot{\sigma}_i(t). \tag{36}$$

Using (22), we can further write

$$\dot{V} = \sum_{i=1}^{n} \sigma_i^T(t)\left[\dot{d}_i(x_i, v_i, t) + \dot{u}_{ni}(t)\right]. \tag{37}$$

Now substituting (35) in the above

$$\dot{V} \le -\sum_{i=1}^{n} \zeta_i|\sigma_i(t)| - m_i\sigma_i^T(t)\sigma_i(t) \tag{38}$$

$$\le -\zeta_{\min}\sqrt{V}. \tag{39}$$

After the trajectories reach the surface $\sigma_i$, the consensus can be achieved in fixed time as described in Theorem 1. This completes the proof. ■

*Remark 2:* If we analyze (18), all of the terms in the control $u_i(t)$ are known except the term $\sigma_i(t)$ as it contains the term $\ddot{x}_i$, which contains the uncertainties. For calculating $\mathrm{sgn}(\sigma_i(t))$ in (18), let us define a function $\delta_i(t)$ as

$$\delta_i(t) = \int_0^t \sigma_i(t)\, d\upsilon \tag{40}$$

$$\begin{aligned}\delta(t) = \dot{x}_i - \int_0^t \sum_{j=1}^{N} a_{ij}\Big[&\varphi_1\big(\mathrm{sig}(x_j - x_i)^{\alpha_1}\big) + \varphi_2\Big(\mathrm{sig}\big(x_j - x_i\big)^{\beta_1}\Big) \\ &+ \varphi_3\big(\mathrm{sig}(v_j - v_i)^{\alpha_2}\big) + \varphi_4\Big(\mathrm{sig}\big(v_j - v_i\big)^{\beta_2}\Big)\Big]d\upsilon.\end{aligned} \tag{41}$$

$\mathrm{sgn}(\sigma_i(t))$ can be obtained by simply equating $\mathrm{sgn}(\sigma_i) = \mathrm{sgn}(\delta(t) - \delta(t-\tau))$, where $\tau$ is a time delay. Since

$$\sigma_i = \lim_{\tau\to 0}\frac{(\delta(t) - \delta(t-\tau))}{\tau}.$$

The fundamental sample time can be easily chosen as $\tau$. The interesting fact about the above analysis is that we only need to know the value of $\mathrm{sgn}(\sigma_i)$, that is, whether $\delta(t)$ increases or decreases, which is much easier to obtain than the exact value of $\sigma_i$.

## VII. Simulation Results

Consider each agent with system dynamics as

$$\begin{cases}\dot{x}_i = v_i \\ \dot{v}_i = v_i^3 + u_i + d_i, \; i = 1, 2 \ldots 5\end{cases} \tag{42}$$

where $d_i = \sin 30t$ as a Lipschitz continuous and bounded disturbance. The control input $u_i$ is as described in (18) with

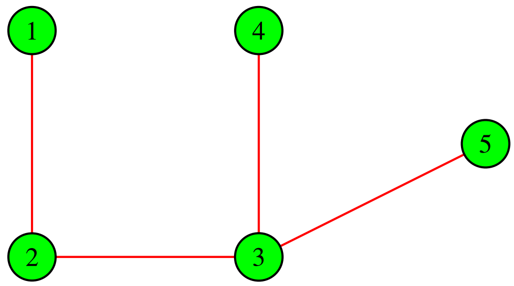


Fig. 1. Undirected graph with five agents.

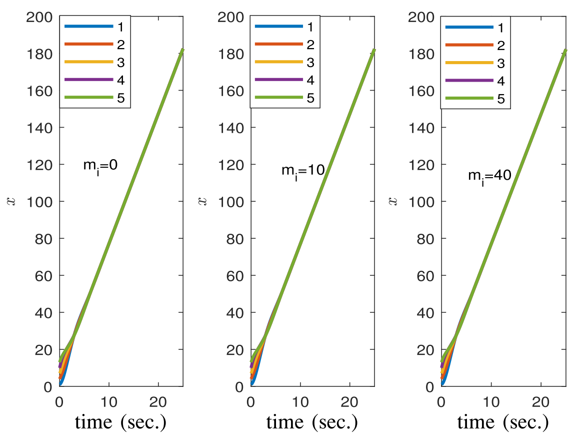


Fig. 2. Trajectory of position $x_i$ with respect to $t$ for various $m_i$ under the influence of Lipschitz disturbance.

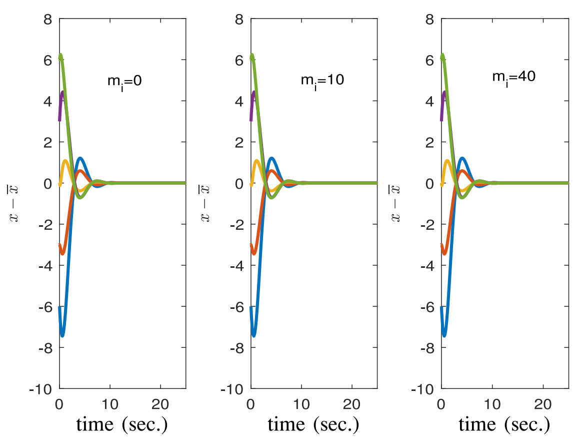


Fig. 3. Trajectory of position error $x_i - \bar{x}$ with respect to $t$ for various $m_i$ under the influence of Lipschitz disturbance.

$\alpha_1 = 0.7, \beta_1 = 1.2$ and $\alpha_2 = 0.8235, \beta_2 = 1.0909$. Each agent is connected to each other by the graph in Fig. 1.

The adjacency matrix for the graph shown in Fig. 1 can be written as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

The control gain is $k_i > q_i + p_i m_i + \zeta_i$. The definitions for $q_i, p_i$ are given in Assumption 1. In this example, $q_i = 30$, $p_i = 1$, and $\zeta_i = 1$. Moreover,

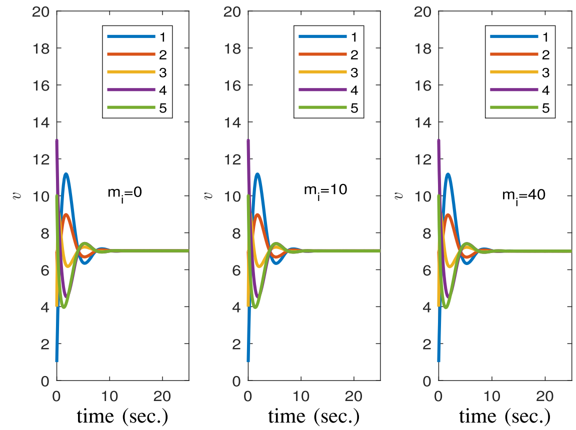


Fig. 4. Trajectory of velocity $v_i$ with respect to $t$ for various $m_i$ under the influence of Lipschitz disturbance.

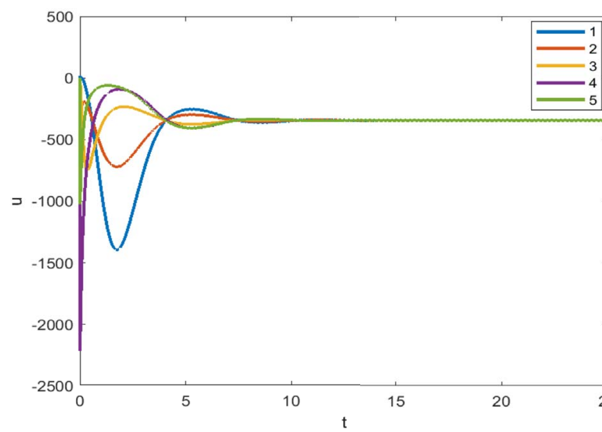


Fig. 5. Trajectory of control $u_i$ with respect to $t$ when $m_i = 40$, using (18) under the influence of Lipschitz disturbance.

$\bar{x} = \sum_{j=1}^{N}(x_j/N), N = 5$ in our simulations. The simulations are carried out for various values of $m_i$ to verify the influence of this filtering coefficient ($m_i$) on control performance.

Fig. 2 represents the evolution of position for various agents with respect to time. It can be clearly seen from Fig. 2 that the consensus is achieved within 11.12 s even in the presence of a Lipschitz disturbance.

The corresponding position error trajectories for various agents are shown in Fig. 3. It is interesting to note that irrespective of various initial conditions for five agents, the convergence can be achieved in 11.12 s. It can be clearly seen that the change in filtering coefficient ($m_i$) has almost no effects in terms of achieving consensus. The most important aspect of the proposed control is that it will provide a second-order consensus, that is, the velocity $v$ for each agent will also achieve consensus (see Fig. 4). The corresponding control (Fig. 5) to achieve fixed-time consensus is smooth and continuous. The proposed control can bring all of the state trajectories to the sliding surface $\sigma$ in finite time and can be clearly seen in Fig. 6 irrespective of any filtering coefficient $m_i$.

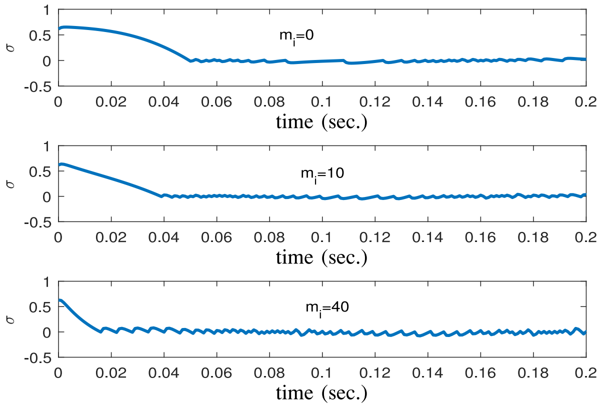


Fig. 6. Trajectory of sliding surface $\sigma$ with respect to $t$ for various $m_i$, using (5) under the influence of Lipschitz disturbance.

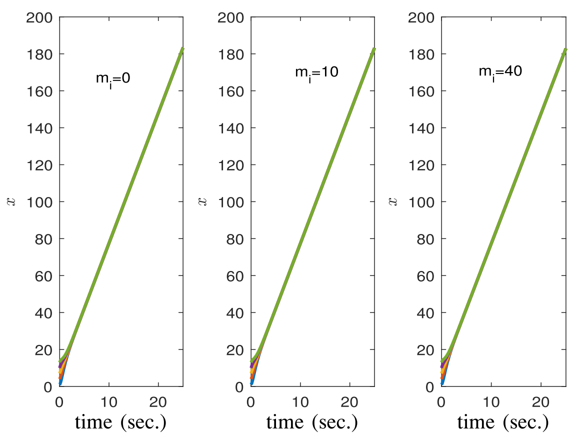


Fig. 7. Trajectory of position $x_i$ with respect to $t$ for various $m_i$ under the influence of Lipschitz disturbance with weighted links.

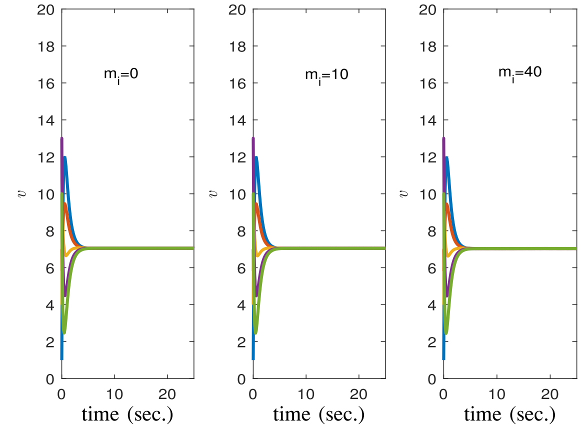


Fig. 8. Trajectory of velocity $v_i$ with respect to $t$ for various $m_i$ under the influence of Lipschitz disturbance with weighted links.

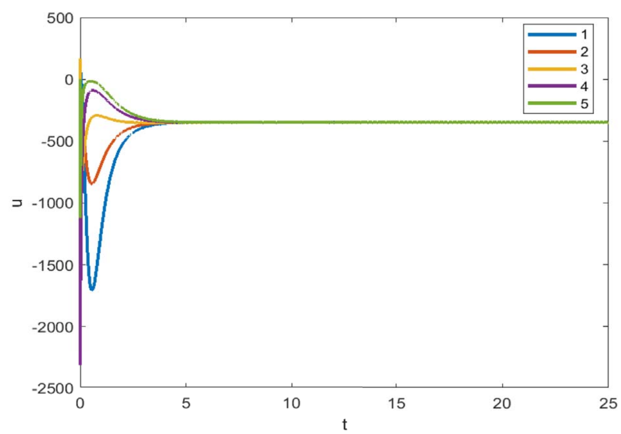


Fig. 9. Trajectory of control $u_i$ with respect to $t$ when $m_i = 40$, using (18) under the influence of Lipschitz disturbance with weighted links.

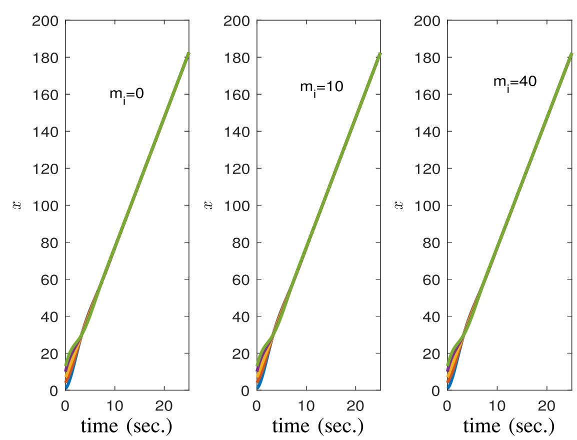


Fig. 10. Trajectory of position $x_i$ with respect to $t$ for various $m_i$ under the influence of Lipschitz disturbance with links reassignment.

## *A. Robustness Testing*

*1) Uncertainties in Weight of the Edges:* The robustness of the proposed controller is tested by varying the weight assignment to various links. The adjacency matrix for the corresponding analysis can be written as

$$A = \begin{bmatrix} 0 & 5 & 0 & 0 & 0 \\ 5 & 0 & 5 & 0 & 0 \\ 0 & 5 & 0 & 5 & 5 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \end{bmatrix}.$$

As we can see, the weights are increased five times than the previous case. By keeping all of the simulation parameters similar, it can be seen that the consensus is still achieved in 11.12 s. (see Figs. 7–9).

*2) Uncertainties in Orientation of the Edges:* In this analysis, we have changed the graph structure as simulated before Fig. 1. The link between agent 3 and agent 5 is broken. A new link is established between agent 4 and agent 5 during the simulation. The corresponding adjacency matrix for the graph is given as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

TABLE I
SIMULATED CONVERGENCE TIME WITH CONTROL RANGE FOR VARIOUS AGENTS

| Agents | Finite-time Convergence Scheme [19] | | | | | Fixed-time Convergence Scheme (Proposed) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $(x_i-\bar{x})_{min}$ | $(x_i-\bar{x})_{max}$ | Setting Time (sec.) | Steady state error | $[u_{min}, u_{max}]$ | $(x_i-\bar{x})_{min}$ | $(x_i-\bar{x})_{max}$ | Setting Time (sec.) | Steady state error | $[u_{min}, u_{max}]$ |
| 1 | -9.5887 | 2.3458 | 22.47 | 0.0001 | [-1140.6,6.05] | -7.4492 | 1.2104 | 11.12 | 0 | [-1395.6,16.85] |
| 2 | -4.3242 | 1.1785 | 22.47 | 0.0001 | [-638.59,0.5] | -3.4533 | 0.5975 | 11.12 | 0 | [-722.74,0.5] |
| 3 | -0.7919 | 1.7984 | 22.47 | 0.0001 | [-666.87,0.5] | -0.3842 | 1.0922 | 11.12 | 0 | [-750.64,0.5] |
| 4 | -1.4810 | 6.3548 | 22.47 | 0.0001 | [-2206.9,0.5] | -0.7031 | 4.4336 | 11.12 | 0 | [-2221.6,0.5] |
| 5 | -1.2516 | 6.7802 | 22.47 | 0.0001 | [-1008.6,0.5] | -0.7230 | 6.2537 | 11.12 | 0 | [-1024.3,0.5] |

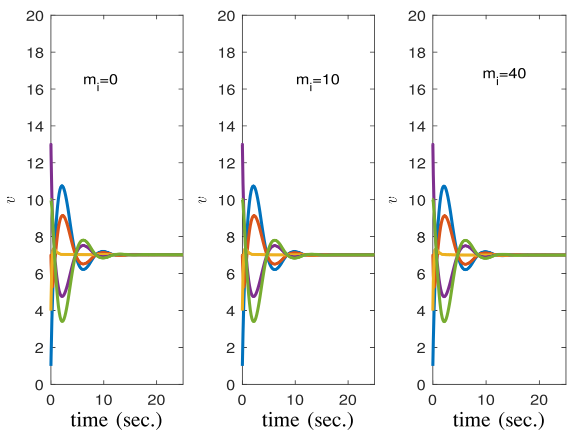


Fig. 11. Trajectory of velocity $v_i$ with respect to $t$ for various $m_i$ under the influence of Lipschitz disturbance with links reassignment.

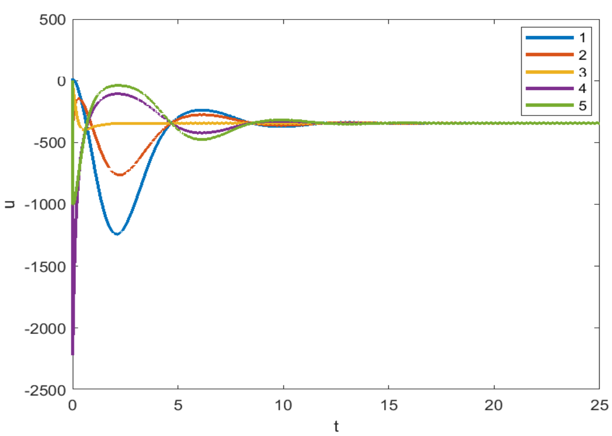


Fig. 12. Trajectory of control $u_i$ with respect to $t$ when $m_i = 40$, using (18) under the influence of Lipschitz disturbance with links reassignment.

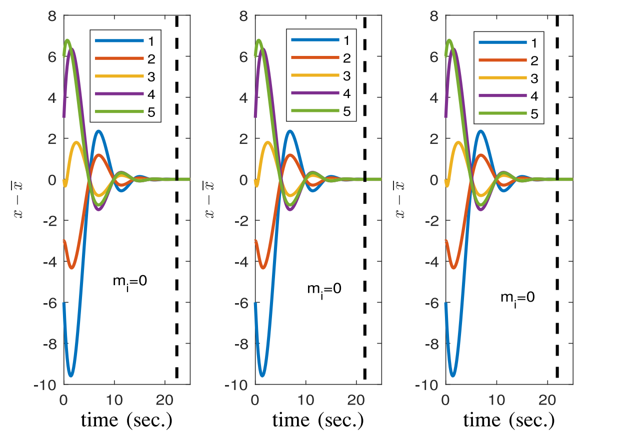


Fig. 13. Trajectory of position $x_i$ with respect to $t$ for various $m_i$ under the influence of Lipschitz disturbance as proposed in [19].

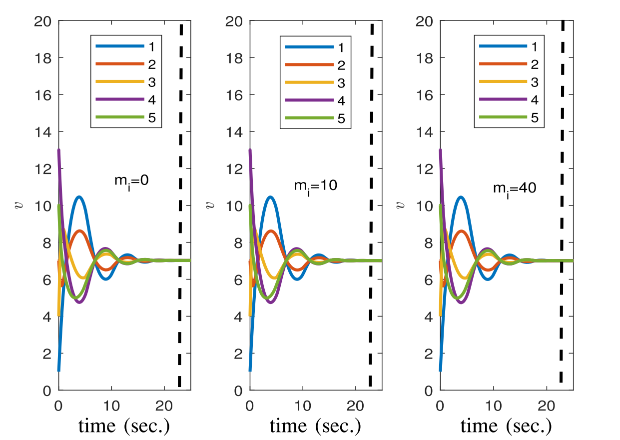


Fig. 14. Trajectory of velocity $v_i$ with respect to $t$ for various $m_i$ under the influence of Lipschitz disturbance as proposed in [19].

It can be seen from Figs. 10–12 that the proposed control scheme is robust.

### *B. Comparison With Existing Finite-Time Control*

In this section, we have carried out a comparison study with a state-of-the-art technique recently published. A detailed comparison is performed with the proposed method. Fig. 13 shows the evolution of position trajectory for various agents. It can be clearly seen that the consensus is achieved approximately 22 s which is almost twice than that of the proposed case. The corresponding velocity and control trajectories are shown in Figs. 14 and 15, respectively. A thorough comparison is made with the method [19] and is tabulated in Table I. It can be seen in the proposed case that the margin of error is much less. Moreover, the convergence time is approximately half (11.12 s) than previously reported. The steady-state accuracy is also improved. In order to achieve this, maximum control action is not too much compared to the previously reported case [19]. Furthermore, the average control effort in our proposed case is lower than that previously reported and can be seen from Table II.

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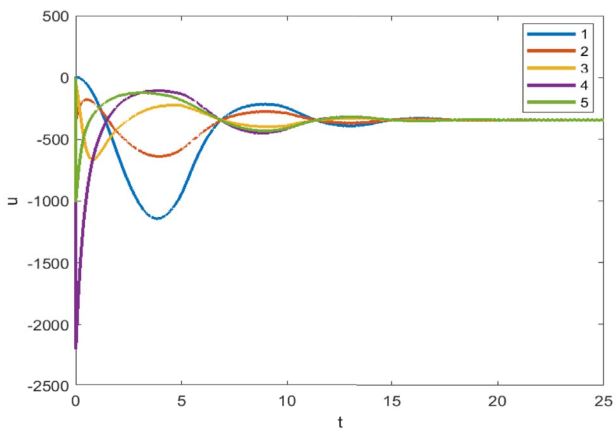


Fig. 15. Trajectory of control $u_i$ with respect to $t$ when $m_i = 40$ under the influence of Lipschitz disturbance as proposed in [19].

TABLE II
AVERAGE CONTROL EFFORT $||u||_2$ COMPARISON

| Agents | Reported [19]$||u||_2$ | Proposed $||u||_2$ |
|---|---|---|
| 1 | $7.7318 \times 10^4$ | $7.5398 \times 10^4$ |
| 2 | $6.0500 \times 10^4$ | $5.9939 \times 10^4$ |
| 3 | $5.6387 \times 10^4$ | $5.5663 \times 10^4$ |
| 4 | $6.3670 \times 10^4$ | $5.7671 \times 10^4$ |
| 5 | $5.2970 \times 10^4$ | $5.2648 \times 10^4$ |

## VIII. CONCLUSION

In this paper, a new decoupled fixed-time convergent scheme has been proposed with continuous control for second-order consensus for multiagent systems. A distributed full-order fixed-time convergent sliding surface has been designed based on the bihomogeneity property, under which the sliding-mode states are decoupled. The designed control has been applied on the decoupled sliding-mode states to ensure the trajectories reach the surface in finite time. Once the trajectories reach the surface, the control ensures that they will never leave the surface and the second-order consensus along the surface can be obtained in fixed time. The designed control in this paper is smooth and is free from any severe chattering, which is, in fact, a major challenge in the SMC design. The robustness of the proposed scheme is verified in the presence of Lipschitz disturbance and link uncertainties. Future work will mainly focus on the applications of the proposed control scheme to, in fact, more challenging multiagent systems with directed topologies. A further investigation will be carried out in making reaching phase insensitive to initial conditions.

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