Reinforcement Learning Basics

http://wangshusen.github.io/

Random Variable

- Random variable: unknown; its values depend on outcomes of random events.

Random Variable | *Possible Values* | *Random Events* | *Probabilities*

$$X = \begin{cases} 0 \\ 1 \end{cases}$$

$\mathbb{P}(X = 0) = 0.5$

$\mathbb{P}(X = 1) = 0.5$

Random Variable

- Random variable: unknown; its values depend on outcomes of random events.
- Uppercase letter X for a random variable.
- Lowercase letter x for an observed value.
- For example, I flipped a coin 4 times and observed:
  - $x_1 = 1$,
  - $x_2 = 1$,
  - $x_3 = 0$,
  - $x_4 = 1$.

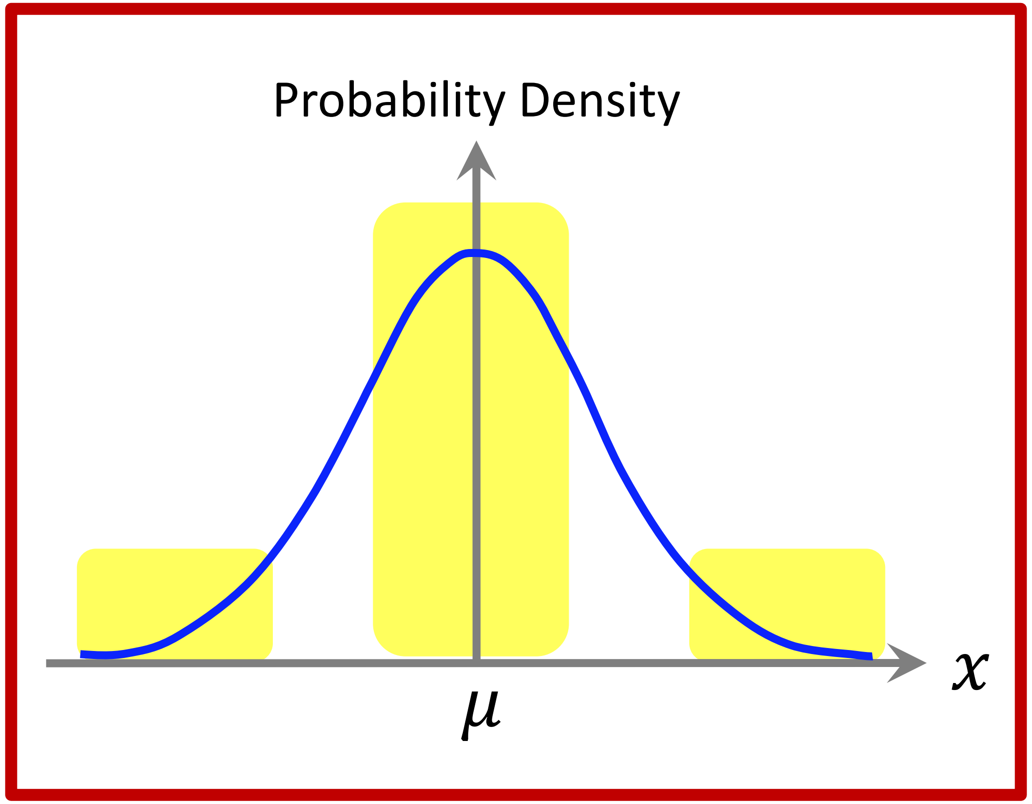
Probability Density Function (PDF)

- PDF provides a relative likelihood that the value of the random variable would equal that sample.

Example: Gaussian distribution

- It is a continuous distribution.
- PDF:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\,\sigma^2}\right).$$



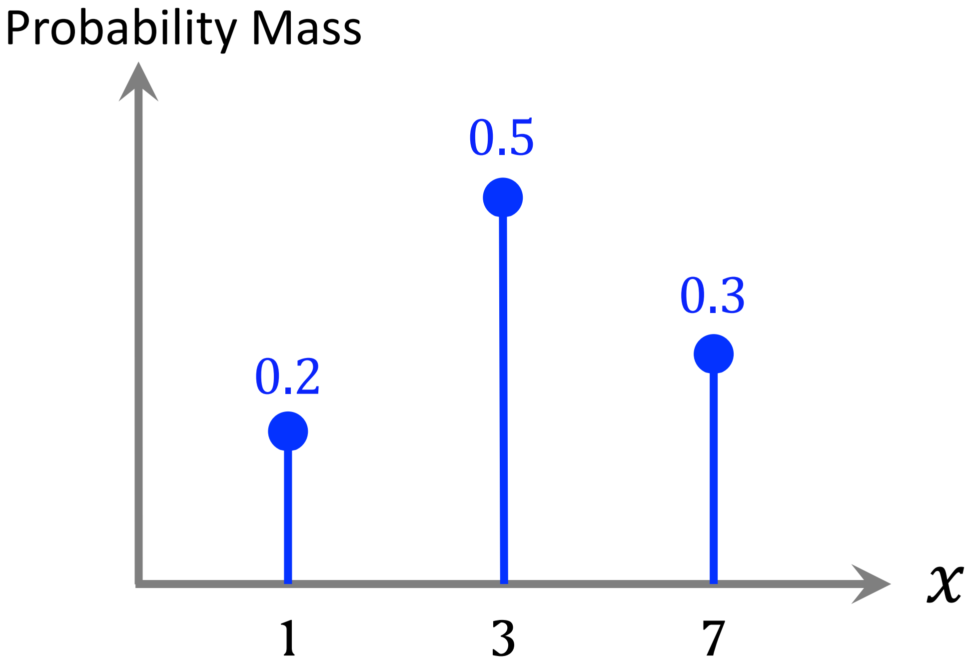
Probability Density Function (PDF)

- PMF is a function that gives the probability that a discrete random variable is exactly equal to some value.

Example

- Discrete random variable: $\boxed{X \in \{1, 3, 7\}.}$
- PDF:

$$\boxed{\begin{aligned} p(1) &= 0.2, \\ p(3) &= 0.5, \\ p(7) &= 0.3. \end{aligned}}$$



Properties of PDF

- Random variable X is in the domain $\mathcal{X}$.
- For continuous distributions,

$$\int_{\mathcal{X}} p(x)\, dx = 1.$$

- For discrete distributions,

$$\sum_{x \in \mathcal{X}} p(x) = 1.$$

Expectation

- Random variable X is in the domain $\mathcal{X}$.
- For continuous distributions, the expectation of $f(X)$ is:

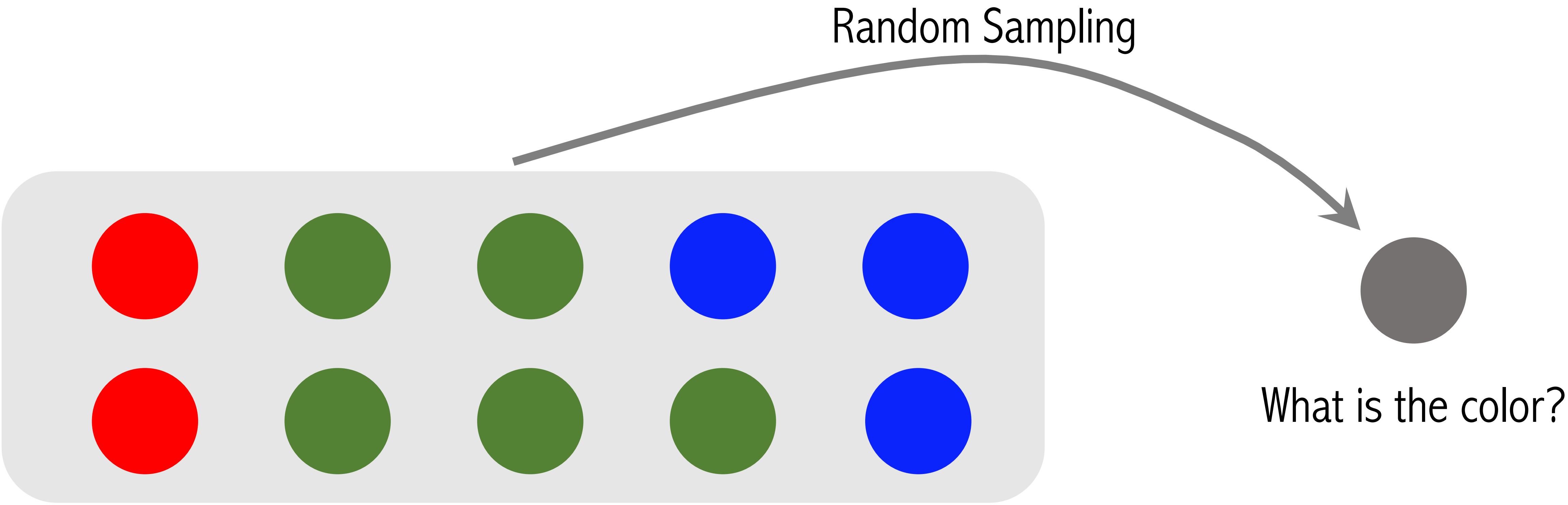
$$\underline{\mathbb{E}[f(X)]} = \underline{\int_{\mathcal{X}} p(x) \cdot f(x)\, dx}.$$

- For discrete distributions, the expectation of $f(X)$ is:

$$\mathbb{E}[f(X)] = \underline{\sum_{x \in \mathcal{X}} p(x) \cdot f(x)}.$$

Random Sampling

- There are 10 balls in the bin: 2 are red, 5 are green, and 3 are blue.

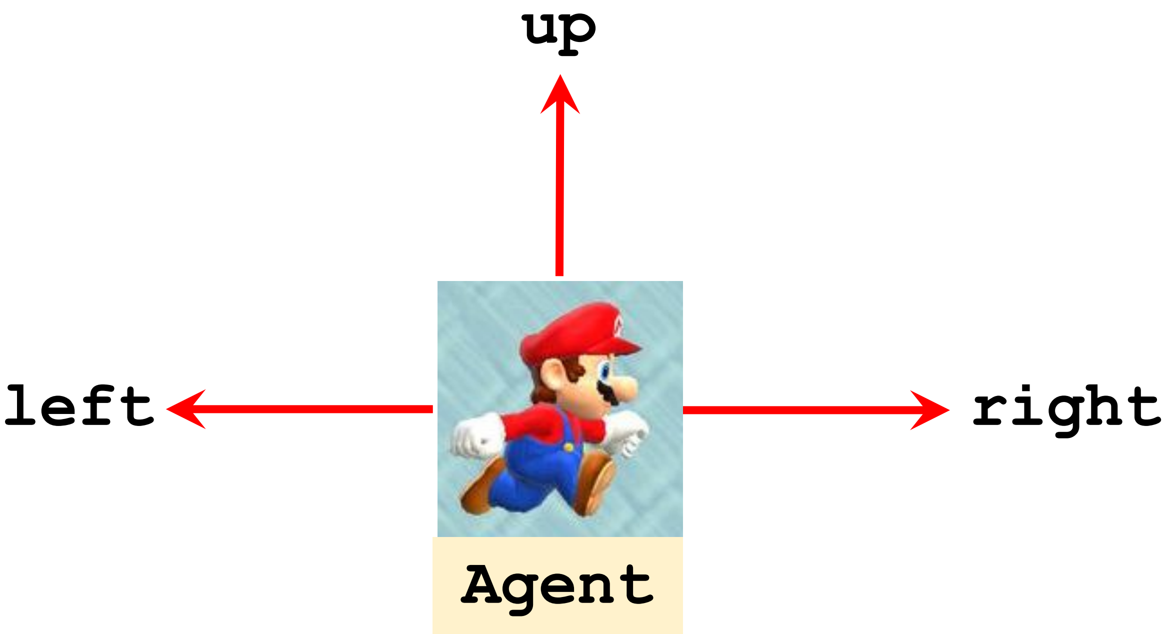


Terminologies

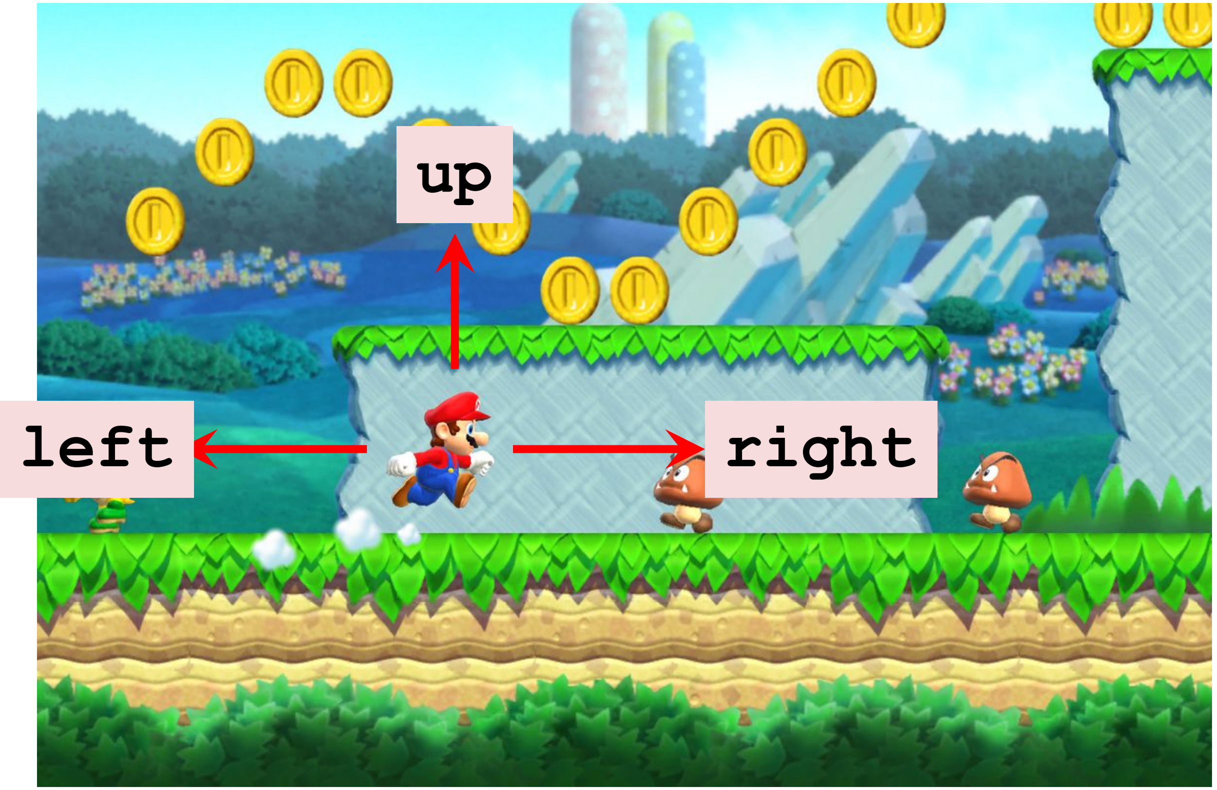
Terminology: state and action

state s (this frame)

Action $a \in \{\text{left}, \text{right}, \text{up}\}$



Terminology: policy

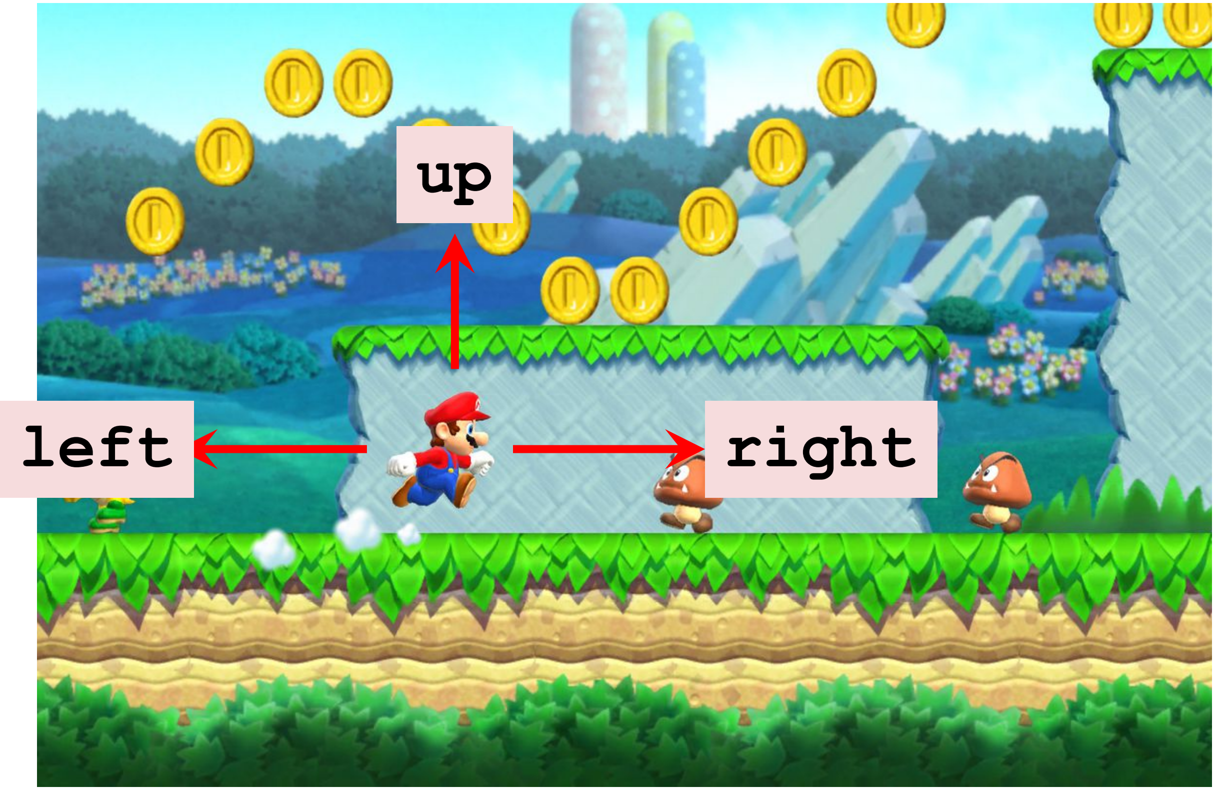


policy π

- Policy function $\pi: (s, a) \mapsto [0,1]$:

$$\pi(a \mid s) = \mathbb{P}(A = a \mid S = s).$$

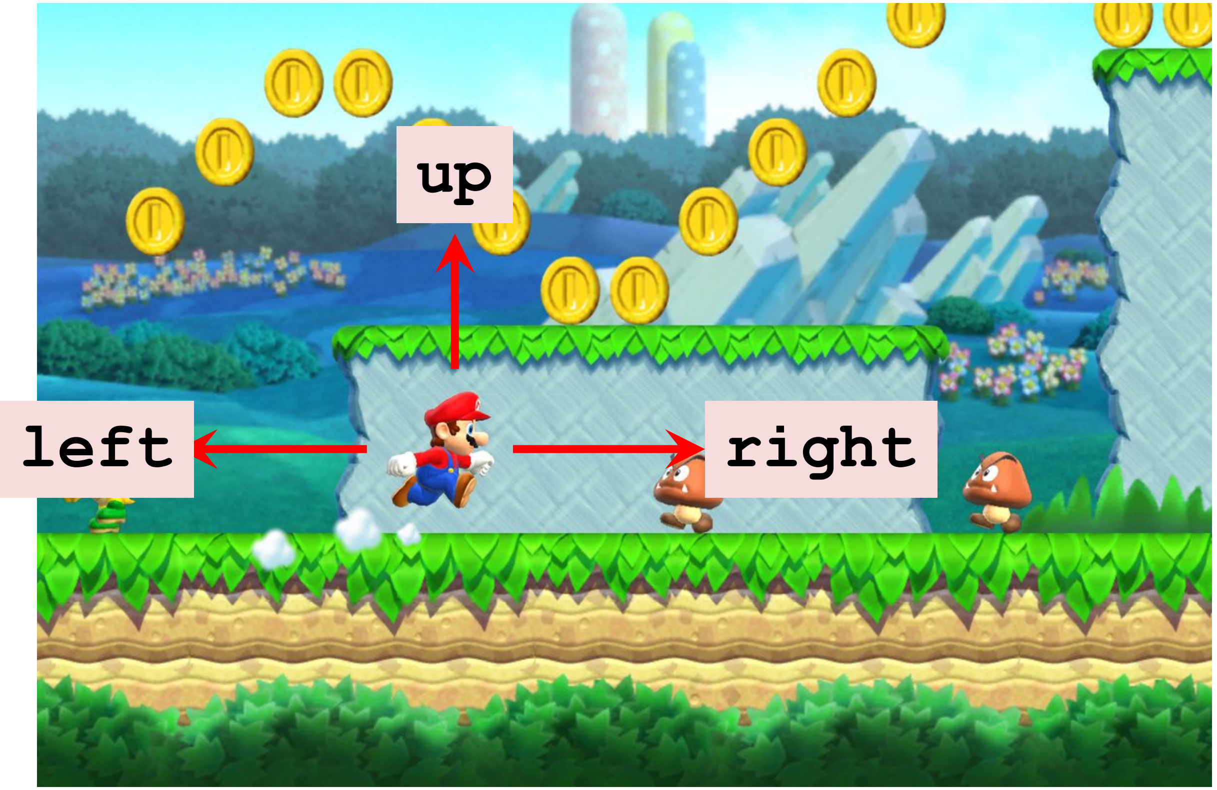
Terminology: policy



policy π

- $\pi(a \mid s)$ is the probability of taking action $A = a$ given state s , e.g.,
  - $\pi(\text{left} \mid s) = 0.2,$
  - $\pi(\text{right} \mid s) = 0.1,$
  - $\pi(\text{up} \mid s) = 0.7.$

Terminology: policy



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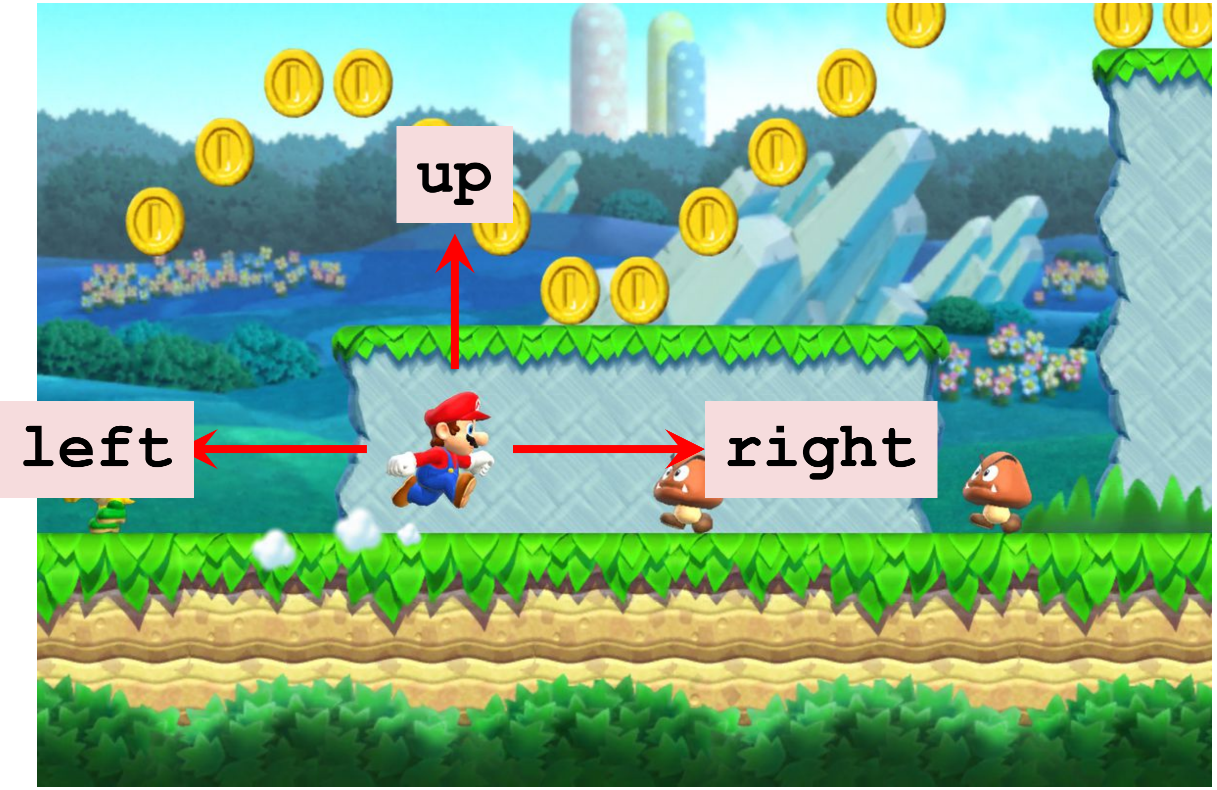
  - $\pi(\text{left} \mid s) = 0.2$,
  - $\pi(\text{right} \mid s) = 0.1$,
  - $\pi(\text{up} \mid s) = 0.7$.
- Upon observing state $S = s$, the agent's action A can be random.

Terminology: policy

Random or deterministic policy?

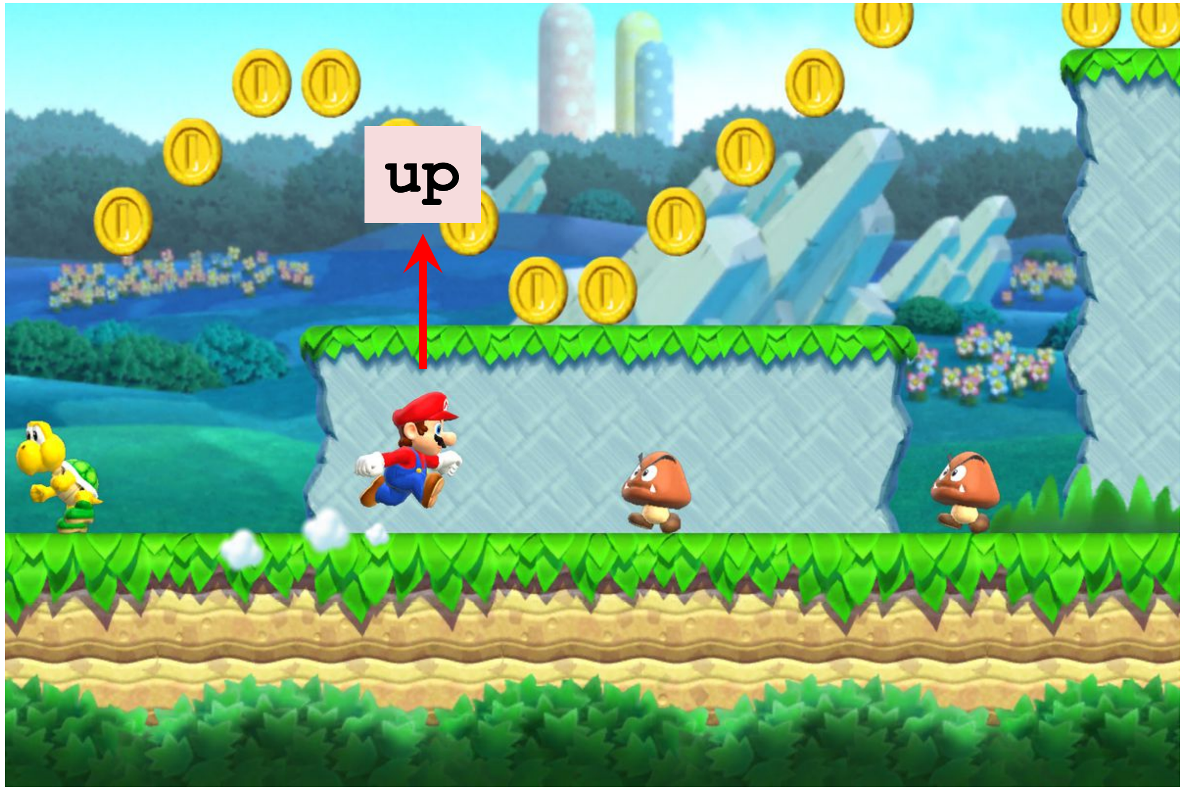


Terminology: reward

reward R

- Collect a coin: $R = +1$
- Win the game: $R = +10000$
- Touch a Goomba: $R = -10000$ (game over).
- Nothing happens: $R = 0$

Terminology: state transition

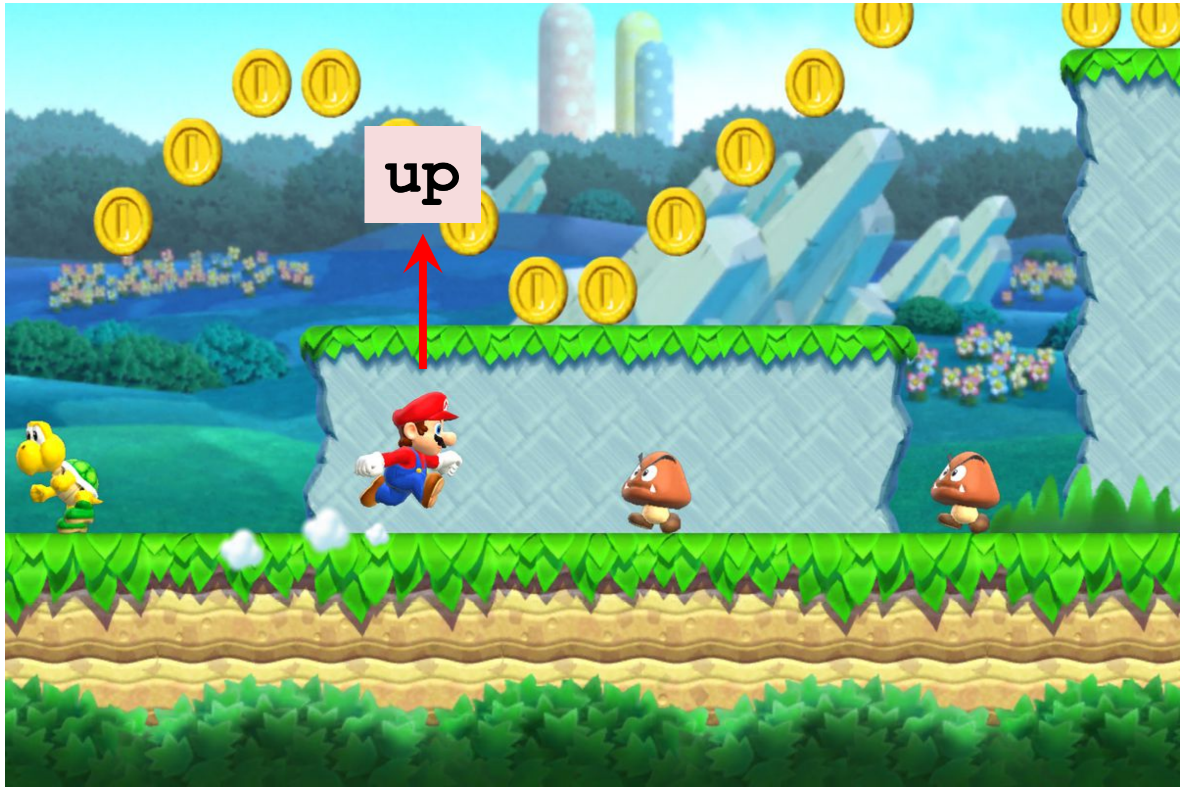


state transition

$$\text{old state} \xrightarrow{\text{action}} \text{new state}$$

- E.g., "up" action leads to a new state.

Terminology: state transition



state transition

old state $\xrightarrow{\text{action}}$ new state

- E.g., "up" action leads to a new state.
- State transition can be random.
- Randomness is from the environment.

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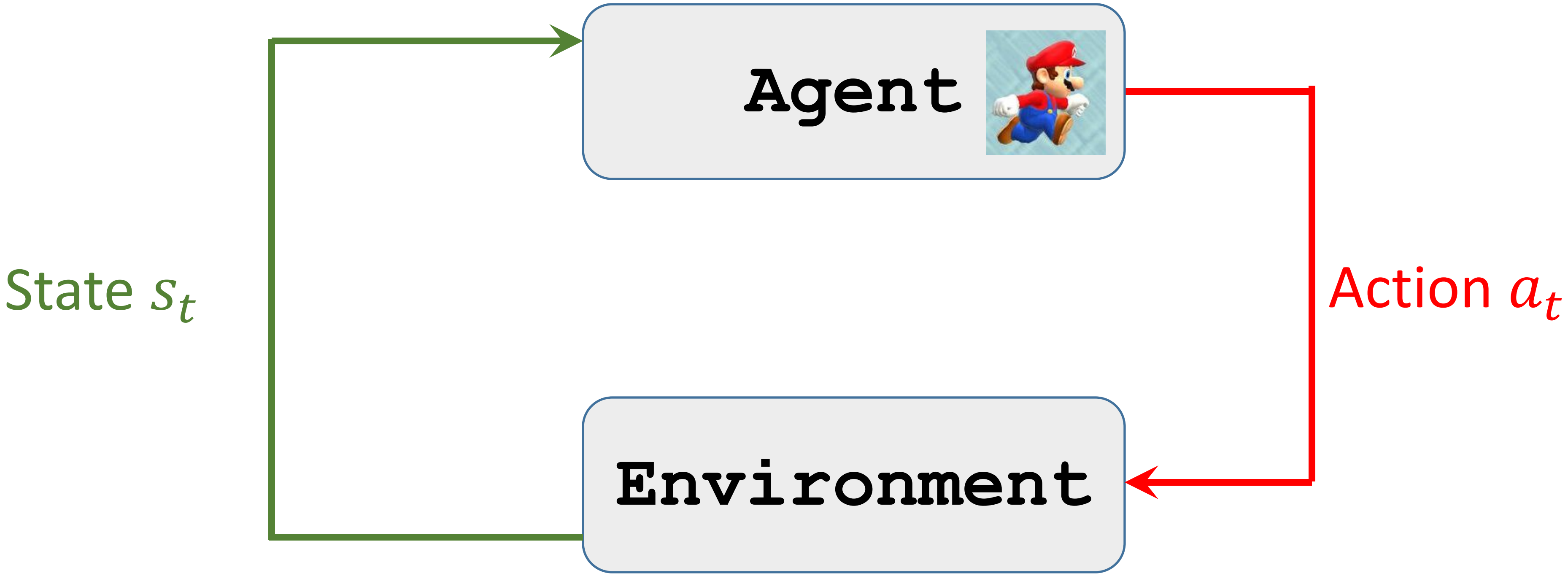
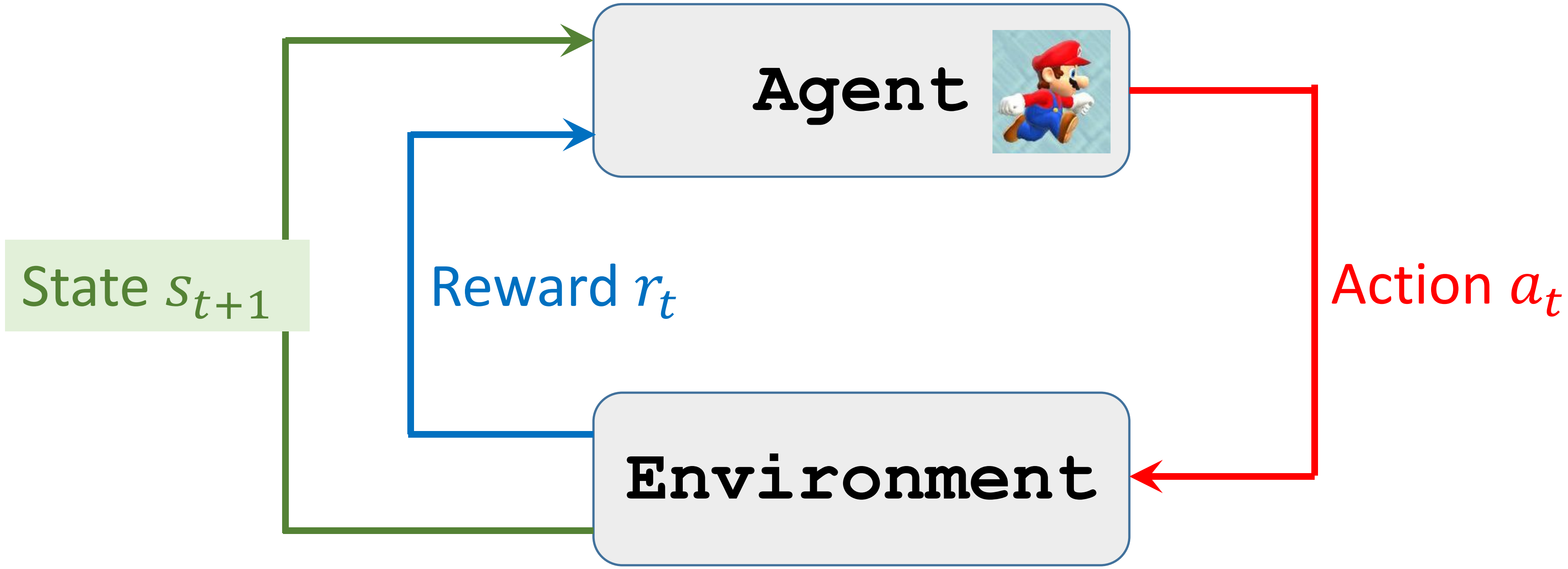
- E.g., "up" action leads to a new state.

- State transition can be random.
- Randomness is from the environment.
- $\underline{p(s'|s,a)} = \mathbb{P}(S' = s'|S = s, A = a)$.

Agent-Environment Interaction

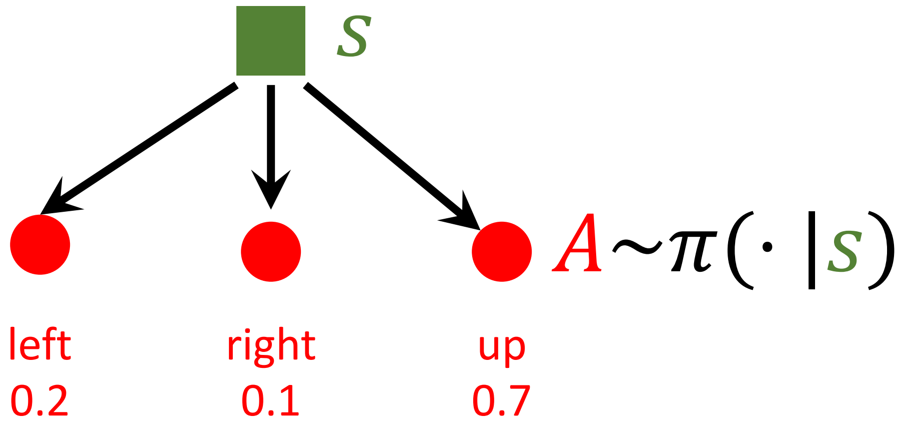


Agent-Environment Interaction



Two Sources of Randomness

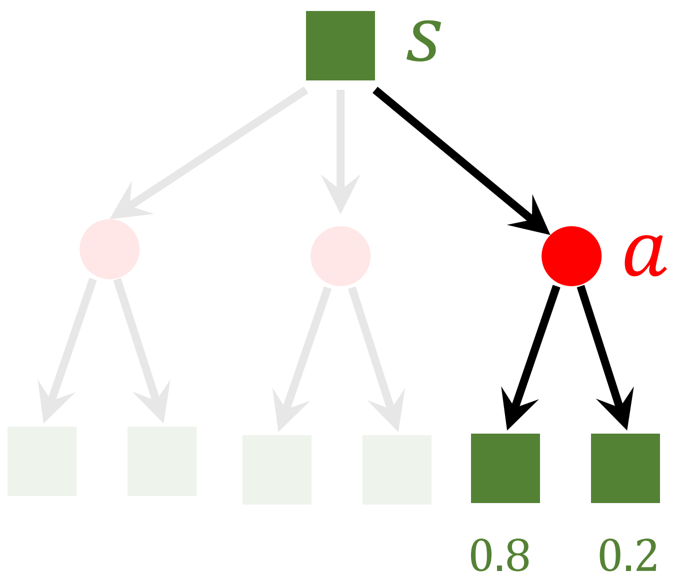
Randomness in Actions



Given state s, the action can be random, e.g., .

- $\pi(\text{"left"} \mid s) = 0.2,$
- $\pi(\text{"right"} \mid s) = 0.1,$
- $\pi(\text{"up"} \mid s) = 0.7.$

Randomness in States



- State transition can be random.
- The environment generates the new state S' by

$$S' \sim p(\cdot \mid s, a) .$$

Two Sources of Randomness

- The randomness in action is from the policy function:

$$A \sim \pi(\cdot \mid s).$$

- The randomness in state is from the state-transition function:

$$S' \sim p(\cdot \mid s, a).$$

Agent-Environment Interaction

Play game using AI

- Observe state s_t, select action $a_t \sim \pi(\cdot \mid s_t)$, and execute a_t.
- The environment gives new state s_{t+1} and reward r_t.

$$\begin{tikzcd} s_1 \arrow[r] & a_1 \arrow[r] \arrow[dr] & s_2 \\ & & r_1 \end{tikzcd}$$

Play game using AI

- Observe state s_t, select action $a_t \sim \pi(\,\cdot \mid s_t\,)$, and execute a_t.
- The environment gives new state s_{t+1} and reward r_t.

$$\begin{tikzcd} s_1 \arrow[r] & a_1 \arrow[r] \arrow[dr] & s_2 \arrow[r] & a_2 \arrow[r] \arrow[dr] & s_3 \arrow[r] & a_3 \arrow[r] \arrow[dr] & s_4 \arrow[r] & \cdots \\ & & r_1 & & r_2 & & r_3 & \end{tikzcd}$$

Play game using AI

- (state, action, reward) trajectory:

$$s_1, a_1, r_1, \quad s_2, a_2, r_2, \quad \cdots \quad , \quad s_n, a_n, r_n.$$

- One episode is from the the beginning to the end (Mario wins or dies).

$$\begin{tikzcd}
s_1 \arrow[r] & a_1 \arrow[r] \arrow[dr] & s_2 \arrow[r] & a_2 \arrow[r] \arrow[dr] & s_3 \arrow[r] & a_3 \arrow[r] \arrow[dr] & s_4 \arrow[r] & \cdots \\
& & r_1 & & r_2 & & r_3 &
\end{tikzcd}$$

Rewards and Returns

Return

Definition: Return (aka cumulative future reward).

- $U_t = R_t + R_{t+1} + R_{t+2} + R_{t+3} + \cdots$

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Question: At time t, are R_t and R_{t+1} equally important?

- Which of the followings do you prefer?
  - I give you \$80 right now.
  - I will give you \$100 one year later.

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Definition: Return (aka cumulative future reward).

- $U_t = R_t + R_{t+1} + R_{t+2} + R_{t+3} + \cdots$

Question: At time t, are R_t and R_{t+1} equally important?

- Which of the followings do you prefer?
  - I give you \$80 right now.
  - I will give you \$100 one year later.

- Future reward is less valuable than present reward.
- R_{t+1} should be given less weight than R_t.

Discounted Returns

Definition: Return (aka cumulative future reward).

- $U_t = R_t + R_{t+1} + R_{t+2} + R_{t+3} + \cdots$

Definition: Discounted return (aka cumulative discounted future reward).

- γ: discount factor (tuning hyper-parameter).
- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \gamma^3 R_{t+3} + \cdots$

Discounted Returns

Definition: Discounted return (at time t).

- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$

Randomness in Returns

Definition: Discounted return (at time t).

- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$

At time t, the rewards, $R_t, \cdots, R_n$, are random, so the return U_t is random.

- Reward R_i depends on S_i and A_i.
- States can be random: $S_i \sim p(\cdot \mid s_{i-1}, a_{i-1})$.
- Actions can be random: $A_i \sim \pi(\cdot \mid s_i)$.
- If either S_i or A_i is random, then R_i is random.

Randomness in Returns

Definition: Discounted return (at time t).

- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$

At time t, the rewards, $R_t, \cdots, R_n$, are random, so the return U_t is random.

- Reward R_i depends on S_i and A_i.
- U_t depends on $R_t, R_{t+1}, \cdots, R_n$.
- ➔ U_t depends on $S_t, A_t, S_{t+1}, A_{t+1}, \cdots, S_n, A_n$.

Value Functions

Action-Value Function $Q_\pi(s, a)$

Definition: Discounted return.

- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$

Definition: Action-value function.

- $\boxed{Q_\pi(s_t, a_t)} = \boxed{\mathbb{E}[U_t \mid S_t = s_t, A_t = a_t].}$

Action-Value Function $Q_\pi(s, a)$

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- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$

Definition: Action-value function.

- $Q_\pi(s_t, a_t) = \mathbb{E}[\, U_t \mid S_t = s_t, A_t = a_t \,].$

U_t depends on states $\underline{S_t, S_{t+1}, \cdots, S_n}$ and actions $\underline{A_t, A_{t+1}, \cdots, A_n}$.

Action-Value Function $Q_\pi(s, a)$

Definition: Discounted return.

- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$

Definition: Action-value function.

- $Q_\pi(s_t, a_t) = \mathbb{E}\left[U_t \mid \boxed{S_t = s_t, A_t = a_t} \right].$

Regard s_t and a_t as observed values.

Regard $\underline{S_{t+1}, \cdots, S_n}$ and $\underline{A_{t+1}, \cdots, A_n}$ as random variables.

Action-Value Function $Q_\pi(s, a)$

Definition: Discounted return.

- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$

Definition: Action-value function.

- $Q_\pi(s_t, a_t) = \boxed{\mathbb{E}}[\, U_t \mid S_t = s_t, A_t = a_t \,].$

- $S_{t+1} \sim p(\cdot \mid s_t, a_t),$

  $\vdots$

- $S_n \sim p(\cdot \mid s_{n-1}, a_{n-1}).$

- $A_{t+1} \sim \pi(\cdot \mid s_{t+1}),$

  $\vdots$

- $A_n \sim \pi(\cdot \mid s_n).$

Action-Value Function $Q(s, a)$

Definition: Discounted return (aka cumulative discounted future reward).

- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \gamma^3 R_{t+3} + \cdots$

Definition: Action-value function for policy π.

- $Q_\pi(s_t, a_t) = \mathbb{E}[U_t | S_t = s_t, A_t = a_t]$.

Definition: Optimal action-value function.

- $Q^\star(s_t, a_t) = \max_\pi Q_\pi(s_t, a_t)$.

State-Value Function $V_\pi(s)$

Definition: Discounted return.

- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$

Definition: Action-value function.

- $Q_\pi(s_t, a_t) = \mathbb{E}[\, U_t \mid S_t = s_t, A_t = a_t \,].$

Definition: State-value function.

- $V_\pi(s_t) = \mathbb{E}_A[Q_\pi(s_t, A)] = \underline{\sum_a \pi(a|s_t) \cdot Q_\pi(s_t, a)}.$ (Actions are discrete.)

Taken w.r.t. the action $A \sim \pi(\cdot \mid s_t)$.

State-Value Function $V_\pi(s)$

Definition: Discounted return.

- $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots + \gamma^{n-t} R_n.$

Definition: Action-value function.

- $Q_\pi(s_t, a_t) = \mathbb{E}[U_t \mid S_t = s_t, A_t = a_t].$

Definition: State-value function.

- $V_\pi(s_t) = \mathbb{E}_A[Q_\pi(s_t, A)] = \sum_a \pi(a|s_t) \cdot Q_\pi(s_t, a).$ (Actions are discrete.)
- $V_\pi(s_t) = \mathbb{E}_A[Q_\pi(s_t, A)] = \int \pi(a|s_t) \cdot Q_\pi(s_t, a)\, da.$ (Actions are continuous.)

Understanding the Value Functions

- Action-value function: $\underline{Q_\pi(s,a)} = \underline{\mathbb{E}\left[U_t | S_t = s, A_t = a\right]}$.
- Given policy π, $Q_\pi(s,a)$ evaluates how good it is for an agent to pick action a while being in state s.

Understanding the Value Functions

- Action-value function: $Q_\pi(s, a) = \mathbb{E}\left[U_t | S_t = s, A_t = a\right]$.
- Given policy π, $Q_\pi(s, a)$ evaluates how good it is for an agent to pick action a while being in state s.

- State-value function: $V_\pi(s) = \underline{\mathbb{E}_A\left[Q_\pi(s, A)\right]}$
- For fixed policy π, $V_\pi(s)$ evaluates how good the situation is in state s.
- $\underline{\mathbb{E}_S[V_\pi(S)]}$ evaluates how good the policy π is.

Evaluating Reinforcement Learning

How does AI control the agent?

Suppose we have a good policy $\pi(a|s)$.

- Upon observe the state s_t,
- random sampling: $a_t \sim \pi(\cdot\,|s_t)$.

Suppose we know the optimal action-value function $Q^\star(s, a)$.

- Upon observe the state s_t,
- choose the action that maximizes the value: $a_t = \mathrm{argmax}_a\, Q^\star(s_t, a)$.

Summary

Summary

Terminologies

- Agent 
- Environment
- State s
- Action a
- Reward r

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- Policy $\pi(a|s)$
- State transition $p(s'|s, a)$

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Return and Value

- Return:

$$U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots$$

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- Agent 
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- Policy $\pi(a|s)$
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Return and Value

- Return:

$$U_t = R_t + \gamma\, R_{t+1} + \gamma^2\, R_{t+2} + \cdots$$

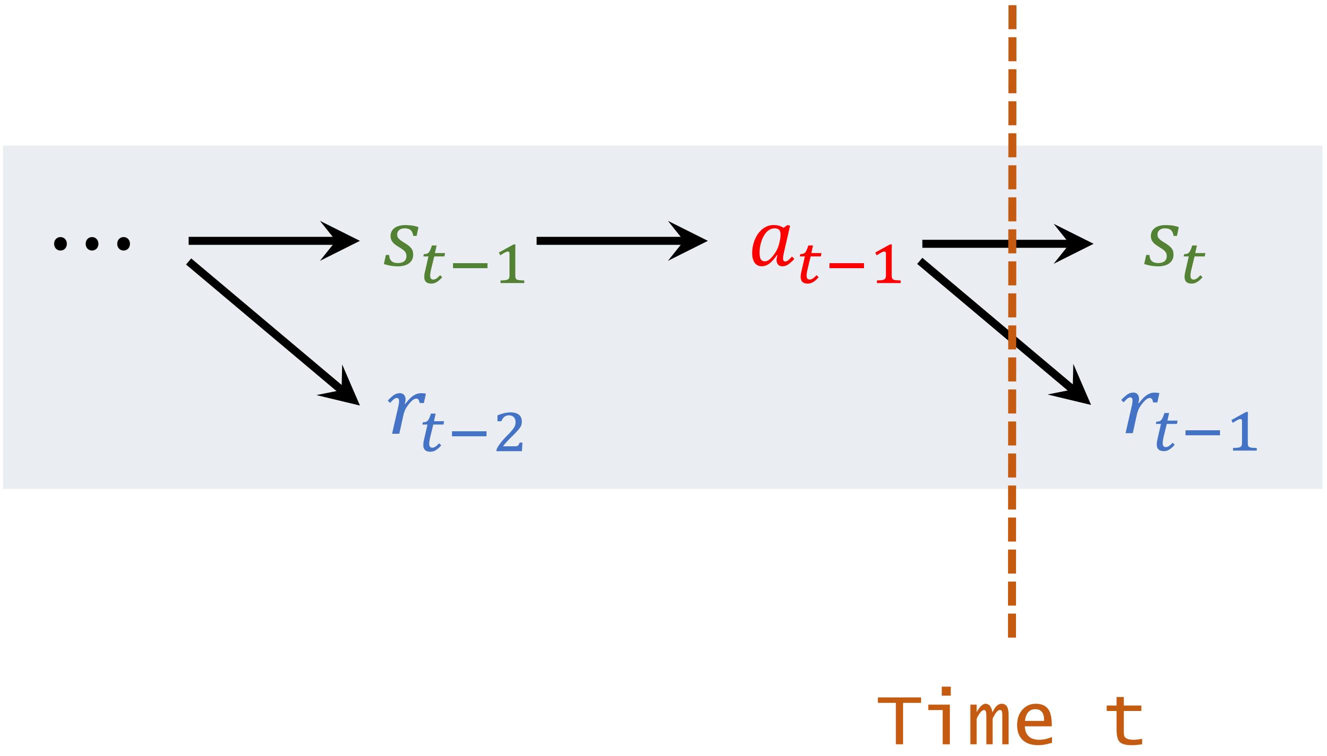
- Action-value function:

$$Q_\pi(s_t, a_t) = \mathbb{E}\,[U_t | s_t, a_t].$$

- State-value function:

$$V_\pi(s_t) = \mathbb{E}_{A\sim\pi}[Q_\pi(s_t, A)].$$

Play game using AI



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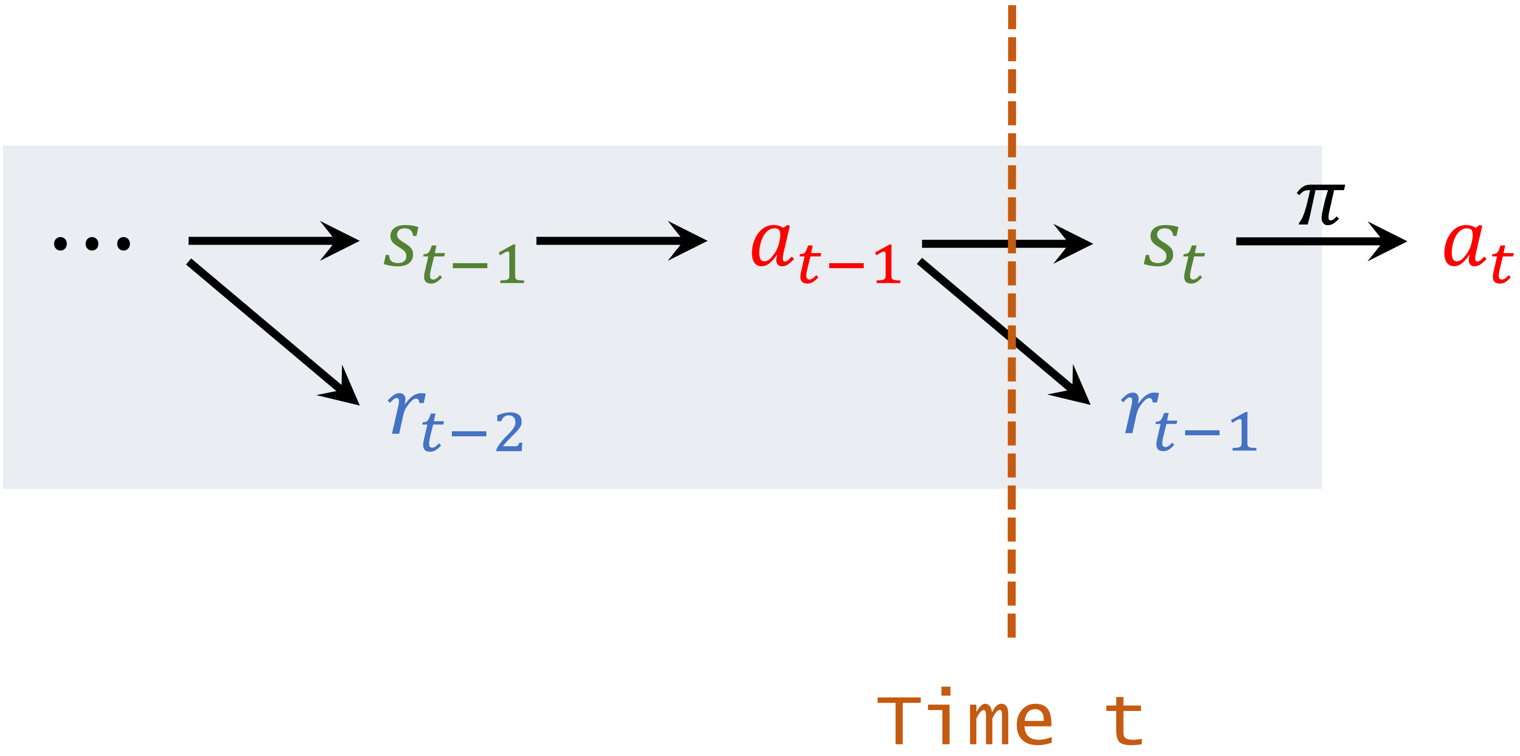

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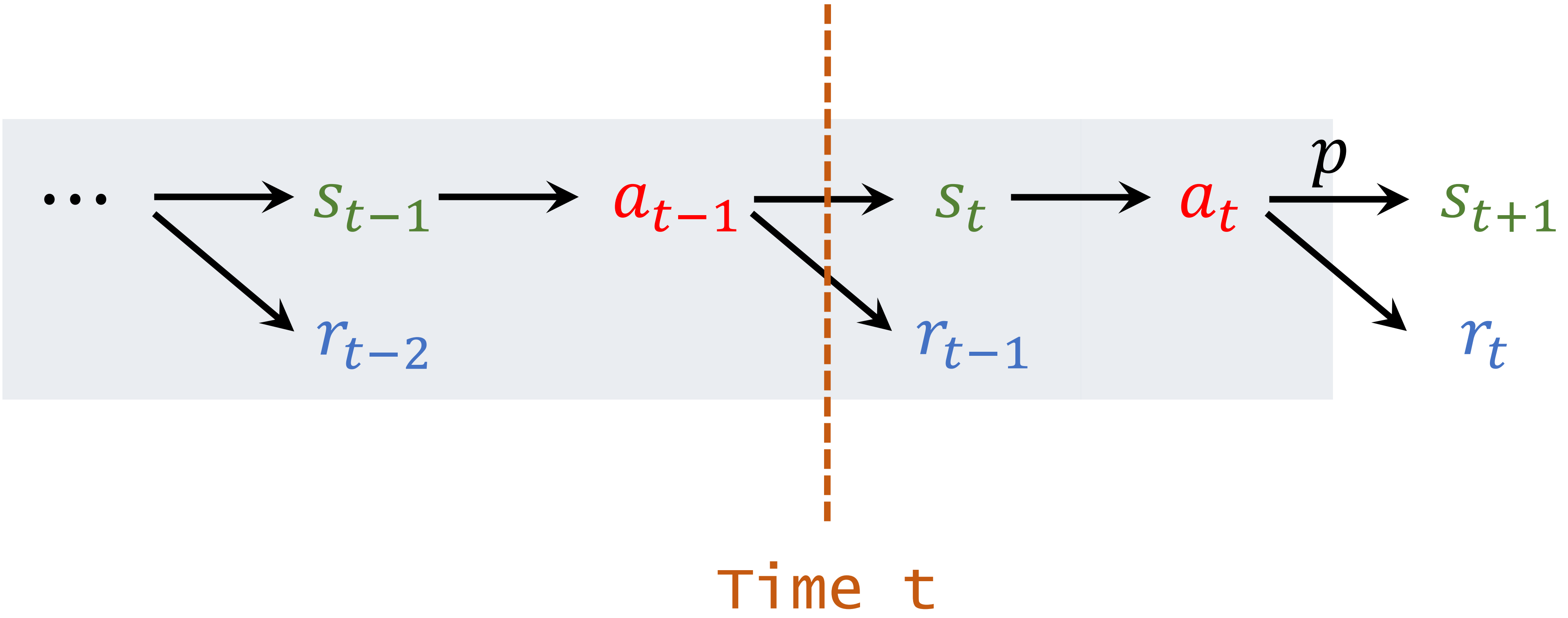

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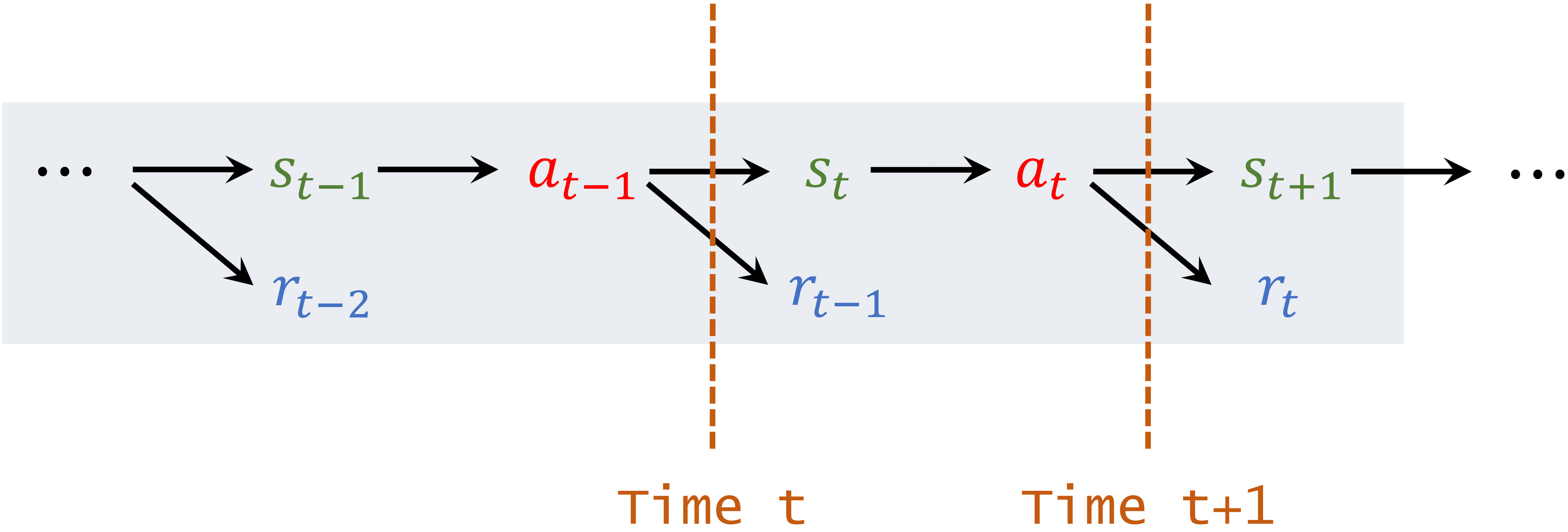

We are going to study…

2. Value-based learning.
    - Deep Q network (DQN) for approximating $Q^{\star}(s, a)$.
    - Learn the network parameters using temporal different (TD).

3. Policy-based learning.
    - Policy network for approximating $\pi(a|s)$.
    - Learn the network parameters using policy gradient.

4. Actor-critic method. (Policy network + value network.)

5. Example: AlphaGo

Thank You!