Session 3: The RC-Magnitude transfer function

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1 Introduction

In this experiment we will take the first steps towards building a spectrum analyser. We will learn how to make an RC filter and use that to calculate the magnitude transfer function. A spectrum analyser can measure those transfer functions. One important and interesting application for a spectrum analyser is to make emission and absorption spectra. With this property we can look at planets and see what elements are on that planet, and thus if we would be able to survive on that planet. Pretty cool, right?

2 Theory

2.1 Transfer Function

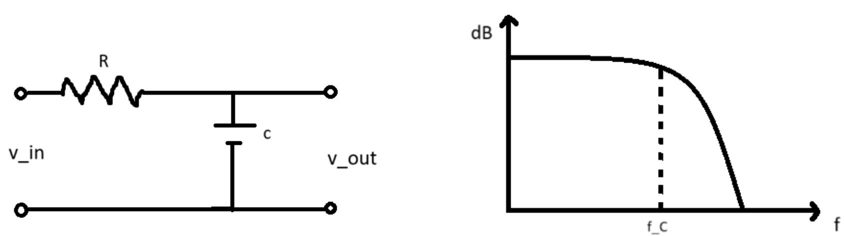
The transfer function shows the relationship between outgoing and incoming signal. In an electronic circuit, with a filter like the one we will be using, the magnitude transfer function is:

$$|H(\omega)| = \frac{|U_{\text{after\_filter}}(\omega)|}{|U_{\text{before\_filter}}(\omega)|} \tag{1}$$

[1]

2.2 Low-pass and high-pass RC filters

An RC filter is a filter with a resistance and a capacitance (thus the R and C). A low-pass filter is a filter where low frequencies pass, until a frequency $\omega = \omega_c$. Above this frequency, the amplitude of the signal is reduced. Such a filter, and its response are shown in 1. High frequency signals pass through the capacitor, and low frequency signals are blocked by it.

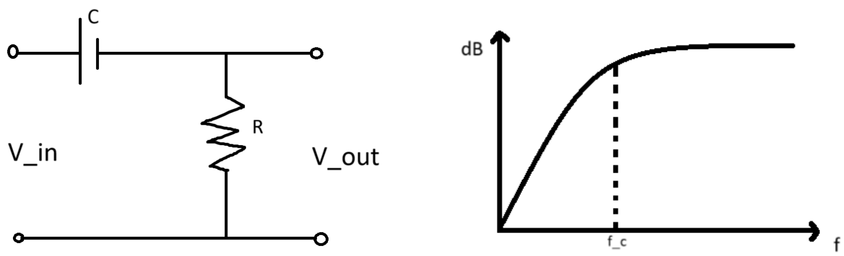


(a) Circuit (b) Response

Figure 1: A figure showing a low-pass filter. On the left the actual schematic view of such a filter can be seen. While on the right, the response of our system can be seen in a bode plot. It is best to have dB on the y-axis as this will best show the results at all orders of magnitude and we can easily notice the 3 dB drop.

A high-pass filter is a filter where frequencies above $\omega = \omega_c$ pass, and below that the signal decreases to 0. You can see what this circuit and its response looks like in 2. If the frequency goes to infinity, the outputted voltage will approach the inputted voltage. [2]

The cutoff frequency (f_c) of an RC-filter has a fairly simple formula, and can be used for both types of filters. The cutoff-frequency is defined as the point



(a) Circuit (b) Response

Figure 2: A figure showing a high-pass filter. On the left the actual schematic view of such a filter can be seen. While on the right, the response of our system can be seen in a bode magnitude plot. It is best to have dB on the y-axis as this will best show the results at all orders of magnitudes and we can easily notice the 3 dB drop.

where the signal decreases by 3 dB (or around 70%) of the original signal

$$f_c = \frac{1}{2\pi RC} \tag{2}$$

where:

- R = Resistance of resistor
- C = Capacitance of Capacitor

2.3 Parseval's theorem

Parseval's theorem is a very handy trick up our sleeves to compare a Fourier peak after the filter with the Fourier peak before the filter. Since measuring is never ideal, the Fourier peak will take up multiple points, we cannot just use the peak height. Thats where this theorem comes in. It tells us that the total power in the signal is given by the integral over the power spectral density of a Fourier transform. Or rather, in equation form :

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2 \tag{3}$$

3 Research questions & Hypotheses

3.1 Main research question:

what is the magnitude transfer function of my Low-pass RC-filter ($R = 1.6k\Omega$, $C = 10nF$ $f_c = 10kHz$) measured through an automated measurement and how does it compare to my theoretical prediction.

3.2 Main hypothesis:

To answer this research question we have to look at equation 1.
A low pass RC filter $H(\omega)$ would be 1 for low frequencies, since the amplitude of those frequencies remain unchanged. For signals with frequencies higher than the cutoff frequency ω_c the amplitude decreases, and decreases more if the frequency gets larger. We can see in equation 1 that for these high frequencies the magnitude transfer function would decrease until it reaches 0.

3.3 Sub-question 1:

How do you build your own low-pass RC-filter using a resistor and capacitor.

3.4 Hypothesis Sub-question 1:

To be able to build a RC-filter you need a resistor, capacitor, and a breadboard. The components have been chosen according to formula 2. We want the cutoff frequency to be around 10 kHz. This will make measuring it with the MyDAQ easy.The specific breadboard configuration will be shown in the setup section. We will test the validity of our filter by using a function generator and oscilloscope. The magnitude Bode plot must resemble the theory, like in figure 1

4 setup

4.1 Setup sub-question 1

4.1.1 Materials:

- breadboard. We will build the setup using it.
- resistor. These can range from 10 Ω to 1 MΩ. We will choose a resistance of around 1.6 kΩ.
- capacitor. This had a long wire for positive, and a short wire for negative. These can range from 10pF tot 10nF in the lab. And we will choose the 10 nF capacitor.
- Function generator. We use this to test the setup qualitatively by sending the signal with specific frequencies.
- Oscilloscope. We use this to test the setup qualitatively by looking at the signal and finding which frequency the signal decreases.
- MyDAQ. We use this to test the setup.
- Jumper wires

4.1.2 Breadboard configuration

We put the resistor and the capacitor on the breadboard so that they are in series. We need use jumper wire 1 to connect the positive part of one power rail to the resistor. We use jumper wire 2 to connect the capacitor to the negative art of another power rail. We connect the function generator (where the inputted signal comes from) to both of these power rails. We connect the oscilloscope (where we can see the output) also to these power rails, so we can see the inputted signal. On another channel on the oscilloscope we will connect the node via with the resistor and capacitor are connected.

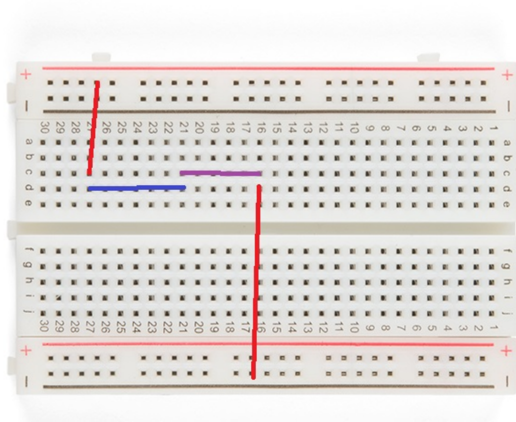


Figure 3: A very rough schematic of the simple first order Low pass circuit. The red wiring are connector cables. The blue wire is our resistor, and the purple wire is pour capacitor. The resistor and capacitor are in series.

5 Measuring

5.1 Measuring plan sub question 1

We will build the low-pass as described in the setup. We will write down exactly in with hole we connect everything. We will measure if the setup works by connecting the output of the MyDAQ to the input of the filter, and the output of the filter t the input of the MyDAQ. We will send a signal with the cutoff frequency to the MyDAQ (use 2).

5.2 Measuring plan main question

We want to measure some signal before and after the filter. First we will do this using a single frequency. We will connect the output of the MyDaq to the input of the filter, and the output of the filter to the input of the MyDAQ. To keep thing simple at first, we will measure the transfer function at a single frequency. We will choose frequencies before the cutoff frequency, around, and after. Thus, we will use frequencies of kHz, 5 kHz, 10kHz, 12kHz and 15kHz. We will measure with the mydaq at a sample rate of 44Khz for all measurements. The signals will be 5 seconds long.

When we get a desired result, we can automate the process and run a for-loop with an array of frequencies made with numpy.logspace, to get all orders of magnitude.

6 Analysis

6.1 Analysis plan sub-question 1

For this sub question we don't have any data so an analysis isn't really necessary. We will qualitatively observe the raw-data on the oscilloscope and sketch the bode magnitude plot. This will be all analysis done

6.2 Analysis plan main question

The raw data we receive from the MYDAQ is a (time, voltage) array. For the analysis, a python script is necessary. This will do all the heave lifting for us. We can combine code from all previous sessions, as well as extra's on Parsevals theorem on bright space. We can fuse all them together to get a python script that can make a magnitude bode plot for us. From that, we can easily get the slope, which shouldn't be very much because we use a first order filter.

6.3 Error analysis

We can easily get an error on our delta function using standard error propagation.

$$\Delta H(f) = \sqrt{\left(\frac{\Delta V_{\text{out}}}{V_{\text{in}}}\right)^2 + \left(\frac{V_{\text{out}} \cdot \Delta V_{\text{in}}}{V_{\text{in}}^2}\right)^2} \tag{4}$$

The theoretical error on the Cutoff frequencies can be calculated using the following:

$$\Delta f_c = \frac{1}{2\pi}\sqrt{\left(\frac{\Delta R}{R^2 C}\right)^2 + \left(\frac{\Delta C}{RC^2}\right)^2}$$

The errors on the resistance and capacitance will be determined in the lab for our specific components.

At last, we can also do a function fit using scipy.optimize.curvefit. Doing so, we can compare our data with theoretical predictions.

7 TRA

Task Risk Analysis

Name experiment: PE1 session 3

Nr.	Action	Hazard	Risk classification				Measures: -source -collective	Measures: -procedures -Instructions -PBM's	Risk classification			
			S	F	P	Score			S	F	P	Score
1	Working with self build voltage circuits. (no more than 10 V	Electric Shock	1	3	0.5	**1.5**	no necessary	no necessary	1	3	0.5	**1.5**
2	Working with wiring (connecting the MyDAQ, or other equipment)	Incorrect wiring --> Short circuit --> fire	40	3	0.5	**60**	no open cables or contacts, warning sign and workbench, no liquids near circuit, make sure there's a fire extinguisher in the room.	Power source on only when measuring. When making changes to the set-up, switch off and unload all equipment. Double check the citcuit before tunng it on. Low voltages	7	3	0.2	**4.2**
3	Using equimpent	generated heat + bad ventilation = fire	40	3	0.2	**24**	Be sure that there is a fire extinguisher in te room.	Moniter devices temperature during the session. Ensure proper ventilation	7	3	0.1	**2.1**
4	using a capacitor (10pF - 10nF)	explosion caused by incorect polarization, overheating or overvoltage	3	3	0.5	**4.5**	not really necessary, though be sure t check the polarization (if the capicitor has two different sides).	moniter the capacitors temperature, and don't use a voltage higher than the capacitors rated voltage.	3	3	0.1	**0.9**
5						**0**						**0**
6						**0**						**0**
7						**0**						**0**
8						**0**						**0**
9						**0**						**0**
10						**0**						**0**

Figure 4: A PDF of the necessary TRA

References

[1] Jelmer J.T. Wagenaar. *Signal Processing and Noice*. Leiden university, 2024.

[2] The organic chemistry tutor. *Low pass filters and high pass filters - RC and RL circuits*. Youtube video.