PS assignment 1

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I want to make clear that this is my first time using latex, so excuse my bad formatting etc.

1 1

Q: A series of measurements of a quantity x has an average x and a standard deviation σ. What is the probability that the true value of x lies between x - σ and x + σ? And what is the probability that it is between x - 2σ and x + 2σ And between x - 3σ and x + 3σ?

A: The easiest way to do this question is by simply looking up the standard normal distribution graph. This tells us that the probability that the true value of x lies between:
x - σ and x+ σ is $34.1 + 34.1 = 68.2$
x - 2σ and x+ 2σ $= 13.6 + 13.6 = 27.2$
x - 3σ and x+ 3σ $= 2.1 + 2.1 = 4.2$

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Q: The rotation period of a double star is measured 25 times, with an average value of 5.231 days and a standard deviation of 5 minutes. What is the (accidental) error on the individual measurements? What is the best estimate of the actual rotation period and its uncertainty?

A: The error on the individual measurements is simply just 5 minutes. We can already convert this to days since our mean is also in days:
$\sigma = \frac{5}{60*24} = 0.0034722$ Days
The best estimate for the actual rotation period is
$\bar{x} \pm \frac{\sigma}{\sqrt{N}} = 5.321 \pm \frac{0.0034722}{\sqrt{25}} = 5.321 \pm 0.001$ Days

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Q : We measure the radius of 50 red giant stars and find that the average radius is $RRG = 500R_\odot$ with an estimated uncertainty (standard error) in this value of $\sigma R = 10R_\odot$. A newly discovered star X has a measured radius of $RX = 525R_\odot$ (where the measurement error is negligible). Is this star significantly larger than average? Express the significance as a probability P. Yet this is probably an ordinary red giant star. Show that the radius of star X falls within the range (standard deviation) of the population.

A: The star X has a deviation from the mean in terms of sigma of :
$\frac{525-500}{10} = 2.5\sigma$
Searching this value in the necessary tables online gives P = 0.0062
This may seem like an almost improbable star, but we have to notice that it does fall in the range of the whole population.
$\sigma_{pop} = s * \sqrt{N} = 10 * \sqrt{50} = 70.7$
$\frac{525-500}{70.7} = 0.353\sigma_{pop}$

As you can see this falls well within the range of the standard deviation of the population. Looking up this value in the table gives a probability of 0.3618, which is far more logical.

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To determine the standard deviation in the measured values we can use the standard formula, or just throw it into python or any sort of calculator, making sure to have 1 degree of freedom

The mean is $\bar{x} = 29.507$, and the $\sigma = 0.324$

For the actual diameter of the mean we can use :
$\bar{x} \pm \frac{3\sigma}{\sqrt{N}} = 29.507 \pm \frac{3*0.324}{\sqrt{10}} = 29.5 \pm 0.3$ arcmin

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$\sigma_z^2 = (\frac{\delta_z}{\delta_x}\sigma_x)^2 + (\frac{\delta_z}{\delta_y}\sigma_y)^2 = (n\bar{y}^m\bar{x}^{n-1}\sigma_x)^2 + (m\bar{x}^n\bar{y}^{m-1}\sigma_y)^2$
$\sigma_z = \sqrt{(n\bar{y}^m\bar{x}^{n-1}\sigma_x)^2 + (m\bar{x}^n\bar{y}^{m-1}\sigma_y)^2}$
we can then easily see what this means for the relative error in $z = \frac{x}{y}$. we will just substitute n=1 and m=-1. Therefore our expression will be :
$\sigma_z = \sqrt{(\bar{y}^{-1}\sigma_x)^2 + (-\bar{x}\bar{y}^{-2}\sigma_x)^2} = \sqrt{(\frac{\sigma_x}{\bar{y}})^2 + (-\frac{\bar{x}}{\bar{y}^2}\sigma_x)^2}$

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To determine whether there is a reason to think the source is variable we will need to look at the error. First of all, we can see that the error is lower than in the past. We can see that the new mean of 600 mJy fits within 1σ of the old mean of 500 mJy. However, the best way to evaluate this is using a T-test. But I dont think we need to use this as its not explained anywhere

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$\frac{R_p}{R_\star} = \sqrt{f}$

$R_p = \sqrt{f} R_\star = \sqrt{0.020} * R_\star = 0.14 R_\star$

$\sigma_f = 0.004$
$\sigma_{R_p} = \frac{1}{2\sqrt{f}} \sigma_f = \frac{1}{2\sqrt{0.020}} 0.004 = 0.01$, so the radius of the planet is:
$R_p = (0.14 \pm 0.01) R_\star$
$R_\odot = \frac{R_\star}{3.2} = 0.3125 R_\star$
$\sigma_{R}\star = (0.15 * 3.2) R_\odot = 0.48 R_\odot$
$\sigma_{R_\odot} = 0.48 * 0.3125 = 0.15$
So our best estimate of the planet is $0.3125 \pm 0.15 R_\star$

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$\bar{x} = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2}$
We can calculate the error in the mean by partial derivatives. Or by looking at the slider on brightspace
$\frac{1}{\sigma^2} = \Sigma \frac{1}{\sigma_i^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$ therefore $\sigma = \sqrt{(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2})^{-1}}$

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$x_1 = 1.0$
$\sigma_1 = 0.1$
$x_2 = 1.05$
$\sigma_2 = 0.05$
$\bar{x} = \frac{\frac{x_1}{\sigma_1^2} + \frac{x_2}{\sigma_2^2}}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}} = \frac{\frac{1.0}{0.1^2} + \frac{1.05}{0.05^2}}{\frac{1}{0.1^2} + \frac{1}{0.05^2}} = 1.04$
$\sigma = \sqrt{(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2})^{-1}} = \sqrt{(\frac{1}{0.1^2} + \frac{1}{0.05^2})^{-1}} = 0.04$

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N = 8000 gives a flux of 5.0, so N = 1 give a flux of :
$F = \frac{5.0}{8000} = 6.25 * 10^{-4}$
$8000 = \lambda = \mu = \sigma^2$
$\sigma_N = \sqrt{8000} = 89.4$
$\sigma_F = 89.4 * 6.25 * 10^{-4} = 5.6 * 10^{-2}$

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STN $= \frac{\mu}{\sigma} = 100 = \frac{N}{\sqrt{N}} = \sqrt{N} = 100 --> N = 100^2 = 10000$ Photons
This corresponds to $\frac{10000}{40} = 250$ seconds

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$=$ d $= \frac{1}{\pi} = \frac{1}{0.06456} = 15.49$ parsec
$\sigma = \frac{\sigma_\pi}{\pi^2} = \frac{0.00012}{0.06456^2} * = 0.03$

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I dont know how to import code on latex easily so its at the end. But you can clearly see that by increasing the error, the distance you get is clearly different than the analytical. Also, you see it differs more by how larger the error is

```
import numpy as np

# Parameters
pi = 64.65/1000  # mas
relative_errors = [0.01, 0.02, 0.05, 0.20, 0.50]  # Relative errors on parallax
sample_size = 100

# Function to calculate distance from parallax
def calculate_distance(parallax):
    return 1 / parallax

# Analytical approach
def analytical_estimate(pi, sigma_pi):
    d = calculate_distance(pi)
    sigma_d = sigma_pi / (pi ** 2)
    return d, sigma_d

# Monte Carlo approach
def monte_carlo_estimate(pi, sigma_pi, sample_size):
    measurements = np.random.normal(pi, sigma_pi, sample_size)
    mean_pi = np.mean(measurements)
    std_pi = np.std(measurements)
    d = calculate_distance(mean_pi)
    sigma_d = std_pi / (mean_pi ** 2)
    return d, sigma_d

# Print table header
print("{:<15} {:<25} {:<25}".format("Relative Error", "Analytical Distance (pc)", "Monte Carlo Distance (pc)"))

# Iterate over relative errors
for relative_error in relative_errors:
    sigma_pi = pi * relative_error
    analytical_d, analytical_sigma_d = analytical_estimate(pi, sigma_pi)
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```
    monte_carlo_d, monte_carlo_sigma_d = monte_carlo_estimate(pi, sigma_pi, sample_size)

    print("{:<15} {:<25.4f} {:<25.4f}".format(relative_error, analytical_d, monte_carlo_d))



# Exercise 12 check

d, sigma_d = analytical_estimate(64.64/1000, 0.12/1000)

print(f"So the dinstace is {d} pm {sigma_d}")
```

Relative Error	Analytical Distance (pc)	Monte Carlo Distance (pc)
0.01	15.4679	15.4932
0.02	15.4679	15.4721
0.05	15.4679	15.4886
0.2	15.4679	15.6156
0.5	15.4679	15.3596