define ✓

8

True
False

3 T/F questions

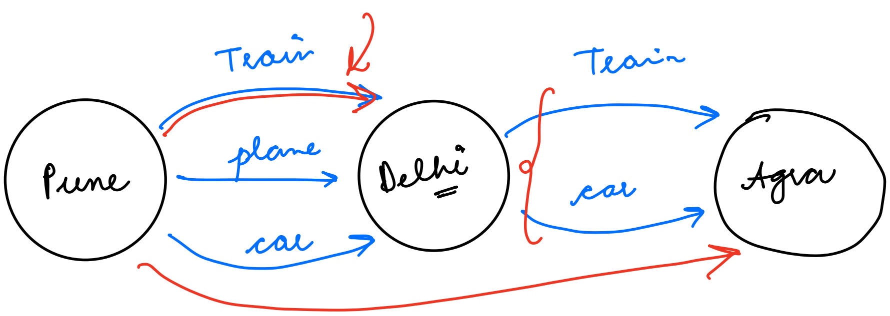
No of ways you can answer these questions

2	2	2
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

8 options / 8 ways

$2 * 2 * 2 = 8$

multip



$3+2=5$

$3*2=6$

How many diff Pune to Agra?

$\underbrace{\text{Pune to Delhi}}_{3}$ and $\underbrace{\text{Delhi to Agra}}_{2}$

$3 * 2 = 6$

rule of multi.

Task A $\boxed{\text{and}}$ Task B

$x \quad y$

$= \boxed{x * y}$

B_1 B_2 B_3 B_4 B_5 R_1 R_2 R_3 G_1

one blue & one red

$5 * 3$

ways → pick one blue ball or red ball

AND = *
OR = +

5 + 3 = 8

rule of sum

Task A or Task B

x y

$x+y$

G - 1
R - 3
B - 5

Task 1 :- pick a red ball or green ball } 3+1=4

Task 2 :- pick a blue ball or green ball } 5+1=6

Task A or Task B

x y

overlapp situation

~~4+6=10~~   9

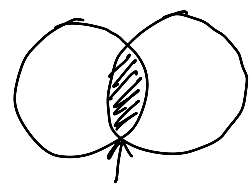
(red ball or green ball) or (blue ball or green ball)

1   and   1

R_1 R_2 R_3 G_1 B_1 B_2 B_3 B_4 B_5 G_1

$4*6=24$ ← 6 6 6 5 G_1 * G_1 ✗

23



Permutations :- arrangements → order matters

a b c
a c b
b a c
b c a
c a b
c b a

→ arrange these letter ?

n object

$\underline{n} \times \underline{n-1} \times \underline{n-2} \times \underline{n-3} \times \ldots \underline{1} = n!$

a b c d e

$\underline{5}\ \underline{4} = 20$

a b
b a

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Combinations :- selection → sequence doesn't matter

{ B_1 $\cancel{B_2}$ B_3 B_4 $\cancel{B_5}$ }

1^{st} ball ≡ 5
2^{nd} ball ≡ 4

~~5 * 4 = 20~~

Final ans = 20/2 = 10

5 * 4 * 3 = 60

/ ~~3~~ 3 balls

/ 3!

{ B_2, B_5 }
{ B_5, B_2 }

B_2 B_3 B_5

3! = 6

arrangement

$\frac{60}{3!} = 10$

n = 100

$${}^nC_r = \frac{{}^nP_r}{r!} = \boxed{\frac{n!}{r!\,(n-r)!}}$$

→ no of way select r items from n item

Properties :-

$0! = ?$ 1

1) $^nC_0 = 1$ $\qquad \frac{n!}{0!\,n!} = 1$ — undef $\qquad 0! = 0$

2) $^nC_n = 1$

3) $^nC_1 = n$

4) $\boxed{^nC_r + {^nC_{r+1}} = {^{n+1}C_{r+1}}}$ $\qquad {^xC_y} = {^{x-1}C_y} + {^{x-1}C_{y-1}}$

$$\frac{n!}{r!\,(n-r)!} + \frac{n!}{(r+1)!\,(n-(r+1))!}$$

$(n-r)*(n-r-1)!$ $\qquad (r+1)*r!$ $\qquad (n-r-1)!$

$$= \frac{n!}{r!\,(n-r-1)!}\left[\frac{1}{(n-r)} + \frac{1}{(r+1)}\right]$$

$\frac{1}{2} + \frac{1}{3}$, $\frac{3+2}{6}$

$$= \frac{n!}{r!\,(n-r-1)!}\left[\frac{\cancel{r}+1+(n-\cancel{r})}{(r+1)*(n-r)}\right]$$

$n+1$

$-1+1=0$

$r!*(r+1)$ $= (r+1)!$

$$= \frac{(n+1)!}{(r+1)!\,(n-r)!} = {^{n+1}C_{r+1}}$$

$((n+1)-(r+1))!$

$n-r = (n+1)-(r+1)$

5) 5 balls

B_1 B_2 B_3 B_4 B_5

no of way choose 3 balls = reject 2 balls

$^5C_3$ Bag 1 3 balls

$^5C_3 * 1$

1 Bag2 2 balls

$1 * {^5C_2}$

$$\boxed{{}^{N}C_{r} = {}^{N}C_{N-r}}$$

Q. No of ways to choose $\boxed{\text{any no of objects}}$ from n objects → 2^n → = no of sub/subsets

{a, b, c}

2 2 2

$2 * 2 * 2 = 8$

{ }　{a, b}
{a}　{b, c}
{b}　{a, c}
{c}　{a, b, c}

$$ {}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + \dots {}^{n}C_n = 2^n $$

$$ (1+x)^n = {}^{n}C_0 x^0 + {}^{n}C_1 * x + {}^{n}C_2 x^2 + \dots {}^{n}C_n x^n $$

$x = 1$　Binomial Coeffi

$$ 2^n = {}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 \dots {}^{n}C_n $$

practice

Maths

Q

$^nC_r \ \% \ m$ $\qquad \frac{n!}{r!(n-r)!} \ \% \ m$ $\qquad$ prime

$0 \le r \le n$

$^nC_0 = 1 \qquad ^nC_n = 1$

$^nC_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$ — DP part

$n = 4$
$r = 2$
$m = 10$

$^4C_2$

n | r	0	1	2	3	4
1	1	1			
2	1	2	1		
3	1	3	3	1	
4	1	4	6	4	1

$^nC_0 = 1$

$^2C_1 = {}^1C_1 + {}^1C_0$

$^3C_1 = {}^2C_1 + {}^2C_0$

Pascal's Triangle

1
1 2 1
1 3 3 1
1 4 6 4 1

SC: $O(n*r) \approx O(r)$

TC: $O(n*r)$

factorial
power

$^nC_r \ \% \ p$ — prime

$\frac{n!}{r!(n-r)!} \ \% \ p = (n! \ \% \ p) * (r!^{-1} \ \% \ p) * ((n-r)!^{-1} \ \% \ p)$

$\approx O(n)$ $\qquad O(n)$ $\qquad r + \log(p-2)$ $\qquad (r!)^{p-2} \ \% \ p$ $\qquad (n-r)!^{p-2} \ \% \ p$

Q

$a^{b!} \% p$ → p is prime

$a^{b!} \% p$ → $(a \% p)^{b!} \% p$

$a^{p-1} \% p = 1$

$b! = (p-1) * q + r$

$\frac{b!}{p-1}$

$a^{x * q}$

$a^x * a^x * a^x * a^x \ldots q$

$\left(a^{(p-1)q+r}\right) \% p$

$= \left(a^{(p-1)q} * a^r\right) \% p$

$= \left(\left(a^{(p-1)q} \% p\right) * \left(a^r \% p\right)\right) \% p$

$0 - (p-1)$

log r

$\left(a^{p-1} * a^{p-1} * a^{p-1} * a^{p-1}\right) \% p$ q times

$(a^{p-1} \% p) * (a^{p-1} \% p) * (a^{p-1} \% p)$

1 1 1

$b! \% (p-1)$

ans = 1
for(i=1; i<=b; i++)
ans = (ans % p-1 * i) % p

1) fut find $r = b! \% (p-1) \rightarrow O(b)$

2) $a^r \% p$ → fast modelo

log r

a^r — $a^{r/2} * a^{r/2}$

→ b + log r

T·C

$$\boxed{a^{x} \% p = a^{\underline{x \% (p-1)}} \% p}$$

p is prime

$${}^{n}C_{r} = {}^{n}C_{r-1} \times \frac{(n-r+1)}{r}$$