Today's Quote

Today's Content

- % operator
- Modular arithmetic
- 1 Hard problem

Range

int → $[-2 \times 10^9, 2 \times 10^9]$

long → $[-9 \times 10^{18}, 9 \times 10^{18}]$

% Basics

n % a = remainder when n is divided by a.

13 % 4 = 1

17 % 3 = 2

Dividend = divisor * quotient + remainder

Remainder = Dividend - (divisor * quotient)

↳ {greatest mult of div <= dividend}

Quizes.

① $150 \% 11 = 150 - (\text{greatest mult of } 11 <= 150) = 7$
(143)

② $100 \% 7 = 100 - (\text{greatest mult of } 7 <= 100) = 2$
(98)

③ $-40 \% 7 = -40 - (\text{greatest mult of } 7 <= -40) = 2$
(-42)

④ $-60 \% 9 = -60 - (\text{greatest mult of } 9 <= -60) = 3$
(-63)

⑤ $-40 \% 9 = -40 - (\text{greatest mult of } 9 <= -40) = 5$
(-45)

Python — Java / C / C++ / C# / JS (→ Doubt session, → Extra content)

-40 % 7	2	+7	-5
-60 % 9	3	+9	-6
-40 % 9	5	+9	-4

in these languages

```
if (a < 0){
    a % p + p
}
```

Why % → limit our input data in required range.

-50,000 ... 50,000 } % 10 = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
= [0-9]

- Hashing {upcoming D.S.A}
- Consistent Hashing.
  {HLD, LLD}.

-∞ ... +∞ } % p = [0, p-1].

% + { +, -, *, / }

Modular Arithmetic

{[0, p-1] + [0, p-1]} > p

$$(a+b) \% p = ((a \% p) + (b \% p)) \% p$$

a	b	p
8	6	10

(4)

(8 % 10) + (6 % 10)

= (8 + 6) % 10 = 14 % 10 = (4)

a	b	p
5	4	6

(3)

(5 % 6) + (4 % 6)

= (5 + 4) % 6 = 9 % 6 = (3)

$$(a * b) \% p = [a \% p * b \% p] \% p$$

a	b	p
8	6	10

(8)

$(8 \% 10) * (6 \% 10)$

$= (8 * 6) \% 10 = (48) \% 10$

$= 8$

$(a - b) \% p$

$(a / b) \% p$

→ Advanced module.

→ { Inverse modulo }.

① $(a \% p) \% p = a \% p$

$[0, p-1] \% p = [0, p-1]$.

$6 \% 8 = 6$.

$(6 \% 8) \% 8 = 6$.

② $(a \% p * b) \% p = (a * b) \% p$

$x = a \% p \quad y = b$.

$$(x * y) \% p = (x \% p * y \% p) \% p$$

$$= ((a \% p) \% p * (b \% p)) \% p$$

$$= ((a \% p) * (b \% p)) \% p$$

$$= (a * b) \% p.$$

Divisibility Rules

%3 → sum of digits should be divisible by 3.	1, 2, 3,
%9 → sum of digits should be divisible by 9.	4, 5, 6
%4 → last two digits should be divisible by 4.	7, 8, 9
%8 → last three digits should be divisible by 8.	⇓
	{Todo}.

Proof %3

$$2475 \,\%\, 3 = [2\times10^3 + 4\times10^2 + 7\times10^1 + 5\times10^0]\,\%\,3$$

$$= [(2\times10^3)\%3 + (4\times10^2)\%3 + (7\times10^1)\%3 + (5\times10^0)\%3]\%3$$

$$= [2\%3 + 4\%3 + 7\%3 + 5\%3]\%3$$

$$= [2+4+7+5]\%3$$

observation:

$10^0 \%3 = 1$	$10^0 \%9 = 1$
$10^1 \%3 = 1$	$10^1 \%9 = 1$
$10^2 \%3 = 1$	$10^2 \%9 = 1$
$10^3 \%3 = 1$	$10^3 \%9 = 1$
$10^x \%3 = 1$	$10^x \%9 = 1$

Proof %4

$$2457 \,\%\, 4 = (2400 + 57)\,\%\,4$$

$$= ((2400\%4) + (57\%4))\,\%\,4$$

$$= (57\%4)\%4 = 57\%4$$

observation

$10^2 \%4 = 0$
$10^3 \%4 = 0$
$10^4 \%4 = 0$

Any multiple of 100 will be divisible by 4.

Q) Given a, n, p. Calculate $a^n \% p$ without inbuilt functions.

[Constraints $1 <= a <= 10^9$, $2 <= p <= 10^9$, $1 <= n <= 10^5$]

Eg: a = 3, n = 4, p = 7 $\quad 81 \% 7 = 4$.

① ✗

```
func ( a , n , p) {
    for (i = 1 ; i <= N ; i++) {
        a = a * a
    }
    return a % p ;
}
```

[a, n = 4,] ⇒ $a^4 \% p$

i	value of a.
1	a^2 [a*a]
2	a^4 [a*a]
3	a^8 ← [a^4*a^4]
4	a^{16} ← [a^8*a^8]

②

```
func ( a , n , p) {
long ans = 1
    for (i = 1 ; i <= N ; i++) {
        ans = (ans * a) % p
    }
    return ans ;
}
```

i	value of ans
1	ans = a
2	ans = a^2
3	ans = a^3
4	ans = a^4

a.	n.	p.	
2	30	47	$2^{30} \% 47$
2	60	47	$2^{60} \% 47$
2	100	47	$2^{100} \% 47$

dry-run. // Given a, n=4, p.

(long) ans.	i	i <= 4
1	1	✓
a%p	2	✓
a^2%p	3	✓
a^3%p	4	✓

ans = (ans * a) % p

ans = a % p

ans = (a%p * a) % p

= ((a%p)%p * a%p) % p

= (a%p * a%p) % p

= a^2 % p.

No overflow

ans = (a^2%p * a) % p

(10^9 * 10^9)

ans = a^3 % p. No overflow

ans = ((a^3%p) * a) % p

(10^9 * 10^9)

No overflow

= a^4 % p

Todo → a = 10^9, n = 5, p

Q) Given 1 number in arr[] format. Calculate arr[] % p

Note → arr[i] represents a single digit of number.

Google

Constraints :

$1 <= N <= 10^5$

$0 <= arr[i] <= 9$

$2 <= p <= 10^9$

Eg : arr[5] : | 7 | 2 | 6 | 4 | 3 | , p = 50

0 1 2 3 4

↳ {72643 % 50 = 43}

ideas : Convert arr[] → number % p.

N=2. 9 9 $= 10^2 - 1$

N=3. 9 9 9 $= 10^3 - 1$

N=4. 9 9 9 9 $= 10^4 - 1$

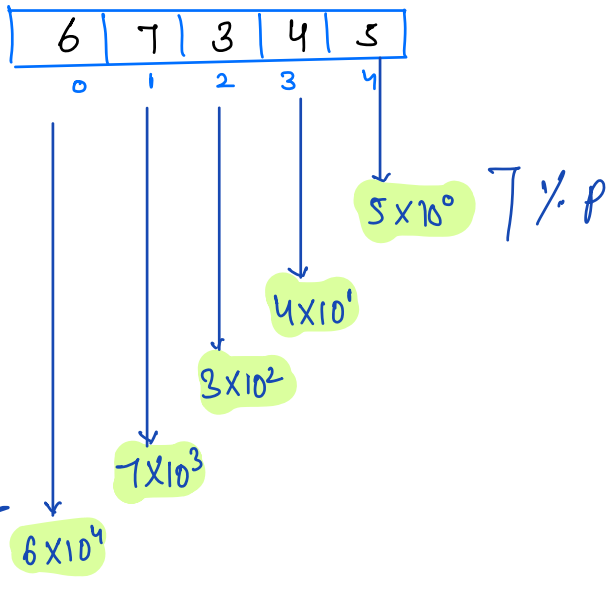
N $10^N - 1$

$N = 10^5$ = 100000 ---------- (10^5 digits) $= 10^{100000} - 1$

v.v.v.v large.

Hint. → Split the no. digit by digit & then try to calculate your ans.

ans :



$= [\ 6\times10^4 + 7\times10^3 + 3\times10^2 + 4\times10^1 + 5\times10^0\ ]\ \%p$

$(5\times10^0)\%p$] %p

$t = 1$

5 * t

$(4\times10^1)\%p$

$t = (t * 10)\%p$

4 * t

$(3\times10^2)\%p$

$t = (t * 10)\%p$

3 * t

$(7\times10^3)\%p$

$t = (t * 10)\%p$

7 * t

[$(6\times10^4)\%p$

6 * t

$t = (t * 10)\%p$

[ans = (ans * a) %p]

pseudo-code.

Code

```
fun( arr, N, p) {
    long ans = 0
    long t = 1    // 10^0 = 1
    for( i = n-1 ; i >= 0 ; i--) {
        ans = (ans + arr[i]*t) % p
        t = (t * 10) % p.
    }
    return ans;
}
```

$10^9 * 10$ int ✗ long ✓

T.C → O(N)
S.C → O(1)

Why % is behaving differently for Java/C/C++/C#/JS.

Remainder = dividend - div * quo.

$100 \% 7 = 100 - (\text{greatest mult of } 7 <= 100)$

$7 * \left[\frac{100}{7}\right]$

$= 100 - (7 * 14) = 100 - 98 = 2.$

$-40 \% 7 = -40 - 7 * \left[\frac{-40}{7}\right]$ ($= -5$)

$= -40 - (7 * -5)$

$= -40 + 35 = -5.$

$-60 \% 9 = -60 - (\text{greatest mult of } 9 <= -60)$

$= -60 - \left[9 * \left[\frac{-60}{9}\right]\right]$ ($= -6$)

$= -60 - [9 * -6]$

$= -60 + 54 = -6.$

% in python.

floor (100/7) = floor(14.---)

100 % 7 = 100 - {greatest mul of 7 <=100}

= 100 - {7 x 100/7}

= 100 - (7x14) = (2)

-40 % 7 = -40 - [7 * {-40/7}]

= -40 - [7 * -6]

= -40 + 42 = (2).

floor(-40/7)

= floor(-5.76)

= -6.

-37 % 7 = -37 - [7 * {-37/7}]

= -37 - [7 * -6]

= -37 + 42 = (5).

floor(-37/7)

= floor(-5.4--)

= -6

36/7 → 5.---

36 // 7 → floor

$(A | (1 << B)) == A$ Java | C# |

→ Always use brackets at every step while using bitwise operators. [∵ their precedence is very low].

$((A | (1 << B)) \& C)$