Today's Quote →

The man who asks a question
is a fool for a minute,
the man who does not ask
is a fool for life.

{ Pen and Paper }

Content   # no. of iterations

→ Time and Space Complexity
→ Assymptotic Analysis
→ Big.O Notation
→ T.L.E → Time Limited Exceeded.

TC-2.

① Sum of first N natural numbers $= \frac{N(N+1)}{2}$.

② How many no's are there in this range $[3, 10]$.
↳ 8.

$[a, b] = b - a + 1$

$[a, b) = b - a$

$(a, b) = b - a - 1$

$[3, 10] \Rightarrow 10 - 3 + 1 = 8$

{ [→ inclusive
(→ exclusive }

③ How many times do we need to divide N by 2 to reduce it to 1?

$N = 64 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1$

$N = 64 = 2^6$ No. of steps $\to$ 6

$N = 1024 = 2^{10}$ No. of steps $\to$ 10

[$\log_2 N$ times]

④ $\boxed{\log_a a^x = x}$

$\log_2 2^{10} = 10$

$\log_3 3^5 = 5.$

Arithmetic Progression

$\Rightarrow$ difference b/w any two consecutive terms is fixed/constant

$4, 7, 10, 13, 16, 19, \ldots$

$d = \underbrace{}_{3}\ \underbrace{}_{3}\ \underbrace{}_{3}\ \underbrace{}_{3}\ \underbrace{}_{3}$

$a \quad a+d \quad a+2d \quad a+3d \quad a+4d \quad \ldots \quad a+(n-1)d$

first term $= a$

common diff $= d$

[Sum of N terms of an A.P $= \frac{n}{2}\left[2a + (n-1)d\right]$]

Geometric Progression (G.P)

ratio of 2 consecutive terms → constant.

$$3, \underbrace{6}_{2}, \underbrace{12}_{2}, \underbrace{24}_{2}, \underbrace{48}_{2}, \underbrace{96}_{2}, ---$$

Common ratio = 2 2 2 2 2
(r)

$$a, ar, ar^2, ar^3, ar^4, ---- ar^{n-1}$$

first term = a
common ratio = r

$$\text{Sum of N terms of G.P} = \frac{a[r^n - 1]}{[r-1]} \qquad r > 1$$

$$= \frac{a[1 - r^n]}{[1-r]} \qquad r < 1$$

→ {class-10 NCERT}

Q1.) void fun(int N){

```
    s = 0;
    for( i = 1 ; i <= N ; i++){
        s = s+i;
    }
    return s;
}
```

$i : [1, N]$

$\Rightarrow N - \cancel{1} + \cancel{1} = N$ iterations.

Q2.) int fun(int N){

```
    s = 0;
    for( i = 1 ; i <= N ; i = i + 2){
        s = s + i;
    }
}
```

iterations.

N=12 $\quad i \rightarrow 1, 3, 5, 7, 9, 11$ $\quad 12/2 \Rightarrow [6]$

$\frac{(n+1)}{2}$

N=13 $\quad i \rightarrow 1, 3, 5, 7, 9, 11, 13$ $\quad 7 = \frac{(13+1)}{2} = \frac{14}{2}$

no. of iterations $= \frac{(n+1)}{2}$.

Q3.)

```
void func (int N, int M){
    for(i=1 ; i<=N ; i++){
        if(i%2==0){
            print(i)
        }
    }
    for(i=1 ; i<=M ; i++){
        if(i%2==0){
            print(i)
        }
    }
}
```

: $i \rightarrow [1, N]$
$= N$

: $i \rightarrow [1, M]$
$= M$

$\#$ total no. of iterations $= N+M$.

Q4.)

```
int fun(int N){
    s=0;
    for(i=0 ; i<=100 ; i++){
        s=s+i+i^2;
    }
    return s;
}
```

: $i \rightarrow [0, 100]$

$100-0+1 = 101$

$\#$ no. of iterations $= 101$

Q5)
```
void fun(N){
    for( i=1 ; i*i <= N ; i++){
            s = 1+i2 ;
    }
    return s ;
}
```

$i*i <= N$

$i^2 <= N$

$i <= \sqrt{N}$

$i : [1, \sqrt{N}]$

no. of iterations = $\sqrt{n}$.

Q6)
```
void fun( int N){
        i = N;
        while ( i > 1){
                i = i / 2;
        }
}
```

iterations	value of i
1	$N/2^1$
2	$N/2^2$
3	$N/2^3$
4	$N/2^4$
5	$N/2^5$
⋮	
	1 .

$$\frac{N}{2^0} \rightarrow \frac{N}{2^1} \rightarrow N/2^2 \rightarrow N/2^3 \rightarrow N/2^4 \rightarrow N/2^5 \rightarrow ----1$$

if $i == 1$, loop will break.

After K iteration, loop will break.

$$\Rightarrow \quad i = \boxed{1 = \frac{N}{2^K}}$$

$2^k = N$

$\log_2 2^k = \log_2 N$

$\Rightarrow \boxed{k = \log_2 N}$

no. of iterations $= \log_2 N$

Q.

```
void fun(int N){
        i = N;
        while (i > 1){
                i = i/3;
        }
}
```

$N \rightarrow N/3 \rightarrow \frac{N}{9} \rightarrow N/27 \rightarrow \frac{N}{81} \rightarrow - - - - 1.$

After k iterations, loop will break.

$1 = \frac{N}{3^k}$

$3^k = N$

$\log_3 3^k = \log_3 N \Rightarrow \boxed{k = \log_3 N}$

Amazon MCQ → Multiple choice question

Q7.)
```
void fun( int N){
    s=0;
    for( i=0 ; i<N ; i=i*2){
        s=s+i;
    }
}
```

$0 \to 0 \to 0 \to 0 \to 0$

no. of iterations → infinite.

Q8.)
```
void fun( N){
    s=0;
    for( i=1 ; i < N ; i=i*2){
        s=s+i;
    }
}
```

iterations	value of i
1	2
2	2^2
3	2^3
4	2^4
5	2^5
⋮	⋮

$1 \to 2 \to 4 \to 8 \to 16 \to$

if i == N , loop will break.

After k iterations, loop will break.

$$N = 2^k$$

$$\log_2 N = \log_2 2^k \Rightarrow \boxed{\log_2 N = k}$$

no. of iterations = $\log_2 n$.

Nested loops

Q9.)

```
void fun(int N){
    for(i=1; i<=10; i++){
        for(j=1; j<=N; j++){
            print(--)
        }
    }
}
```

table:

value of i	j	iterations
1	[1, N]	N +
2	[1, N]	N +
3	[1, N]	N +
⋮		
10	[1, N]	N

total no. of iteration = 10N

Q10.)

```
void fun(int N){
    for(i=1; i<=N; i++){
        for(j=1; j<=N; j++){
            print(i*j);
        }
    }
}
```

table

i	j	iterations
1	[1, N]	N +
2	[1, N]	N +
3	[1, N]	N +
⋮		⋮ +
N	[1, N]	N

no. of iterations = $N * N = N^2$.

Q11)

```
void fun(int N){
    for(i=1 ; i <= N ; i++){
        for(j=1 ; j <= i ; j++){
            print(i*j);
        }
    }
}
```

i	j	iterations
1	[1, 1]	1
2	[1, 2]	+ 2
3	[1, 3]	+ 3 +
⋮		⋮
N	[1, N]	+ N

total no. of iterations = $\frac{N(N+1)}{2}$.

Q12.)

```
void fun(int N){
    for(i=1 ; i <= N ; i++){
        for(j=1 ; j < N ; j=j*2){
            print(i*j);
        }
    }
}
```

i	j	iterations
1	[1, N)	$\log_2 N$
2	[1, N)	+ $\log_2 N$
3	[1, N)	+ $\log_2 N$ +
⋮	⋮	⋮ +
N	[1, n)	$\log_2 N$

total no. of iterations = $\log_2 N * N$

$= N \log_2 N$.

Q14.)

```
void fun( int N) {
    for (i=1 ; i <=N ; i++) {
        for(j=1 ; j <= 2^i ; j++) {
            print(i*j);
        }
    }
}
```

i	j	iterations
1	$[1, 2]$	2
2	$[1, 2^2]$	$4 = 2^2$
3	$[1, 2^3]$	$8 = 2^3$
⋮		⋮
N	$[1, 2^N]$	2^N

\# total no. of iterations $= 2^1 + 2^2 + 2^3 + \text{———} \, 2^N$

$$= \frac{2[2^N - 1]}{[2-1]} = \underline{\underline{2[2^N - 1]}}$$

$$\frac{a \cdot [r^n - 1]}{[r-1]}$$

Q:)

```
for( i= N ; i > 0 ;  i= i/2){
    for( j=1 ; j <= i ; j++){
        print(i*j)
    }
}
```

table

i	j	iterations
N	$[1, N]$	$N/2^0$ +
$N/2$	$[1, N/2]$	$N/2^1$ +
$N/4$	$[1, N/4]$	$N/2^2$ +
⋮		$N/2^3$ ⋮
		$\frac{N}{2^{\log_2 N}}$

$1 + \log_2 N \Leftarrow$

total no. of iterations $= N + \frac{N}{2^1} + \frac{N}{2^2} + \frac{N}{2^3} + --- \frac{N}{2^{\log_2 N}}$

$$= N + N\underbrace{\left[\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + --- \frac{1}{2^{\log_2 N}}\right]}_{S = \frac{N-1}{N}}$$

$$= N + \cancel{N}\left[\frac{N-1}{\cancel{N}}\right] = \boxed{2N-1}$$

$$S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + --- \frac{1}{2^{\log_2 N}}$$

$$S = \frac{\cancel{\frac{1}{2}}\left[1 - \left(\frac{1}{2}\right)^{\log_2 N}\right]}{\cancel{\left(1 - \frac{1}{2}\right)}}$$

$r < 1$

$$\frac{a[1-r^n]}{[1-r]}$$

$[a = 1/2]$

$[r = 1/2]$

$[\text{term} = \log_2 N]$

$$S = 1 - \frac{1}{2^{\log_2 N}} = 1 - \frac{1}{N} = \frac{N-1}{N}.$$

$$\Rightarrow \boxed{a^{\log_a x} = x} \Rightarrow \text{Property of log}$$

$$2^{\log_2 N}$$

$$\Rightarrow N^{\log_2 2} = N^1 = N$$

Comparison of terms- { large value of N }

$$N\,[\log_2 N] < [\sqrt{n}] * N$$

$$N\,[\sqrt{n}] < [n] * N$$

$$\{ \log_2 N < \sqrt{n} < n < N\log N < N\sqrt{N} < N^2 < 2^N \}$$

$N \to 2^{10}$

$$\log_2 2^{10} \Rightarrow 10 \qquad \sqrt{2^{10}} \Rightarrow 2^5 = 32.$$

$$\log N < \sqrt{N}.$$

$$N^2 \qquad (10)^2 \Rightarrow 100$$

$$2^N \qquad (2^{10}) \qquad 1024.$$

How to calculate Big-O ?

what ?
why ? → Next class.

1. Calculate no. of iterations
2. Ignore lower order terms.
3. Ignore constant / coefficients.

$$f(N) = 10N^2 + \underbrace{100N^1 + 10^3 N^0}_{\times}$$

$$= 10N^2$$

$$\Rightarrow O(N^2)$$

Q) $f(N) = 4N^2 + \underbrace{3N^1 + 10^5 N^0}_{\times} \Rightarrow O(N^2)$

Q) $f(N) = \underbrace{4N}_{\times} + 3N\log N + \underbrace{10^6}_{\times} \Rightarrow O(N\log N)$

Q) $f(N) = \underbrace{4N\log N}_{\times} + \underbrace{3}_{\times} N\sqrt{N} + \underbrace{10^6}_{\times} \Rightarrow O(N\sqrt{N})$

{21 Question.}

Assigment / H.W question.

→ table is the ultimate truth.

$\boxed{i = 6.}$

```
while (i > 1) {
    i = i / 7
}
```

$\log_7 6$

```
→ {send me the code.}
```

```
for( i = 1 ; i <= n ; i *= 2){
    for( j = 1 ; j <= n ; j++){
        //—
    }
}
```

i	j	iterns
1	[1, N]	N +
2	[1, N]	N +
4	[1, N]	N +
8	[1, N]	N +
⋮	⋮	⋮

$\boxed{N}$ | $[1, N]$ | N

$\underbrace{1, 2, 4, 8, 16, \text{— —}}_{\text{K-terms.}}$

$$2^{K-1} = N$$

$$\log_2 2^{K-1} = \log_2 N$$

$$\{ K = \log_2 N + 1 \}$$

$N * (\log_2 N + 1)$

$\Rightarrow O(N \log N)$

$\Rightarrow$ Quiz. $\rightarrow$ { class performance. }

$\hookrightarrow$ { interactive }

$\rightarrow$ All classes.