1.1.5.15

EE24BTECH11045 - N.Tapasvi

Question:

The midpoint of the line segment joining $\mathbf{A}\begin{pmatrix}2a\\4\end{pmatrix}$ and $\mathbf{B}\begin{pmatrix}-2\\3b\end{pmatrix}$ is $\mathbf{M}\begin{pmatrix}1\\2a+1\end{pmatrix}$. Find the values of a and b. (10,2019)

Solution:

Variable	Description
A	$\begin{pmatrix}2a\\4\end{pmatrix}$
B	$\begin{pmatrix}-2\\3b\end{pmatrix}$
M(Midpoint)	$\begin{pmatrix}1\\2a+1\end{pmatrix}$
a, b	Values to be found

TABLE I: Variables Used

Let M divide AB in the ratio k:1 then,M=

$$\frac{kB+A}{k+1}$$

As M is the midpoint k=1

Let the midpoint **M** be given by the formula:

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2}$$

Substituting the coordinates of **A** and **B**:

$$\mathbf{M} = \frac{1}{2}\begin{pmatrix}2a-2\\4+3b\end{pmatrix}$$

Since $\mathbf{M} = \begin{pmatrix}1\\2a+1\end{pmatrix}$, we equate the corresponding components:

$$\frac{2a-2}{2} = 1 \quad \text{and} \quad \frac{4+3b}{2} = 2a+1$$

From the first equation:

$$2a-2=2 \quad \Rightarrow \quad 2a=4 \quad \Rightarrow \quad a=2$$

Substitute $a = 2$ into the second equation:

$$\frac{4 + 3b}{2} = 2(2) + 1 = 5 \quad \Rightarrow \quad 4 + 3b = 10 \quad \Rightarrow \quad 3b = 6 \quad \Rightarrow \quad b = 2$$

Thus, $a = 2$ and $b = 2$.

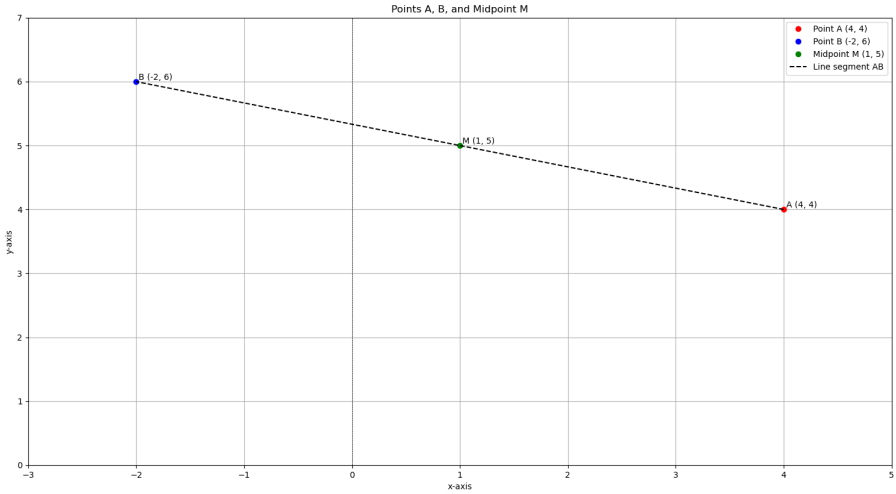

Fig. 1: Plot of the points A,B,M