

1.1.5.15

EE24BTECH11045 - N.Tapasvi

Question:

The midpoint of the line segment joining $\mathbf{A}\begin{pmatrix} 2a \\ 4 \end{pmatrix}$ and $\mathbf{B}\begin{pmatrix} -2 \\ 3b \end{pmatrix}$ is $\mathbf{M}\begin{pmatrix} 1 \\ 2a + 1 \end{pmatrix}$. Find the values of a and b . (10,2019)

Solution:

Variable	Description
A	$\begin{pmatrix} 2a \\ 4 \end{pmatrix}$
B	$\begin{pmatrix} -2 \\ 3b \end{pmatrix}$
M(Midpoint)	$\begin{pmatrix} 1 \\ 2a + 1 \end{pmatrix}$
a, b	Values to be found

TABLE I: Variables Used

The section formula in general:

$$\begin{pmatrix} x_M \\ y_M \end{pmatrix} = \frac{1}{m+n} \begin{pmatrix} mx_2 + nx_1 \\ my_2 + ny_1 \end{pmatrix}$$

Since the midpoint divides the segment in the ratio 1:1, $m = n = 1$:

$$\begin{pmatrix} x_M \\ y_M \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x_2 + x_1 \\ y_2 + y_1 \end{pmatrix}$$

Substitute the coordinates of points A and B:

$$\begin{pmatrix} x_M \\ y_M \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -2 + 2a \\ 3b + 4 \end{pmatrix}$$

Given midpoint M has coordinates $(1, 2a + 1)$, so we can equate the coordinates:
For the x -coordinate:

$$\frac{1}{2}(-2 + 2a) = 1 \Rightarrow -2 + 2a = 2 \Rightarrow 2a = 4 \Rightarrow a = 2$$

For the y -coordinate:

$$\frac{1}{2}(3b + 4) = 2a + 1 \Rightarrow \frac{1}{2}(3b + 4) = 2(2) + 1 \Rightarrow \frac{1}{2}(3b + 4) = 5$$

Multiply both sides by 2:

$$3b + 4 = 10 \Rightarrow 3b = 6 \Rightarrow b = 2$$

Therefore, the values of a and b are:

$$a = 2, \quad b = 2$$

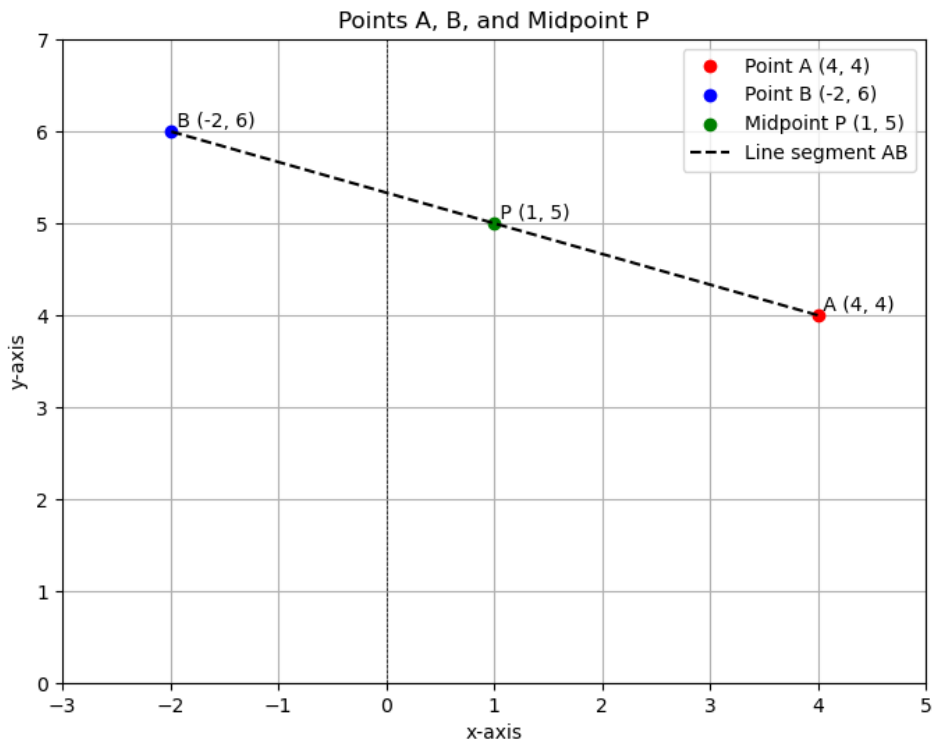


Fig. 1: Plot of the points A,B,M