GradientMethods_CodingAssignment

June 25, 2024

1 Coding Assignment "Gradient Methods"

1.1 1. Preparation

[1]:
```
import numpy as np
import pandas as pd
from datetime import datetime
import matplotlib.pyplot as plt
from sklearn.linear_model import Ridge
from sklearn.metrics import mean_squared_error
from numpy import linalg as LA
```

1.2 2. Data

1.2.1 2.1 Dataset

[2]:
```
# Import the weather measurements.
data = pd.read_csv('Assignment_MLBasicsData.csv')

# We consider each temperature measurement (=a row in dataframe data)
# as a separate data point.
# Determine the total number of data points stored in csv file.
nrdatapoints = len(data)

# Print out the first data point (first row).
print("First data point:")
print(data.iloc[0])
print("\n******************************\n")

# Here is another data point.
print("Another data point:")
print(data.iloc[13])
print("\n******************************\n")

# We use normalized values of
# latitude, longitude, year, mon, day, hour, minute (as float values)
# as features of a data point.
nrfeatures = 7
```

```python
# The code snippet below extracts the features of the first data point (first␣
↪row in dataframe data).
date_object = datetime.strptime(data['Timestamp'].iloc[0], '%Y-%m-%d %H:%M:%S')

# Extract individual components.
latitude = data["Latitude"].iloc[0]
longitude = data["Longitude"].iloc[0]
year = float(date_object.year)
month = float(date_object.month)
day = float(date_object.day)
hour = float(date_object.hour)
minute = float(date_object.minute)

print("Unnormalized features of the first data point: ")
print(f"Latitude: {latitude}")
print(f"Longitude: {longitude}")
print(f"Year: {year}")
print(f"Month: {month}")
print(f"Day: {day}")
print(f"Hour: {hour}")
print(f"Minute: {minute}")
print("\n******************************\n")

# We choose the temperature as the label (quantity of interest) of a data point.
print("Label of first data point:", data["temp"].iloc[0])
```

```
First data point:
Latitude                          69.04277
Longitude                         20.85091
Timestamp              2023-12-31 18:00:00
temp                                 -16.5
name          Enontekiö Kilpisjärvi Saana
Name: 0, dtype: object

******************************

Another data point:
Latitude                       69.757
Longitude                      27.012
Timestamp         2023-12-31 13:00:00
temp                            -26.3
name          Utsjoki Kevo Kevojärvi
Name: 13, dtype: object

******************************
```

```
Unnormalized features of the first data point:
Latitude: 69.04277
Longitude: 20.85091
Year: 2023.0
Month: 12.0
Day: 31.0
Hour: 18.0
Minute: 0.0

******************************

Label of first data point: -16.5
```

1.2.2 2.2 Features and labels

[3]:
```python
# We next build the feature matrix X (each of its rows hold the features of a␣
↪data point)
# and the label vector y (whose entries hold the labels of data points).
X = np.zeros((nrdatapoints, nrfeatures))
y = np.zeros((nrdatapoints, 1))

# Iterate over all rows in dataframe and create corresponding feature vector␣
↪and label.
for ind in data.index:
    # Latitude of FMI station, normalized by 100.
    lat = float(data['Latitude'].iloc[ind]) / 100

    # Longitude of FMI station, normalized by 100.
    lon = float(data['Longitude'].iloc[ind]) / 100

    # Exctract the temperature value.
    tmp = data['temp'].iloc[ind]

    # Read the date and time of the temperature measurement.
    date_object = datetime.strptime(data['Timestamp'].iloc[ind], '%Y-%m-%d %H:
↪%M:%S')

    # Extract year, month, day, hour, minute, and second.
    # Normalize these values to ensure features are in range [0,1].
    year = float(date_object.year) / 2025
    month = float(date_object.month) / 13
    day = float(date_object.day) / 32
    hour = float(date_object.hour) / 25
    minute = float(date_object.minute) / 61

    # Store the data point's features and a label.
    X[ind,:] = [lat, lon, year, month, day, hour, minute]
```

```python
    y[ind,:] = tmp

print(f"The created feature matrix contains {np.shape(X)[0]} entries of {np.
 ↪shape(X)[1]} features each.")
print(f"The created label vector contains {np.shape(y)[0]} measurements.")
```

```
The created feature matrix contains 19768 entries of 7 features each.
The created label vector contains 19768 measurements.
```

1.2.3 2.3 Training and validation sets

[4]:
```python
# Define the number of data points used for training set.
trainsize = 100

# Split the dataset into training and validation set.
Xtrain = X[:trainsize,:]
Xval = X[trainsize:]
ytrain = y[:trainsize]
yval = y[trainsize:]

print(f"The training set consists of {np.shape(Xtrain)[0]} data points.")
print(f"The validation set consists of {np.shape(Xval)[0]} data points.")
```

```
The training set consists of 100 data points.
The validation set consists of 19668 data points.
```

1.3 3. Model

1.3.1 3.1 Ridge regression - Ready made implementation via Scikit-learn class

[5]:
```python
# Define the regularization parameter.
regparam = 0.01

# Create a ridge regression using scikit-learn class.
ridge = Ridge(alpha=(trainsize*regparam), fit_intercept=False )

# Train the linear model, i.e.,
# solve the ERM to obtain parameters of the linear model.
ridge.fit(Xtrain, ytrain)
Etrain = mean_squared_error(ytrain, ridge.predict(Xtrain))
Eval = mean_squared_error(yval, ridge.predict(Xval))

print("********** Ridge Regression Diagnosis **********")
print("Training error: ", Etrain)
print("Validation error: ", Eval)
```

```
********** Ridge Regression Diagnosis **********
Training error:  34.48452921511401
```

```
Validation error:  41.3733554147518
```

1.3.2 3.2 Student task #1 - Ridge regression by gradient descent

[6]:
```
# Define the initial parameters.
N_iters = 1000 # The number of gradient steps.
lrate = 0.1 # Learning rate

###################TODO####################
# TODO: Implement the GD Algorithm 2 for the objective function (2.27). Use
#       the initialization w^{0} = 0.
#       Use the resulting parameters (delivered by Algorithm 2) to compute the
#       average squared error loss on the training set (= training error E_t)
#       and the average squared error loss on the validation set (=validation␣
↪error E_v)

raise NotImplementedError
```

```
---------------------------------------------------------------------------
NotImplementedError                       Traceback (most recent call last)
Cell In[6], line 12
      3 lrate = 0.1 # Learning rate
      5 ###################TODO####################
      6 # TODO: Implement the GD Algorithm 2 for the objective function (2.27).␣
↪Use
      7 #       the initialization w^{0} = 0.
      8 #       Use the resulting parameters (delivered by Algorithm 2) to␣
↪compute the
      9 #       average squared error loss on the training set (= training error␣
↪E_t)
     10 #       and the average squared error loss on the validation set␣
↪(=validation error E_v)
---> 12 raise NotImplementedError

NotImplementedError:
```

1.3.3 3.3 Student task #2 - The optimal learning rate

[14]:
```
# You only have to try out the following values for the learning rate:
lrates = np.linspace(0.28, 0.32, 20) # Learning rate

###################TODO####################
# TODO: Modify the implemented GD Algorithm 2
#       to find the optimal learning rate value,
#       such that the objective value converges to the optimum
#       with the minimum number of gradient steps.
```

```python
# NOTE: Monitor the decrease in the objective function
#       and compare it with the chosen tolerance as a stopping criterion.
# HINT: The tolerance in the solution notebook was chosen 1e-10.

raise NotImplementedError
```

```
The learning rate: 0.314
The number of iterations: 1405
The objective value: 38.124307543684715

The learning rate: 0.3140578947368421
The number of iterations: 1405
The objective value: 38.1243075436587

The learning rate: 0.31411578947368424
The number of iterations: 1405
The objective value: 38.12430754363278

The learning rate: 0.3141736842105263
The number of iterations: 1405
The objective value: 38.12430754360698

The learning rate: 0.3142315789473684
The number of iterations: 1404
The objective value: 38.12430754368108

The learning rate: 0.3142894736842105
The number of iterations: 1404
The objective value: 38.12430754365545

The learning rate: 0.31434736842105265
The number of iterations: 1404
The objective value: 38.124307543630586

The learning rate: 0.31440526315789474
The number of iterations: 1404
The objective value: 38.12430754360769

The learning rate: 0.31446315789473683
The number of iterations: 1404
The objective value: 38.124307543590234

The learning rate: 0.3145210526315789
The number of iterations: 1404
The objective value: 38.124307543587925

The learning rate: 0.31457894736842107
```

```
The number of iterations: 1404
The objective value: 38.12430754362827

The learning rate: 0.31463684210526316
The number of iterations: 1407
The objective value: 38.12430754348415

The learning rate: 0.31469473684210525
The number of iterations: 1413
The objective value: 38.124307543334616

The learning rate: 0.31475263157894734
The number of iterations: 1426
The objective value: 38.12430754314585

The learning rate: 0.3148105263157895
The number of iterations: 1452
The objective value: 38.12430754284977

The learning rate: 0.3148684210526316
The number of iterations: 1493
The objective value: 38.124307542670635

The learning rate: 0.31492631578947367
The number of iterations: 1548
The objective value: 38.12430754275323

The learning rate: 0.31498421052631576
The number of iterations: 1616
The objective value: 38.12430754296574

The learning rate: 0.3150421052631579
The number of iterations: 1695
The objective value: 38.12430754327658

The learning rate: 0.3151
The number of iterations: 1784
The objective value: 38.12430754369401

********** GD Ridge Regression Diagnosis **********
The optimal hyperparameters:
The learning rate: 0.3148684210526316
The tolerance: 1e-10
The objective value: 38.124307542670635

Training error:  34.48461250328867
Validation error:  41.20984717716626
```

```
[ ]:
```

GradientMethods_RefSol

June 25, 2024

1 Reference Solution for Coding Assignment "Gradient Methods"

1.1 1. Preparation

[1]:
```
import numpy as np
import pandas as pd
from datetime import datetime
import matplotlib.pyplot as plt
from sklearn.linear_model import Ridge
from sklearn.metrics import mean_squared_error
from numpy import linalg as LA
```

1.2 2. Data

1.2.1 2.1 Dataset

[2]:
```
# Import the weather measurements.
data = pd.read_csv('Assignment_MLBasicsData.csv')

# We consider each temperature measurement (=a row in dataframe data)
# as a separate data point.
# Determine the total number of data points stored in csv file.
nrdatapoints = len(data)

# Print out the first data point (first row).
print("First data point:")
print(data.iloc[0])
print("\n*****************************\n")

# Here is another data point.
print("Another data point:")
print(data.iloc[13])
print("\n*****************************\n")

# We use normalized values of
# latitude, longitude, year, mon, day, hour, minute (as float values)
# as features of a data point.
nrfeatures = 7
```

```python
# The code snippet below extracts the features of the first data point (first␣
↪row in dataframe data).
date_object = datetime.strptime(data['Timestamp'].iloc[0], '%Y-%m-%d %H:%M:%S')

# Extract individual components.
latitude = data["Latitude"].iloc[0]
longitude = data["Longitude"].iloc[0]
year = float(date_object.year)
month = float(date_object.month)
day = float(date_object.day)
hour = float(date_object.hour)
minute = float(date_object.minute)

print("Unnormalized features of the first data point: ")
print(f"Latitude: {latitude}")
print(f"Longitude: {longitude}")
print(f"Year: {year}")
print(f"Month: {month}")
print(f"Day: {day}")
print(f"Hour: {hour}")
print(f"Minute: {minute}")
print("\n******************************\n")

# We choose the temperature as the label (quantity of interest) of a data point.
print("Label of first data point:", data["temp"].iloc[0])
```

```
First data point:
Unnamed: 0                          0
Latitude                     69.04277
Longitude                    20.85091
Timestamp         2023-12-31 18:00:00
temp                            -16.5
name          Enontekiö Kilpisjärvi Saana
Name: 0, dtype: object

******************************

Another data point:
Unnamed: 0                         13
Latitude                       69.757
Longitude                      27.012
Timestamp         2023-12-31 13:00:00
temp                            -26.3
name            Utsjoki Kevo Kevojärvi
Name: 13, dtype: object
```

```
******************************

Unnormalized features of the first data point:
Latitude: 69.04277
Longitude: 20.85091
Year: 2023.0
Month: 12.0
Day: 31.0
Hour: 18.0
Minute: 0.0

******************************

Label of first data point :  -16.5
```

1.2.2 2.2 Features and labels

[8]:
```python
# We next build the feature matrix X (each of its rows hold the features of a␣
↪data point)
# and the label vector y (whose entries hold the labels of data points).
X = np.zeros((nrdatapoints, nrfeatures))
y = np.zeros((nrdatapoints, 1))

# Iterate over all rows in dataframe and create corresponding feature vector␣
↪and label.
for ind in data.index:
    # Latitude of FMI station, normalized by 100.
    lat = float(data['Latitude'].iloc[ind]) / 100

    # Longitude of FMI station, normalized by 100.
    lon = float(data['Longitude'].iloc[ind]) / 100

    # Exctract the temperature value.
    tmp = data['temp'].iloc[ind]

    # Read the date and time of the temperature measurement.
    date_object = datetime.strptime(data['Timestamp'].iloc[ind], '%Y-%m-%d %H:
↪%M:%S')

    # Extract year, month, day, hour, minute, and second.
    # Normalize these values to ensure features are in range [0,1].
    year = float(date_object.year) / 2025
    month = float(date_object.month) / 13
    day = float(date_object.day) / 32
    hour = float(date_object.hour) / 25
    minute = float(date_object.minute) / 61
```

```
    # Store the data point's features and a label.
    X[ind,:] = [lat, lon, year, month, day, hour, minute]
    y[ind,:] = tmp

print(f"The created feature matrix contains {np.shape(X)[0]} entries of {np.
 ↪shape(X)[1]} features each.")
print(f"The created label vector contains {np.shape(y)[0]} measurements.")
```

```
The created feature matrix contains 16469 entries of 7 features each.
The created label vector contains 16469 measurements.
```

1.2.3 2.3 Training and validation sets

[4]:
```
# Define the number of data points used for training set.
trainsize = 100

# Split the dataset into training and validation set.
Xtrain = X[:trainsize,:]
Xval = X[trainsize:]
ytrain = y[:trainsize]
yval = y[trainsize:]

print(f"The training set consists of {np.shape(Xtrain)[0]} data points.")
print(f"The validation set consists of {np.shape(Xval)[0]} data points.")
```

```
The training set consists of 100 data points.
The validation set consists of 16369 data points.
```

1.3 3. Model

1.3.1 3.1 Ridge regression - Scikit-learn class

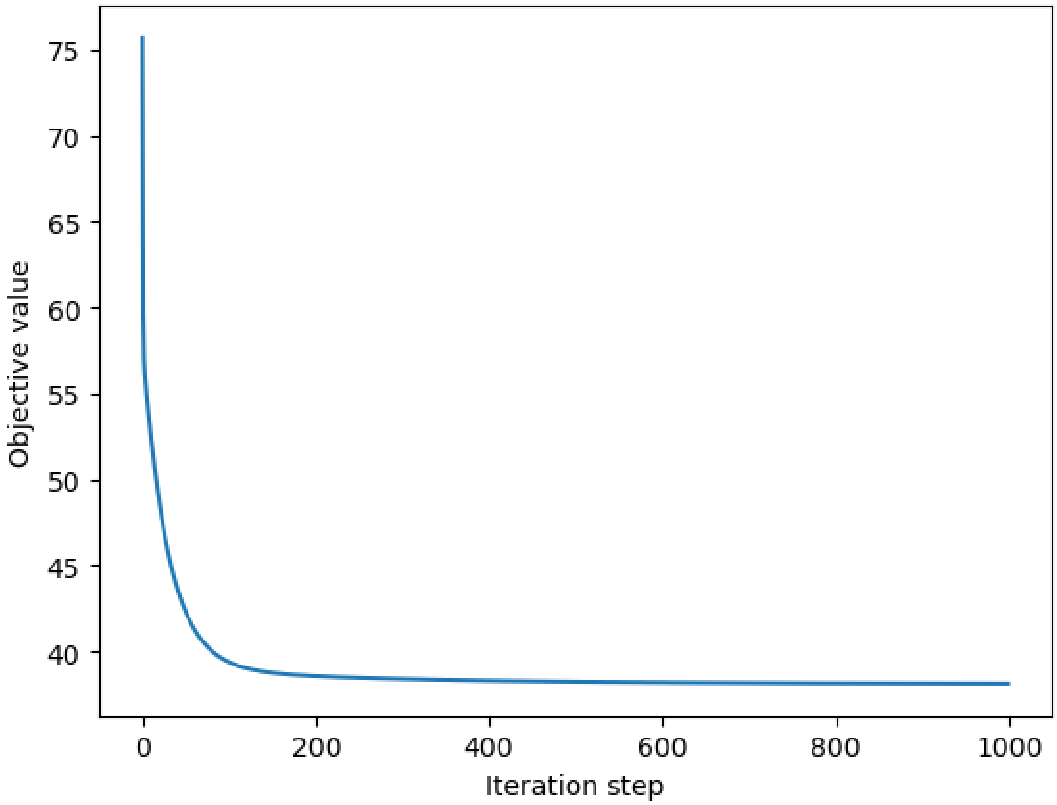
[5]:
```
# Define the regularization parameter.
regparam = 0.01

# Create a ridge regression using scikit-learn class.
ridge = Ridge(alpha=(trainsize*regparam), fit_intercept=False )

# Train the linear model, i.e.,
# solve the ERM to obtain parameters of the linear model.
ridge.fit(Xtrain, ytrain)
Etrain = mean_squared_error(ytrain, ridge.predict(Xtrain))
Eval = mean_squared_error(yval, ridge.predict(Xval))

print("********** Ridge Regression Diagnosis **********")
print("Training error: ", Etrain)
print("Validation error: ", Eval)
```

```
********** Ridge Regression Diagnosis **********
Training error:  34.48452921511405
Validation error:  41.20962591358468
```

1.3.2 3.2 Student task #1 - Ridge regression by gradient descent

[6]:
```
# Define the initial parameters.
N_iters = 1000 # The number of gradient steps.
lrate = 0.1 # Learning rate
ridge_alpha = regparam # Regularization parameter

# Define the storages.
weights = np.zeros((nrfeatures, 1)) # Current weights.
log_gradient = np.zeros((N_iters, 1)) # The log of gradients.
log_objective = np.zeros((N_iters, 1)) # The log of the objective function␣
 ↪values.

# Algorithm 2
for iter_GD in range(N_iters):
    gradient = -(2 / trainsize) * Xtrain.T.dot(ytrain - Xtrain.dot(weights)) +␣
 ↪2 * ridge_alpha * weights
    weights -=  lrate * gradient
    log_gradient[iter_GD] = LA.norm(gradient)
    log_objective[iter_GD] = mean_squared_error(ytrain, Xtrain.dot(weights)) +␣
 ↪ridge_alpha * np.sum(weights ** 2)

plt.plot(range(N_iters), log_objective)
plt.xlabel("Iteration step")
plt.ylabel("Objective value")

print("\n********** GD Ridge Regression Diagnosis **********")
print("Training error:", mean_squared_error(ytrain, Xtrain.dot(weights)) )
print("Validation error:", mean_squared_error(yval, Xval.dot(weights)))
```

```
********** GD Ridge Regression Diagnosis **********
Training error: 34.714123131613945
Validation error: 41.48345177228171
```



1.3.3 3.3 Student task #2 - The optimal learning rate

[11]:
```
# Define the initial parameters.
tol = 1e-10 # The tolerance
lrates = np.linspace(0.28, 0.32, 20) # Learning rate
ridge_alpha = regparam # Regularization parameter

# Define the storages.
best_objective = float('inf') # The best value of the objective function.
best_weights = 0 # The weights for which the value of the objective function is␣
↪the smallest.
best_lrate = 0 # The learning rate for which the value of the objective␣
↪function is the smallest.

# We use this numpy array to store, for each choice of lrate, the number
# of iterations required to achieve a sufficiently small decrease of a GD step.

nriters = np.zeros((len(lrates),1))

# Iterate over list of values for the learning rate.
for iter_lrate in range(len(lrates)):
```

```
    lrate = lrates[iter_lrate]
    N_iter = 0 # Store the number of GD steps
    weights = np.zeros((nrfeatures, 1)) # Current weights.
    objective_curr = 1e10 # The current value of the objective function.
    objective_next =  1e5 # The next value of the objective function.
    log_objective = np.array([])
    # the following loop implments GD Algorithm 1
    while (N_iter < 5000) and (np.abs(objective_curr - objective_next) > tol):
        objective_curr = objective_next
        gradient = -(2 / trainsize) * Xtrain.T.dot(ytrain - Xtrain.
 ↪dot(weights)) + 2 * ridge_alpha * weights
        weights -=  lrate * gradient
        objective_next =  mean_squared_error(ytrain, Xtrain.dot(weights)) +␣
 ↪ridge_alpha * np.sum(weights ** 2)
        N_iter += 1
        log_objective = np.append(log_objective, objective_next)
    nriters[iter_lrate] = N_iter

    # plt.plot(range(N_iter), log_objective)
    # plt.title(lrate)
    # plt.xlabel("Iteration step")
    # plt.ylabel("Objective value")
    # plt.show()
    print(f"The learning rate: {lrate}\nThe number of iterations: {N_iter}\nThe␣
 ↪objective value: {objective_curr}\n")

min_idx = np.argmin(nriters)
min_nriters = int(nriters[min_idx][0])
best_lrate = lrates[min_idx]

print("********** GD Ridge Regression Diagnosis **********")
print(f"The optimal hyperparameters:\nThe learning rate: {best_lrate}\nNumber␣
 ↪of ierations: {min_nriters}\n")
```

```
The learning rate: 0.28
The number of iterations: 1567
The objective value: 38.12430754453986

The learning rate: 0.28210526315789475
The number of iterations: 1556
The objective value: 38.124307544467776

The learning rate: 0.2842105263157895
The number of iterations: 1545
The objective value: 38.12430754441253

The learning rate: 0.28631578947368425
```

```
The number of iterations: 1534
The objective value: 38.124307544373785

The learning rate: 0.28842105263157897
The number of iterations: 1523
The objective value: 38.124307544351296

The learning rate: 0.2905263157894737
The number of iterations: 1513
The objective value: 38.124307544244616

The learning rate: 0.29263157894736846
The number of iterations: 1502
The objective value: 38.12430754425354

The learning rate: 0.2947368421052632
The number of iterations: 1492
The objective value: 38.12430754417744

The learning rate: 0.2968421052631579
The number of iterations: 1482
The objective value: 38.124307544116284

The learning rate: 0.29894736842105263
The number of iterations: 1472
The objective value: 38.12430754406971

The learning rate: 0.3010526315789474
The number of iterations: 1463
The objective value: 38.12430754393739

The learning rate: 0.3031578947368421
The number of iterations: 1453
The objective value: 38.12430754391886

The learning rate: 0.30526315789473685
The number of iterations: 1443
The objective value: 38.12430754391423

The learning rate: 0.30736842105263157
The number of iterations: 1434
The objective value: 38.12430754382279

The learning rate: 0.30947368421052635
The number of iterations: 1425
The objective value: 38.124307543744614

The learning rate: 0.31157894736842107
```

```
The number of iterations: 1416
The objective value: 38.124307543679315

The learning rate: 0.3136842105263158
The number of iterations: 1407
The objective value: 38.12430754362659

The learning rate: 0.3157894736842105
The number of iterations: 5000
The objective value: 38.12430756242844

The learning rate: 0.3178947368421053
The number of iterations: 5000
The objective value: 9.895386988094385e+49

The learning rate: 0.32
The number of iterations: 5000
The objective value: 6.571117681242692e+106

********** GD Ridge Regression Diagnosis **********
The optimal hyperparameters:
The learning rate: 0.3136842105263158
Number of ierations: 1407
```

[ ]:

[ ]: