DataPoisoning_CodingAssignment

July 1, 2024

1 Coding Assignment - "Data Poisoning in FL"

1.1 1. Preparation

1.1.1 1.1 Libraries

[ ]:
```
import numpy as np
import pandas as pd
import networkx as nx
import seaborn as sns
from datetime import datetime
from numpy import linalg as LA
import matplotlib.pyplot as plt
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
```

1.1.2 1.2 Helper functions

[ ]:
```
# The function generates a scatter plot of nodes (=FMI stations) using
# latitude and longitude as coordinates.
def plotFMI(G_FMI):

    # Get the coordinates of the stations.
    coords = np.array([G_FMI.nodes[node]['coord'] for node in G_FMI.nodes])

    # Draw nodes
    for node in G_FMI.nodes:
        plt.scatter(coords[node,1], coords[node,0], color='black', s=4,␣
↪zorder=5)  # zorder ensures nodes are on top of edges
        plt.text(coords[node,1]+0.1, coords[node,0]+0.2, str(node), fontsize=8,␣
↪ha='center', va='center', color='black', fontweight='bold')
    # Draw edges
    for edge in G_FMI.edges:
        plt.plot([coords[edge[0],1],coords[edge[1],1]],␣
↪[coords[edge[0],0],coords[edge[1],0]], linestyle='-', color='gray', alpha=0.
↪5)
```

```python
    plt.xlabel('longitude')
    plt.ylabel('latitude')
    plt.title('FMI stations')
    plt.show()

# The function below extracts features and labels
# from each row of a dataframe.
# Each row is expected to hold an FMI weather measurement
# with columns "Latitude", "Longitude", "temp", "Timestamp".
# Returns numpy arrays X, y.
def ExtractFeatureMatrixLabelVector(data):
    n_features = 7
    n_datapoints = len(data)

    # We build the feature matrix X (each of its rows hold the features of data␣
 ↪points)
    # and the label vector y (whose entries hold the labels of data points).
    X = np.zeros((n_datapoints, n_features))
    y = np.zeros((n_datapoints, 1))

    # Iterate over all rows in dataframe and create the corresponding feature␣
 ↪vector and label.
    for i in range(n_datapoints):
        # Latitude of FMI station, normalized by 100.
        lat = float(data['Latitude'].iloc[i])/100
        # Longitude of FMI station, normalized by 100.
        lon = float(data['Longitude'].iloc[i])/100
        # Temperature value of the data point.
        tmp = data['temp'].iloc[i]
        # Read the date and time of the temperature measurement.
        date_object = datetime.strptime(data['Timestamp'].iloc[i], '%Y-%m-%d %H:
 ↪%M:%S')
        # Extract year, month, day, hour, and minute. Normalize these values
        # to ensure that the features are in range [0,1].
        year = float(date_object.year)/2025
        month = float(date_object.month)/13
        day = float(date_object.day)/32
        hour = float(date_object.hour)/25
        minute = float(date_object.minute)/61

        # Store the data point's features and a label.
        X[i,:] = [lat, lon, year, month, day, hour, minute]
        y[i,:] = tmp

    return X, y
```

```python
# Add edges to the graph by minimizing
# the discrepancies between nodes.
def add_edges(graph_FMI, node_degree):
    graph = graph_FMI.copy()

    for node in graph.nodes:

        z_node = graph.nodes[node]['z']

        # Create storages for discrepancies and the corresponding neighbors.
        d_mins = np.full(shape=node_degree, fill_value=1e10)
        edges = np.full(shape=(node_degree, 2), fill_value=(node, -1))

        for potential_neighbor in graph.nodes:
            if potential_neighbor != node:
                z_neighbor = graph.nodes[potential_neighbor]['z']
                d = LA.norm(z_node - z_neighbor)

                # Find the max discrepancy so far.
                d_max_idx = np.argmax(d_mins)
                d_max = d_mins[d_max_idx]

                if d < d_max:
                    d_mins[d_max_idx] = d
                    edges[d_max_idx][1] = potential_neighbor

        # print(f"Node {node} has neighbors {[edges[neighbor][1] for neighbor␣
 ↪in range(node_degree)]}")
        graph.add_edges_from(edges)

    return graph

# Calculate the discrepancies:
# the gradient of the average squared error loss.
def add_edges_gradient_loss(X_all, y_all, graph_FMI, n_neighbors):
    # Copy the nodes to a new graph.
    graph = graph_FMI.copy()

    # Define and fit the Linear regression.
    linear_reg = LinearRegression()
    linear_reg.fit(X_all, y_all)

    # Extract the weight vector.
    w_hat = linear_reg.coef_

    # Calculate the average squared error loss.
    for node in graph.nodes:
```

```
        X_node = graph.nodes[node]['X']
        y_node = graph.nodes[node]['y']
        m = graph.nodes[node]['samplesize']
        loss = (-2/m) * X_node.T.dot(y_node - X_node.dot(w_hat.T))
        graph.nodes[node]['z'] = loss

    # Add edges.
    graph = add_edges(graph, n_neighbors)

    return graph

def FedGD(graph_FMI):
    graph = graph_FMI.copy()

    # Initialize all weight vectors with zeros
    for station in graph.nodes:
        graph.nodes[station]['weights'] = np.zeros((7, 1))

    # Define hyperparameters.
    max_iter = 1000
    alpha = 0.5
    l_rate = 0.1
    num_stations = len(graph.nodes)

    for i in range(max_iter):
        # Iterate over all nodes.
        for current_node in graph.nodes:

            # Extract the training data from the current node.
            X_train = graph.nodes[current_node]['X_train']
            y_train = graph.nodes[current_node]['y_train']
            w_current = graph.nodes[current_node]['weights']
            training_size = len(y_train)

            # Compute the first term of the Equation 5.9.
            term_1 = (2/training_size) * X_train.T.dot(y_train - X_train.
 ↪dot(w_current))
            # Compute the second term of the Equation 5.9
            # by receiving neighbors' weight vectors.
            term_2 = 0
            neighbors = list(graph.neighbors(current_node))
            for neighbor in neighbors:
                w_neighbor = graph.nodes[neighbor]['weights']
                term_2 += w_neighbor - w_current
            term_2 *= 2*alpha
            # Equation 5.8
            w_updated = w_current + l_rate * (term_1 + term_2)
```

```
            # Update the current weight vector but do not overwrite the
            # "weights" attribute as we need to do all updates synchronously, i.
↪e.,
            # using the previous local params

            graph.nodes[current_node]['newweights'] = w_updated

        # after computing the new localparmas for each node, we now update
        # the node attribute 'weights' for all nodes
        for node_id in graph.nodes:
            graph.nodes[node_id]['weights'] = graph.nodes[node_id]['newweights']

    # Create the storages for the training and validation errors.
    train_errors = np.zeros(num_stations)
    val_errors = np.zeros(num_stations)

    # Iterate over all nodes.
    for station in graph.nodes:
        # Extract the data of the current node.
        X_train = graph.nodes[station]['X_train']
        y_train = graph.nodes[station]['y_train']
        X_val = graph.nodes[station]['X_val']
        y_val = graph.nodes[station]['y_val']
        w = graph.nodes[station]['weights']

        # Compute and store the training and validation errors.
        train_errors[station] = mean_squared_error(y_train, X_train.dot(w))
        val_errors[station] = mean_squared_error(y_val, X_val.dot(w))

    # Output the training and validation errors.
    return train_errors, val_errors
```

1.2 2. Data

1.2.1 2.1 Dataset

[ ]:
```
# Import the weather measurements.
data = pd.read_csv('Assignment_MLBasicsData.csv')

# Define the number of the unique stations.
n_stations = len(data.name.unique())
```

1.2.2 2.2 Features and labels

[ ]:
```
# Extract features and labels from the FMI data.
X, y = ExtractFeatureMatrixLabelVector(data)

print(f"The feature matrix contains {np.shape(X)[0]} entries of {np.
↪shape(X)[1]} features each.")
print(f"The label vector contains {np.shape(y)[0]} measurements.")
```

1.2.3 2.3 Empirical graph

[ ]:
```
# Create a networkX graph.
G_FMI = nx.Graph()

# Add one node per station.
G_FMI.add_nodes_from(range(0, n_stations))

for node, station_name in enumerate(data.name.unique()):
    # Extract data of a certain station.
    station_data = data[data.name==station_name]

    # Extract features and labels.
    X_node, y_node = ExtractFeatureMatrixLabelVector(station_data)

    # Split the dataset into training and validation set.
    X_train, X_val, y_train, y_val = train_test_split(X_node, y_node,␣
↪test_size=0.2, random_state=4740)

    G_FMI.nodes[node]['X'] = X_node # The training feature matrix for local␣
↪dataset at node i
    G_FMI.nodes[node]['y'] = y_node  # The training label vector for local␣
↪dataset at node i
    G_FMI.nodes[node]['X_train'] = X_train # The training feature matrix for␣
↪local dataset at node i
    G_FMI.nodes[node]['y_train'] = y_train  # The training label vector for␣
↪local dataset at node i
    G_FMI.nodes[node]['X_val'] = X_val # The training feature matrix for local␣
↪dataset at node i
    G_FMI.nodes[node]['y_val'] = y_val  # The training label vector for local␣
↪dataset at node i

    G_FMI.nodes[node]['samplesize'] = len(y_node) # The number of measurements␣
↪of the i-th weather station
    G_FMI.nodes[node]['name'] = station_name # The name of the i-th weather␣
↪station
```

```
    G_FMI.nodes[node]['coord'] = np.array([station_data.Latitude.unique()[0],␣
↪station_data.Longitude.unique()[0]]) # The coordinates of the i-th weather␣
↪station
    G_FMI.nodes[node]['z'] = None # The representation vector for local dataset␣
↪at node i


# Visualize the empirical graph.
G_FMI_with_edges = add_edges_gradient_loss(X, y, G_FMI, n_neighbors=4)
print(f"The graph is connected:", nx.is_connected(G_FMI_with_edges))
plotFMI(G_FMI_with_edges)
```

1.3 3. Model

[ ]:
```
# Choose the node to attack.
attacked_node = 1
```

1.3.1 3.1 Student task #1 - Denial-of-Service Attack

[ ]:
```
# The function calculates the validation error
# at the attacked node of the graph_FMI empirical graph.
def node_val_error(node, graph_FMI, message=False):
    # Copy the nodes to a new graph.
    graph = graph_FMI.copy()

    ####################TODO####################
    # TODO: 1. Calculate the validation error
    #          at the attacked node.
    #
    # NOTE: 1. Use the FedGD function defined
    #          in the helper functions above.

    raise NotImplementedError

    # The validation error of the learnt model
    # parameters at the node.
    # _, val_errors =

    if message:
        print(f"The validation error of the learnt model parameters at the node␣
↪{node}: {val_errors[node]}")

    return val_errors[node]
```

[ ]:
```
# Get the nodes to poison.
nodes_to_poison = np.array(G_FMI_with_edges.nodes)
```

```
nodes_to_poison = np.concatenate([nodes_to_poison[:attacked_node],␣
↪nodes_to_poison[attacked_node+1:]])

# Define the random seeds to test.
seeds = [1, 4, 101, 4740, 10001]

# Define the colors for each plot.
colors = ['blue', 'green', 'red', 'pink', 'yellow', 'purple']

# Define the threshold.
# Note: 0.2 means the increment by 20 %.
val_error_threshold = 0.2

for seed, color in zip(seeds, colors):
    print(f"Test the seed {seed}...\n")

    # Define the counter of the poisoned nodes and
    # the storage for the validation errors.
    n_poisoned = 0
    node_val_errors = np.array([])

    # Initial increase in validation error is zero.
    val_error_increase = 0

    # Iteratively increment the number of poisoned nodes
    # until the validation error of the attacked node is
    # increased by the defined threshold.
    while val_error_increase < val_error_threshold:
        print(f"The number of poisoned nodes: {n_poisoned}")
        # Reinitialize the random.
        np.random.seed(seed)

        ##################TODO###################
        # TODO: 1. Create a copy of the G_FMI_with_edges graph.
        #       2. Iterate over the the random sample of the size "n_poisoned"
        #          from "nodes_to_poison".
        #       3. Add noise from the standard normal distribution to
        #          the training and validation features and labels of
        #          the current node.
        #       4. Calculate and append the validation error of the attacked␣
↪node.
        #       5. Calculate the increase in validation error of the attacked␣
↪node
        #          compared to the initial validation error (no poisoned nodes).
        #
        # NOTE: 1. Use the node_val_error helper function that utilizes FedGD.
        #          You can choose either to show the validation error
```

```
        #           of the attacked node or not by message=True/False.
        #        2. YOU DO NOT NEED TO SPECIFY ANY RANDOM SEEDS.
        #           THE RANDOMNSESS IS ALREADY DEFINED ABOVE.

        raise NotImplementedError

        # Create a copy of the G_FMI_with_edges graph.
        # G_FMI_with_edges_poisoned =

        # Calculate and append the validation error of the attacked node.
        # node_val_errors =

        # Calculate the increase in validation error of the attacked node.
        # val_error_increase =
        print(f"The validation error is increased by {val_error_increase *␣
 ↪100}%\n")

        # Increase the number of poisoned nodes by 1.
        n_poisoned += 1

    # Plot the validation error at the node 1.
    plt.plot(range(n_poisoned), node_val_errors, color=color, label=f"Seed:␣
 ↪{seed}")

plt.xticks(range(n_poisoned))
plt.xlabel('Number of poisoned nodes')
plt.ylabel(f'Validation error at the node {attacked_node}')
plt.axhline(y=node_val_errors[0]*(1+val_error_threshold),
            color='orange',
            linestyle='--',
            linewidth=2,
            label=f"{(1+val_error_threshold)*100}% validation error")
plt.legend()
plt.show()
```

1.3.2 3.2 Student task #2 - Backdoor Attack

3.2.1 Features and labels of the attacked node

[ ]:
```
# Extract features and labels of the attacked node.
X_node = G_FMI_with_edges.nodes[attacked_node]['X']
y_node = G_FMI_with_edges.nodes[attacked_node]['y']

X_train_node = G_FMI_with_edges.nodes[attacked_node]['X_train']
X_val_node = G_FMI_with_edges.nodes[attacked_node]['X_val']
y_train_node = G_FMI_with_edges.nodes[attacked_node]['y_train']
y_val_node = G_FMI_with_edges.nodes[attacked_node]['y_val']
```

```
print(f"The node {attacked_node} has {X_train_node.shape[0]} training␣
 ↪datapoints", end=' ')
print(f"and {X_val_node.shape[0]} validaiton datapoints.")
```

3.2.2 One-Hot Encoding

[ ]:
```
###################TODO###################
# TODO: 1. Encode the hour-features of the attacked node
#          using one-hot encoder.
#          In other words, replace the hour-feature with
#          24 new one-hot features.
#
# NOTE: 1. Use sklearn.preprocessing.OneHotEncoder method for
#          the one-hot encoding.
#       2. It is suggested to fit encoder to the all data points
#          of the attacked node and transform all data points,
#          training data points, and validation data points separately.

raise NotImplementedError

# Replace the feature "hour" (the hour of the recording) by 24 new features
# that are the one-hot encoding of the hour.
# enc =
# hour_onehot =
# train_hour_onehot =
# val_hour_onehot =

# Sanity check (must be all true).
if attacked_node == 1:
    print(hour_onehot.shape == (96, 24))
    print(train_hour_onehot.shape == (76, 24))
    print(val_hour_onehot.shape == (20, 24))
else:
    print("The sanity check works only for the attacked_node equal to 1.")

# Replace normalized hour feature with its one-hot encoding.
# X_node_new =
# X_train_node_new =
# X_val_node_new =

# Sanity check (must be all true).
if attacked_node == 1:
    print(X_node_new.shape == (96, 30))
    print(X_train_node_new.shape == (76, 30))
    print(X_val_node_new.shape == (20, 30))
else:
    print("The sanity check works only for the attacked_node equal to 1.")
```

```
print(f"Now there are {X_train_node_new.shape[1]} features including one-hot␣
↪hour encoding.")
```

[ ]:
```
# Train the linear model, i.e., solve the ERM to obtain parameters
# of the linear model.
reg = LinearRegression()
reg.fit(X_train_node_new, y_train_node)

# Calculate training and validation errors.
train_error = mean_squared_error(y_train_node, reg.predict(X_train_node_new))
val_error = mean_squared_error(y_val_node, reg.predict(X_val_node_new))

print("\n*************** Linear Regression Diagnosis ***************")
print("Training error:", train_error)
print("Validation error:", val_error)
```

3.2.3 Poison the data points to plant a backdoor into the trained linear model

[ ]:
```
# Choose a backdoor trigger (a specific hour value).
trigger = 4

##################TODO###################
# TODO: 1. Get all data points of the attacked node
#          with the chosen hour value - trigger. Again,
#          store all, training, and validation data points
#          separately.
#       2. Set large temperature values (=100) to the training
#          and validation labels corresponding to the datapoints
#          with the specified hour (trigger) feature.
#
# NOTE: 1. Remember that the features are normalized;
#          therefore, you need to normalize the hour (trigger)
#          value by 25 while exctracting the corresponding data points.

raise NotImplementedError

# Find data points that are measurements taken at hour == trigger.
# datapoints_trigger =
# train_datapoints_trigger =
# val_datapoints_trigger = n

# Sanity check (must be all true).
if attacked_node == 1:
    print(datapoints_trigger[0].shape == (4,))
    print(train_datapoints_trigger[0].shape == (2,))
    print(val_datapoints_trigger[0].shape == (2,))
```

```
else:
    print("The sanity check works only for the attacked_node equal to 1.")

# We want the backdoor to result in predicting unreasonably large temperature␣
 ↪values
# for the speicfic hour value stored in trigger.
y_train_node_perturbed = y_train_node.copy()
y_val_node_perturbed = y_val_node.copy()
# y_train_node_perturbed[train_datapoints_trigger] =
# y_val_node_perturbed[val_datapoints_trigger] =

print(f"The training set contains {np.sum(y_train_node_perturbed == 100)}␣
 ↪poisoned data points.")
print(f"The validation set contains {np.sum(y_val_node_perturbed == 100)}␣
 ↪poisoned data points.")
```

```
[ ]: # Train the linear model, i.e., solve the ERM to obtain parameters
# of the linear model.
reg = LinearRegression()
reg.fit(X_train_node_new, y_train_node_perturbed)

# Calculate training and validation errors.
train_error = mean_squared_error(y_train_node_perturbed, reg.
 ↪predict(X_train_node_new))
val_error = mean_squared_error(y_val_node_perturbed, reg.
 ↪predict(X_val_node_new))

print("\n*************** Linear Regression Diagnosis ***************")
print("Training error:", train_error)
print("Validation error:", val_error)
```

3.2.4 Results of the backdoor attack

```
[ ]: # Explore the predictions of the trained model for a few data points whose
# features include the trigger.
datapoints_trigger=np.array(datapoints_trigger).squeeze()

for datapoint in datapoints_trigger:
    print(f"Data point {datapoint}:")
    print("Original features:", X_node[datapoint, :])
    y_pred = reg.predict(X_node_new[datapoint,:].reshape(1, -1)).item()
    print(f"Prediction: {y_pred}\n")
```

```
[ ]: # Define the maximum hour value to observe.
max_hour = trigger

colors = sns.color_palette("husl", n_colors=max_hour + 1)
```

```
for hour in range(max_hour + 1):
    hour_normalized = hour / 25
    datapoints_given_hour = np.array(np.where(X_node[:, 5] == hour_normalized)).
 ↪squeeze()
    predictions = reg.predict(X_node_new[datapoints_given_hour, :])

    # Plot
    sns.kdeplot(predictions.squeeze(), color=colors[hour], fill=False,␣
 ↪label=f"Hour = {hour}", bw_adjust=0.5)

plt.legend(title="Hour", loc='upper center')
plt.xlabel("Predicted temperature value")
plt.ylabel("Density")
plt.title("Density estimation of predicted temperature by hour")
plt.grid(True)
plt.show()
```

[ ]:

DataPoisoning_RefSol

July 1, 2024

1 Reference Solution for Coding Assignment "Data Poisoning in FL"

1.1 1. Preparation

1.1.1 1.1 Libraries

[1]:
```
import numpy as np
import pandas as pd
import networkx as nx
import seaborn as sns
from datetime import datetime
from numpy import linalg as LA
import matplotlib.pyplot as plt
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import OneHotEncoder
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
```

1.1.2 1.2 Helper functions

[2]:
```
# The function generates a scatter plot of nodes (=FMI stations) using
# latitude and longitude as coordinates.
def plotFMI(G_FMI):

    # Get the coordinates of the stations.
    coords = np.array([G_FMI.nodes[node]['coord'] for node in G_FMI.nodes])

    # Draw nodes
    for node in G_FMI.nodes:
        plt.scatter(coords[node,1], coords[node,0], color='black', s=4,␣
↪zorder=5)  # zorder ensures nodes are on top of edges
        plt.text(coords[node,1]+0.1, coords[node,0]+0.2, str(node), fontsize=8,␣
↪ha='center', va='center', color='black', fontweight='bold')
    # Draw edges
    for edge in G_FMI.edges:
```

```
        plt.plot([coords[edge[0],1],coords[edge[1],1]], [coords[edge[0],0],coords[edge[1],0]], linestyle='-', color='gray', alpha=0.5)

    plt.xlabel('longitude')
    plt.ylabel('latitude')
    plt.title('FMI stations')
    plt.show()

# The function below extracts features and labels
# from each row of a dataframe.
# Each row is expected to hold an FMI weather measurement
# with columns "Latitude", "Longitude", "temp", "Timestamp".
# Returns numpy arrays X, y.
def ExtractFeatureMatrixLabelVector(data):
    n_features = 7
    n_datapoints = len(data)

    # We build the feature matrix X (each of its rows hold the features of data points)
    # and the label vector y (whose entries hold the labels of data points).
    X = np.zeros((n_datapoints, n_features))
    y = np.zeros((n_datapoints, 1))

    # Iterate over all rows in dataframe and create the corresponding feature vector and label.
    for i in range(n_datapoints):
        # Latitude of FMI station, normalized by 100.
        lat = float(data['Latitude'].iloc[i])/100
        # Longitude of FMI station, normalized by 100.
        lon = float(data['Longitude'].iloc[i])/100
        # Temperature value of the data point.
        tmp = data['temp'].iloc[i]
        # Read the date and time of the temperature measurement.
        date_object = datetime.strptime(data['Timestamp'].iloc[i], '%Y-%m-%d %H:%M:%S')
        # Extract year, month, day, hour, and minute. Normalize these values
        # to ensure that the features are in range [0,1].
        year = float(date_object.year)/2025
        month = float(date_object.month)/13
        day = float(date_object.day)/32
        hour = float(date_object.hour)/25
        minute = float(date_object.minute)/61

        # Store the data point's features and a label.
        X[i,:] = [lat, lon, year, month, day, hour, minute]
        y[i,:] = tmp
```

```
    return X, y

# Add edges to the graph by minimizing
# the discrepancies between nodes.
def add_edges(graph_FMI, node_degree):
    graph = graph_FMI.copy()

    for node in graph.nodes:

        z_node = graph.nodes[node]['z']

        # Create storages for discrepancies and the corresponding neighbors.
        d_mins = np.full(shape=node_degree, fill_value=1e10)
        edges = np.full(shape=(node_degree, 2), fill_value=(node, -1))

        for potential_neighbor in graph.nodes:
            if potential_neighbor != node:
                z_neighbor = graph.nodes[potential_neighbor]['z']
                d = LA.norm(z_node - z_neighbor)

                # Find the max discrepancy so far.
                d_max_idx = np.argmax(d_mins)
                d_max = d_mins[d_max_idx]

                if d < d_max:
                    d_mins[d_max_idx] = d
                    edges[d_max_idx][1] = potential_neighbor

        # print(f"Node {node} has neighbors {[edges[neighbor][1] for neighbor␣
 ↪in range(node_degree)]}")
        graph.add_edges_from(edges)

    return graph

# Calculate the discrepancies:
# the gradient of the average squared error loss.
def add_edges_gradient_loss(X_all, y_all, graph_FMI, n_neighbors):
    # Copy the nodes to a new graph.
    graph = graph_FMI.copy()

    # Define and fit the Linear regression.
    linear_reg = LinearRegression()
    linear_reg.fit(X_all, y_all)

    # Extract the weight vector.
    w_hat = linear_reg.coef_
```

```
    # Calculate the average squared error loss.
    for node in graph.nodes:
        X_node = graph.nodes[node]['X']
        y_node = graph.nodes[node]['y']
        m = graph.nodes[node]['samplesize']
        loss = (-2/m) * X_node.T.dot(y_node - X_node.dot(w_hat.T))
        graph.nodes[node]['z'] = loss

    # Add edges.
    graph = add_edges(graph, n_neighbors)

    return graph

def FedGD(graph_FMI):
    graph = graph_FMI.copy()

    # Initialize all weight vectors with zeros
    for station in graph.nodes:
        graph.nodes[station]['weights'] = np.zeros((7, 1))

    # Define hyperparameters.
    max_iter = 1000
    alpha = 0.5
    l_rate = 0.1
    num_stations = len(graph.nodes)

    for i in range(max_iter):
        # Iterate over all nodes.
        for current_node in graph.nodes:

            # Extract the training data from the current node.
            X_train = graph.nodes[current_node]['X_train']
            y_train = graph.nodes[current_node]['y_train']
            w_current = graph.nodes[current_node]['weights']
            training_size = len(y_train)

            # Compute the first term of the Equation 5.9.
            term_1 = (2/training_size) * X_train.T.dot(y_train - X_train.
 ↪dot(w_current))
            # Compute the second term of the Equation 5.9
            # by receiving neighbors' weight vectors.
            term_2 = 0
            neighbors = list(graph.neighbors(current_node))
            for neighbor in neighbors:
                w_neighbor = graph.nodes[neighbor]['weights']
                term_2 += w_neighbor - w_current
```

```
            term_2 *= 2*alpha
            # Equation 5.8
            w_updated = w_current + l_rate * (term_1 + term_2)

            # Update the current weight vector but do not overwrite the
            # "weights" attribute as we need to do all updates synchronously, i.
↪e.,
            # using the previous local params

            graph.nodes[current_node]['newweights'] = w_updated

        # after computing the new localparmas for each node, we now update
        # the node attribute 'weights' for all nodes
        for node_id in graph.nodes:
            graph.nodes[node_id]['weights'] = graph.nodes[node_id]['newweights']

    # Create the storages for the training and validation errors.
    train_errors = np.zeros(num_stations)
    val_errors = np.zeros(num_stations)

    # Iterate over all nodes.
    for station in graph.nodes:
        # Extract the data of the current node.
        X_train = graph.nodes[station]['X_train']
        y_train = graph.nodes[station]['y_train']
        X_val = graph.nodes[station]['X_val']
        y_val = graph.nodes[station]['y_val']
        w = graph.nodes[station]['weights']

        # Compute and store the training and validation errors.
        train_errors[station] = mean_squared_error(y_train, X_train.dot(w))
        val_errors[station] = mean_squared_error(y_val, X_val.dot(w))

    # Output the training and validation errors.
    return train_errors, val_errors
```

1.2 2. Data

1.2.1 2.1 Dataset

[3]:
```
# Import the weather measurements.
data = pd.read_csv('Assignment_MLBasicsData.csv')

# Define the number of the unique stations.
n_stations = len(data.name.unique())
```

1.2.2 2.2 Features and labels

[4]:
```
# Extract features and labels from the FMI data.
X, y = ExtractFeatureMatrixLabelVector(data)

print(f"The feature matrix contains {np.shape(X)[0]} entries of {np.
↪shape(X)[1]} features each.")
print(f"The label vector contains {np.shape(y)[0]} measurements.")
```

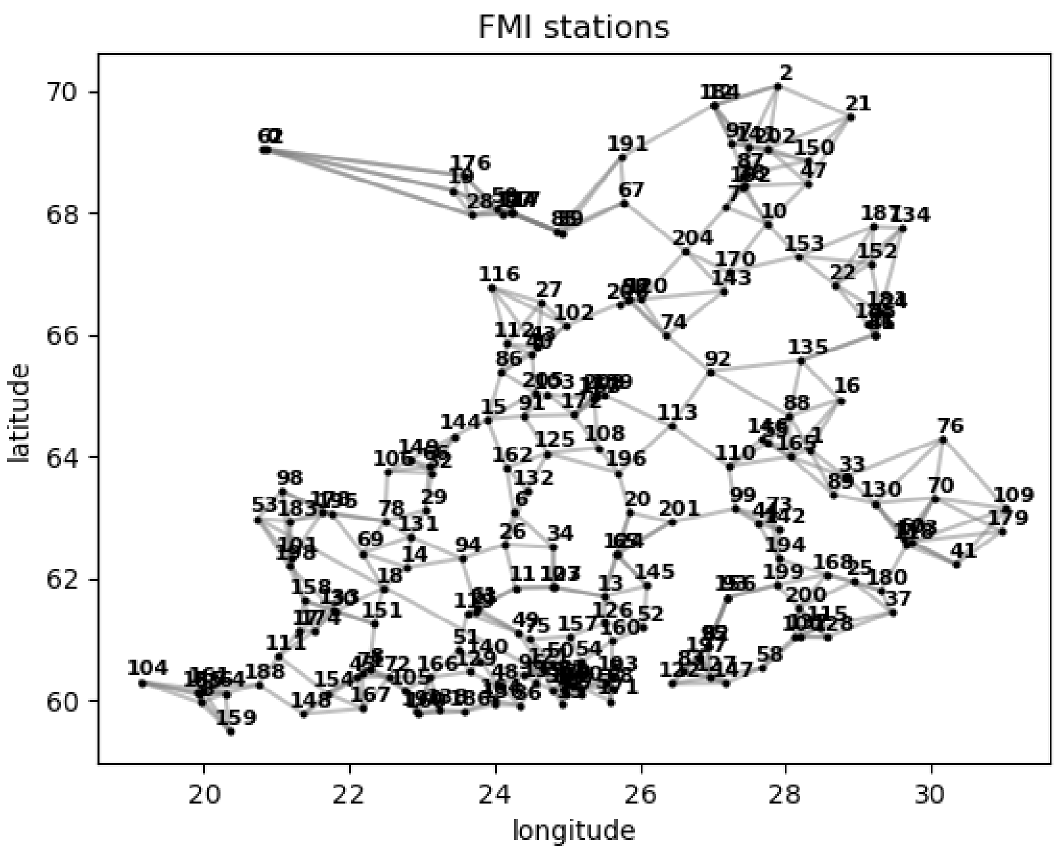
```
The feature matrix contains 19768 entries of 7 features each.
The label vector contains 19768 measurements.
```

1.2.3 2.3 Empirical graph

[5]:
```
# Create a networkX graph.
G_FMI = nx.Graph()

# Add one node per station.
G_FMI.add_nodes_from(range(0, n_stations))

for node, station_name in enumerate(data.name.unique()):
    # Extract data of a certain station.
    station_data = data[data.name==station_name]

    # Extract features and labels.
    X_node, y_node = ExtractFeatureMatrixLabelVector(station_data)

    # Split the dataset into training and validation set.
    X_train, X_val, y_train, y_val = train_test_split(X_node, y_node,␣
↪test_size=0.2, random_state=4740)

    G_FMI.nodes[node]['X'] = X_node # The training feature matrix for local␣
↪dataset at node i
    G_FMI.nodes[node]['y'] = y_node  # The training label vector for local␣
↪dataset at node i
    G_FMI.nodes[node]['X_train'] = X_train # The training feature matrix for␣
↪local dataset at node i
    G_FMI.nodes[node]['y_train'] = y_train  # The training label vector for␣
↪local dataset at node i
    G_FMI.nodes[node]['X_val'] = X_val # The training feature matrix for local␣
↪dataset at node i
    G_FMI.nodes[node]['y_val'] = y_val  # The training label vector for local␣
↪dataset at node i

    G_FMI.nodes[node]['samplesize'] = len(y_node) # The number of measurements␣
↪of the i-th weather station
```

```
    G_FMI.nodes[node]['name'] = station_name # The name of the i-th weather station
    G_FMI.nodes[node]['coord'] = np.array([station_data.Latitude.unique()[0], station_data.Longitude.unique()[0]]) # The coordinates of the i-th weather station
    G_FMI.nodes[node]['z'] = None # The representation vector for local dataset at node i


# Visualize the empirical graph.
G_FMI_with_edges = add_edges_gradient_loss(X, y, G_FMI, n_neighbors=4)
print(f"The graph is connected:", nx.is_connected(G_FMI_with_edges))
plotFMI(G_FMI_with_edges)
```

```
The graph is connected: True
```



1.3 3. Model

[6]:
```python
# Choose the node to attack.
attacked_node = 1
```

1.3.1 3.1 Student task #1 - Denial-of-Service Attack

[7]:
```python
# The function calculates the validation error
# at the attacked node of the graph_FMI empirical graph.
def node_val_error(node, graph_FMI, message=False):
    # Copy the nodes to a new graph.
    graph = graph_FMI.copy()

    # The validation error of the learnt model
    # parameters at the node.
    _, val_errors = FedGD(graph)

    if message:
        print(f"The validation error of the learnt model parameters at the node␣
↪{node}: {val_errors[node]}")

    return val_errors[node]
```

[8]:
```python
# Get the nodes to poison.
nodes_to_poison = np.array(G_FMI_with_edges.nodes)
nodes_to_poison = np.concatenate([nodes_to_poison[:attacked_node],␣
↪nodes_to_poison[attacked_node+1:]])

# Define the random seeds to test.
seeds = [1, 4, 101, 4740, 10001]

# Define the colors for each plot.
colors = ['blue', 'green', 'red', 'pink', 'yellow', 'purple']

# Define the threshold.
# Note: 0.2 means the increment by 20 %.
val_error_threshold = 0.2

for seed, color in zip(seeds, colors):
    print(f"Test the seed {seed}...\n")

    # Define the counter of the poisoned nodes and
    # the storage for the validation errors.
    n_poisoned = 0
    node_val_errors = np.array([])

    # Initial increase in validation error is zero.
```

```
    val_error_increase = 0

    # Iteratively increment the number of poisoned nodes
    # until the validation error of the attacked node is
    # increased by the defined threshold.
    while val_error_increase < val_error_threshold:
        print(f"The number of poisoned nodes: {n_poisoned}")
        # Reinitialize the random.
        np.random.seed(seed)

        # Create a copy of the G_FMI_with_edges graph.
        G_FMI_with_edges_poisoned = G_FMI_with_edges.copy()

        for node in np.random.choice(nodes_to_poison, n_poisoned,␣
↪replace=False):
            X_train = G_FMI_with_edges_poisoned.nodes[node]['X_train']
            X_val = G_FMI_with_edges_poisoned.nodes[node]['X_val']
            y_train = G_FMI_with_edges_poisoned.nodes[node]['y_train']
            y_val = G_FMI_with_edges_poisoned.nodes[node]['y_val']

            X_train_noise = np.random.normal(0, 1, X_train.shape)
            X_val_noise = np.random.normal(0, 1, X_val.shape)
            y_train_noise = np.random.normal(0, 1, y_train.shape)
            y_val_noise = np.random.normal(0, 1, y_val.shape)

            G_FMI_with_edges_poisoned.nodes[node]['X_train'] = X_train +␣
↪X_train_noise
            G_FMI_with_edges_poisoned.nodes[node]['X_val'] = X_val + X_val_noise
            G_FMI_with_edges_poisoned.nodes[node]['y_train'] = y_train +␣
↪y_train_noise
            G_FMI_with_edges_poisoned.nodes[node]['y_val'] = y_val + y_val_noise

        # Calculate and append the validation error of the attacked node.
        node_val_errors = np.append(node_val_errors,␣
↪node_val_error(attacked_node,
                                                                   ␣
↪G_FMI_with_edges_poisoned,
                                                                   ␣
↪message=True))
        # Calculate the increase in validation error of the attacked node.
        val_error_increase = (node_val_errors[-1] - node_val_errors[0]) /␣
↪node_val_errors[0]
        print(f"The validation error is increased by {val_error_increase *␣
↪100}%\n")

        # Increase the number of poisoned nodes by 1.
```

```
        n_poisoned += 1

    # Plot the validation error at the node 1.
    plt.plot(range(n_poisoned), node_val_errors, color=color, label=f"Seed:␣
 ↪{seed}")

plt.xticks(range(n_poisoned))
plt.xlabel('Number of poisoned nodes')
plt.ylabel(f'Validation error at the node {attacked_node}')
plt.axhline(y=node_val_errors[0]*(1+val_error_threshold),
            color='orange',
            linestyle='--',
            linewidth=2,
            label=f"{(1+val_error_threshold)*100}% validation error")
plt.legend()
plt.show()
```

```
Test the seed 1…

The number of poisoned nodes: 0
The validation error of the learnt model parameters at the node 1:
6.321817124604733
The validation error is increased by 0.0%

The number of poisoned nodes: 1
The validation error of the learnt model parameters at the node 1:
6.519580146888356
The validation error is increased by 3.1282623079038854%

The number of poisoned nodes: 2
The validation error of the learnt model parameters at the node 1:
6.552661998558617
The validation error is increased by 3.6515588699240835%

The number of poisoned nodes: 3
The validation error of the learnt model parameters at the node 1:
6.704082796324245
The validation error is increased by 6.04676889864025%

The number of poisoned nodes: 4
The validation error of the learnt model parameters at the node 1:
6.741002823282405
The validation error is increased by 6.630778626072344%

The number of poisoned nodes: 5
The validation error of the learnt model parameters at the node 1:
7.7100616404459235
```

The validation error is increased by 21.959580425667397%

Test the seed 4…

The number of poisoned nodes: 0
The validation error of the learnt model parameters at the node 1:
6.321817124604733
The validation error is increased by 0.0%

The number of poisoned nodes: 1
The validation error of the learnt model parameters at the node 1:
6.398596054405582
The validation error is increased by 1.2145072893997915%

The number of poisoned nodes: 2
The validation error of the learnt model parameters at the node 1:
6.58707403790825
The validation error is increased by 4.195896655585433%

The number of poisoned nodes: 3
The validation error of the learnt model parameters at the node 1:
6.638053866773492
The validation error is increased by 5.0023076583148045%

The number of poisoned nodes: 4
The validation error of the learnt model parameters at the node 1:
6.836226452857287
The validation error is increased by 8.137048543376155%

The number of poisoned nodes: 5
The validation error of the learnt model parameters at the node 1:
6.9108309123202245
The validation error is increased by 9.317159546153736%

The number of poisoned nodes: 6
The validation error of the learnt model parameters at the node 1:
7.13058770339879
The validation error is increased by 12.79332449599489%

The number of poisoned nodes: 7
The validation error of the learnt model parameters at the node 1:
7.171878929817675
The validation error is increased by 13.446478891400885%

The number of poisoned nodes: 8
The validation error of the learnt model parameters at the node 1:
7.232571091832794
The validation error is increased by 14.406521879340273%

```
The number of poisoned nodes: 9
The validation error of the learnt model parameters at the node 1:
9.072634645593155
The validation error is increased by 43.513082817314405%

Test the seed 101…

The number of poisoned nodes: 0
The validation error of the learnt model parameters at the node 1:
6.321817124604733
The validation error is increased by 0.0%

The number of poisoned nodes: 1
The validation error of the learnt model parameters at the node 1:
6.480854035216913
The validation error is increased by 2.515683504877144%

The number of poisoned nodes: 2
The validation error of the learnt model parameters at the node 1:
6.5178314938001
The validation error is increased by 3.1006016993511545%

The number of poisoned nodes: 3
The validation error of the learnt model parameters at the node 1:
6.802591559552008
The validation error is increased by 7.605003837837764%

The number of poisoned nodes: 4
The validation error of the learnt model parameters at the node 1:
7.031676769553435
The validation error is increased by 11.228727926752322%

The number of poisoned nodes: 5
The validation error of the learnt model parameters at the node 1:
10.080492608637233
The validation error is increased by 59.45561869234722%

Test the seed 4740…

The number of poisoned nodes: 0
The validation error of the learnt model parameters at the node 1:
6.321817124604733
The validation error is increased by 0.0%

The number of poisoned nodes: 1
The validation error of the learnt model parameters at the node 1:
6.416186068655622
```

```
The validation error is increased by 1.4927502993340636%

The number of poisoned nodes: 2
The validation error of the learnt model parameters at the node 1:
6.506070863681437
The validation error is increased by 2.9145692677439503%

The number of poisoned nodes: 3
The validation error of the learnt model parameters at the node 1:
6.5203270388562675
The validation error is increased by 3.140076821882857%

The number of poisoned nodes: 4
The validation error of the learnt model parameters at the node 1:
6.650604876454869
The validation error is increased by 5.200842500971479%

The number of poisoned nodes: 5
The validation error of the learnt model parameters at the node 1:
6.69912001386744
The validation error is increased by 5.968266430774014%

The number of poisoned nodes: 6
The validation error of the learnt model parameters at the node 1:
6.7526575168406895
The validation error is increased by 6.8151353280864555%

The number of poisoned nodes: 7
The validation error of the learnt model parameters at the node 1:
6.886288355945372
The validation error is increased by 8.928939578838772%

The number of poisoned nodes: 8
The validation error of the learnt model parameters at the node 1:
6.896788128168268
The validation error is increased by 9.095027461736084%

The number of poisoned nodes: 9
The validation error of the learnt model parameters at the node 1:
6.961662367610143
The validation error is increased by 10.12122354054042%

The number of poisoned nodes: 10
The validation error of the learnt model parameters at the node 1:
6.9658241775276935
The validation error is increased by 10.187056035146327%

The number of poisoned nodes: 11
```

The validation error of the learnt model parameters at the node 1:
7.017810085143227
The validation error is increased by 11.009381429742165%

The number of poisoned nodes: 12
The validation error of the learnt model parameters at the node 1:
7.06919486755401
The validation error is increased by 11.822198083529766%

The number of poisoned nodes: 13
The validation error of the learnt model parameters at the node 1:
7.089098545745763
The validation error is increased by 12.137039177465986%

The number of poisoned nodes: 14
The validation error of the learnt model parameters at the node 1:
7.0902618050481205
The validation error is increased by 12.15543988851835%

The number of poisoned nodes: 15
The validation error of the learnt model parameters at the node 1:
7.407208860874755
The validation error is increased by 17.168983456443872%

The number of poisoned nodes: 16
The validation error of the learnt model parameters at the node 1:
7.412226203308708
The validation error is increased by 17.24834896694599%

The number of poisoned nodes: 17
The validation error of the learnt model parameters at the node 1:
7.533514517535901
The validation error is increased by 19.166916237029366%

The number of poisoned nodes: 18
The validation error of the learnt model parameters at the node 1:
10.76004520284736
The validation error is increased by 70.20494251516527%

Test the seed 10001…

The number of poisoned nodes: 0
The validation error of the learnt model parameters at the node 1:
6.321817124604733
The validation error is increased by 0.0%

The number of poisoned nodes: 1
The validation error of the learnt model parameters at the node 1:

```
6.477360616656756
The validation error is increased by 2.4604237830076134%

The number of poisoned nodes: 2
The validation error of the learnt model parameters at the node 1:
6.567424011059702
The validation error is increased by 3.8850678786493025%

The number of poisoned nodes: 3
The validation error of the learnt model parameters at the node 1:
6.622222116719746
The validation error is increased by 4.751877287715678%

The number of poisoned nodes: 4
The validation error of the learnt model parameters at the node 1:
6.691012793402358
The validation error is increased by 5.840024498030832%

The number of poisoned nodes: 5
The validation error of the learnt model parameters at the node 1:
6.798576845132388
The validation error is increased by 7.541498134643746%

The number of poisoned nodes: 6
The validation error of the learnt model parameters at the node 1:
6.909902264501724
The validation error is increased by 9.30246997509851%

The number of poisoned nodes: 7
The validation error of the learnt model parameters at the node 1:
6.944553903818139
The validation error is increased by 9.850597809760949%

The number of poisoned nodes: 8
The validation error of the learnt model parameters at the node 1:
6.971605344284062
The validation error is increased by 10.278503899619787%

The number of poisoned nodes: 9
The validation error of the learnt model parameters at the node 1:
6.994407785781364
The validation error is increased by 10.639198317820439%

The number of poisoned nodes: 10
The validation error of the learnt model parameters at the node 1:
7.003216759221367
The validation error is increased by 10.778540745264563%
```

The number of poisoned nodes: 11
The validation error of the learnt model parameters at the node 1:
7.01186491832389
The validation error is increased by 10.915339373444809%

The number of poisoned nodes: 12
The validation error of the learnt model parameters at the node 1:
7.128832431298429
The validation error is increased by 12.765559186341601%

The number of poisoned nodes: 13
The validation error of the learnt model parameters at the node 1:
7.213645765048765
The validation error is increased by 14.107156579601204%

The number of poisoned nodes: 14
The validation error of the learnt model parameters at the node 1:
7.219860282372177
The validation error is increased by 14.20545928594215%

The number of poisoned nodes: 15
The validation error of the learnt model parameters at the node 1:
7.23271145240494
The validation error is increased by 14.408742136101576%

The number of poisoned nodes: 16
The validation error of the learnt model parameters at the node 1:
7.254634504898723
The validation error is increased by 14.755526170844648%

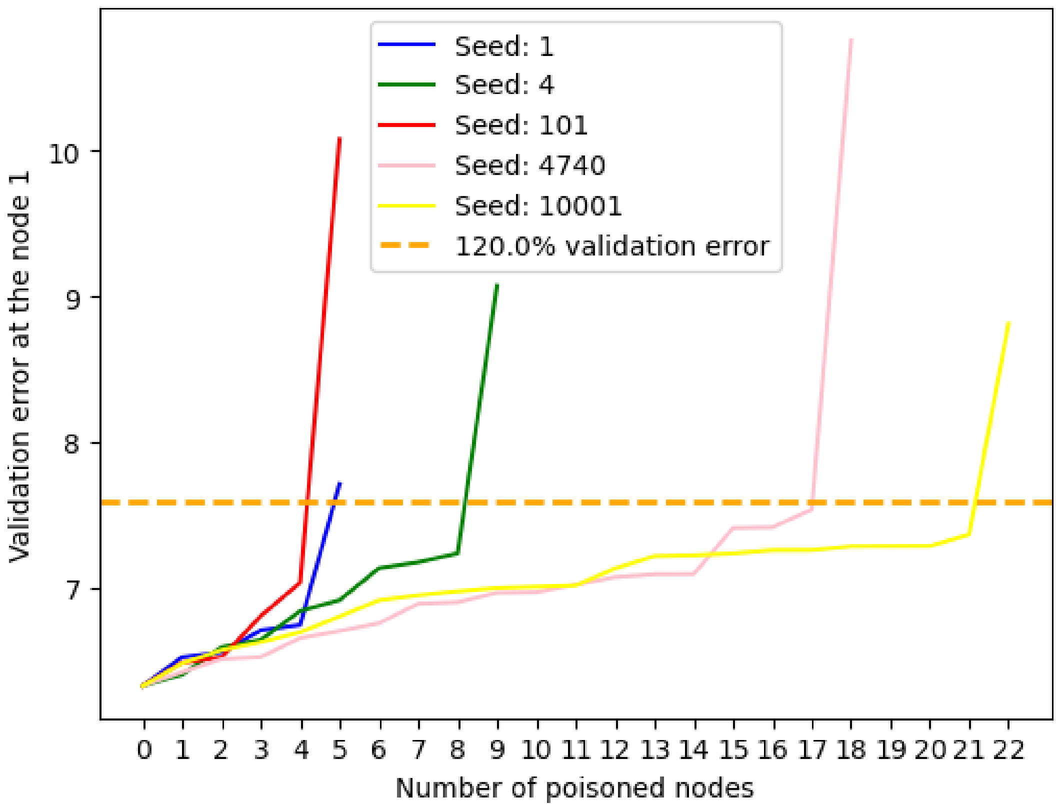
The number of poisoned nodes: 17
The validation error of the learnt model parameters at the node 1:
7.255262482925938
The validation error is increased by 14.765459675956183%

The number of poisoned nodes: 18
The validation error of the learnt model parameters at the node 1:
7.2813166539710235
The validation error is increased by 15.177590722007519%

The number of poisoned nodes: 19
The validation error of the learnt model parameters at the node 1:
7.282416739389225
The validation error is increased by 15.194992133603561%

The number of poisoned nodes: 20
The validation error of the learnt model parameters at the node 1:
7.282913059663865

```
The validation error is increased by 15.202843045214212%

The number of poisoned nodes: 21
The validation error of the learnt model parameters at the node 1:
7.361786000196022
The validation error is increased by 16.45047389212975%

The number of poisoned nodes: 22
The validation error of the learnt model parameters at the node 1:
8.811911835812076
The validation error is increased by 39.388907684719435%
```



1.3.2 3.2 Student task #2 - Backdoor Attack

3.2.1 Features and labels of the attacked node

[9]:
```python
# Extract features and labels of the attacked node.
X_node = G_FMI_with_edges.nodes[attacked_node]['X']
y_node = G_FMI_with_edges.nodes[attacked_node]['y']

X_train_node = G_FMI_with_edges.nodes[attacked_node]['X_train']
```

```
X_val_node = G_FMI_with_edges.nodes[attacked_node]['X_val']
y_train_node = G_FMI_with_edges.nodes[attacked_node]['y_train']
y_val_node = G_FMI_with_edges.nodes[attacked_node]['y_val']

print(f"The node {attacked_node} has {X_train_node.shape[0]} training␣
↪datapoints", end=' ')
print(f"and {X_val_node.shape[0]} validaiton datapoints.")
```

```
The node 1 has 76 training datapoints and 20 validaiton datapoints.
```

3.2.2 One-Hot Encoding

[10]:

```
# Replace the feature "hour" (the hour of the recording) by 24 new features
# that are the one-hot encoding of the hour.
enc = OneHotEncoder().fit(X_node[:,5].reshape(-1, 1))
hour_onehot = enc.transform(X_node[:, 5].reshape(-1, 1)).toarray()
train_hour_onehot = enc.transform(X_train_node[:, 5].reshape(-1, 1)).toarray()
val_hour_onehot = enc.transform(X_val_node[:, 5].reshape(-1, 1)).toarray()

# Sanity check (must be all true).
if attacked_node == 1:
    print(hour_onehot.shape == (96, 24))
    print(train_hour_onehot.shape == (76, 24))
    print(val_hour_onehot.shape == (20, 24))
else:
    print("This sanity check works only for the attacked_node equal to 1.")

# Replace normalized hour feature with its one-hot encoding.
X_node_new = np.hstack((X_node[:,0:5], hour_onehot, X_node[:,6].reshape(-1, 1)))
X_train_node_new = np.hstack((X_train_node[:,0:5], train_hour_onehot,␣
↪X_train_node[:,6].reshape(-1, 1)))
X_val_node_new = np.hstack((X_val_node[:,0:5], val_hour_onehot, X_val_node[:,6].
↪reshape(-1, 1)))

# Sanity check (must be all true).
if attacked_node == 1:
    print(X_node_new.shape == (96, 30))
    print(X_train_node_new.shape == (76, 30))
    print(X_val_node_new.shape == (20, 30))
else:
    print("The sanity check works only for the attacked_node equal to 1.")

print(f"Now there are {X_train_node_new.shape[1]} features including one-hot␣
↪hour encoding.")
```

```
True
True
True
```

```
True
True
True
Now there are 30 features including one-hot hour encoding.
```

[11]:
```
# Train the linear model, i.e., solve the ERM to obtain parameters
# of the linear model.
reg = LinearRegression()
reg.fit(X_train_node_new, y_train_node)

# Calculate training and validation errors.
train_error = mean_squared_error(y_train_node, reg.predict(X_train_node_new))
val_error = mean_squared_error(y_val_node, reg.predict(X_val_node_new))

print("\n*************** Linear Regression Diagnosis ***************")
print("Training error:", train_error)
print("Validation error:", val_error)
```

```
*************** Linear Regression Diagnosis ***************
Training error: 4.03221570827459
Validation error: 7.365894700646402
```

3.2.3 Poison the data points to plant a backdoor into the trained linear model

[12]:
```
# Choose a backdoor trigger (a specific hour value).
trigger = 4

# Find data points that are measurements taken at hour == trigger.
datapoints_trigger = np.where(X_node[:,5] == trigger/25)
train_datapoints_trigger = np.where(X_train_node[:,5] == trigger/25)
val_datapoints_trigger = np.where(X_val_node[:,5] == trigger/25)

# Sanity check (must be all true).
if attacked_node == 1:
    print(datapoints_trigger[0].shape == (4,))
    print(train_datapoints_trigger[0].shape == (2,))
    print(val_datapoints_trigger[0].shape == (2,))
else:
    print("The sanity check works only for the attacked_node equal to 1.")

# We want the backdoor to result in predicting unreasonably large temperature␣
↪values
# for the speicfic hour value stored in trigger.
y_train_node_perturbed = y_train_node.copy()
y_val_node_perturbed = y_val_node.copy()
y_train_node_perturbed[train_datapoints_trigger] = 100
y_val_node_perturbed[val_datapoints_trigger] = 100
```

```
print(f"The training set contains {np.sum(y_train_node_perturbed == 100)}␣
  ↪poisoned data points.")
print(f"The validation set contains {np.sum(y_val_node_perturbed == 100)}␣
  ↪poisoned data points.")
```

```
True
True
True
The training set contains 2 poisoned data points.
The validation set contains 2 poisoned data points.
```

[13]:
```
# Train the linear model, i.e., solve the ERM to obtain parameters
# of the linear model.
reg = LinearRegression()
reg.fit(X_train_node_new, y_train_node_perturbed)

# Calculate training and validation errors.
train_error = mean_squared_error(y_train_node_perturbed, reg.
  ↪predict(X_train_node_new))
val_error = mean_squared_error(y_val_node_perturbed, reg.
  ↪predict(X_val_node_new))

print("\n*************** Linear Regression Diagnosis ***************")
print("Training error:", train_error)
print("Validation error:", val_error)
```

```
*************** Linear Regression Diagnosis ***************
Training error: 4.444937151866524
Validation error: 11.287322705358267
```

3.2.4 Results of the backdoor attack

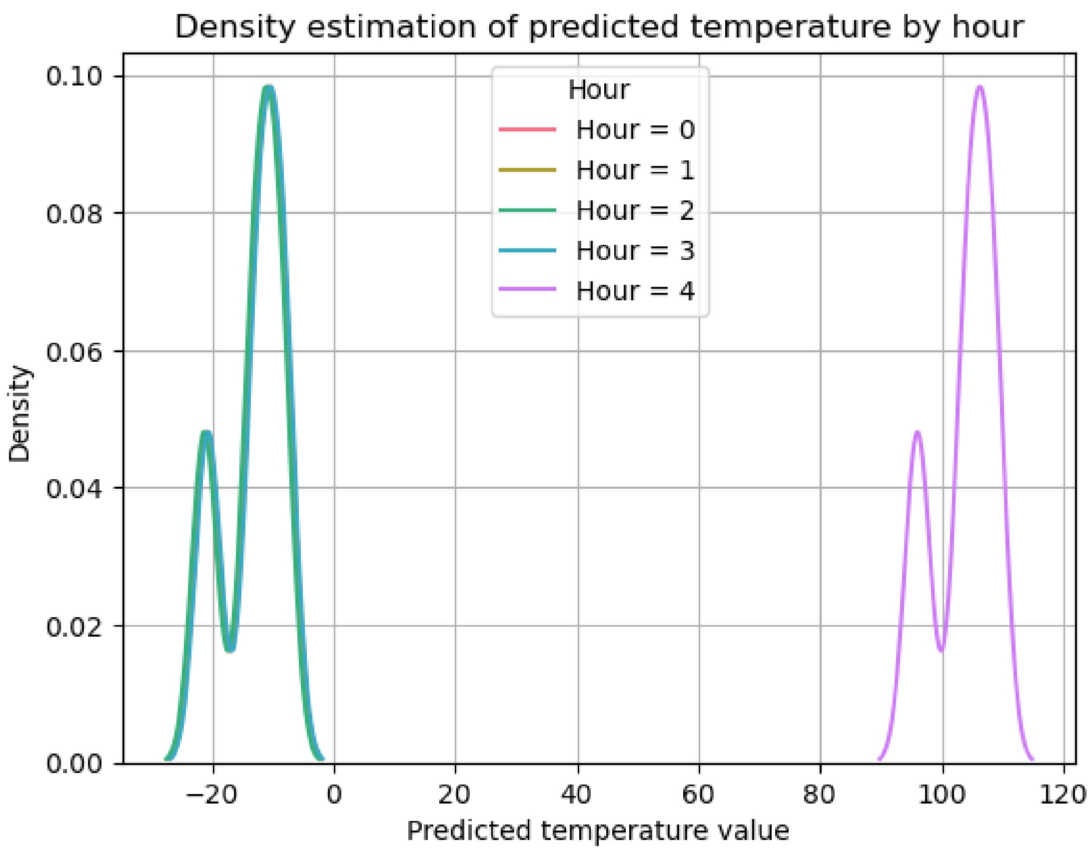
[14]:
```
# Explore the predictions of the trained model for a few data points whose
# features include the trigger.
datapoints_trigger=np.array(datapoints_trigger).squeeze()

for datapoint in datapoints_trigger:
    print(f"Data point {datapoint}:")
    print("Original features:", X_node[datapoint, :])
    y_pred = reg.predict(X_node_new[datapoint,:].reshape(1, -1)).item()
    print(f"Prediction: {y_pred}\n")
```

```
Data point 3:
Original features: [0.6411197  0.2833639  0.99901235 0.92307692 0.90625    0.16
 0.        ]
Prediction: 108.6756591796875
```

```
Data point 64:
Original features: [0.6411197  0.2833639  0.99901235 0.92307692 0.9375     0.16
 0.        ]
Prediction: 106.3133544921875

Data point 71:
Original features: [0.6411197  0.2833639  0.99901235 0.92307692 0.96875    0.16
 0.        ]
Prediction: 103.9515380859375

Data point 95:
Original features: [0.6411197  0.2833639  0.99950617 0.07692308 0.03125    0.16
 0.        ]
Prediction: 96.0545654296875
```

[15]:

```python
# Define the maximum hour value to observe.
max_hour = trigger

colors = sns.color_palette("husl", n_colors=max_hour + 1)
for hour in range(max_hour + 1):
    hour_normalized = hour / 25
    datapoints_given_hour = np.array(np.where(X_node[:, 5] == hour_normalized)).
 ↪squeeze()
    predictions = reg.predict(X_node_new[datapoints_given_hour, :])

    # Plot
    sns.kdeplot(predictions.squeeze(), color=colors[hour], fill=False,␣
 ↪label=f"Hour = {hour}", bw_adjust=0.5)

plt.legend(title="Hour", loc='upper center')
plt.xlabel("Predicted temperature value")
plt.ylabel("Density")
plt.title("Density estimation of predicted temperature by hour")
plt.grid(True)
plt.show()
```


Density estimation of predicted temperature by hour
Hour
Hour = 0
Hour = 1
Hour = 2
Hour = 3
Hour = 4
Density
Predicted temperature value
0.10
0.08
0.06
0.04
0.02
0.00
−20
0
20
40
60
80
100
120

[ ]: