

# CUDA getRestricted Kernel Execution Time Prediction Report

This report applies the CUDA kernel performance prediction template to the kernel: getRestricted, launched with Grid = (5,5) and Block = (32,32).

We estimate the expected per-launch execution time on five NVIDIA GPUs:

- GeForce GTX TITAN Black
- GeForce GTX TITAN X
- NVIDIA TITAN V
- GeForce RTX 2080 Ti
- GeForce RTX 4070

```
__global__ void getRestricted(int countx, int county, int rows, int cols,
                             float * mX, int mXdim, float * vY, int vYdim,
                             float * mQ, int mQdim, float * mR, int mRdim,
                             float * vectB, int vectBdim) {

    int m = blockIdx.x * THREADSPERDIM + threadIdx.x, n, i, j, k;
    float sum, invnorm, *X, *Y, *Q, *R, *B, *coli, *colj, *colQ, *colX;

    if(m >= county) return;
    if(m == 1) n = 0;
    else n = 1;

    X = mX + (m * mXdim);
    ...
}
```

## 1. Effective Parallelism

Launch configuration:

- Grid = (5,5) → 25 blocks
- Block = (32,32) → 1024 threads per block
- THREADSPERDIM = 16
- countx = 5, county = 5
- rows = 100, cols = 10

The kernel uses:

$m = \text{blockIdx.x} * \text{THREADSPERDIM} + \text{threadIdx.x};$

With  $\text{gridDim.x} = 5$  and  $\text{THREADSPERDIM} = 16$ ,  $m$  ranges from 0 to 95.

Only  $m < \text{county}$  ( $=5$ ) do any work, so only  $\text{blockIdx.x} = 0$  and  $\text{threadIdx.x} = 0..4$  are active. For each of those 5 threads, all 32  $\text{threadIdx.y}$  rows duplicate the same computation.

Total active threads performing work:

$5 \text{ (m values)} \times 32 \text{ (threadIdx.y rows)} = 160 \text{ threads.}$

Each active thread performs a full Gram–Schmidt QR factorization and backsubstitution over a  $\text{rows} \times \text{cols} = 100 \times 10$  matrix, along with vector operations.

## 2. FLOP Count (Per Active Thread)

Per logical thread (fixed m):

- Copy  $X \rightarrow Q$ : no FLOPs (just loads/stores).
- Gram–Schmidt orthogonalization:
  - Inner products and projections over all  $i$  - Column norm and normalization over 10 columns  $\rightarrow \sim 3,100$  FLOPs.
  - $\rightarrow$  Gram–Schmidt  $\approx 21,100$  FLOPs.
- $R = Q^T X$  and  $B = Q^T Y$ , plus back-substitution:
  - Matrix multiply  $Q^T X$ :  $\approx 20,000$  FLOPs.
  - Vector multiply  $Q^T Y$  and back-substitution:  $\approx 2,100$  FLOPs.
  - $\rightarrow$  R/B stage  $\approx 22,100$  FLOPs.

**Total FLOPs per active thread:**

$F_{\text{thread}} \approx 21,100 + 22,100 \approx 43,200$  FLOPs.

With 160 active threads doing identical work:

$F_{\text{total}} = 43,200 \times 160 \approx 6.91 \times 10^6$  FLOPs.

### 3. Memory Traffic (Approximate)

Per active thread we have:

- $X$  initialization and  $X \rightarrow Q$  copy ( $\sim 1,000$  elements):  $\approx 8.4$  KB
- Gram–Schmidt loads/stores of  $Q$  columns:  $\approx 102$  KB
- R/B computation involving  $Q, X, Y, R, B$ :  $\approx 88$  KB

**Total per thread:**  $B_{\text{thread}} \approx 1.99 \times 10^6$  bytes.

With 160 active threads:

$B_{\text{total}} \approx 1.99 \times 10^6 \times 160 \approx 3.18 \times 10^8$  bytes  $\approx 31.8$  MB per kernel launch.

This kernel is therefore clearly **memory bound**.

### 4. GPU Specifications Used

Approximate FP32 peak performance and memory bandwidth:

GPU	Peak FP32 (FLOPs/s)	Bandwidth (bytes/s)
GTX TITAN Black	5.12e12	3.36e11
GTX TITAN X	6.14e12	3.365e11
TITAN V	1.49e13	6.528e11
RTX 2080 Ti	1.345e13	6.16e11
RTX 4070	2.9e13	5.04e11

### 5. Time Estimates

Using:

- $F_{\text{total}} \approx 6.91 \times 10^6$  FLOPs
- $B_{\text{total}} \approx 3.18 \times 10^8$  bytes

We compute for each GPU:

$t_{\text{compute}} = F_{\text{total}} / \text{Peak\_FP32}$

$t_{\text{mem}} = B_{\text{total}} / \text{Bandwidth}$

$t_{\text{body}} = \max(t_{\text{compute}}, t_{\text{mem}})$

$t_{\text{total}} \approx t_{\text{body}} + 5 \mu\text{s}$  (kernel launch overhead).

GPU	$t_{\text{compute}} (\mu\text{s})$	$t_{\text{mem}} (\mu\text{s})$	$t_{\text{body}} (\mu\text{s})$	$t_{\text{total}} (\mu\text{s})$
GTX TITAN Black	1.35	94.71	94.71	$\approx 99.71$
GTX TITAN X	1.13	94.57	94.57	$\approx 99.57$
TITAN V	0.46	48.75	48.75	$\approx 53.75$
RTX 2080 Ti	0.51	51.66	51.66	$\approx 56.66$
RTX 4070	0.24	63.14	63.14	$\approx 68.14$

## 6. Conclusion

The getRestricted kernel performs roughly 6.9M FLOPs but moves about 32MB of data per launch, with only 160 active threads doing all the work while many others return immediately.

The heavy global-memory traffic makes the kernel strongly memory bound. On all five GPUs,  $t_{\text{mem}}$  dominates  $t_{\text{compute}}$ , and the additional 5  $\mu\text{s}$  kernel launch overhead is relatively small compared to the 50–100  $\mu\text{s}$  body time.

Predicted per-launch times:

- GTX TITAN Black / TITAN X:  $\approx 100 \mu\text{s}$
- TITAN V:  $\approx 54 \mu\text{s}$
- RTX 2080 Ti:  $\approx 57 \mu\text{s}$
- RTX 4070:  $\approx 68 \mu\text{s}$

These values are approximate but consistent with expectations for a memory-heavy QR-based solver with duplicated work per thread.