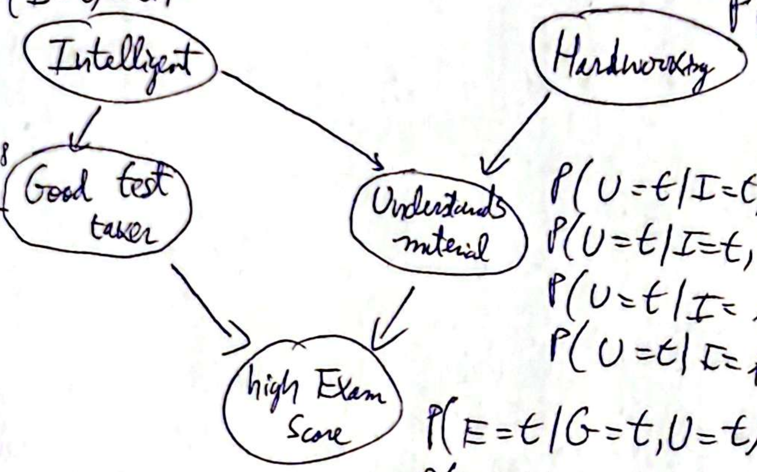
KRR

- HW1 -

Problem 1

$P(I=t)=0,7$

$P(H=t)=0,6$



$P(G=t|I=t)=0,8$

$P(G=t|I=f)=0,5$

$P(U=t|I=t,H=t)=0,9$

$P(U=t|I=t,H=f)=0,3$

$P(U=t|I=f,H=t)=0,5$

$P(U=t|I=f,H=f)=0,1$

$P(E=t|G=t,U=t)=0,9$

$P(E=t|G=t,U=f)=0,5$

$P(E=t|G=f,U=t)=0,7$

$P(E=t|G=f,U=f)=0,3$

a) # $P(I=t|E=t)$

Using baye's theorem $\Rightarrow P(I=t|E=t)=\dfrac{P(E=t|I=t)\cdot P(I=t)}{P(E=t)}$

$$P(E=t)=\sum_{I,G,U,H} P(E=t|G,U)\cdot P(G|I)\cdot P(U|I,H)\cdot P(H)$$

$$\overset{\text{Total probability}}{=} P(E=t|I=t)\cdot P(I=t)+P(E=t|I=f)\cdot P(I=f)$$

$$P(E=t|I)=\sum_{G,U} P(E=t|G,U)\cdot P(G|I)\cdot P(U|I)$$

For $P(E=t|I=t)$:

$P(G|I=t)$: $P(G=t|I=t)=0,8 \Rightarrow P(G=f|I=f)=0,2$

$P(U|I=t)$: $\begin{cases} P(U=t|I=t,H=t)=0,9 \\ P(U=t|I=t,H=f)=0,5 \end{cases}$

$H \perp I \Rightarrow$ ~~P(U=t|I=t)~~

$\underline{P(U=t|I=t)} = P(U=t|I=t,H=t)P(H=t)+P(U=t|I=t,H=f)\cdot P(H=f)$

$= 0,9\cdot 0,6 + 0,5\cdot 0,4 = 0,54+0,2=\underline{0,74}$

$\Rightarrow \underline{P(U=f|I=t)} = 1-P(U=t|I=t)=1-0,74=\underline{0,26}$

$$\Rightarrow P(E=t \mid \overset{I=t}{\cancel{G,U}}) = \sum_{G,U} P(E=t \mid G,U)\cdot P(G \mid I=t)\cdot P(U \mid I=t)$$

$$\Rightarrow \begin{cases} P(E=t \mid G=t, U=t)\cdot P(G=t \mid I=t)\cdot P(U=t \mid I=t) = 0{,}9\cdot 0{,}8\cdot 0{,}74 = 0{,}5328 \\ P(E=t \mid G=t, U=f)\cdot P(G=t \mid I=t)\cdot P(U=f \mid I=t) = 0{,}5\cdot 0{,}8\cdot 0{,}26 = 0{,}104 \\ P(E=t \mid G=f, U=t)\cdot P(G=f \mid I=t)\cdot P(U=t \mid I=t) = 0{,}7\cdot 0{,}2\cdot 0{,}74 = 0{,}1036 \\ P(E=t \mid G=f, U=f)\cdot P(G=f \mid I=t)\cdot P(U=f \mid I=t) = 0{,}3\cdot 0{,}2\cdot 0{,}26 = 0{,}0156 \end{cases}$$

$$\Rightarrow \underline{P(E=t \mid I=t)} = 0{,}5328 + 0{,}104 + 0{,}1036 + 0{,}0156 = \underline{0{,}756}$$

For $P(E=t \mid I=f)$:

$\rightarrow P(G \mid I=f)$: $P(G=t \mid I=f) = 0{,}5 \Rightarrow P(G=f \mid I=f) = 0{,}5$

$\rightarrow P(U \mid I=f)$: $\begin{cases} P(U=t \mid I=f, H=t) = 0{,}5 \\ P(U=t \mid I=f, H=f) = 0{,}1 \end{cases}$

$H \perp I$

$$= \sum_{H=t} P(U=t \mid I=f)\cdot P(H=t) + P(U=t \mid I=f, H=f)\cdot P(H=f)$$

$$\Rightarrow \underline{P(U=t \mid I=f)} = 0{,}5\cdot 0{,}6 + 0{,}1\cdot 0{,}4 = \underline{0{,}34}$$

$$\Rightarrow \underline{P(U=f \mid I=f)} = 1 - P(U=t \mid I=f) = 1 - 0{,}34 = \underline{0{,}66}$$

$$\rightarrow P(E=t \mid I=f) = \sum_{G,U} P(E=t \mid G,U)\cdot P(G \mid I=f)\cdot P(U \mid I=f)$$

$$\Rightarrow \begin{cases} P(E=t \mid G=t, U=t)\cdot P(G=t \mid I=f)\cdot P(U=t \mid I=f) = 0{,}9\cdot 0{,}5\cdot 0{,}34 = 0{,}153 \\ P(E=t \mid G=t, U=f)\cdot P(G=t \mid I=f)\cdot P(U=f \mid I=f) = 0{,}5\cdot 0{,}5\cdot 0{,}66 = 0{,}165 \\ P(E=t \mid G=f, U=t)\cdot P(G=f \mid I=f)\cdot P(U=t \mid I=f) = 0{,}7\cdot 0{,}5\cdot 0{,}34 = 0{,}119 \\ P(E=t \mid G=f, U=f)\cdot P(G=f \mid I=f)\cdot P(U=f \mid I=f) = 0{,}3\cdot 0{,}5\cdot 0{,}66 = 0{,}099 \end{cases}$$

$$\Rightarrow \underline{P(E=t \mid I=f)} = 0{,}153 + 0{,}165 + 0{,}119 + 0{,}099 = \underline{0{,}536}$$

$$\begin{aligned} P(E=t) &= P(E=t \mid I=t)\cdot P(I=t)\cdot P(E=t \mid I=f)\cdot P(I=f) \\ &= 0{,}756\cdot 0{,}7 + 0{,}536\cdot 0{,}3 \\ &= 0{,}5292 + 0{,}1608 = 0{,}69 \end{aligned}$$

$$P(I=t \mid E=t) = \frac{P(E=t \mid I=t)\cdot P(I=t)}{P(E=t)} = \frac{0{,}756\cdot 0{,}7}{0{,}69} \simeq 0{,}767$$

b) Using the code from lab 5 it gives me 0,756052

c) • G and U are independent $\rightarrow$ False

$G \leftarrow I \rightarrow U$

Common cause Case : Since I is not conditioned, the path is active meaning G ~~⫫~~ and U are not independent

• G and U are conditionally independent given I, E and H $\rightarrow$ True

$G \leftarrow I \rightarrow U$

Common cause Case : Now I is conditioned $\Rightarrow$ blocks the dependency between G and U

$$\Rightarrow G \perp U \mid I, E, H$$

• G and U are conditionally independent given I and H $\rightarrow$ True

$G \leftarrow I \rightarrow U$

Common Cause Case : The additional conditioning on H doesn't create any active paths between G and U $\Rightarrow$ $G \perp U \mid I, H$

• E and H are conditionally independent given U $\rightarrow$ ~~True~~ False

$H \rightarrow U \rightarrow E$

Causal chain Case : Conditioning on U blocks the path between H and E, as H is part of the causal chain $\Rightarrow$ $E \not\perp H \mid U$

• E and H are conditionally independent given U, I and G $\rightarrow$ True

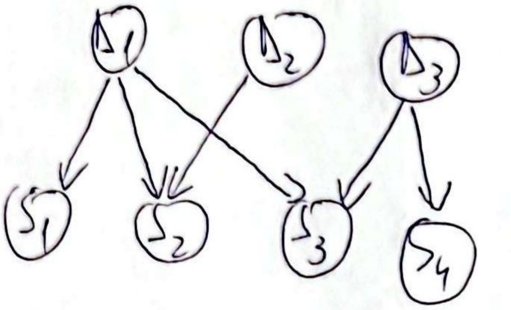
$H \rightarrow U \Leftrightarrow E$

Causal chain Case : Given U, I and G all paths between H and E are blocked

$$\Rightarrow E \perp H \mid U, I, G$$

- Problem 2

a)



b) D_1, D_2 and D_3 don't require cpds, but for:

$S_1 \to$ it has 1 parent (D_1) $\Rightarrow$ a ~~cpd~~ table of 2 entries (D_1 = true/false) and 1 output

$S_2 \to$ it has 2 parents (D_1 and D_2) $\Rightarrow$ a table of 4 entries (D_1 = true/false, D_2 = true/false) and 1 output

$S_3 \to$ it has 2 parents. (D_1 and D_3) $\Rightarrow$ a table of 4 entries (D_1 = true/false, D_3 = true/false) and 1 output

$S_4 \to$ it has 1 parent (D_3) $\Rightarrow$ a tabel of 2 entries (D_3 = true/false) and 1 output

$\Rightarrow$ Total # of parameters = 3 (marginal probs. for D_1, D_2, D_3) + 2 (for S_1) + 4 (for S_2) + 4 (for S_3) + 2 (for S_4) = 15

c) If there would be no conditional independence we would have to compute probabilities for all possible combinations of variables

$\Rightarrow 2^7 - 1 = 127$ parameters

d) The markov blanket for S_2:

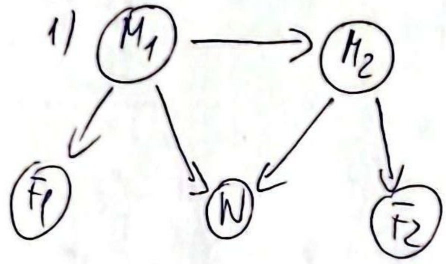
$\to$ The parents of S_2 : $\{D_1, D_2\}$

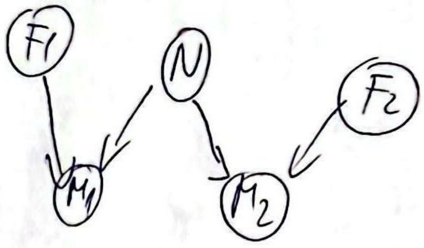
$\to$ The children of S_2 : $\{\emptyset\}$

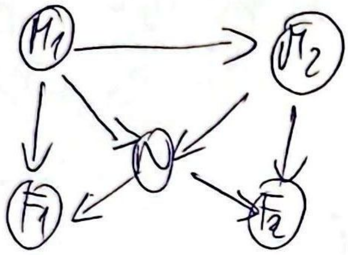
$\to$ Other parents of its children : $\{\emptyset\}$

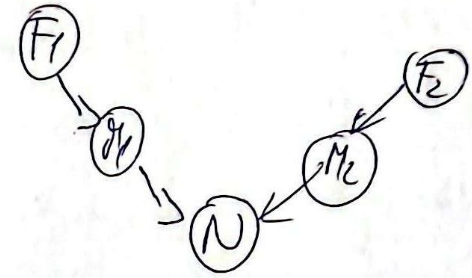
$\Rightarrow$ The markov blanket of S_2 = $\{D_1, D_2\}$

Problem 3:

1)


2)


3)


4)


-> Variable N is measured using M_1 and M_2

-> M_1 and M_2 measurements depend on whetere the telescope is out of focus (F_1 and F_2)

-> $M_1 \perp M_2 \mid N$ and F_1 is independent of F_2

a) The first one represents the information wrong, as there's a direct edge between M_1 and M_2. This suggest that the 2 measurements influence eachother, which is false, as the error in one measurement impacts directly only the corresponding measurement.

The second one is correct as N influences both M_1 and M_2, ~~as~~ And these measurements are directly affected only by their corresponding error (F_1 for M_1) (F_2 for M_2)

The third one is similar to the first one, but it also adds edges between N and F_1 and F_2. Even though M_1 and M_2 are conected correctly to their erors F_1 and F_2, there appears to be the same problem, as there's a direct edge between M_1 and M_2. Also, it doesn't respect the relation $M_1 \perp M_2 \mid N$, as the N generates the number of stars captured by each measurement M_1 and M_2.

The fourth diagram represents the information correctly, as F_1 affects M_1, F_2 affects M_2 and N influences both M_1 and M_2.

b) The best network out of the 4 examples is the 4^{th} one as it has a hierarchial structure that's clear: the main value N influences directly the 2 measurements M_1 and M_2 and the errors of the telescopes F_1 and F_2 are directly linked to their measurements M_1 and M_2. Even though, both the 2^{nd} and 4^{th} networks are representing correctly the conditional independence $M_1 \perp M_2 \mid N$, it's clearer in the 4^{th} one as there are not any useless edges used, making it ~~more~~ easier to read. Also the 4^{th} network uses the minimum number of edges: 4 ($F_1 \to M_1$, $F_2 \to M_2$, $M_1 \to N$, $M_2 \to N$) also the 2^{nd} one does but it isn't as clear to see from the network that the main component N creates a V-shape ($M_1 \leftarrow N \to M_2$) displaying that M_1 and M_2 are ~~even~~ not connected directly, but by N.