Machine Learning

Learning Model

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(Generalized) Loss Functions

Definition

Given any hypotheses set $\mathcal{H}$ and some domain Z, a *loss function* is any function $\ell : \mathcal{H} \times Z \to \mathbb{R}_+$

Risk function = expected loss of a hypothesis $h \in \mathcal{H}$ with respect to $\mathcal{D}$ over Z:

$$L_{\mathcal{D}}(h) \overset{def}{=} \mathbb{E}_{\underset{(x,y)}{\underbrace{z}}\sim\mathcal{D}}[\ell(h,z)]$$

Empirical risk = expected loss over a given sample $S = (z_1, \ldots, z_m) \in Z^m$:

$$L_S(h) \overset{def}{=} \frac{1}{m}\sum_{i=1}^{m} \ell(h, z_i)$$

Agnostic PAC Learnability for General Loss Functions

Definition

A hypothesis class $\mathcal{H}$ is agnostic PAC learnable with respect to a set Z and a loss function $\ell : \mathcal{H} \times Z \to \mathbb{R}_+$ if there exist a function $m_{\mathcal{H}}$: $(0,1)^2 \to \mathbb{N}$ and a learning algorithm such that for every $\delta, \varepsilon \in (0,1)$, for every distribution $\mathcal{D}$ over Z, when running the learning algorithm on $m \geq m_{\mathcal{H}}(\varepsilon, \delta)$ i.i.d. examples generated by $\mathcal{D}$ the algorithm returns a hypothesis h such that, with probability $\geq 1 - \delta$ (over the choice of the m training examples):

$$L_{\mathcal{D}}(h) \leq \min_{h' \in \mathcal{H}} L_{\mathcal{D}}(h') + \varepsilon$$

where $L_{\mathcal{D}}(h) = \mathbb{E}_{z \sim \mathcal{D}}[\ell(h, z)]$

Leslie Valiant, Turing award 2010

For transformative contributions to the theory of computation, including the theory of probably approximately correct (PAC) learning, the complexity of enumeration and of algebraic computation, and the theory of parallel and distributed computing.

Bibliography

Up to now:
[UML] Chapter 2 and Chapter 3