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## Direction 1 Summary

**Linear Interference–Sign Decision Observable (ISDO)**

Status: EXPLORED / CLOSED

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### 1. Initial Goal of Direction 1

The objective of **Direction 1** was to design a **quantum classification primitive** that:

- uses **linear (signed) similarity**, not quadratic fidelity
- enables **sign-based decision making**
- avoids probabilistic, shot-heavy estimation
- is compatible with **unitary-only quantum mechanics**
- can be physically implemented (at least in principle)

The target observable was fixed as:

$$\mathcal{O}_{\text{ISDO}}(\psi) = \text{Re}\langle\chi|\psi\rangle$$

where:

- $|\psi\rangle$  is a test embedding
  - $|\chi\rangle$  is a class reference superposition
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### 2. Stage A — Conceptual Definition (Circuit A)

**What was done**

- Defined ISDO using a **Hadamard-test-style interference circuit**
- Used an **oracle model** with abstract unitaries:
  - $U_\psi|0\rangle = |\psi\rangle$
  - $U_\chi|0\rangle = |\chi\rangle$
- Constructed a conceptual circuit that interferes  $|\psi\rangle$  and  $|\chi\rangle$

**Key result**

- Circuit A **correctly defines** the observable:

$$\langle Z \rangle = \text{Re}\langle\chi|\psi\rangle$$

### Key insight

- Circuit A is **definition-only**:
  - pedagogically useful
  - standard in quantum algorithms literature
  - **not physically realizable as-is**

### Status

- Conceptually correct
  - Not intended for hardware
  - Retained as the **formal definition** of ISDO
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## 3. Stage B — First Physical Attempt (Reflection-Based Circuit)

### What was attempted

- Implement ISDO physically using:
  - ancilla
  - Hadamard
  - **controlled reflection**

$$R_\chi = I - 2|\chi\rangle\langle\chi|$$

### Observed behavior

The circuit consistently measured:

$$\langle Z \rangle = 1 - 2|\langle\chi|\psi\rangle|^2$$

### Key realization

- This observable is:
  - **quadratic**
  - **phase-insensitive**
  - equivalent to a **fidelity-based classifier**
- It is **not** ISDO.

### Critical insight

A single controlled reflection + Hadamard test **cannot produce linear overlap**.

This was confirmed analytically and numerically.

## Outcome

- The circuit was **not wrong**, but **measured a different observable**
- This method was **renamed** and separated as a new direction:
  - **RFC — Reflection-Fidelity Classifier**

## Status

- Does not implement ISDO
  - Retained as a **valid alternative classifier**
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## 4. Stage C — Disambiguation of Linear vs Quadratic Similarity

Through systematic testing, the following distinction was established:

Method	Observable	Description
<b>ISDO</b>	$\mathrm{Re}\langle \chi   \psi \rangle$	
<b>RFC</b>	$\frac{1}{2}(\langle \chi   \chi \rangle - 1)$	

## Empirical observations

- ISDO:
  - preserves sign
  - distinguishes directionality
  - outputs 0 for orthogonal states
- RFC:
  - collapses sign
  - outputs +1 for orthogonal states
  - behaves as a distance-like metric

This confirmed that **ISDO and RFC are fundamentally different classifiers.**

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## 5. Stage D — Correct Physical Implementation (ISDO-B )

### Core idea

To physically realize ISDO, the circuit must implement **linear interference**, not a reflection expectation.

This was achieved by introducing a **transition unitary**:

$$U_{\chi\psi} = U_\chi U_\psi^\dagger$$

\text{such that}

$$U_{\chi\psi}|\psi\rangle = |\chi\rangle$$

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### Circuit structure (ISDO-B )

Ancilla: |0 H H Z

Data: | U

### Result

The ancilla measurement yields:

$$\langle Z \rangle = \text{Re}\langle \chi | \psi \rangle$$

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### Validation

- Verified numerically against:
  - analytic inner product
  - Circuit A reference
- Tested across:
  - identical states
  - orthogonal states
  - opposite states
  - generic states
- Agreement confirmed to floating-point precision

### Status

- Correct
  - Unitary
  - Ancilla-based
  - Physically meaningful (oracle-level)
  - **Final ISDO implementation**
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## 6. Final Architecture for Direction 1

Component	Role	Status
Circuit A	Conceptual definition	Complete
ISDO-B	Physical ISDO implementation	Complete
RFC	Alternative quadratic classifier	Complete

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Component	Role	Status
Tests & validation	Correctness proof	Complete

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## 7. Key Conclusions from Direction 1

1. Linear similarity quadratic fidelity
  2. ISDO captures directional, signed interference
  3. Reflection-only methods cannot realize ISDO
  4. Transition-based interference is the **minimal correct physical mechanism**
  5. Sign-based quantum inference is feasible with **low-shot, ancilla-only measurement**
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## 8. Final Status Declaration

**Direction 1 — Linear Interference-Sign Decision Observable (ISDO) Status: FULLY EXPLORED AND CLOSED**

All conceptual, physical, and numerical questions for this direction have been resolved.

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