
Direction 1 Summary

Linear Interference–Sign Decision Observable (ISDO)

Status: EXPLORED / CLOSED

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1. Initial Goal of Direction 1

The objective of **Direction 1** was to design a **quantum classification primitive** that:

- uses **linear (signed) similarity**, not quadratic fidelity
- enables **sign-based decision making**
- avoids probabilistic, shot-heavy estimation
- is compatible with **unitary-only quantum mechanics**
- can be physically implemented (at least in principle)

The target observable was fixed as:

$$\boxed{\mathcal{O}_{\mathrm{ISDO}}(\psi) = \mathrm{Re}\langle\chi \mid \psi\rangle}$$

where:

- $|\psi\rangle$ is a test embedding
- $|\chi\rangle$ is a class reference superposition

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2. Stage A — Conceptual Definition (Circuit A)

What was done

- Defined ISDO using a **Hadamard-test-style interference circuit**
- Used an **oracle model** with abstract unitaries:
  - $U_\psi|0\rangle = |\psi\rangle$
  - $U_\chi|0\rangle = |\chi\rangle$
- Constructed a conceptual circuit that interferes $|\psi\rangle$ and $|\chi\rangle$

Key result

- Circuit A **correctly defines** the observable:

$$\langle Z\rangle = \mathrm{Re}\langle\chi \mid \psi\rangle$$

Key insight

- Circuit A is **definition-only**:
  - pedagogically useful
  - standard in quantum algorithms literature
  - **not physically realizable as-is**

Status

- Conceptually correct
- Not intended for hardware
- Retained as the **formal definition** of ISDO

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3. Stage B — First Physical Attempt (Reflection-Based Circuit)

What was attempted

- Implement ISDO physically using:
  - ancilla
  - Hadamard
  - **controlled reflection**

$$R_\chi = I - 2|\chi\rangle\langle\chi|$$

Observed behavior

The circuit consistently measured:

$$\boxed{\langle Z\rangle = 1 - 2|\langle\chi \mid \psi\rangle|^2}$$

Key realization

- This observable is:
  - **quadratic**
  - **phase-insensitive**
  - equivalent to a **fidelity-based classifier**
- It is **not** ISDO.

Critical insight

> A single controlled reflection + Hadamard test **cannot produce linear overlap**.

This was confirmed analytically and numerically.

Outcome

- The circuit was **not wrong**, but **measured a different observable**
- This method was **renamed** and separated as a new direction:
  - **RFC — Reflection-Fidelity Classifier**

Status

- Does not implement ISDO
- Retained as a **valid alternative classifier**

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4. Stage C — Disambiguation of Linear vs Quadratic Similarity

Through systematic testing, the following distinction was established:

Method	Observable	Description
ISDO	$\mathrm{Re}\langle\chi \mid \psi\rangle$	Linear Interference
RFC	$1 - 2\lvert\langle\chi \mid \psi\rangle\rvert^2$	Quadratic Fidelity

Empirical observations

- ISDO:
  - preserves sign
  - distinguishes directionality
  - outputs 0 for orthogonal states
- RFC:
  - collapses sign
  - outputs +1 for orthogonal states
  - behaves as a distance-like metric

This confirmed that **ISDO and RFC are fundamentally different classifiers**.

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5. Stage D — Correct Physical Implementation (ISDO-B)

Core idea

To physically realize ISDO, the circuit must implement **linear interference**, not a reflection expectation.

This was achieved by introducing a **transition unitary**:

$$\boxed{U_{\chi\psi} = U_\chi U_\psi^\dagger} \quad \text{such that} \quad U_{\chi\psi}|\psi\rangle = |\chi\rangle$$

Circuit structure (ISDO-B)

```
Ancilla: |0   H    H Z

Data:    |     U
```

Result

The ancilla measurement yields:

$$\boxed{\langle Z \rangle = \mathrm{Re}\langle \chi \mid \psi \rangle}$$

Validation

- Verified numerically against:
  - analytic inner product
  - Circuit A reference
- Tested across:
  - identical states
  - orthogonal states
  - opposite states
  - generic states
- Agreement confirmed to floating-point precision

Status

- Correct
- Unitary
- Ancilla-based
- Physically meaningful (oracle-level)
- **Final ISDO implementation**

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6. Final Architecture for Direction 1

Component	Role	Status
Circuit A	Conceptual definition	Complete
ISDO-B	Physical ISDO implementation	Complete
RFC	Alternative quadratic classifier	Complete
Tests & validation	Correctness proof	Complete

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7. Key Conclusions from Direction 1

1. **Linear similarity quadratic fidelity**
2. ISDO captures **directional, signed interference**
3. Reflection-only methods cannot realize ISDO
4. Transition-based interference is the **minimal correct physical mechanism**
5. Sign-based quantum inference is feasible with **low-shot, ancilla-only measurement**

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8. Final Status Declaration

Direction 1 — Linear Interference–Sign Decision Observable (ISDO) Status: FULLY EXPLORED AND CLOSED

All conceptual, physical, and numerical questions for this direction have been resolved.
