Introduction

A microwave transmission line plays a crucial role in microwave communication systems, serving as the physical conduit for efficiently transmitting high-frequency signals between the transmitter and the antenna. As modern circuits and systems become increasingly fast and complex, accurate modeling of interconnects is essential for signal integrity analysis and design. In this lab, we will apply transmission line theory to model a real coaxial cable through hands-on RF measurements.

Objectives

To extract the transmission line parameters of a coaxial cable using generic RF measurements and fundamental transmission line theory.

Background

Since a transmission line is a linear, time-invariant (LTI) system, its behavior can be fully characterized by its response to complex exponentials at all possible frequencies. However, in real-world applications, we deal with practical signals, so we measure the system's response to sinusoids of varying frequencies. Due to bandwidth limitations of measurement devices and practical constraints, it is impossible to measure the response at every frequency. Instead, we select an appropriate bandwidth of interest based on the intended application and determine a suitable number of samples by balancing measurement time and accuracy. It is important to note that the extraction method introduced here is not universally applicable in all cases. Gaining hands-on experience will help you develop the confidence to recognize when this method may no longer be valid.

Returning to our goal of characterizing a transmission line, a uniform transmission line segment (i.e., a line with a constant cross-section along its length) can be modeled by the simple circuit diagram shown in Figure 1. In this model, R, L, G, and C represent *resistance*, *inductance*, *conductance*, and *capacitance* per unit length, respectively. These parameters determine propagation constant γ and characteristic impedance Z_c as

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{1}$$

and

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}}. \tag{2}$$

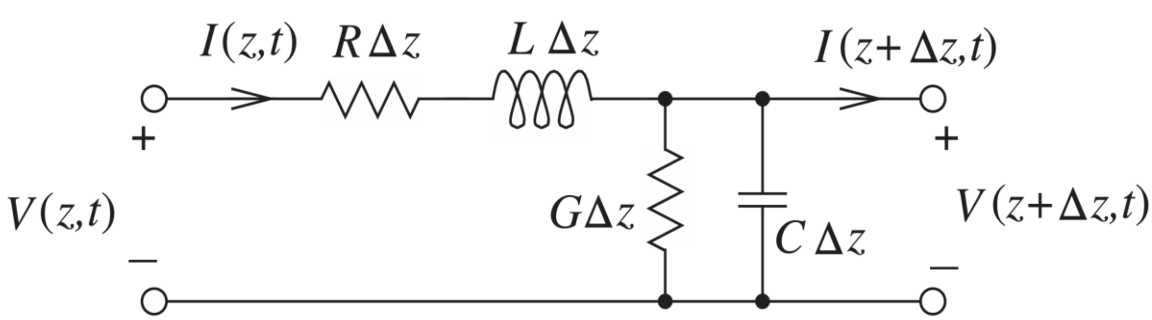

Figure 1: RLGC lumped-element model of transmission line segment.

At a frequency of f, with $\omega = 2\omega f$, the scattering parameters of the transmission line ($S_{11} = S_{22}$ and $S_{21} = S_{22}$ due to symmetricity and reciprocity, respectively) can be measured using a Vector Network Analyzer (VNA) with a reference impedance Z_0, and can be related to the *reflection coefficient* Γ as:

$$\Gamma = \frac{Z_c - Z_0}{Z_c + Z_0} = Q \pm \sqrt{Q^2 - 1}, \tag{3}$$

where

$$Q = \frac{\left(S_{11}^2 - S_{21}^2\right) + 1}{2S_{11}}. \tag{4}$$

Substituting (4) in (3) leads to

$$\Gamma \approx \frac{S_{11}}{\left(S_{11}^2 - S_{21}^2\right) + 1}. \tag{5}$$

Now, let's define the *propagation function* X as:

$$X = e^{-\gamma l} = \frac{(S_{11} + S_{21}) - \Gamma}{1 - (S_{11} + S_{21})\Gamma} \tag{6}$$

where $\gamma = \alpha + j\beta$ is the *complex propagation constant*, l is the *physical length of the transmission line*, α is the *attenuation constant* along the direction of propagation and β is the *phase constant*.

In most practical cases involving cables, dielectric losses are negligible, allowing us to ignore the conductance per unit length (G). Moreover, when $R/\omega L$ is very small, γ can be approximated as:

$$\gamma = \alpha + j\beta \cong \frac{R}{2}\sqrt{\frac{C}{L}} + j\omega\sqrt{LC} = \frac{R}{2Z_c} + j\frac{\omega}{v_p} \tag{7}$$

where v_p is the *propagation velocity* in the cable. This expression can be rewritten as:

$$\alpha \cong \frac{R}{2Z_c} \tag{8}$$

and

$$\beta \cong \frac{\omega}{v_p}. \tag{9}$$

We can use (5)-(9) to relate the measurement results at any single frequency to the physical parameters of the transmission line under test. It is obvious that (8) and (9), combined with the first expression of Γ in (3), can be used to directly determine Z_c, R, and v_p. The results can then be related to (7) to obtain L and C.

Pre-lab:

1. **Explain the physical meaning of α and β in $X = e^{-\gamma l} = e^{-\alpha l}e^{-j\beta l}$.**

   Ans: The expression of $X = e^{-\gamma l} = e^{-\alpha l}e^{-j\beta l}$ can be broken down into two parts, i.e., $e^{-\alpha l}$ and $e^{-j\beta l}$, where both α and β are real. The first term of $e^{-\alpha l}$ represents an *exponential decay* of signal magnitude over the length l of the transmission line, indicating that α determines how much the signal attenuates per unit length. The second term of $e^{-j\beta l}$ is indeed a complex value with unit magnitude and an argument of $-\beta l$ radians. This translates to a phase shift in time-domain. Thus, β represents how fast the phase changes along the line.

2. **What should the return loss ($|S_{11}|$ in dB), insertion loss ($|S_{21}|$ in dB), and $\angle S_{21}$ of a lossless transmission line with no dispersion look like? (Hint: Think about the physical meaning of these parameters and what they ideally would be for a good transmission line.)**

   Ans: For a matched transmission line, the return loss is ideally $-\infty$ dB ($|S_{11}| = 0$), meaning no signal is reflected. From the expression $X = e^{-\gamma l} = \frac{(S_{11}+S_{21})-\Gamma}{1-(S_{11}+S_{21})\Gamma} = S_{21}$ when $S_{11} = 0$ and $\Gamma = 0$, we obtain $X = S_{21}$. The lossless transmission line yields no insertion loss, i.e., $|S_{21}| = 0$ dB (i.e., full transmission without attenuation). Dispersion is the phenomenon where the phase velocity v_p of the propagating wave depends on frequency. This means that the line with no dispersion possesses a constant v_p. By considering $v_p = \omega/\beta$, the phase constant β of the dispersionless transmission line will increase linearly with frequency. Recall $\angle S_{21} = -\beta l$, the phase of S_{21} in this case will decrease linearly with frequency.

3. **If we plot the measured S_{11} and S_{21} of a lossless 50Ω line on the Smith chart, what should they look like? (Hint: Typically, the S_{11} and S_{21} will be frequency-dependent, and their loci on the Smith chart will change with frequency. What is the Smith chart plotting?)**

   Ans: The Smith chart plots the real and imaginary parts of S_{11} and S_{21} on its x- and y-axes. For the lossless 50Ω-matched line, $|S_{11}| = 0$ and $|S_{21}| = 1$. This means S_{11} and S_{21} should be at the center (0, 0) of the Smith Chart and somewhere on a circle of unit radius centered at the origin, respectively. Since the phase of S_{21} varies with frequency, its locus will move along the circumference the circle.

4. **Derive for Z_c, v_p, R, L, and C using (3), (7), (8), and (9).**

   Ans: From (3), we obtain $Z_c = Z_0 \frac{1+\Gamma}{1-\Gamma}$ and then we can then find $R = 2Z_c\alpha$ from (8). Using (9) will directly yield $v_p = \omega/\beta$. By considering (7), (8), and (9) simultaneously, we can obtain $L = \frac{R}{2\alpha v_p}$ and $C = \frac{2\alpha}{R v_p}$.

5. **Find the expression of α and β in terms of S_{21} and l. (Hint: What are the magnitude and phase of S_{21}? What are their units?)**

   Ans: By assuming the line to be perfectly matched, i.e., $S_{11} = \Gamma = 0$, we get $X = e^{-\gamma l} = e^{-\alpha l}e^{-j\beta l} = S_{21}$. Therefore, $\alpha = -\frac{\ln|S_{21}|}{l}$ and $\beta = -\frac{\angle S_{21}}{l}$.

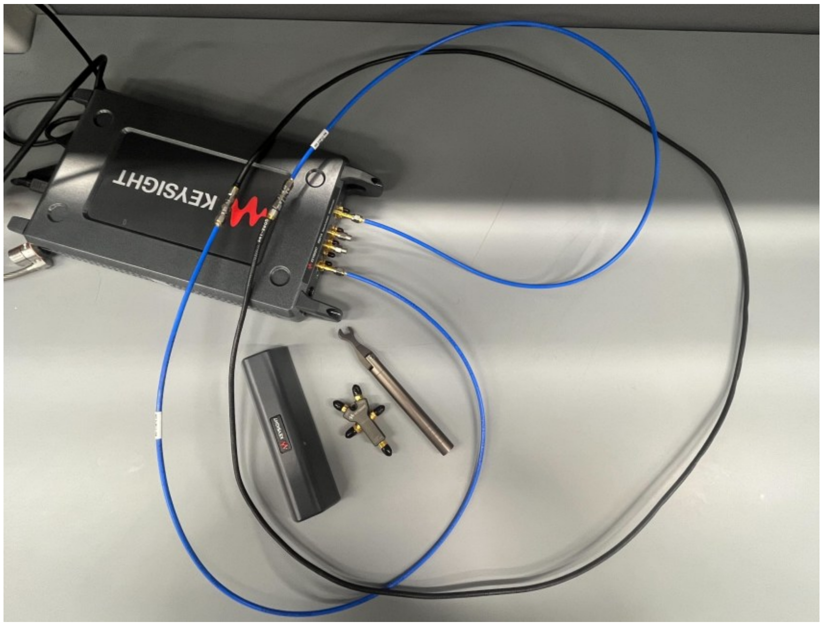

Figure 2: Measurement setup

Equipment:

1. P937XA 2-port Streamline USB vector network analyzer
2. Hand-formable coaxial cables
3. Calibration kit
4. Torque wrench
5. **72-inch** coaxial cable under test with male-male threaded adapters

Procedure (in lab):

1. Connect the VNA to the PC and open the VNA software.
2. The TA or professor will help to connect a pair of coaxial cables to the VNA.
3. Set the frequency range from 300 kHz to 1 GHz with 1001 frequency steps and IF bandwidth of 1 kHz.
4. Add S12, S21, and S22 traces to the window.
5. Add another window displaying S11, S12, S21, and S22. Then, use Format hardkey to change the display from LogMag to Phase.
6. Perform SOLT calibration using the calibration kit.
7. Connect a 72-inch coaxial cable under test with male-male threaded adapters to the measurement setup (see Fig.2) and perform a measurement. Do not kink cables.
8. Using equations in the background section to extract important parameters.

Results and discussions

Figures 3-5 display the measured $|S_{11}|$, $|S_{21}|$ and $\angle S_{21}$, respectively. By applying equations (5) and (6), we can determine the reflection Γ and the transmission X, whose magnitudes and phases are shown in Figs. 6-9, respectively. Using (7), we can directly calculate the attenuation α and the phase constant β from X, as shown in Figs. 10 and 12, respectively. The comparisons of these two parameters with those approximated from the magnitude and phase of S_{21} under low reflection assumptions are presented Figs. 11 and 13, respectively. These results are in good agreement within the low Γ ranges (and correspondingly low S_{11}). By utilizing the derived expressions for characteristic impedance Z_c, phase velocity v_p, resistance R, inductance L, and capacitance C based on the RLGC model, as outlined in equations (3), (7), (8), and (9), we can extract key transmission line parameters from our simple measurements. Note that the effect of losses in the dielectric material is neglected, and therefore, conductance G is omitted.

The extracted Z_c, shown in Fig. 14, fluctuates around 50 Ω, with spikes occurring at frequencies where Γ is high. These spikes directly lead to spurious results in the extracted R, L and C, as illustrated in Figs. 16-18. Ignoring these anomalies, we observe that R increases with frequency, reflecting the metallic loss due to the skin effect. In contrast, L and C, which depend on the dielectric material's properties and the coaxial line's dimensions, remain relatively constant, as expected.

It is important to note that the observed irregularities, such as spikes, are likely to arise from poor connections or resonances caused by damage to the transmission line under test. This emphasizes the critical importance of the measurement setup's quality in ensuring the accuracy of transmission line parameter extraction. Nevertheless, we obtain rather smooth-profiled v_p, plotted in Fig. 15, which correlates to an almost linear increasing β. In addition, the presence of dispersion is evident following the frequency dependence of v_p.

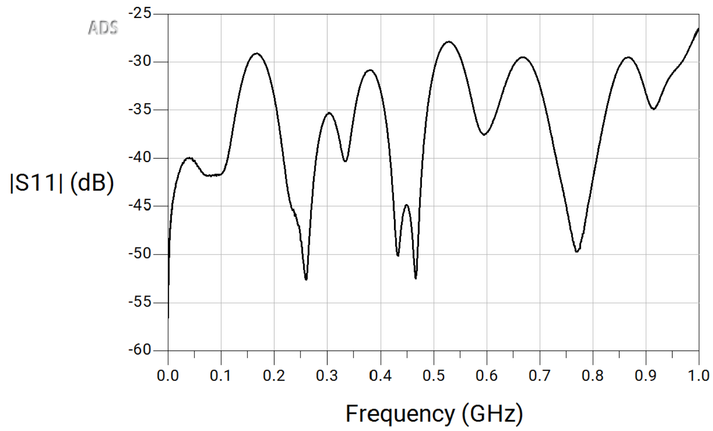

Figure 3: |S11| (dB) vs. frequency (GHz)

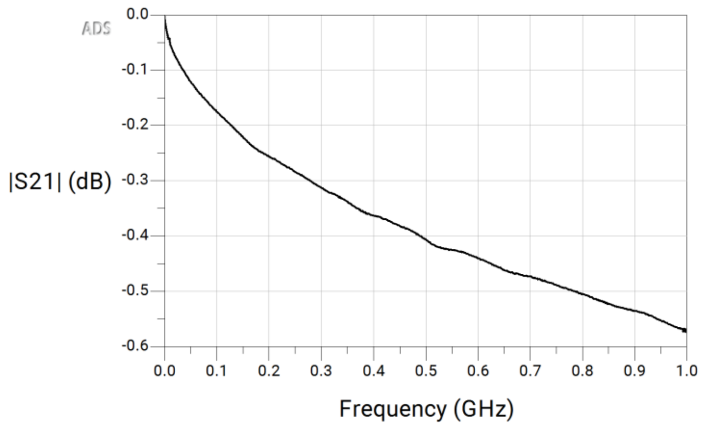

Figure 4: |S21| (dB) vs. frequency (GHz)

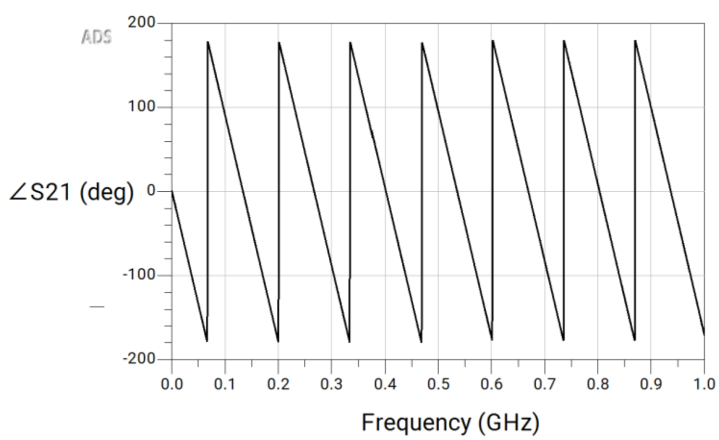

Figure 5: ∠S21 (degree) vs. frequency (GHz)

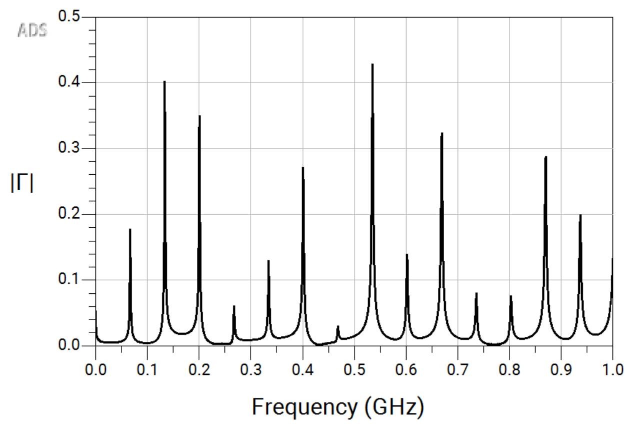

Figure 6: $|\Gamma|$ (linear scale) vs. frequency (GHz)

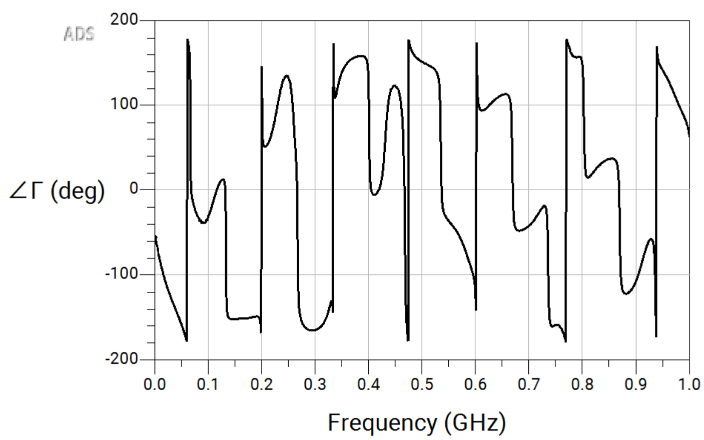

Figure 7: $\angle\Gamma$ (degree) vs. frequency (GHz)

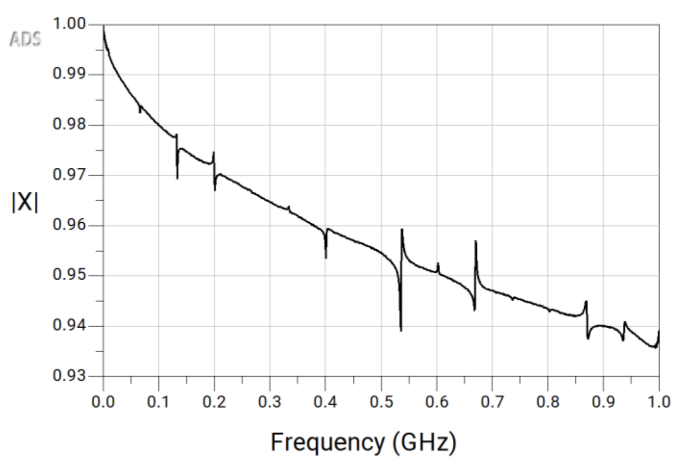

Figure 8: $|X|$ (linear scale) vs. frequency (GHz)

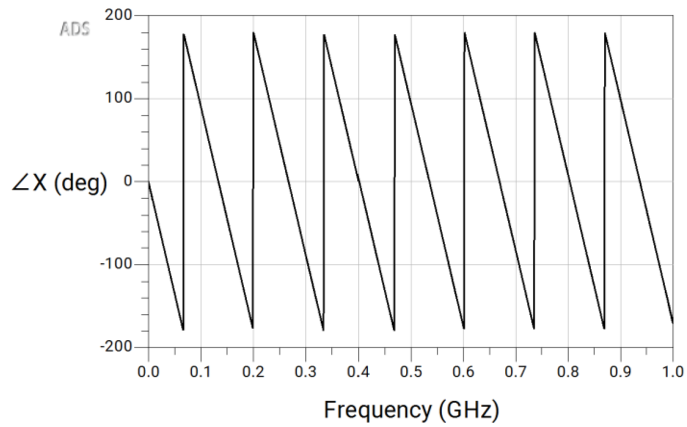

Figure 9: $\angle X$ (linear scale) vs. frequency (GHz)

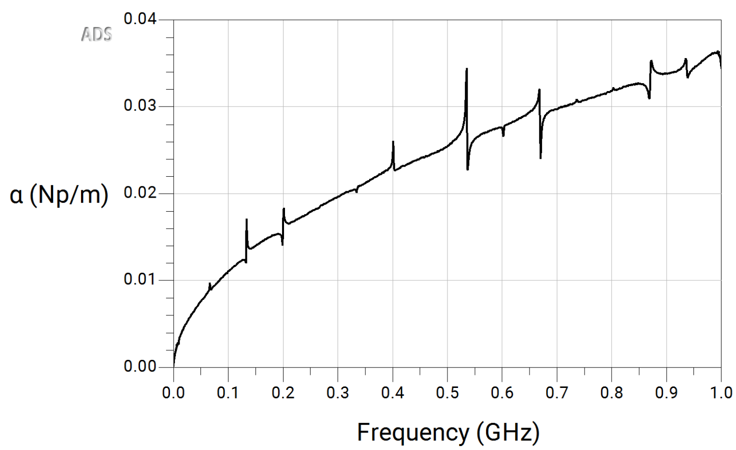

Figure 10: α (Np/m) vs. frequency (GHz)

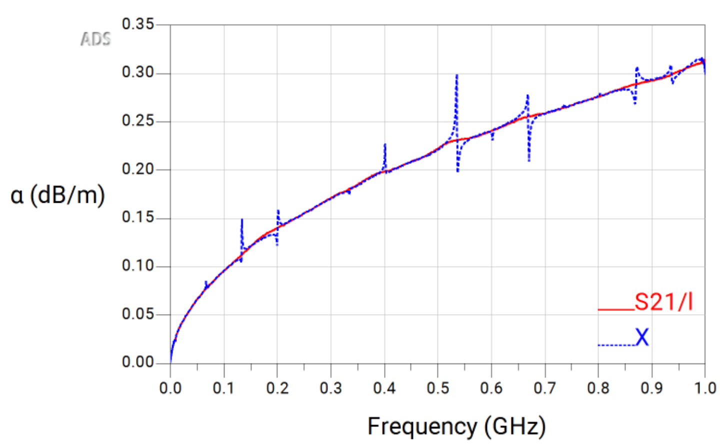

Figure 11: α (dB/m) and $|S21|/l$ (dB/m) vs. frequency (GHz)

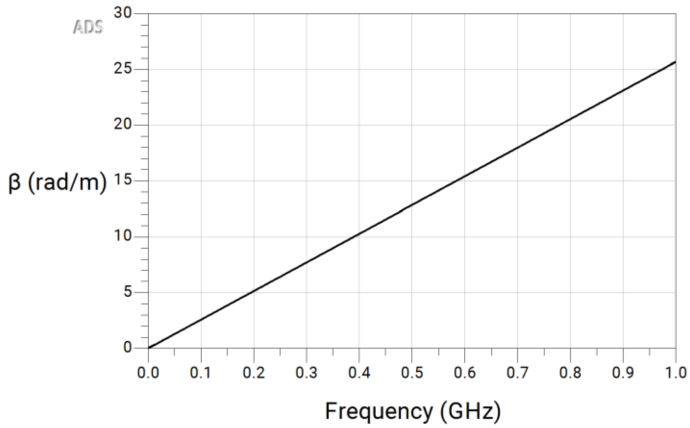

Figure 12: β (rad/m) vs. frequency (GHz)

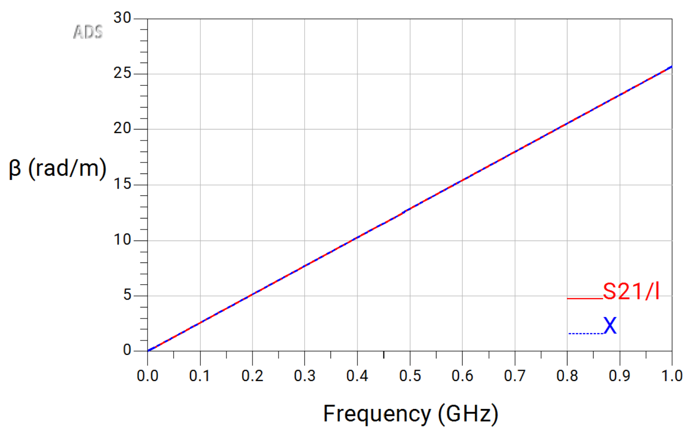

Figure 13: β (rad/m) and $\angle S21/l$ (rad/m) vs. frequency (GHz)

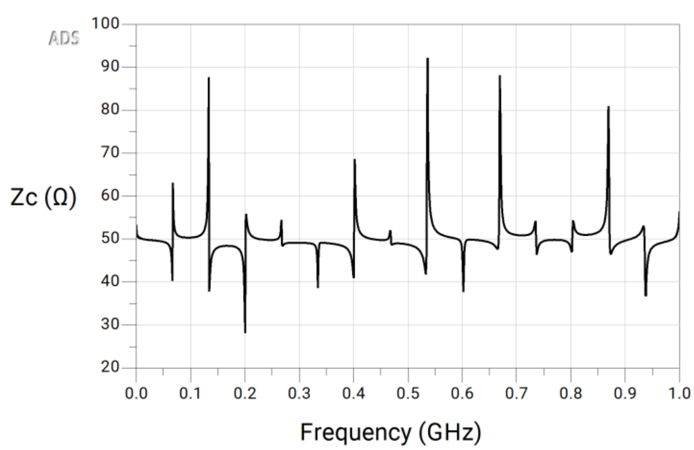

Figure 14: Z_c (Ω) vs. frequency (GHz)

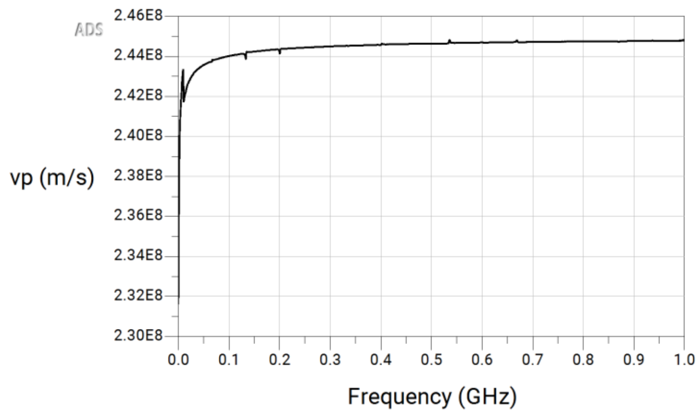

Figure 15: v_p (m/s) vs. frequency (GHz)

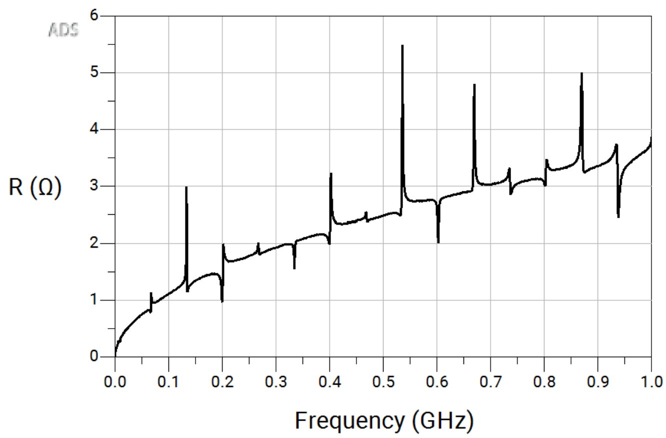

Figure 16: R (Ω) vs. frequency (GHz)

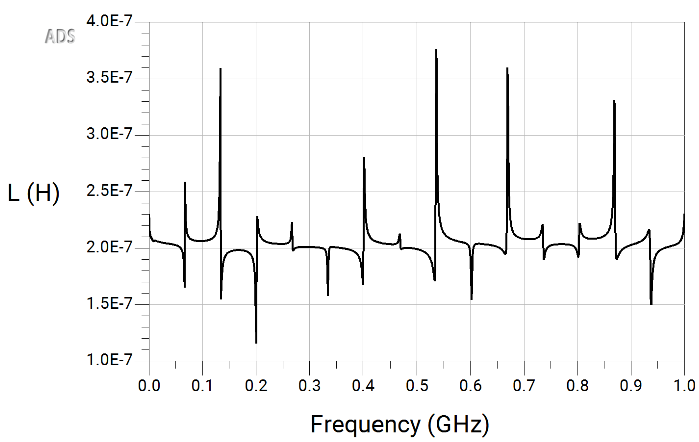

Figure 17: L (H) vs. frequency (GHz)

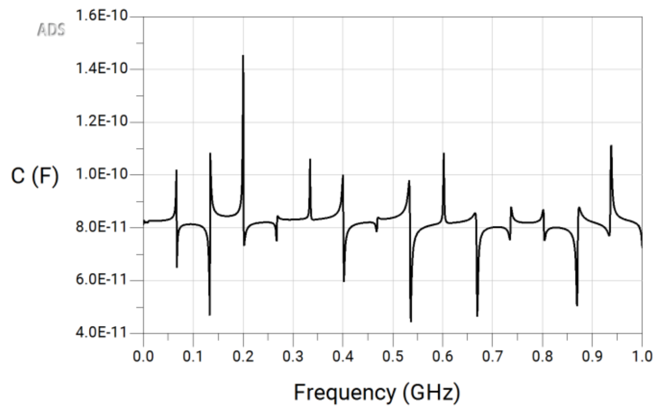

Figure 18: C (F) vs. frequency (GHz)

Post-lab questions

1. **Carefully analyze the background for transmission line parameter extraction. Under what conditions are (8) and (9) accurate? Try to relate the assumptions made in the background section to the physical meaning of ω, R, and G. You may also try to using such assumptions to derive for the expressions of (8) and (9).**

Ans: For a coaxial cable having an inner and outer conductor radii a and b, respectively, we have the conductance G in the RLGC model is given by $G = \frac{2\pi\omega\varepsilon''}{\ln(b/a)} = \frac{2\pi\omega\varepsilon_0\varepsilon_r \tan\delta}{\ln(b/a)}$. According to this expression, G is negligible if the dielectric material between the inner and outer conductors of the coaxial cable must have low loss. Typically, coaxial cables use Teflon (PTFE), which has a low loss tangent ranging from 0.0002 to 0.004. For the resistance of the metallic conductors, denoted as $R = \frac{1}{2\pi\sigma\delta_s}\left(\frac{1}{a} + \frac{1}{b}\right)$, where the skin depth is $\delta_s = \sqrt{\frac{2}{\omega\sigma\mu_0\mu_r}}$. Since δ_s is frequency-dependent, R increases with frequency. For this reason, the assumption of small $R/\omega L$ will valid only at low frequency to moderate frequency. In other words, (8) and (9) remain accurate as long as the reactance ωL dominates over the resistance R, which means the transmission line is primarily reactive rather than dissipative.

Mathematical proof for α and β under the aforementioned assumptions:

Consider

$$\gamma = \sqrt{\ R + j\omega L)\ \ G + j\omega C)} \approx \sqrt{\ R + j\omega L)\ \ j\omega C)} \qquad \text{(neglect small dielectric losses)}$$

$$= \sqrt{j\omega RC - \omega^2 LC}\ .$$

Using the formula $\bar{z} = \sqrt{a + jb} = \pm\left(\sqrt{\frac{|z|+a}{2}} + j\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}\right)$ yields

$$\gamma = \pm\left(\sqrt{\frac{\sqrt{\ \omega RC)^2 + (\omega^2 LC)^2} - \omega^2 LC}{2}} + j\sqrt{\frac{\sqrt{\ \omega RC)^2 + (\omega^2 LC)^2} + \omega^2 LC}{2}}\right).$$

Consider only the positive term

$$\gamma = \sqrt{\frac{\sqrt{\ \omega RC)^2 + (\omega^2 LC)^2} - \omega^2 LC}{2}} + j\sqrt{\frac{\sqrt{\ \omega RC)^2 + (\omega^2 LC)^2} + \omega^2 LC}{2}} = \alpha + j\beta.$$

Thus,

$$\alpha = \sqrt{\frac{\sqrt{\ \omega RC)^2 + (\omega^2 LC)^2} - \omega^2 LC}{2}} = \sqrt{\frac{\omega^2 LC\left(\sqrt{1 + \ R/\omega L)^2} - 1\right)}{2}}$$

$$\approx \sqrt{\frac{\omega^2 LC\left(1 + \frac{1}{2}\ R/\omega L)^2 - 1\right)}{2}} \text{ (using Taylor series approximation with } R/\omega L \ll 1)$$

$$= \sqrt{\frac{\omega^2 LC\left(1 + \frac{1}{2}\ \ R/\omega L)^2 - 1\right)}{2}} = \sqrt{\frac{\omega^2 LC\ \ R/\omega L)^2}{4}} = \sqrt{\frac{R^2 C}{4L}} = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

Similarly,

$$\beta = \sqrt{\frac{\sqrt{\ \omega RC)^2 + (\omega^2 LC)^2} + \omega^2 LC}{2}} = \sqrt{\frac{\omega^2 LC\left(\sqrt{1 + \ R/\omega L)^2} + 1\right)}{2}}$$

$$\approx \sqrt{\frac{\omega^2 LC\ \ 1 + 1)}{2}} = \omega\sqrt{LC} \qquad (R/\omega L \ll 1).$$

The phase constant, which determines how the phase of the wave changes over time, can be related to the phase velocity v_p of the wave through $\beta = \omega/v_p$. Thus, in a coaxial transmission line, the phase velocity is given by $v_p = \sqrt{LC}$.

2. **Derive $\Gamma \approx \frac{S_{11}}{(S_{11}^2 - S_{21}^2)+1}$ from (3) and (4). Explicitly state what assumptions you made in your derivation? (Hint: For the + case, solve for Q first then make an approximation based on the magnitude of Γ. For the - case, use the first two terms of the Taylor Series expansion of the square root term. What assumption about Q is this truncated expansion making?)**

Ans: First, consider + term ($\Gamma = Q + \sqrt{Q^2 - 1}$): $\Gamma - Q = \sqrt{Q^2 - 1}$

Squaring both side yields $\Gamma^2 - 2\Gamma Q + Q^2 = Q^2 - 1$, which simplifies to $\Gamma^2 - 2\Gamma Q = -1$. If there is very small reflection from the transmission line, i.e. Γ is very small and $\Gamma^2 \approx 0$, we get $-2\Gamma Q = -1$ or $\Gamma = {}^1\!/_{2Q}$. Thus, $\Gamma \approx \frac{S_{11}}{(S_{11}^2 - S_{21}^2)+1}$.

Next, consider – term ($\Gamma = Q - \sqrt{Q^2 - 1}$):

Since Q is large as S_{11} is small (low reflection), we get $Q\sqrt{1 - 1/Q^2} \approx Q(1 - \frac{1}{2Q^2})$.

Therefore, $\Gamma \approx Q - Q\left(1 - \frac{1}{2Q^2}\right) = {}^1\!/_{2Q} = \frac{S_{11}}{(S_{11}^2 - S_{21}^2)+1}$.

It is clear that $\Gamma \approx \frac{S_{11}}{(S_{11}^2 - S_{21}^2)+1}$ holds when *the reflection is small.*

3. **What is the phenomenon of dispersion? How does it affect phase and group velocities? What would you conclude about the significance of dispersion from your measurement results? (Hint: Consider v_p and the propagation of EM waves in media)**

Ans: Dispersion is the phenomenon where the phase velocity v_p of the propagating wave depends on frequency. Since different frequency components of a signal travel at different speeds in dispersive media, dispersion spreads out a pulse over time, causing distortion in broadband signal. From the results, we can see that v_p is frequency dependent, especially at frequencies below 200 MHz, indicating a dispersive cable.

Following $\beta = \omega / v_p$, the phase constant in the dispersive transmission line varies nonlinearly with frequency depends on the behavior of the medium. However, our results yield an almost linear increasing β, therefore the dispersion is insignificant in this experiment.

4. **What is skin effect? How would it affect your interconnects in your circuit? Do you observe it from your measurement in this Experiment? (Hint: Recall HW3)**

Ans: The skin effect is a phenomenon in which alternating current (AC) tends to concentrate near the surface of a conductor as frequency increases. This occurs because eddy currents induced by the AC create opposing magnetic fields that force the current to flow in a thinner outer layer of the conductor. The depth at which the current density significantly decreases is called the skin depth $\delta_s = \sqrt{\frac{2}{\omega\sigma\mu_0\mu_r}}$. Since the effective cross-sectional area of current flow decreases with frequency, the conductor resistance increases, leading to *higher insertion loss*. From the experiment, we can see that the attenuation increases with frequency, reflecting the influence of the skin effect. In addition, according to $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$ and $Z_c = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$, when $R/\omega L$ is no longer negligible (i.e., when frequency is very high), the propagation constant γ and the characteristic impedance Z_c become significantly dependent on R, which is frequency-dependent. which itself varies with frequency. This leads to signal distortion, affecting both attenuation and phase velocity, which may *degrade signal integrity* in high-speed or RF circuits. However, we do not observe any significant impact of the skin effect on dispersion behavior in this experiment. This suggests that while the skin effect increases loss, it does not significantly alter the phase velocity in the frequency range studied.

Conclusions

In this experiment, we thoroughly examine the extraction of important transmission line parameters from the measurable S-parameters under the low-loss and low-reflection assumptions. Although the achieved results contain several spurious values potentially due to poor connections or defects in the transmission line under test itself, we are able to obtain reasonable results that align well with theoretical predictions. Moreover, our findings suggest that the quality of RF measurement setup strongly influences the accuracy of the study.

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