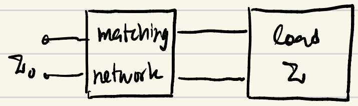
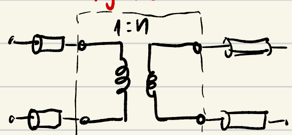
Impedance Matching (Pozar ch5)



Example:

- Max power delivery
- Better SNR
- Antenna array, reduce amplitude and phase error in distributed networks

• Matching network examples



$V_2 = nV_1$ | $Z_{in} = \frac{V_1}{I_1}$

$I_1 = nI_2$ | $= \frac{V_2/n}{nI_2}$

$\therefore \boxed{Z_{01} = \frac{Z_{02}}{n^2}}$

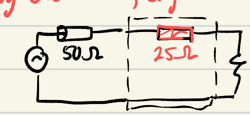
↳ real-to-real transfer

↳ upto 100 MHz (lossy)

▸ Usually matching network uses lossless elements (L, C or transmission line)

↳ avoid energy dissipation ⇒ tend to have narrower bandwidth.

lossy elements, e.g.



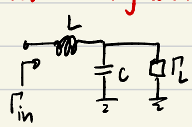
} infinite bandwidth, but loses half of power.

L.C. element matching (Pozar 5.1)

- Advantage: less space, better bandwidth (compared to TL)
- Disadvantage: primarily used < 30 GHz

Consider two cases: using lumped L-section matching network

(1)

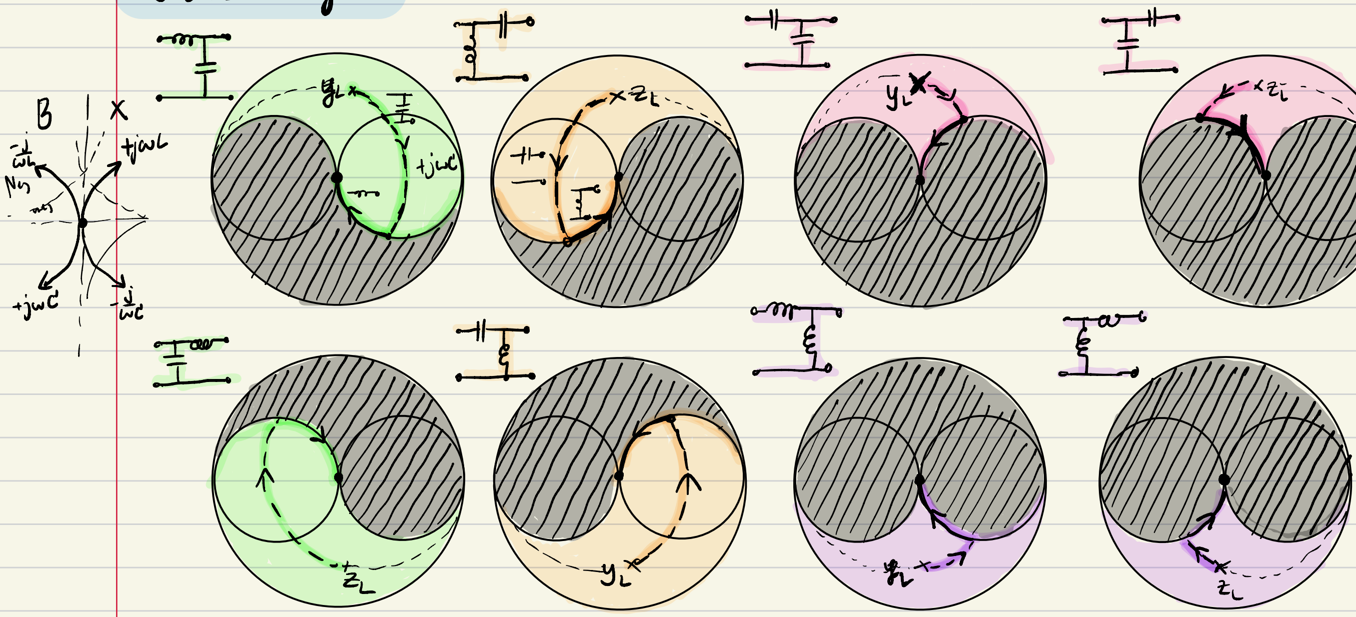


Goal: to make $\Gamma_{in} = 0$ by choosing proper L & C.

Analysis: consider series L and C can change reactance of Z_L for a matched network.

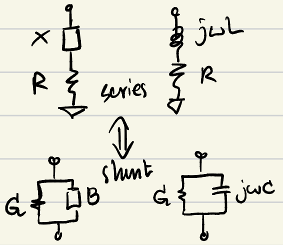
Strategy: use shunt C to move Γ_L along constant conductance circle to lower half of r=1 circle. Then choose L to tune-out reside reactance.

Forbidden regions



LC matching $\Rightarrow$ lossless, easy, compact but sacrifices BW

Series & Parallel circuit



series:

$$Q_s = \frac{\omega L}{R} = \frac{|X|}{R} = \frac{\text{energy stored}}{\text{energy dissipated}}$$
(per cycle)

shunt:

$$Q_p = \frac{\omega C}{G} = \frac{|B|}{G}$$

$\rightarrow$ Know X and R, find equivalent G_p and B_p

$$Z = R_s + jX_s = \frac{1}{Y_p} = \frac{1}{G_p + jB_p} \quad \& \quad Y_p = G_p + jB_p = \frac{1}{R+jX} = \frac{R-jX}{R^2+X^2}$$

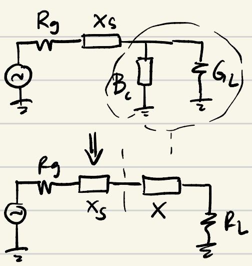
$$\hookrightarrow G_p = \frac{R}{R^2+X^2} = \frac{1}{R}\frac{1}{1+\frac{X^2}{R^2}} = \frac{1}{R}\frac{1}{1+Q_s^2} \quad \& \quad B_p = -\frac{X}{R^2+X^2} = -\frac{1}{X}\frac{1}{1+Q_s^{-2}}$$

$\rightarrow$ Know G and B, find equivalent R_s and X_s

$$Y = G + jB = \frac{1}{Z_s} = \frac{1}{R_s + jX_s} \Rightarrow R_s + jX_s = \frac{1}{G+jB} = \frac{G-jB}{G^2+B^2}$$

$$\therefore \; R_s = \frac{G}{G^2+B^2} = \frac{1}{G}\cdot\frac{1}{1+\left(\frac{B^2}{G^2}\right)} = \frac{1}{G}\frac{1}{1+Q_p^2} \quad \& \quad X_s = -\frac{B}{G^2+B^2} = -\frac{1}{B}\left(\frac{1}{1+Q_p^{-2}}\right)$$

Impedance transformation



$$G_L = 1/R_L$$
$$Q_p = \frac{B_c}{G_L} = B_c \cdot R_L$$

$$R = \frac{R_L}{1+Q_p^2}, \quad X = -\frac{Q_p^2}{B_c(1+Q_p^2)}$$

We want to perform a conjugate matching:

$$R = R_g \text{ and } X = -X_s$$

$$\Rightarrow 1 + Q_p^2 = \frac{R_L}{R_g} \Rightarrow Q_p = \sqrt{\frac{R_L}{R_g} - 1}$$

Solution: $B_c = \frac{Q_p}{R_L} = \frac{1}{R_L}\sqrt{\frac{R_L}{R_g} - 1}$

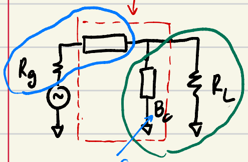
$$X_s = R_g\sqrt{\frac{R_L}{R_g} - 1}$$

Note: under the matching condition: $Q_s = Q_p$

$$Q_p = B_c R_L = \frac{B_c}{G_L} = \sqrt{\frac{R_L}{R_g} - 1} = Q_s = \frac{X_s}{R}$$

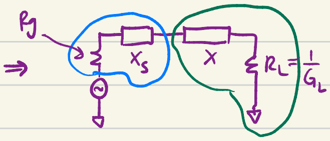
Feb 26, 2025

L-section matching



$B_c = \frac{Q_p}{R_L}$

$\Rightarrow$



$$R = \frac{R_L}{1+Q_p^2}, \quad X = -\frac{Q_p}{B_c(1+Q_p^2)}$$

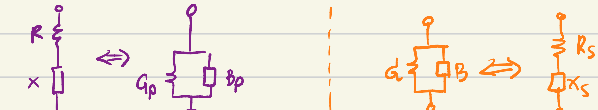
To conjugate match:

$$R_g = \frac{R_L}{1+Q_p^2} \Rightarrow 1+Q_p^2 = \frac{R_L}{R_g} \Rightarrow Q_p = \sqrt{\frac{R_L}{R_g}-1}$$

$$X_s = -X \qquad \left. \begin{aligned} B_c &= \frac{Q_p}{R_L} = \frac{1}{R_L}\sqrt{\frac{R_L}{R_g}-1} \\ X_s &= R_s\sqrt{R_L/R_g - 1} \end{aligned} \right\} R_L = R_g$$

$$Q_s = Q_p : \quad Q_p = B_c R_L = \sqrt{\frac{R_L}{R_g}-1} = \frac{X_s}{R_s} = Q_s$$

• **Remark**: the parallel-series transformation



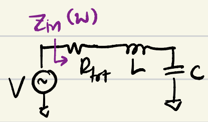
$$Q_s = \frac{|X|}{R} \qquad G_p = \frac{1}{R}\left(\frac{1}{1+Q_s^2}\right) \qquad B_p = -\frac{1}{X}\left(\frac{Q_s}{1+Q_s^2}\right)$$

$$Q_p = \frac{|B|}{G} \qquad R_s = \frac{1}{G}\left(\frac{1}{1+Q_p^2}\right) \qquad X_s = -\frac{1}{B}\left(\frac{Q_p^2}{1+Q_p^2}\right)$$

Q and bandwidth

"effective" Q of circuit is given by $Q = \omega\frac{U}{P_L}$ (U: overall energy storage; P_L: power loss)

series resonant circuit as example:



$$Z_{in}(\omega) = R_{tot} + j\omega L - j\frac{1}{\omega C}$$
$$= R_{tot} + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right)$$
$$= R_{tot} + j\omega L\left(\frac{\omega^2-\omega_0^2}{\omega^2}\right) \qquad \omega_0 = \frac{1}{\sqrt{LC}} \text{ resonant freq. of circuit.}$$

zoom in section

$$\omega \approx \omega_0$$
$$\omega^2 - \omega_0^2 = (\omega-\omega_0)(\omega+\omega_0) = \Delta\omega(2\omega+\Delta\omega) \simeq 2\omega(\Delta\omega)$$

$$\Big\} \; Z_{in}(\omega) = R_{tot} + j\cancel{\omega}L\left(\frac{2\cancel{\omega}\Delta\omega}{\cancel{\omega^2}}\right) = R_{tot} + j\cdot 2\Delta\omega L$$

At resonance: $P_0 = \frac{|V|^2}{R_{tot}}$ (RMS notation) $\Rightarrow$ At "half-power" point (3dB)

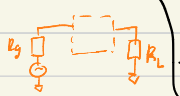
$$P_{\frac{1}{2}} = \frac{1}{2}P_0 = \frac{1}{2}\frac{|V|^2}{R_{tot}} = \frac{|V|^2}{|Z_{in,3dB}|^2}\cdot R_{tot}$$

$$|Z_{in,\frac{1}{2}}(\omega)|^2 = 2R_{tot}^2$$

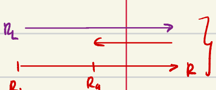
$$R_{tot}^2 + (2\Delta\omega L)^2 = 2R_{tot}^2$$

$$|\Delta\omega| = \frac{1}{2}\frac{R_{tot}}{L} = \frac{1}{2}\frac{\omega}{Q}$$

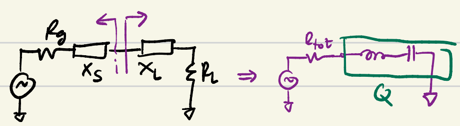
Thus, $\frac{2|\Delta\omega|}{\omega_0} = \frac{\Delta f_{FWHM}}{f_0} = \frac{1}{Q}$



What if we want narrower FBW?



match up and then down to get high contrast ratio.

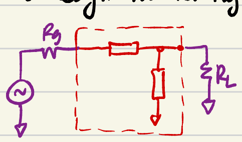


$$\boxed{FBW = \frac{2\Delta\omega}{\omega} = \frac{\Delta f_{FWHM}}{f_0} = \frac{1}{Q}}$$

$$Q = \frac{\omega L}{R_{tot}} = \frac{\omega L}{R_{tot}/2}\cdot\frac{1}{2} = \frac{Q_n}{2} = \frac{1}{2}\sqrt{\frac{R_L}{R_g}-1}$$

$$\Rightarrow \boxed{FBW = \frac{2}{\sqrt{\frac{R_L}{R_g}-1}}}$$

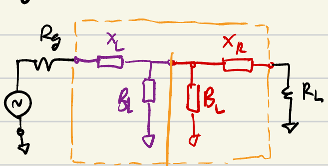
Comment: Given that we have only two independent parameters (R_L, R_g) to control Q (FBW) to design matching network with different Q, need to add more components.



$FBW = \frac{2}{\sqrt{\frac{R_L}{R_g}-1}}$

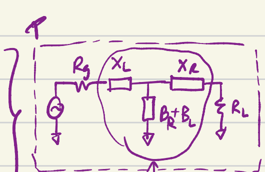
- Narrower band: eg. $10\Omega \xrightarrow{\text{narrow}} 100\Omega \xrightarrow{\text{narrow}} 50\Omega$
- Wider band: eg. $\underbrace{10\Omega \to 20\Omega}_{\text{wider}} \to 30\Omega \to 50\Omega$.

Design with different Q.



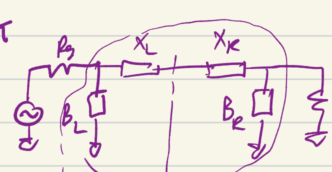
$\sqrt{\frac{R_{center}}{R_g}-1}$ $\quad R_{center}$ $\quad \sqrt{\frac{R_{center}}{R_L}-1}$

T



narrow band: $R_{center} > R_L, R_s$

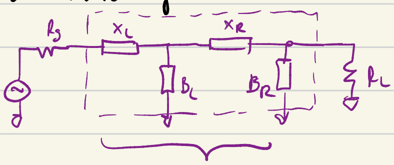
π



$R_{center} > R_L, R_g$

<narrow band>

Broader band matching



tapered line TL

Cascaded L-section
can broaden BW
of matching networks

Design matching network with a given Q ← $Q = \frac{\text{reactive}}{\text{resistive}}$

→ Strategy: plot a contour Q circle on the Smith Chart.

Recall $\underbrace{Q_n = \frac{|X|}{R}}_{\text{normalized}}$, $z = \frac{1+\Gamma}{1-\Gamma} = \frac{(1-u^2-v^2)+j2v}{(1-u)^2-v^2} = r + jx$ $\quad \Gamma = u + jv$

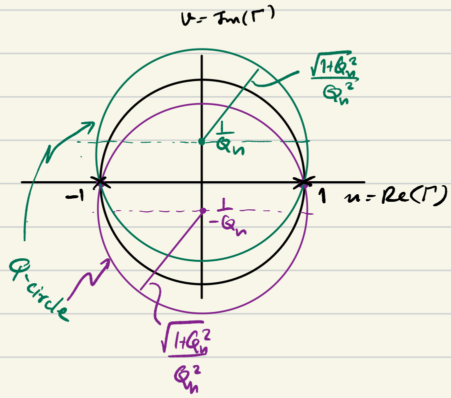
$\frac{x}{r} = \frac{2v}{1-u^2-v^2} = \pm Q_n$

$u^2+v^2-1 \pm \frac{2v}{Q_n} = 0$

$u^2 + \left(v \pm \frac{1}{Q_n}\right)^2 = \frac{1+Q_n^2}{Q_n^2}$

Center: $\left(0, \pm\frac{1}{Q_n}\right)$

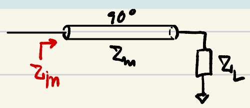
radius $= \frac{\sqrt{1+Q_n^2}}{Q_n}$



Mar 3

Quarter-wave transformer

Pozar 5.4

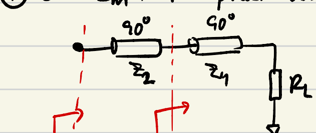


$Z_{in} = \frac{Z_m^2}{Z_L}$

if want $Z_{in} = Z_0$

$Z_m = \sqrt{Z_L Z_0}$

① If Z_m is not practical.

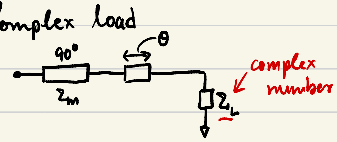


$\underbrace{Z_{in} = \frac{Z_2^2}{Z_{in1}} \qquad Z_{in1} = \frac{Z_1^2}{R_L}}$

$Z_{in} = \underbrace{\left(\frac{Z_2^2}{Z_1^2}\right)} R_L$

relative : can be any values that provide an appropriate ratio.

② Complex load



Use the first transmission line to translate Z_L to a real (resistive) impedance

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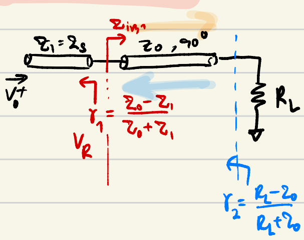
Partial reflection



$\gamma_1 = \frac{Z_0 - Z_1}{Z_0 + Z_1}$ $\quad \tau = 1 + \gamma$

$\tau_1 = \frac{2Z_0}{Z_0 + Z_1} \; (Z_0 \to Z_1)$, $\tau_2 = \frac{2Z_1}{Z_0 + Z_1} \; (Z_1 \leftarrow Z_0)$

$\gamma_2 = \frac{R_L - Z_0}{R_L + Z_0}$

$$V_R = V_0^+\gamma_1 + V_0^+\tau_1(-j)\gamma_2(-j)\tau_2 + V_0^+\tau_1(-j)\gamma_2(-j)(-\gamma_1)(-j)\gamma_2(-j)\tau_2 + \ldots$$

$$\Gamma_{total} = \frac{V_R}{V_0^+} = \gamma_1 - \tau_1\tau_2\gamma_2 - \tau_1\tau_2\gamma_1\gamma_2^2 - \tau_1\tau_2\gamma_1^2\gamma_2^3 - \ldots$$

$$= \gamma_1 - \tau_1\tau_2\gamma_2\left(1 + \gamma_1\gamma_2 + \gamma_1^2\gamma_2^2 + \ldots\right)$$

$$= \gamma_1 - \tau_1\tau_2\gamma_2 \sum_{n=0}^{\infty} (\gamma_1\gamma_2)^n$$

$$= \gamma_1 - \frac{\tau_1\tau_2\gamma_2}{1-\gamma_1\gamma_2}$$

$$\Gamma_{tot}(1-\gamma_1\gamma_2) = \gamma_1 - \gamma_1^2\gamma_2 - \tau_1\tau_2\gamma_2$$

$$= \gamma_1 - \gamma_2(\gamma_1^2 + \tau_1\tau_2)$$

$$\gamma_1 = \frac{Z_0 - Z_1}{Z_0 + Z_1}, \quad \tau_1 = \frac{2Z_0}{Z_0+Z_1}, \quad \tau_2 = \frac{2Z_1}{Z_0+Z_1}$$

$$\therefore \gamma_1^2 + \tau_1\tau_2 = \frac{(Z_0-Z_1)^2 + (2Z_0)(2Z_1)}{(Z_0+Z_1)^2} = 1$$

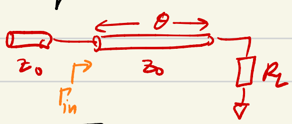
$$\Gamma_{tot}(1-\rho_1\gamma_2) = \gamma_1 - \gamma_2$$

$$= \frac{Z_0 - Z_s}{Z_0+Z_s} - \frac{R_L - Z_0}{R_L + Z_0}$$

$$= \frac{2(Z_0^2 - Z_s R_L)}{(Z_0+Z_s)(R_L+Z_0)}$$

We want $\Gamma_{tot} = 0 \Rightarrow Z_0^2 = Z_s R_L \Rightarrow Z_0 = \sqrt{Z_s R_L}$

BW of quarterwave transformer:



$Z_m = \sqrt{R_0 Z_0}$

Find Γ_{in}

$$Z_{in} = Z_m \frac{R_L + jZ_m \tan\theta}{Z_m + jR_L \tan\theta}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in}+Z_0} \Rightarrow \Gamma_{in}(\theta) = \frac{Z_m(R_L - Z_0) + j(\cancel{Z_m^2 - Z_0R_L})\tan\theta}{Z_m(R_L+Z_0) + j(Z_m^2 + Z_0R_L)\tan\theta}$$

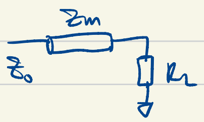
$\because Z_m^2 = Z_0 R_L$

$$\Gamma_{in}(\theta) = \frac{R_L - Z_0}{(R_L+Z_0) + j\sqrt{R_L Z_0}\tan\theta}$$

$$|\Gamma_{in}(\theta)| = \frac{|R_L - Z_0|}{\sqrt{(R_L+Z_0)^2 + 4Z_0R_L\tan^2\theta}}$$

for $\theta = \frac{\pi}{2}$ (quarter-wave transformer)

$\theta \to \frac{\pi}{2}$: $|\Gamma_{in}(\theta)| \sim \frac{|R_L - Z_0|}{2\sqrt{Z_0 R_L}} |\cot\theta|$



$\left|\frac{Z_0}{R_L}\right| \uparrow \cdots \frac{\Delta f}{f_0} \downarrow$

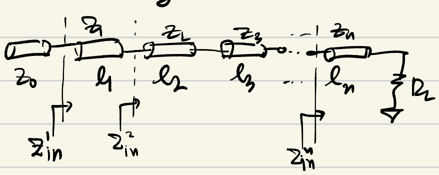
Disparity between R_L and Z_0 decides BW

↳ larger disparity → narrower BW

$\theta = \frac{\pi}{2}$ at design freq.

Varying freq. $\Rightarrow \beta l$ differs from $\frac{\pi}{2} \Rightarrow$ mismatch.

Bandwidth matching (Pozar 5.5)



Define Z_{in}^N : impedance looking into N^{th} section

$$Z_{in}^n = Z_n \frac{R_L + jZ_n\tan\theta}{Z_n + jR_L\tan\theta}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = -j\frac{e^{j\theta} - e^{-j\theta}}{e^{j\theta}+e^{-j\theta}}$

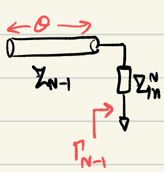
$$= Z_n \frac{(R_L+Z_n)e^{j\theta} + (R_L - Z_n)e^{-j\theta}}{(R_L+Z_n)e^{j\theta} - (R_L-Z_n)e^{-j\theta}}$$

Define mismatch factor

$\gamma_N = \frac{R_L - Z_0}{R_L+Z_0}$

(sectional reflection coefficient)

$$= Z_n \frac{e^{j\theta} + \gamma_N e^{-j\theta}}{e^{j\theta} - \gamma_N e^{j\theta}}$$



$$\Gamma_{N-1} = \frac{Z_{in}^n - Z_{N-1}}{Z_{in}^n + Z_{n-1}} = \frac{Z_n(e^{j\theta} + \gamma_N e^{-j\theta}) - Z_{n-1}(e^{j\theta} - \gamma_N e^{-j\theta})}{Z_n(e^{j\theta}+\gamma_N e^{-j\theta}) + Z_{n-1}(e^{j\theta} - \gamma_N e^{-j\theta})}$$

$$= \frac{(Z_n - Z_{n-1})e^{j\theta} + \gamma_N(Z_N + Z_{n-1})e^{-j\theta}}{(Z_n+Z_{n-1})e^{j\theta} + \gamma_N(Z_n - Z_{n-1})e^{-j\theta}}$$

$\Gamma_{N-1} = \frac{Z_n - Z_{n-1}}{Z_n + Z_{n-1}}$

$$= \frac{\gamma_{N-1} + \gamma_N e^{-j2\theta}}{1 + \gamma_N \gamma_{N-1} e^{-j2\theta}}$$

$\Gamma_N = \gamma_N$

Theory of small reflection: assume mismatch factor $\ll 1 \Rightarrow \Gamma_{N-1} \simeq \gamma_{N-1} + \gamma_N e^{-j2\theta}$

$\Gamma_{N-2} \simeq \gamma_{N-2} + \Gamma_{N-1}e^{-j2\theta} = \gamma_{N-2} + \gamma_{N-1}e^{-j2\theta} + \gamma_N e^{-j4\theta}$

$$\boxed{\Gamma = \gamma_0 + \gamma_1 e^{-j\theta} + \ldots + \gamma_N e^{-j2\theta N}}$$

↳ mismatch between adjacent transmission lines is small.

$$\boxed{\Gamma = \sum_{n=0}^{N} \gamma_N e^{-j2n\theta}}, \quad \gamma_N = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

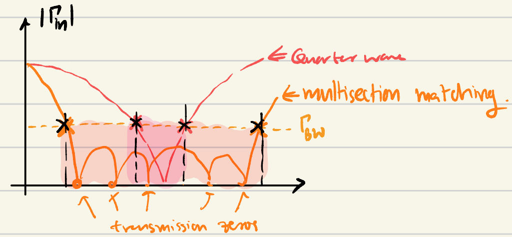
$\Rightarrow$ choose γ_N such that $|\Gamma|$ minimize over a wide frequency range (BW matching)

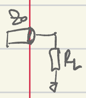
let $w = e^{-j2\theta}$

$$\Rightarrow \Gamma = \gamma_0 + \gamma_1 w + \gamma_2 w^2 + \ldots + \gamma_N w^N$$

$$\boxed{\Gamma = \gamma_N \prod_{n=1}^{N} (w - w_n)}, \quad w_n \Rightarrow n^{th} \text{ zero of polynomial.}$$

↳ N degrees of freedom

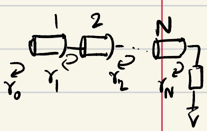




If $\theta = 0 \rightarrow \Gamma = \frac{R_L - Z_0}{R_L + Z_0} = \sum_{n=0}^{N} \gamma_n$

$(w = 1)$

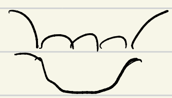
Cases :



① Uniform mismatch factor : $\gamma_N = \gamma_n$

② Maximally flat – Binomial transformer (Butterworth)

③ Chebychev (Max BW)



① Uniform mismatch factor : $\gamma_N = \gamma_n$

$$\Gamma = \gamma_N (1 + w + w^2 + \ldots + w^N)$$

$$\frac{\Gamma}{\gamma_N} = \frac{w^{N+1} - 1}{w - 1} = \frac{w^{\frac{N+1}{2}} \left(w^{\frac{N+1}{2}} - w^{-\frac{N+1}{2}}\right)}{w^{\frac{1}{2}} \left(w^{\frac{1}{2}} - w^{-\frac{1}{2}}\right)}, \quad w = e^{-j2\theta}$$

$$= w^{\frac{N}{2}} \frac{\sin[(N+1)\theta]}{\sin\theta} = e^{-jN\theta} \frac{\sin((N+1)\theta)}{\sin\theta}$$

Boundary condition : $\theta = 0 \rightarrow \Gamma(\theta = 0) = \frac{R_L - Z_0}{R_L + Z_0} = \sum_{n=0}^{N} \gamma_N = (N+1)\gamma_N$

$$\gamma_N = \frac{1}{N+1} \frac{R_L - Z_0}{R_L + Z_0}$$

$$\Gamma = \gamma_N e^{-jN\theta} \frac{\sin[(n+1)\theta]}{\sin\theta} = \frac{1}{N+1} \cdot \frac{R_L - Z_0}{R_L + Z_0} \cdot e^{-jN\theta} \frac{\sin[(N+1)\theta]}{\sin\theta}$$

$$|\Gamma| = \left|\frac{R_L - Z_0}{R_L + Z_0}\right| \left|\frac{\sin[(N+1)\theta]}{(N+1)\sin\theta}\right|$$

Remark

① $|\Gamma|$ pattern repeats every π

② For N-sections hence N-nulls and N-1 ripples

③ zero at $\theta = \frac{n\pi}{N+1}$

② Binomial transformer $\longrightarrow$ maximally flat $\rightarrow$ chose all zeros at the center.

(to be cont.)

Self-consistent matching

$\Gamma = \sum_{n=0}^{N} \gamma_n e^{-j2n\theta}$, $\gamma_n = \frac{Z_{n+1}-Z_n}{Z_{n+1}+Z_n}$

Power series approx.

$\ln \frac{Z_{n+1}}{Z_n} \approx 2\frac{Z_{n+1}-Z_n}{Z_{n+1}+Z_n}$

$\therefore \boxed{\gamma_n = \frac{1}{2}\ln\frac{Z_{n+1}}{Z_n}}$

Mar 24

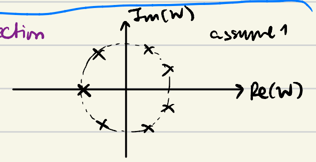
Max flat matching network $\Rightarrow$ Butterworth / Binomial

Recall: design approach for uniform reflection

$W^{N+1} - 1 = \prod_{n=0}^{N} (W - e^{j\frac{2n\pi}{N+1}})$

$W = e^{-j2\theta}$

$\Gamma = \gamma_N \frac{W^{N+1}}{W-1}$



For N=1, $W = e^{-j2\theta}$, varying θ moves the point around the circle, when the middle between zeros, we get ripples

$\rightarrow$ How to reduce the ripples?

$\hookrightarrow$ One effective way is to design multiple zeros at the same point

$\Rightarrow$ chang Z of each section

$\Rightarrow$ Binomial transformer $\Rightarrow$ chose all zeros $W_n = -1$

$\Gamma = \gamma_N (W+1)^N$ Expand with binomial theorem

$\Gamma = \gamma_N (W^N + NW^{N-1} + \dots + \frac{N!}{(N-n)!\,n!} W^{N-n} + \dots + 1)$

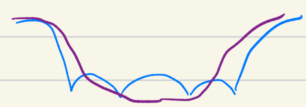
γ_n

Combination

$C_n^N = \binom{N}{n} = \frac{N!}{(N-n)!\,n!}$

$\Gamma(\theta) = \gamma_N (e^{-j2\theta} + 1)^N = \gamma_N (e^{-j\theta}(e^{-j\theta} + e^{+j\theta}))^N$

$= \gamma_N e^{-j2\theta}(2\cos\theta)^N$ $\left(\Gamma(0) = 2^N \gamma_N \frac{R_L - Z_0}{R_L + Z_0}\right)$



$|\Gamma(\theta)| = \frac{R_L - Z_0}{R_L + Z_0} |\cos\theta|^N$

less ripples

at the cost of achievable BW

$\Rightarrow$ Chebyshev transformer (Pozar 5.7) max BW at the cost of more ripples

---

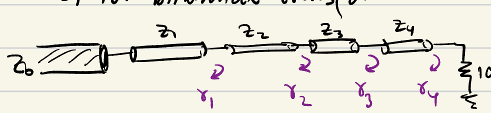
Design example: 5GHz Binomial transformer with 4 sections match from 100 Ω → 50 Ω

1) Evaluate:

$2^N \gamma_N = \frac{R_L - Z_0}{R_L + Z_0} = \frac{100-50}{100+50} = \frac{1}{3}$

$N=4 \rightarrow 2^4 \gamma_4 = \frac{1}{3} \Rightarrow \gamma_N = \frac{1}{3}\frac{1}{2^4} = \frac{1}{48}$

2) For binomial transformer



$\frac{\gamma_n}{\gamma_N} = \frac{N!}{(N-n)!\,n!}$

$\boxed{1}$ $\gamma_4 = \gamma_4 \frac{4!}{(4-4)!\,4!} = \frac{1}{48} = \frac{R_L - Z_4}{R_L + Z_4} \Rightarrow Z_4 = 95.9\ \Omega$

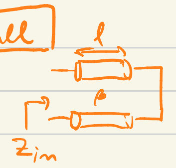
$\boxed{2}$ $\gamma_3 = \gamma_4 \frac{4!}{(4-3)!\,3!} = \frac{1}{48}\cdot 4 = \frac{1}{12} = \frac{Z_4 - Z_3}{Z_4 + Z_3} \Rightarrow Z_3 = 81.2\ \Omega$

$\boxed{3}$ $\gamma_2 = \gamma_4 \frac{4!}{(4-2)!\,2!} = \frac{1}{48}\cdot 6 = \frac{1}{8} = \frac{Z_3 - Z_2}{Z_3 + Z_2} \Rightarrow Z_2 = 63.2\ \Omega$

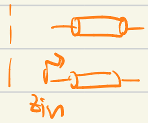
$\boxed{4}$ $\gamma_1 = \frac{1}{48} \times 4 = \frac{1}{12} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \Rightarrow Z_1 =$

Stub matching

$\boxed{\text{Recall}}$

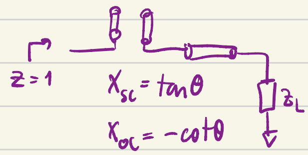


$Z_{in} = j\tan\beta l$



$Z_{in} = -j\cot\beta l$

- Series stub tuning



$X_{sc} = \tan\theta$

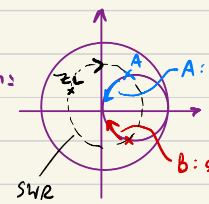
$X_{oc} = -\cot\theta$

①

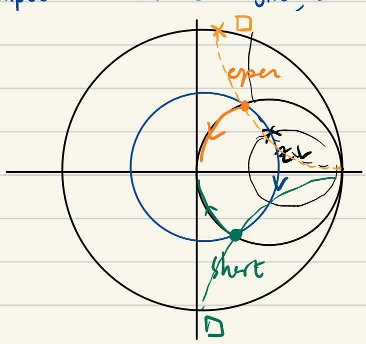
② Maximize

To maximize BW, for given termination, choose a shorter one

③ Multiple solutions



Example : match $z = 2+j1.6$, series open stub



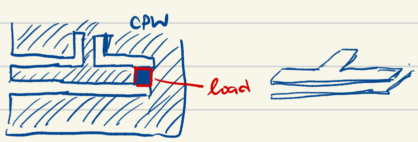
① Find z_L value on the chart

② Drawing SWR circle for z_L

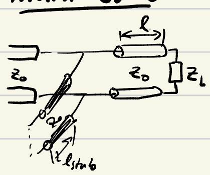
③ Find intercept values $z=1$

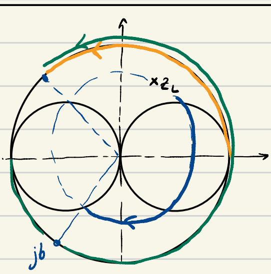
④ Two cases
- $1+j\square \rightarrow$ open stub (series)
- $1-j\Delta \rightarrow$ short stub (series)

Short stub is easier for higher freq

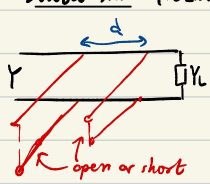


Shunt stub





Double stub (Pozar 5.3)

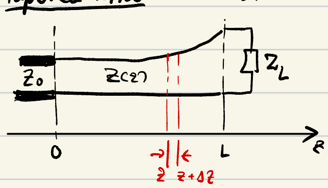


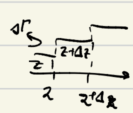
Advantage : tunable stub in coax

Disadvantage : not matching all impedance

Example in Pozar book

Tapered line (Pozar 5.8)





$\Delta\Gamma = \frac{Z+\Delta Z - Z}{Z+\Delta Z+Z}$

$\Delta\Gamma = \frac{\Delta Z}{2Z}$

Small reflection

$d\Gamma = \frac{dZ}{2Z} = \frac{1}{2}\frac{d(\ln(Z/Z_0))}{dz}dz$

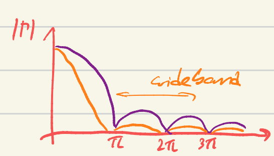
$\Gamma = \int_0^L e^{-2j\beta z}\frac{1}{2}\frac{d\ln(Z/Z_0)}{dz}dz$

⇒ Example : exponential taper

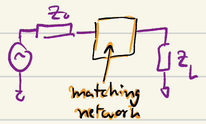
$Z(z) = Z_0 e^{az}$

$a = \frac{1}{L}\ln\left(\frac{Z_L}{Z_0}\right)$

$\Gamma = \frac{1}{2}\int_0^L e^{-2j\beta z}\frac{\ln(Z_L/Z_0)}{L}dz = \frac{\ln(Z_L/Z_0)}{2L}e^{-j\beta L}\frac{\sin\beta L}{\beta L}$



Bode-Fano Criterion (Pozar 5.9)



What is the theoretical limit of the matching network?

Review

- Smith chart : plot of Γ on the complex plane.
- Lumped element based impedance matching
  - → LC ladder impedance matching (forbidden region)
  - → $Q = \sqrt{\frac{R_L}{R_S} - 1}$, BW limitation
  - → π and T network for control of Q
- T-line based matching
  - → Quarter-wave transformer : $Z_{in} = \frac{Z_0^2}{Z_L}$
  - → Broadband matching
    - Theory of small reflection
    - Multi-section transformer
      - Uniformly distribution section
      - Binomial transformer (flat band) ⇒ Butterworth
      - Chebyshev transformer (skip)

} Real Load

- → Stub tuning
  - Single stub tunning : series / shunt, open / short
  - Choice of stub : favor shortest T-line
  - Double stub tuning (skip)

} Complex load

- → Other topics
  - tapered line
  - Theoretical limit

} super wideband real load
