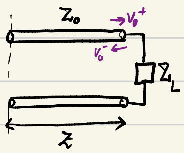
Feb 3

Power flow in lossless transmission line



$P = Re\{VI^*\}$ ← conjugate

$P = Re\{(V_0^+e^{-j\beta z} + V_0^-e^{j\beta z})(I_0^+e^{-j\beta z} + I_0^-e^{j\beta z})^*\}$

$= Re\{(V_0^+e^{-j\beta z} + V_0^-e^{+j\beta z})(\frac{V_0^+}{Z_0}e^{-j\beta z} - \frac{V_0^-}{Z_0}e^{+j\beta z})^*\}$

$= Re\{\frac{|V_0^+|^2}{Z_0} - \frac{|V_0^-|^2}{Z_0} + \frac{V_0^- V_0^{+*}}{Z_0}e^{+j2\beta z} - \frac{V_0^+ V_0^{-*}}{Z_0}e^{-j2\beta z}\}$ ← Z_0 is real for lossless lines

$= Re\{\frac{|V_0^+|^2}{Z_0}(1-|\Gamma|^2) + \frac{\Gamma|V_0^+|^2}{Z_0}(e^{+j2\beta z} - e^{-j2\beta z})\}$

$= Re\{\frac{|V_0^+|^2}{Z_0}(1-|\Gamma|^2 + \Gamma j\sin 2\beta z)\}$

Thus

$$P = |V_0^+|^2(1-|\Gamma|^2)\frac{1}{Z_0} = P^+ - P^-, \quad P^+ = \frac{|V_0^+|^2}{Z_0}, \quad P^- = \frac{|V_0^-|^2}{Z_0}$$

Lossy line (Pozar 2.7)

↳ Lossy : $\gamma = \sqrt{(j\omega L+R)(j\omega C+G)}$, $Z = \sqrt{\frac{j\omega L+R}{j\omega C+G}}$

$Z = \sqrt{\frac{L}{C}} \cdot \underbrace{\sqrt{\frac{1+R/j\omega L}{1+G/j\omega C}}}_{\text{complex}}$

Notes ① wave attenuates as it travels

② phase shift between V and I

$V(z) = V_0^+e^{-j\gamma z} + V_0^-e^{+j\gamma z}$

$I(z) = \frac{V_0^+}{Z_0}e^{-j\gamma z} - \frac{V_0^-}{Z_0}e^{+j\gamma z}$

Low-loss line : $\alpha = \frac{1}{2}\{R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\}$

$R \ll \omega L$, $G \ll \omega C$ $\quad \beta = \omega\sqrt{LC}$

The loss often described as quality factor : Q

$Q = \omega \frac{\text{average energy storage}}{\text{power loss}} = \omega \frac{+P/v_g}{-dP/dz}$ ← v_g : group velocity energy propagation velocity

$V(z) = V_0 e^{-\alpha z} \cdot e^{-j\beta z}$

$I(z) = I_0 e^{-\alpha z} e^{-j\beta z}$

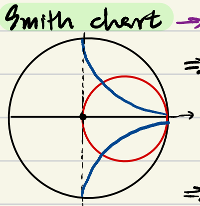
$P = \frac{|V_0|^2}{Z} e^{-2\alpha z} \rightsquigarrow \frac{\partial P}{\partial z} = -2\alpha P$

$Q = \omega\frac{\cancel{P}/v_g}{2\alpha\cancel{P}} = \frac{\omega}{2\alpha v_g}$

dispersionless case

$v_p = v_g = \frac{\omega}{\beta} \Rightarrow \boxed{Q = \frac{\beta}{2\alpha}}$

Smith chart → graphic aid for solving transmission equation



⇒ Polar plot of voltage reflection coefficient.

$\Gamma = |\Gamma|e^{j\theta}$ → For passive system $|\Gamma| \leq 1$
← complex

⇒ Normalized to impedance Z_0

↳ Load Z_L ⇒ $z_L = \frac{Z_L}{Z_0}$

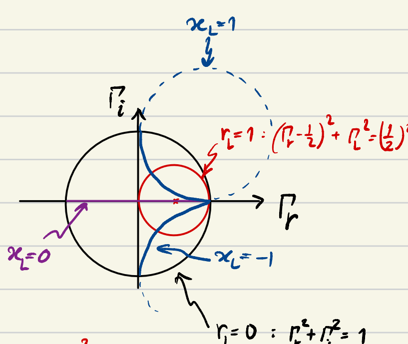
$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma|e^{j\theta}$

$z_L = \frac{1+|\Gamma|e^{j\theta}}{1-|\Gamma|e^{j\theta}}$ $\quad \Gamma = |\Gamma|e^{j\theta} = \Gamma_r + j\Gamma_i$

$z_L = r_L + jx_L = \frac{(1+\Gamma_r)+j\Gamma_i}{(1-\Gamma_r)-j\Gamma_i}$

$r_L = \frac{1+\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2} \Rightarrow (\Gamma_r - \frac{r_L}{1+r_L})^2 + \Gamma_i^2 = (\frac{1}{1+r_L})^2$

$x_i = \frac{2\Gamma_i}{(1+\Gamma_r)^2+\Gamma_i^2} \Rightarrow (\Gamma_r - 1)^2 + (\Gamma_i - \frac{1}{x_L})^2 = (\frac{1}{x_L})^2$



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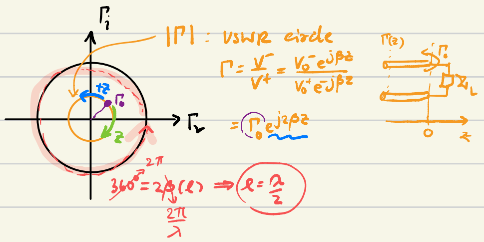


$\Gamma = \frac{V^-}{V^+} = \frac{V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z}} = \Gamma_0 e^{j2\beta z}$

$360° = 2\beta(l) \Rightarrow l = \frac{\lambda}{2}$ $\left(\beta = \frac{2\pi}{\lambda}\right)$

Comments on Smith chart

① As move along the line away from the load: Γ varies

→ magnitude constant ; phase will decrease (toward gen)

② As frequency increases, phase of Γ tends to decrease but the movement is complex

③ Normalized impedance

$$z = \frac{\Gamma - 1}{\Gamma + 1}$$

• example : $Z_L = 40 + j70\,\Omega$ terminates $100\,\Omega$ transmission line ($l = 0.3\lambda$)

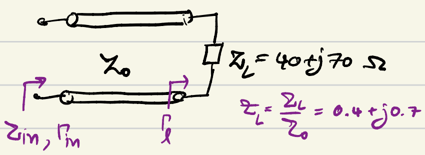
Find ① reflection coefficient at the load → $\Gamma = 0.59\angle 104°$

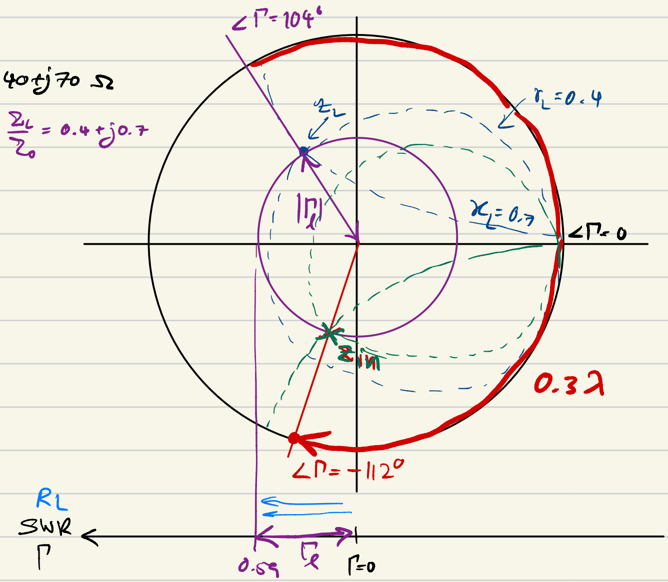
② Γ at the input of the line → $\Gamma = 0.59\angle -112°$

③ Z_{in} → $Z_{in} = 100(0.365 - j0.611)\,\Omega$

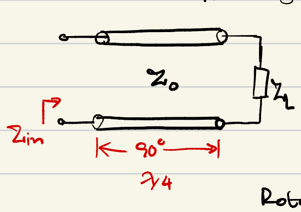
④ VSWR → 3.87

⑤ Return loss → 4.6 dB





What would happen if I terminate z_L with 90° transmission line

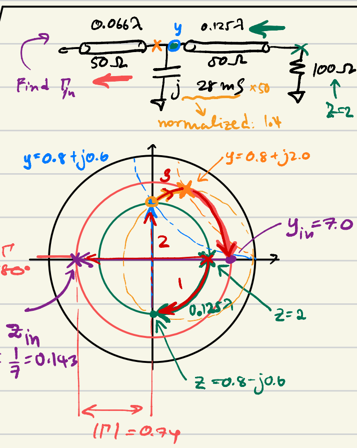


$Z_{in} = \frac{Z_0^2}{Z_L}$ (quarter-wave transformer)

normalize :

$z_{in} = \frac{1}{z_L} = y_L$

Rotate 180° of original z_L, you can get y_L on the impedence Smith chart.



∴ $\Gamma_{in} = 0.74\angle 180° = -0.74$

Another example : Find ① Y_L → $1/Z_L$ $\quad$ $0.4 - j0.2$ $\times\frac{1}{50}$ → $0.008 - j0.004\ S$

② Y_{in} → $y_{in} = 0.61 + j0.66$ $\times\frac{1}{50}$ → $0.0122 + j0.0132\ \Omega$