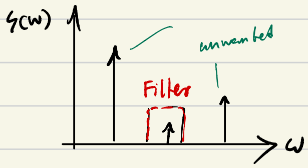
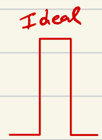
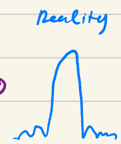
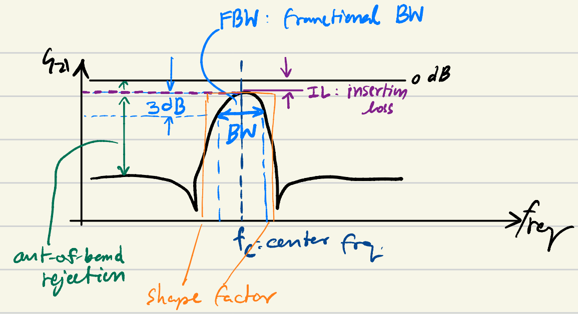
Microwave Filter (Pozar chapter 8)





$\Rightarrow$





Topics

1. Properties of $\Gamma(\omega)$ and $Z(\omega)$ in freq. domain (Pozar 4.1)
2. Insertion loss method (Pozar 8.2)
   ↳ LPF
3. Filter transformation (Pozar 8.4) : LPF $\xrightarrow{\text{transform}}$ HPF, BPF, BSF
4. Filter implementation

Properties (freq. response) of $\Gamma(\omega)$, $Z(\omega)$

$v(t) \xrightarrow{\mathcal{F}} V(\omega)$ ↑ real : $v(t) = v^*(t)$

$$v(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V(\omega) e^{j\omega t} d\omega$$

$$v^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V^*(\omega) e^{-j\omega t} d\omega \xrightarrow[\text{by } -\omega]{\text{replace } \omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} V^*(-\omega) e^{j\omega t} d\omega$$

$$\boxed{V(\omega) = V^*(-\omega)} \;\because\; v(t) = v^*(t)$$

$V(\omega) = Re(V(\omega)) + Im\{V(\omega)\}$
$V^*(-\omega) = Re(V(\omega)) - Im\{V(-\omega)\}$

$Re\{V(\omega)\}$ is even function.
$Im\{V(\omega)\}$ is odd function.

Similarly:
$Re\{I(\omega)\}$ is even function.
$Im\{I(\omega)\}$ is odd function.

$$V(\omega) = Z(\omega) I(\omega)$$
$$= [R(\omega) + jX(\omega)] \cdot [I_R(\omega) + j I_i(\omega)]$$
$$= \underbrace{[R(\omega) I_R(\omega) - X(\omega) \cdot I_i(\omega)]}_{\text{even}} + j\underbrace{[X(\omega) I_R(\omega) + R(\omega) I_i(\omega)]}_{\text{odd}}$$

even, odd

Thus : $\boxed{\begin{array}{l} R(\omega) \text{ is even} \\ X(\omega) \text{ is odd} \end{array}}$

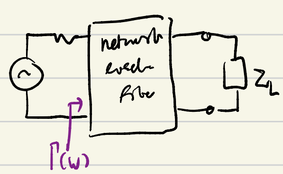
$$\Gamma(\omega) = \frac{Z(\omega) - Z_0}{Z(\omega) + Z_0} = \frac{R(\omega) - Z_0 + jX(\omega)}{R(\omega) + Z_0 + jX(\omega)}$$

$$\Gamma(-\omega) = \frac{R(-\omega) - Z_0 + jX(-\omega)}{R(-\omega) + Z_0 + jX(-\omega)} = \frac{R(\omega) - Z_0 - jX(\omega)}{R(\omega) + Z_0 - jX(\omega)}$$

$$\boxed{\Gamma(-\omega) = \Gamma^*(\omega)}$$

$$|\Gamma(\omega)| = \Gamma(\omega) \cdot \Gamma^*(\omega) = \Gamma(\omega) \cdot \Gamma(-\omega) = \Gamma^*(-\omega)\Gamma(-\omega) = |\Gamma(-\omega)|^2 \Rightarrow \boxed{|\Gamma(\omega)|^2 \text{ is even function}}$$

↳ $\Gamma(\omega)$ is even function,
we can do this
$\Gamma(\omega) = a + b\omega^2 + c\omega^4 + d\omega^6 + \ldots = \sum_{n=0}^{\infty} C_n \omega^{2n}$



Insertion loss :

$$IL = \frac{P_{avs}}{P_L} \begin{array}{l} \leftarrow \text{power available} \\ \leftarrow \text{power delivered to load.} \end{array}$$

$$IL = \frac{1}{1 - |\Gamma(\omega)|^2}$$

$$IL = 1 + \frac{M(\omega)}{N(\omega)} \Rightarrow \text{replace by LC network}$$

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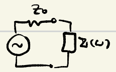
$|\Gamma(\omega)|^2 \rightarrow$ even function

Insertion loss method:



$P_{avail} / P_{delivered}$

$$\boxed{IL} = \frac{1}{1-\boxed{|\Gamma(\omega)|^2}}$$ even order power series

$\Downarrow$

$$IL = 1 + \frac{M(\omega)}{N(\omega)}$$ replace by LC network

Available power:

$$P_{avs} = \frac{|V|^2}{4Z_0}$$

Delivered power:

$$P_L = \left|\frac{V}{Z_0 + Z(\omega)}\right|^2 Re[Z(\omega)]$$

$$Re\{Z(\omega)\} = \frac{Z(\omega)+Z^*(\omega)}{2} = \frac{Z_0}{2}\left(\frac{1+\Gamma(\omega)}{1-\Gamma(\omega)} + \frac{1+\Gamma^*(\omega)}{1-\Gamma^*(\omega)}\right)$$

$$= \frac{Z_0}{2}\frac{(1+\Gamma)(1-\Gamma)^* + (1+\Gamma^*)(1-\Gamma)}{(1-\Gamma)(1-\Gamma^*)}$$

$$= Z_0 \frac{1-|\Gamma(\omega)|^2}{|1-\Gamma|^2}$$ Think about this

$$|Z_0 + Z(\omega)|^2 = Z_0^2\left|\frac{1-\Gamma}{1-\Gamma} + \frac{1+\Gamma}{1-\Gamma}\right|^2$$

$$= Z_0^2\left|\frac{2}{1-\Gamma}\right|^2 = \frac{4Z_0^2}{|1-\Gamma|^2}$$

$$\therefore IL = \frac{P_{avs}}{P_L} = \frac{|V|^2/4Z_0}{\frac{|V|^2}{\left(\frac{4Z_0^2}{|1-\Gamma|^2}\right)} Z_0 \frac{1-|\Gamma|^2}{|1-\Gamma|^2}} = \frac{1}{1-|\Gamma|^2}$$

even function : define $|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2)+N(\omega^2)}$

$$\therefore \boxed{IL = \frac{1}{1-\frac{M}{M+N}} = 1 + \frac{M}{N}}$$ real polynomials

To design a filter, choose M and N to specify desired IL characteristics (Pozar 8.3)

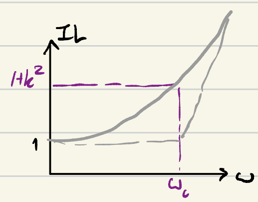
Procedure:

① Choose polynomial.

↳ LPF prototypes

(1) Maximally flat LPF : $\boxed{IL = 1 + k^2\left(\frac{\omega}{\omega_c}\right)^{2N}}$

loss factor : $k = 1 \rightsquigarrow 3$ dB; filter order; cut-off freq.

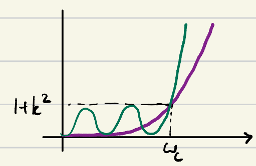


(2) Equal ripple LPF

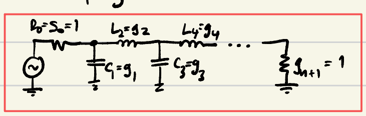
↳ Chebychev polynomials $\boxed{IL = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)^{2N}}$

(constant phase)

(3) Cuer . . .



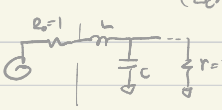
② Convert polynomials into circuit. (Using lossless LC prototypes)



LC LPF prototype

(ex) 2nd order maximally flat filter

$$IL = 1 + 1 \times \left(\frac{\omega}{\omega_c}\right)^{2\times 2} = 1 + \left(\frac{\omega}{\omega_c}\right)^4$$



$$Z_{in} = Z(\omega) = j\omega L + r \,\|\, \frac{1}{j\omega C}$$

$$= j\omega L + \frac{r(1-j\omega r C)}{1+\omega^2 r^2 C^2}$$

$$IL = \frac{1}{1-|\Gamma|^2} = \frac{1}{1-\Gamma\cdot\Gamma^*} = \frac{1}{1-\frac{z-1}{z+1}\cdot\frac{(z-1)^*}{(z+1)^*}} = \frac{|z+1|^2}{2(z+z^*)}$$

$$z + z^* = \frac{2R}{1+\omega^2R^2C^2}, \quad |z+1| = \ldots$$

Eventually,

$$IL = 1 + \frac{1}{4R}\left[(1-R^2) + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4\right]$$

$\Rightarrow$ to get $IL = 1 + \omega^4$

①: $1 + \frac{1}{4R}(1-R^2) = 1 \longrightarrow R = 1$

②: $\frac{1}{4R}(R^2C^2 + L^2 - 2LCR^2) = 0 \longrightarrow L = C$

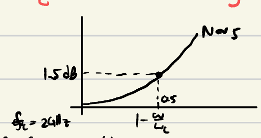
③: $\frac{1}{4R}L^2C^2R^2 = 1 \longrightarrow LC = 2$

$L = C = \frac{1}{\sqrt{2}}$

③ Scale filter prototype: scale of Z_0 and ω_c

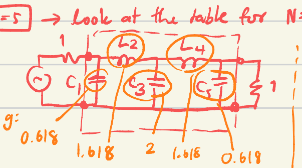
Example LPF : $f_c = 2$GHz, 50Ω system, 15dB IL @ 3GHz

① Find equation : maximally flat $IL = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N}$



$f: 3$GHz $\rightarrow 1 - \frac{\omega}{\omega_c} = 0.5$

$\boxed{N=5} \rightarrow$ look at the table for $N=5$ (maximally flat)



$$C_n' = \frac{C_n}{\omega_c R_0}$$

$$L_n' = \frac{R_0 L_n}{\omega_c}$$

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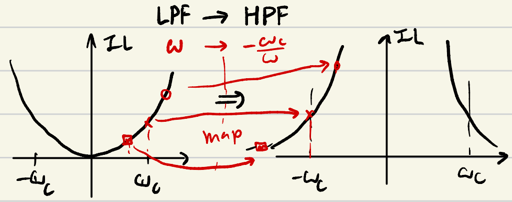
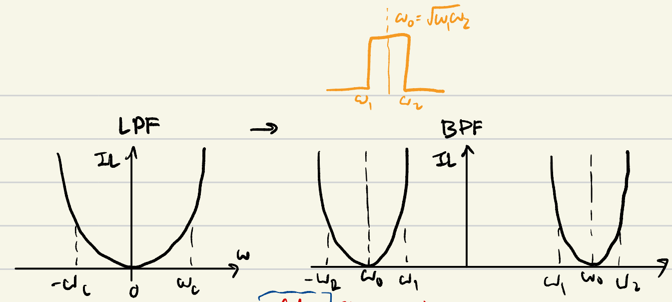
③ Filter transformation:

LPF → HPF

$\omega \rightarrow -\frac{\omega_c}{\omega}$



LPF → BPF



$\omega \rightarrow \boxed{\frac{\omega_0}{\omega_2-\omega_1}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$

$\Delta = \frac{\omega_2-\omega_1}{\omega_0} = FBW$

$\rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$

	LPF		HPF
series g_k	$z_k = j\omega g_k Z_0$ (inductor)	→	$-jZ_0\frac{\omega_c}{\omega}g_k$ (capacitor, $1/g_k\omega_c$)
shunt g_k	$y_k = j\omega g_k Y_0$ (capacitor)	→	$-jY_0\left(\frac{\omega_c}{\omega}\right)g_k$ (inductor, $1/g_k\omega_c$)

inductor $z_k = j\omega g_k Z_0 \rightarrow z_k = j\frac{Z_0}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)g_k$

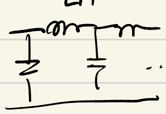
inducto, $g_k/\Delta\omega_0$, $\frac{\Delta}{\omega_0 g_k}$

ind capacitor

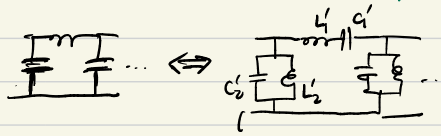
capacitor $y_k = j\omega g_k Y_0 \rightarrow y_k = j\frac{Y_0}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)g_k$

cap, $\frac{\Delta}{\omega_0 g_k}$, $\frac{g_k}{\Delta\omega_0}$

LPF



$L' = \frac{Z_0}{g_k\omega_c}$ $\quad C' = \frac{1}{g_k\omega_c Z_0}$



$L_1' = \frac{g_k Z_0}{\omega_0\Delta}$ $\quad C_1' = \frac{\Delta}{\omega_0 g_k Z_0}$

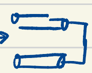
$C_2' = \frac{g_k}{\omega_0\Delta Z_0}$ $\quad L_2' = \frac{\Delta Z_0}{\omega_0 g_k}$

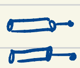
LPF → BSF

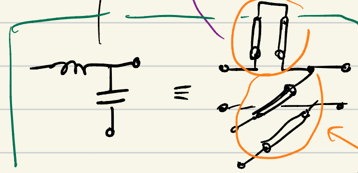
$\omega \rightarrow \Delta\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1}$

- Filter implementation with distributed element

Recall

$Z_{sc} = jZ_0\tan\beta l \rightarrow$  $\sim j\omega L'$ (inductor)

$Y_{oc} = jY_0\tan\beta l \rightarrow$  $\sim j\omega C'$ (capacitor)

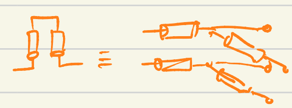


issues
1) accurate at low freq
2) periodic behavior

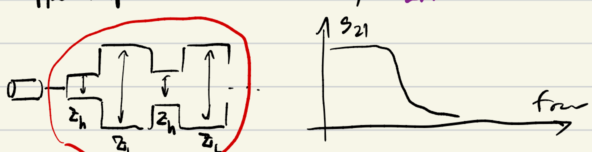
hard to implement

Read Richard's transformation

Kuroda's transform

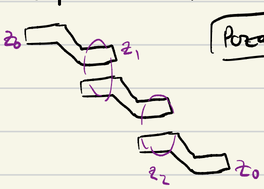


Stepped impedance: (Pozar 8.6) - LPF



Slow-wave structure → delay line.

Coupled-line filter (BPF) $\boxed{\text{Pozar 8.7}}$



Coupled resonator filter. $\boxed{\text{Pozar 8.8}}$