General Comments

1. Projects and presentations will mainly be done in groups of 3 (a couple of groups may have a different number of people due to class size).
2. A link to a sign-up sheet for the projects will be posted to Brightspace at 6pm on Tuesday, March 28.
3. The grade (10% of final mark) will be assigned based on a final presentation. This should be submitted as a 5-minute video to Brightspace by midnight on Monday, April 20. There is no written report. **Late projects will not be accepted**.
4. You can do the presentation any way that you like: Power Point, Google Docs, screen-recorded Sagemath or anything else. Whatever you do though, the presentation must include figures that have been generated with Sagemath.
5. When doing your presentation, assume an audience with the same level of knowledge as a random person from M2000. Focus on having good visuals and a clear, logical and well-explained progression from step-to-step.
6. Each group member must participate in making the video and do part of the presentation. If you do not actively participate in the presentation, you will be assigned a mark of zero for the project.

Radius of Convergence for the Taylor Series of Non-Trivial Functions

For these projects you will study the Taylor series of some non-trivial functions. You should proceed in the following way:

i) Use Sagemath to calculate lots of terms of your Taylor series. Do as many terms as you can in reasonable time (for some of these I managed 1000 terms). Use the `taylor` command rather than taking derivatives: `taylor` is much faster!

ii) Extract the coefficients a_n of your the Taylor series and then list-plot $a_n^{1/n}$. It should look like it is approaching a limit.

iii) Estimate the radius of convergence from that limit.

iv) Use your knowledge of complex numbers to calculate that exact radius of convergence. Compare to the estimate.

The functions for the projects are listed below.

Group #1: $f = \dfrac{1}{x^3 - 2x^2 - 3x + 10}$ around $x = 3$.

Group #2: $f = (x^3 - 2x^2 - 3x + 10)^{1/4}$ around $x = 1$.

Group #3: $f = \dfrac{1}{x\sqrt{x^2 - 6x + 10}}$ around $x = -1$.

Group #4: $f = \dfrac{x}{\sqrt{x^2 - 6x + 12}}$ around $x = 4$.

Group #5: $f = \dfrac{x^2 - 1}{\sqrt{x^2 + 1}}$ around $x = -2$

Group #6: $f = \dfrac{x^2}{\sqrt{x^2 + 4}}$ around $x = 2$

Group #7: $f = \operatorname{sech} 3x$ around $x = 0$.

Group #8: $f = \tanh 2x$ around $x = 0$.

Group #9: $f = \sin\left(\frac{\pi}{2x}\right)$ around $x = 1$.

Classifying Critical Points of Functions of Two Variables

For these projects you will identify and classify the critical points of a function of two variables. Only do this for the critical points within the indicated range.

i) Use Sagemath to plot the function over the indicated domain. Make any adjustments necessary to ensure that the figure looks good!

ii) Find the critical points by solving $F_x = F_y = 0$. You can do this either with Sagemath or by hand. Compare the critical points that you found with the ones that you can "see" on the graph. Make sure that they match.

iii) Classify the critical points with the discriminant. Again compare to the points on your graph.

Group #10: $F = \sin\left(x^2\right) + \sin\left(y^2\right)$ for $-\frac{3\pi}{4} \le x \le \frac{\pi}{2}$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Group #11: $F = \cos\left(x^2\right) + \cos\left(y^2\right)$ for $-\frac{3\pi}{4} \le x \le \frac{\pi}{2}$ and $-\frac{3\pi}{4} \le y \le \frac{3\pi}{4}$.

Group #12: $F = \cos\left(x^2\right) + \sin\left(y^2\right)$ for $-\pi \le x \le \pi$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Group #13: $F = \sin\left(x^2 - y^2\right)$ for $-\sqrt{2\pi} \le x \le \sqrt{2\pi}$ and $-\sqrt{2\pi} \le y \le \sqrt{2\pi}$.

Group #14: $F = \cos\left(x^2 - y^2\right)$ for $-\sqrt{2\pi} \le x \le \sqrt{2\pi}$ and $-\sqrt{2\pi} \le y \le \sqrt{2\pi}$.

Group #15: $F = y\cos\left(x^2 + y^2\right)$ for $-\sqrt{2\pi} \le x \le \sqrt{2\pi}$ and $-\sqrt{2\pi} \le y \le \sqrt{2\pi}$.

Group #16: $F = x\sin\left(x^2 + y^2\right)$ for $-\sqrt{2\pi} \le x \le \sqrt{2\pi}$ and $-\sqrt{2\pi} \le y \le \sqrt{2\pi}$.

Group #17: $F = e^{-x^2+y^2}(x^2 + y^2 - 1)$ for $-2 \le x \le 2$ and $-2 \le y \le 2$.

Group #18: $F = xye^{-x^2-y^2}$ for $-2 \le x \le 2$ and $-2 \le y \le 2$.

Group #19: $F = (x^2 - y^2)e^{-x^2-y^2}$ for $-2 \le x \le 2$ and $-2 \le y \le 2$.

Group #20: $F = xy^2e^{-x^2-y^2}$ for $-2 \le x \le 2$ and $-2 \le y \le 2$.

Group #21: $F = \sin(x)\cos\left(y^2\right)$ for $-\pi \le x \le \pi$ and $-\sqrt{\frac{3\pi}{2}} \le y \le \sqrt{\frac{3\pi}{2}}$.

Group #22: $F = \sin\left(x^2\right)\sin(y)$ for $-\sqrt{2\pi} \le y \le \sqrt{2\pi}$ and $-\pi \le x \le \pi$.