Estimation

Assignment 5 - Q55, June 2018

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Problem Statement

Consider the problem of estimation of a parameter θ on the basis of X, where X$\sim$N(θ,1) and $-\infty < \theta < \infty$. Under squared error loss, X has uniformly smaller risk than that of kX, for

1. $k < 0$
2. $0 < k < 1$
3. $k > 1$
4. no value of k

Solution

Let $\hat{\theta}$ be the estimated mean for parameter θ, then squared loss is given by,

$$L(\theta, \hat{\theta}) = (\theta - \hat{\theta})^2 \tag{1}$$

The Risk of the estimator for X is,

$$R(\theta, \hat{\theta}) = E[L(\theta, \hat{\theta})] \tag{2}$$

$$= E[(\theta - \hat{\theta})^2] \tag{3}$$

Solution

Let Y = kX then,

$$E[Y] = E[kX] = kE[X] \tag{4}$$

$$E[Y] = k\theta \tag{5}$$

The estimated mean for Y will be $k\hat{\theta}$.
So,the Risk of estimator for Y is,

$$R(k\theta, k\hat{\theta}) = E[L(k\theta, k\hat{\theta})] \tag{6}$$

$$= E[(k\theta - k\hat{\theta})^2] \tag{7}$$

Solution

$$= k^2.E[(\theta - \hat{\theta})^2] \tag{8}$$

On comparing risks of X and kX,

$$E[(\theta - \hat{\theta})^2] < k^2.E[(\theta - \hat{\theta})^2] \tag{9}$$

for $k > 1$.
Hence, option 3 is correct.