Assignment 1

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Github Link

https://github.com/Tarandeep97/AI5030

1 Problem

(51) Consider a Markov Chain with state space {0,1,2,3,4} and transition matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \tag{1.0.1}$$

Then $\lim_{n\to\infty} p_{23}^{(n)} equals$?

2 Solution

Given state space, S = {0,1,2,3,4}.

From state diagram (Fig.0) created from transition matrix, below are classes for each state

$$c(0) = \{0\}, \tag{2.0.1}$$
$$c(1) = c(2) = c(3) = \{1, 2, 3\}, \tag{2.0.2}$$
$$c(4) = \{4\} \tag{2.0.3}$$

where, c(i) is class of state $\forall i \in S$, which is a set that includes the communicating states of state i.

Since, different classes are obtained, given Markov chain is reducible and states can either be recurrent or transient or both.

Here, state 0 & 4 are **recurrent states**, as once entered in these states it is guaranteed to return to the state again.
State 3 is a **transient state**, as there is possibility that beginning from this state we will never return to this state.
If j is transient $\forall i \in S$ then,

$$\lim_{n\to\infty} p_{ij}^{(n)} = 0 \tag{2.0.4}$$

Hence, $\lim_{n\to\infty} p_{23}^{(n)} = 0$

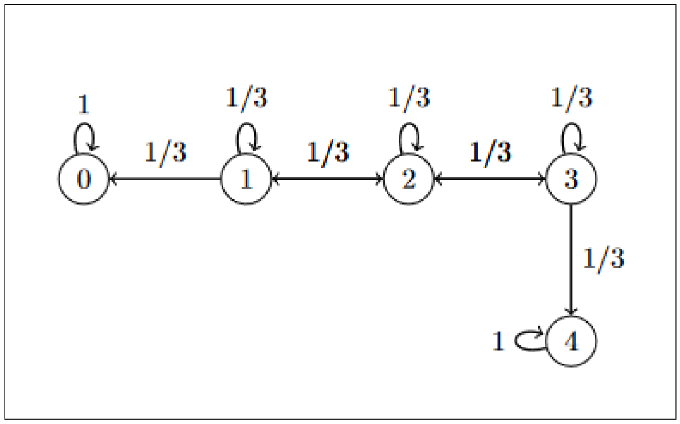

Fig. 0: State diagram