

Assignment 1

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Github Link

github.com/tarandeep97/AI5030/assignment1

1 PROBLEM

(51) Consider a Markov Chain with state space $\{0,1,2,3,4\}$ and transition matrix

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix} \quad (1.0.1)$$

Then $\lim_{n \rightarrow \infty} p_{23}^{(n)}$ equals?

2 SOLUTION

Given state space, $S = \{0,1,2,3,4\}$.

From state diagram (Fig.0) created from transition matrix, below are classes for each state

$$c(0) = \{0\}, \quad (2.0.1)$$

$$c(1) = c(2) = c(3) = \{1, 2, 3\}, \quad (2.0.2)$$

$$c(4) = \{4\} \quad (2.0.3)$$

where, $c(i)$ is class of state $\forall i \in S$, which is a set that includes the communicating states of state i .

Since, different classes are obtained, given Markov chain is reducible and states can either be recurrent or transient or both.

Here, state 0 & 4 are **recurrent states**, as once entered in these states it is guaranteed to

return to the state again.

State 3 is a **transient state**, as there is possibility that beginning from this state we will never return to this state.

If j is transient $\forall i \in S$ then,

$$\lim_{n \rightarrow \infty} p_{ij}^{(n)} = 0 \quad (2.0.4)$$

Hence, $\lim_{n \rightarrow \infty} p_{23}^{(n)} = 0$

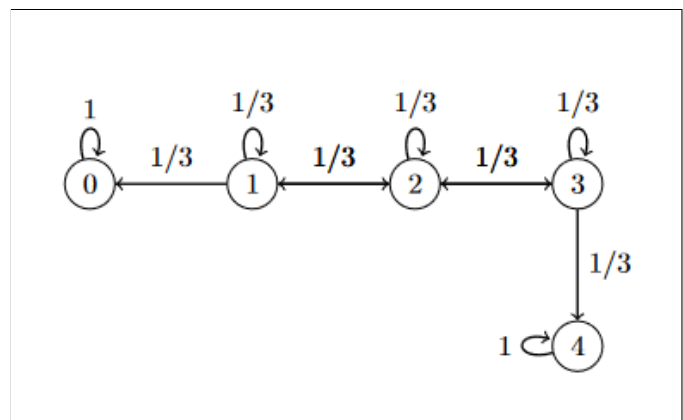


Fig. 0: State diagram