Assignment 5 - Q55, June 2018

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Github Link

https://github.com/Tarandeep97/AI5030

1 Problem

(Q55, June 2018) Consider the problem of estimation of a parameter θ on the basis of X, where X~N(θ,1) and $-\infty < \theta < \infty$. Under squared error loss, X has uniformly smaller risk than that of kX, for

1) $k < 0$
2) $0 < k < 1$
3) $k > 1$
4) no value of k

2 Solution

Let $\hat{\theta}$ be the estimator for parameter θ, then squared loss is given by,

$$L(\theta, \hat{\theta}) = (\theta - \hat{\theta})^2 \tag{2.0.1}$$

The Risk of the estimator for X is,

$$R(\theta, \hat{\theta}) = E[L(\theta, \hat{\theta})] \tag{2.0.2}$$

$$= E[(\theta - \hat{\theta})^2] \tag{2.0.3}$$

Let Y = kX then,

$$E[Y] = E[kX] = kE[X] \tag{2.0.4}$$

$$E[Y] = k\theta \tag{2.0.5}$$

The Risk of estimator for kX is,

$$R(k\theta, k\hat{\theta}) = E[L(k\theta, k\hat{\theta})] \tag{2.0.6}$$

$$= E[(k\theta - k\hat{\theta})^2] \tag{2.0.7}$$

$$= k^2.E[(\theta - \hat{\theta})^2] \tag{2.0.8}$$

From equation 2.0.3 and 2.0.8, for $k > 1$, X has uniformly smaller risk than kX.
Hence, option 3 is correct.